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Macro-Modeling of Reinforced Concrete Structural Walls: State-of-the-Art

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During the past decades, various analytical macroscopic models of structural walls have been developed for simulating the seismic behavior of reinforced concrete (RC) walls. Due to the inherently complicated characteristics of RC walls, macroscopic models that can capture all the important response characteristics with good accuracy and applicability are very challenging to establish. A thorough review of the four main types of mathematical macro models of RC walls, i.e., the vertical-line-element-model, the 2-D shear panel element model, the equivalent truss model and the fiber-based model, is presented to discuss the methodology behind each model and examine the corresponding merits and disadvantages. Suggestions are also made for the further research of the macro modeling of structural walls.

Keywords Reinforced Concrete; Wall; Macro Model; Nonlinear Analysis

1. Introduction

Over the past decades, reinforced concrete (RC) structural walls have been widely used in tall buildings located in seismic regions all around the world in that structural walls are effective in providing resistance and stiffness against lateral loads induced by wind or earthquake excitations. The significance of structural walls in ensuring the structural safety under natural or anthropogenic hazards has become more and more recognized and verified by extensive experimental research. However, in order to better the understanding on the complicated nonlinear responses RC structural walls, experimental research cannot be the only means due to the limitations of cost and time. As a consequence, analytical modeling techniques for walls have attracted the attention of researchers and engineers in the field of seismic performance of complex structures consisting of structural walls during the last 50 years.

In the 1970's and 1980's, considerable research, both experimental and analytical, have been carried out in order to predict and/or simulate the nonlinear responses of the RC structural system with structural walls, such as frame-wall structures. It was turned out that

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most analytical models could hardly describe the nonlinear features of those structural systems. Some studies [Aktan and Bertero, 1984; Aktan and Nelson, 1988] indicated that one of the primary reasons for the poor correlation between predicted and measured responses was the unsuitable analytical model for structural walls.

According to Paulay and Priestley [1992], the main failure modes of structural walls, largely depending on the geometric characteristics (or aspect ratio), can be basically divided into flexural failure, diagonal tension failure, diagonal compression failure, sliding shear failure, out-of-plane buckling failure, and bond failure along lapped slices or anchorages. In general, RC walls with an aspect ratio (height to length) more than 3.0 are classified as slender walls which are mainly controlled by the flexural behavior, while when the aspect ratio is less than 2.0, structural walls are regarded as squat walls generally governed by the shear behavior. The nonlinear behavior of walls with aspect ratios between 2.0 and 3.0 is normally governed by both flexure and shear. In ASCE/SEI 41-13 [2014], actions of structural components can be classified as deformation controlled or force controlled. For squat walls, it is suggested to be analyzed either as displacement-controlled components with low ductility capacities or as force-controlled components. In addition, due to the distinctive internal force patterns of the web region, boundary elements and flanges of the structural wall subjected to axial load, flexural moment, and shear, the analysis and simulation of structural walls are much more complicated than frame members. In order to thoroughly account for the inelastic responses of RC structural walls, the following modeling issues need to be addressed in the analysis models: (1) the fluctuation of the neutral axis of the wall cross-section; (2) the rocking of the wall at the base, as depicted in Fig. 1; (3) the axial load-flexure interaction; (4) the interaction between flexure and shear behavior (more detailed description is provided in the following section); (5) the outriggering interaction with the connecting frame elements (the effect of the structural elements connected to the wall boundaries), as illustrated in Fig. 2; and (6) the out-of-plane stiffness, in-plane and out-of-plane coupling.

Presently, the analytical modeling of the inelastic responses of RC wall systems can be classified into two main groups. One is the microscopic modeling, which adopts finite element method and focuses on the local behavior of structural walls. The other group is the macroscopic modeling, which describes the overall structural wall behavior by means of an analogous structural idealization. Microscopic finite element models, using 2-D shell elements or 3-D solid elements, can provide a refined and detailed interpretation of the local response. However, the microscopic modeling requires tremendous computational cost,

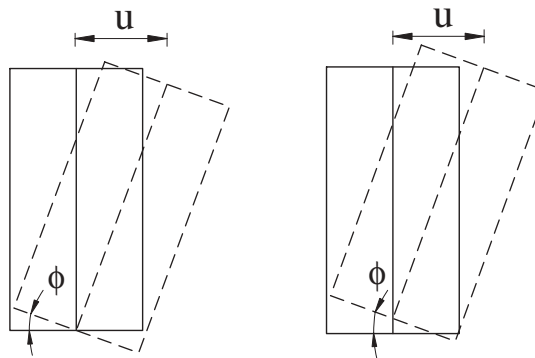


FIGURE 1 Rocking of a wall.

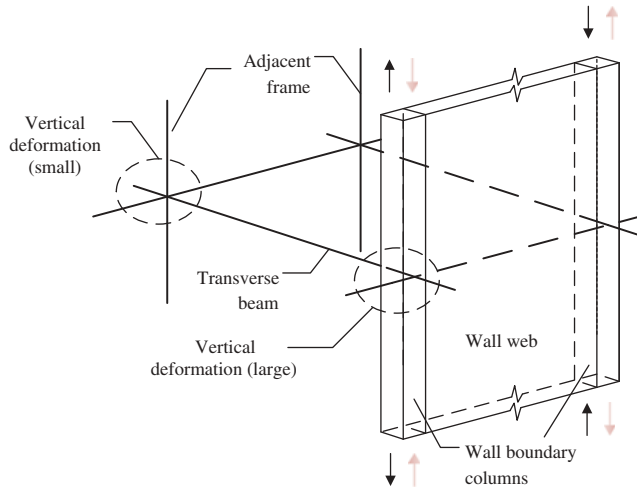


FIGURE 2 Outriggering interaction with connecting frame elements.

which may be infeasible and unnecessary for the comprehensive simulation of complex high-rise buildings. Therefore, the application of microscopic approach is basically to the analysis of isolated wall piers or simple structural systems. In comparison with microscopic models, macroscopic models are relatively simple and computationally efficient. Requiring less numerical effort, the macroscopic approach is more practical for simulating the nonlinear responses of RC structural walls, despite their analysis results are less accurate largely due to the assumptions and simplifications on both elements' topological and material properties. During the past decades, many types of macroscopic models have been developed and applied to the analysis of RC structural walls. However, since these macro models were established based on different assumptions and theories, the models' applicability, accuracy and computing cost significantly vary from one to another, which causes a great difficulty to researchers and practicing engineers and limits their confidence in the analysis results. Aiming to provide a guideline for numerical modeling of RC frame elements for the nonlinear structural analysis, Huang and Kwon [2015] evaluated the performance of five representative frame element models for RC elements, including the force-based beam-column element and elastic element with plastic hinges. Similar work on the macro modeling of structural walls, however, has rarely been reported.

In this article, the main progress and recent development of the macro modeling of RC structural walls are reviewed. Emphasis is placed on four categories of macro models, i.e. the vertical-line-element models (VLEM), the 2-D shear panel element models (2-D SPEM), the equivalent truss models (ETM), and the fiber-based models. By reviewing the different types of macro models, merits and disadvantages of each important macro model are compared and discussed. Suggestions are provided for the further implementation.

2. Shear-Flexure Interaction of RC Walls

The interaction between nonlinear flexural and shear behaviors in RC walls was observed from experiments [Oosterle *et al.*, 1979; Vallenat *et al.*, 1979; Hiraishi, 1984; Thomsen and Wallace, 1995; Tran and Wallace, 2012]. Experimental data indicated that although shear force is constant over the height in the case of a cantilevered wall subjected to lateral load at the top of the wall, the shear strains are not evenly distributed over the height [Beyer

and Priestley, 2011]. Based on the experimental data from Thomsen and Wallace [1995], Massone and Wallace [2004] further revealed that although the shear capacity of a wall is near twice as the applied story shear, inelastic shear behavior still occurs, particularly at wall regions where flexural yielding is developed. It was then pointed out that the shear-flexure interaction (SFI) is a generic wall response to lateral load even for relatively slender walls and that the inelastic flexural deformation is often accompanied by the inelastic shear deformation. It is certain that SFI has more substantial effect on the deformation response of squat walls, which are commonly used in many industrial and nuclear facilities, as well as at the lower stories of tall buildings [Wallace, 2007]. Although having been widely recognized for a long time, until recently the SFI has been often ignored for the simplification of the analysis, which may result in the overestimation of the seismic resistance of walls. Therefore, macro models with the consideration of SFI of walls [Chen and Kabeyasawa, 2000; Massone *et al.*, 2006, 2009; Panagiotou *et al.*, 2012; Fischinger *et al.*, 2012; Lu *et al.*, 2014; Kolozvari *et al.*, 2015] are also reviewed in the following sections.

3. Vertical-Line-Element Model (VLEM)

3.1. One-Vertical-Line-Element Model (OVLEM)

One of the early-developed macro models for structural walls in frame-wall structural system is called the equivalent beam model [Vulcano *et al.*, 1988], also referred to as the equivalent column model [Kabeyasawa *et al.*, 1983] or the beam-column element model [Fischinger *et al.*, 1990]. In this type of macro model the entire wall is represented by only one element corresponding to the centroid axis of the wall and connected to the frame beams or coupling beams by rigid links (Fig. 3). For this topological feature, this type of model is referred as the one-vertical-line-element model (OVLEM) in this article.

The OVLEMs can be further divided into one-component model [Giberson, 1969], two-component model [Clough *et al.*, 1965], multiple spring model [Takayanagi 1977], etc. The most commonly used OVLEM is the one-component model (Fig. 4a), consisting of a perfect elastic element and two inelastic rotational springs at two ends of the

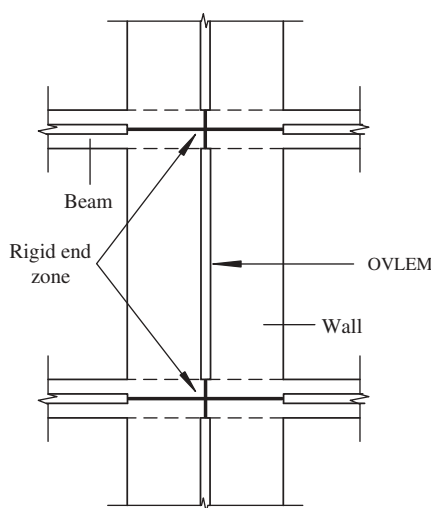


FIGURE 3 Typical OVLEM.

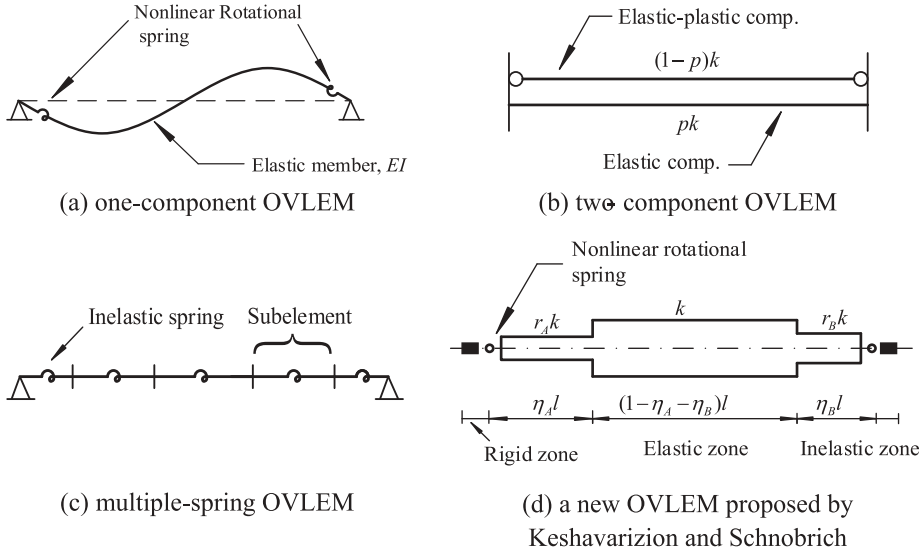


FIGURE 4 Four types of OVLEM.

element, i.e., the member's inelasticity is lumped into the two rotational springs, and the rest part of the member is assumed elastic. It is also assumed that the plastic hinges at two ends are ideal with zero length. Keshavarzian and Schnobrich [1984] compared four different OVLEMs, including the one-component model (Fig. 4a), two-component model (Fig. 4b), multiple-spring model (Fig. 4c), and one new model (Fig. 4d) consisting of an elastic central region and the inelastic zones with variable length at the ends of the member.

The multiple-spring model (Fig. 4c) was developed and verified by Takayanagi [1977] based on the test results of a ten story RC coupled wall structure [Aristizabal-Ochoa and Sozen, 1976]. In order to model the spread of plasticity along a partial length of the wall, the element is divided into several sub-elements. The flexural, axial, and shear rigidities are assigned to each sub-element. The number of sub-elements for a wall member decreases with the increase of story height since the inelastic behavior mainly concentrated at the base region. Although axial rigidity is only related to the curvature and axial strain of the section, correlation of bending moment and axial force is taken into account. To consider the flexure and shear interaction, it is assumed that the ratio of inelastic to elastic shear rigidities is equal to that inelastic to elastic flexural rigidities, or

$$GA_i = \frac{EI_i}{EI_e} GA_e, \quad (1)$$

where GA_e is the elastic shear rigidity; GA_i is the inelastic shear rigidity; EI_e is the elastic flexural rigidity; and EI_i is the inelastic flexural rigidity.

The simplicity of OVLEMs can provide a lot of convenience in computation at the price of accuracy. Research effort aiming to improve the accuracy of OVLEMs has been made (e.g., Mergos and Beyer, 2012). However, the application of OVLEMs may be limited since some important experimental observations cannot be effectively captured. In OVLEMs the wall section is assumed to rotate about the centroid axis of the wall and thus the location of neutral axis remains unchanged during analysis. However, the tests

on three-story walls with connecting beams [Hiraishi *et al.*, 1981] have indicated that the neutral axis of wall section will migrate because of the large elongation of the boundary element on the tension side and the relatively small shortening of the boundary element on the compression side. In addition, behaviors like the rocking of the wall and outriggering interaction with the frame members connected to wall boundaries are also neglected in OVLEMs.

3.2. Three-Vertical-Line-Element Model (TVLEM)

A full-scale test on a seven-story RC frame-wall structure was conducted by Kabeyasawa *et al.* [1983], as part of a U.S.-Japan Cooperative Research Program. To overcome the disadvantages of the conventional OVLEMs, three-vertical-line-element model (TVLEM) was proposed. The wall is idealized as three vertical line elements with infinite rigid beams at the top and bottom floor levels (Fig. 5). The two exterior vertical elements represent the axial stiffness of boundary elements while the interior element represents the wall web region. The interior element is composed of vertical, horizontal, and rotational springs representing the axial, shear, and flexural behavior of the wall web, respectively. It is worth mentioning that in this model the moment distribution along the story height is assumed uniform and equal to the moment at wall critical section.

The axial rigidities of the two exterior elements and the interior element are calculated in the same way as generic RC columns. The rotational stiffness of the interior element can be defined corresponding to the wall area bounded by the inner faces of boundary elements. Equation to calculate the initial elastic shear stiffness is:

$$K_s = \frac{GA_w}{xh}, \quad (2)$$

where G is the elastic shear modulus; A_w is the area of shear wall section; x is the shape factor for shear deformation, and $x = 3(1 + u)[1 - u^2(1 - \nu)]/4[1 - u^3(1 - \nu)]$; h is the story height; u and ν are geometrical parameters defined in Fig. 6.

Kabeyasawa *et al.* [1983] introduced an axial-stiffness hysteresis model (ASHM) to account for the axial hysteretic behavior of all the vertical springs in TVLEM. The hysteresis behaviors of the horizontal and rotational springs were considered by the origin-oriented hysteresis model (OOHM). Shear stiffness degradation was taken into account without the consideration of concrete cracking effects.

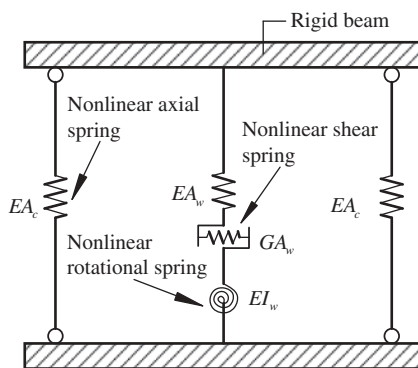


FIGURE 5 Wall idealization in TVLEM.

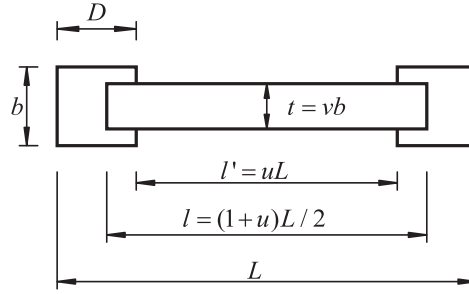


FIGURE 6 Geometric properties of wall section in TVLEM.

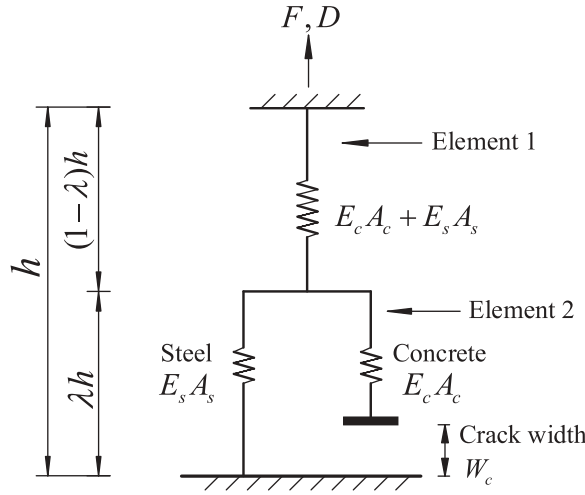


FIGURE 7 Two-axial-element-in-series model.

By introducing the two exterior elements, the TVLEM can describe the main features of the experimental observations, including the variation of neutral axis of wall cross section, rocking and outriggering interaction with the connecting frame elements. However, since only three vertical elements are used, the depth of the compression zone of wall cross section is inaccurate; consequently, the variation of the neutral axis location can only be roughly reflected. In addition, the deformation defined by the elongation and shortening of the two exterior springs accounting for the boundary elements can hardly be compatible with that defined by the vertical and rotational springs of interior element accounting for the wall web region. This incompatibility problem may be even worse since the property of the rotational spring is very difficult to calibrate. Therefore, considerable research has been carried out to improve the simulation effectiveness of TVLEM. Vulcano and Bertero [1986, 1987] replaced the ASHM by the two-axial-element-in-series model (AESM). The AESM consists of two elements instead of the one-component element. As shown in Fig. 7, element 1 is a single spring to account for steel and uncracked concrete, and element 2 with two springs represents the rest part of steel and cracked concrete with bond deterioration. A bond degradation parameter λ was introduced to define the lengths of the two elements. It is suggested that the value of λ should be treated carefully because of its sensitivity. Moreover, although this modified TVLEM still adopted the OOHM to describe the shear hysteretic behavior, it was pointed out that there were discrepancies between predicted and

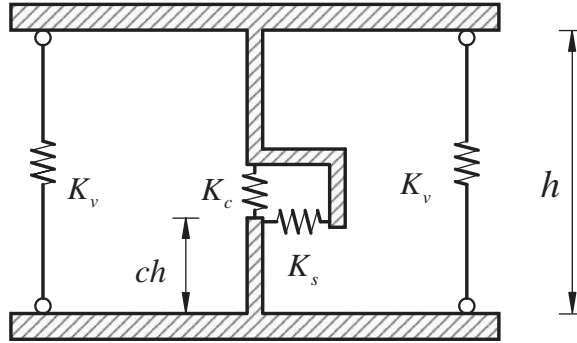


FIGURE 8 Modified TVLEM by Linde.

measured results for squat walls under high stresses. As for compatibility problem mentioned above, although softening stiffness properties are assigned to the rotational spring, this method is still not reliable and efficient enough. Attempt was also made by Linde and Bachmann [1994] to solve the incompatibility problem. Omitting the rotational spring of the interior element, the modified model used only three vertical springs to meet the satisfaction of both axial and flexural behavior (Fig. 8), and a horizontal spring to represent the shear stiffness of the wall.

3.3. Multiple-Vertical-Line-Element Model (MVLEM)

The most accepted modification to TVLEM led to the development of the multiple-vertical-line element model (MVLEM) proposed by Vulcano *et al.* [1988]. The MVLEM, also called the multi-component-in parallel model, has been widely used during the last twenty years. It was developed based on the geometry modification of the TVLEM and the adoption of more refined hysteretic rules. The two exterior elements still represent the axial stiffness (K_1 and K_2) of the boundary elements. The additional interior elements (at least two with axial stiffness $K_3 \dots K_m$) represent the axial and flexural behavior of the web region of walls (Fig. 9). Like the four-spring model proposed by Linde and Bachmann [1994], the rotational spring is omitted herein due to the difficulty in satisfying the compatibility requirement, while the horizontal spring with stiffness K_h is still kept. Similar to the model introduced by Vulcano and Bertero [1987], the relative rotation of the wall between top and bottom levels is assumed at the point where the height is ch measured from the bottom. The parameter c , ranging from 0–1, is determined based on the expected curvature

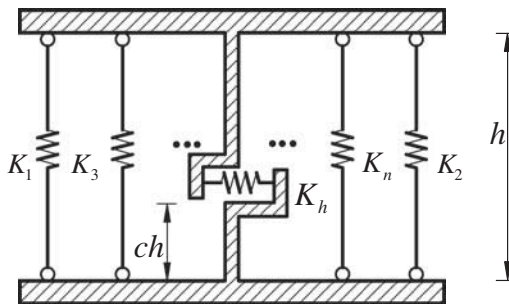


FIGURE 9 Multiple-vertical-line element model.

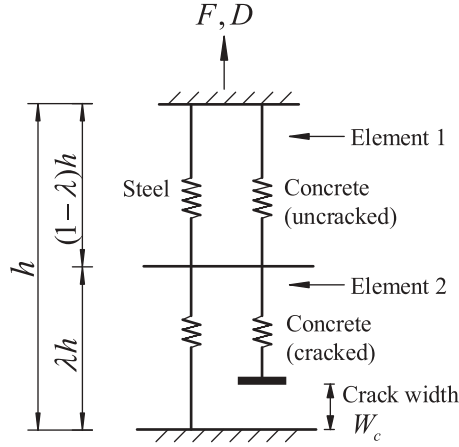


FIGURE 10 Modified ASME.

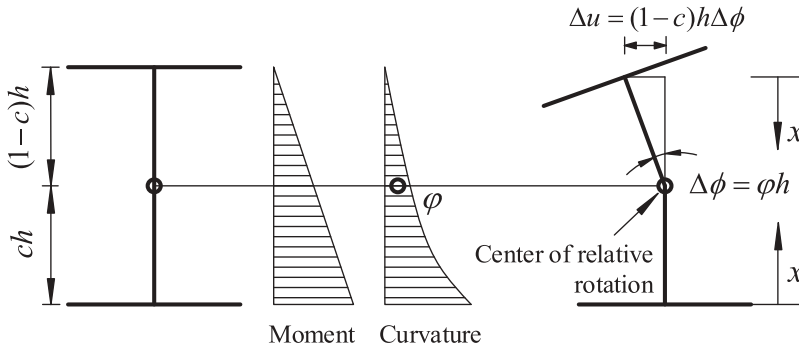


FIGURE 11 Relative rotation and flexural displacement in MVLEM.

distribution along the height of the wall. For the hysteretic model, the element 1 of original ASME (Fig. 7) changes into two parallel components to account for the mechanical behavior of steel and the uncracked concrete, while the rest remains the same (Fig. 10).

When adopting MVLEM to model a structural wall, the selection of rotation center height ch can be very challenging. The dimensionless parameter c is decided based on the curvature distribution along the height of the wall, and the value of ch represents the distance between the centroid of the curvature distribution curve and the base (Fig. 11). The following equations express the definition of parameter c :

$$\Delta u = \int_0^h \varphi x' dx' = - \int_h^0 \varphi (h-x) dx = h \int_0^h \varphi dx - \int_0^h \varphi x dx \quad (3)$$

$$\Delta \phi = \int_0^h \varphi dx \quad (4)$$

$$ch = \frac{\int_0^h \varphi x dx}{\int_0^h \varphi dx} = \frac{h \int_0^h \varphi dx - \Delta u}{\int_0^h \varphi dx} = \frac{h \Delta \varphi - \Delta u}{\Delta \varphi} \quad (5)$$

$$\Delta u = \Delta \phi h(1 - c), \quad (6)$$

where φ is the center of the curvature distribution and $\Delta \phi$ is the rotation at the center of the element. If bending moment along the height of the element is uniformly distributed, c equals 0.5. If it is a triangular distribution, the value of c changes into 0.3. However, since the actual moment distribution is difficult to predict, the selection of the value of parameter c seems to be empirical or even arbitrary. Vulcano and Bertero [1987] proved that $c = 0.4$ would give the best correlation between the predicted and measured responses in their parametric studies. Fajfar and Fischinger [1990] suggested stacking multiple MVLEMs (Fig. 12) to reduce the influence of parameter c . Massone and Wallace [2004] examined experimental data and found that an average value of 0.4 for the parameter c is reasonable to determine the center of rotation.

By using several interior elements instead of an independent axial spring and rotational spring at the centroid of the web, the MVLEM avoids the incompatibility problem of the TVLEM. The variation of neutral axis of wall cross section can be more accurately

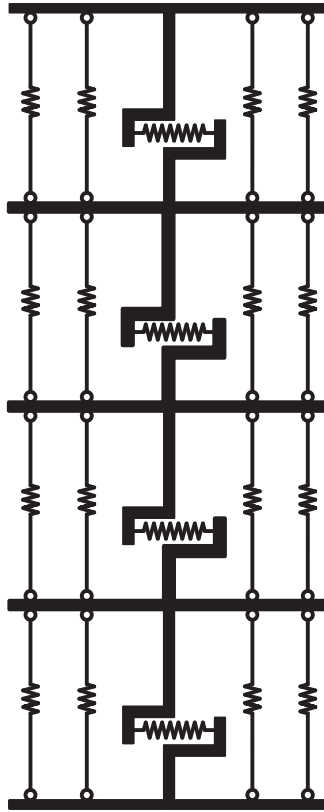


FIGURE 12 Stack of MVLEMs.

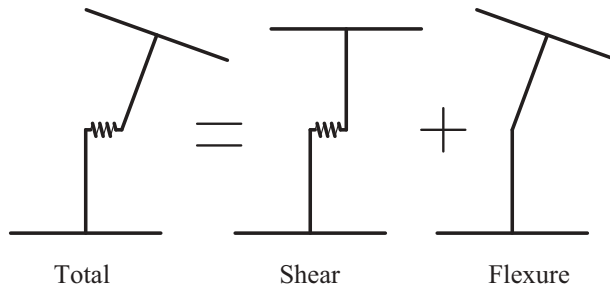


FIGURE 13 Uncoupling mode of deformation of MVLEM.

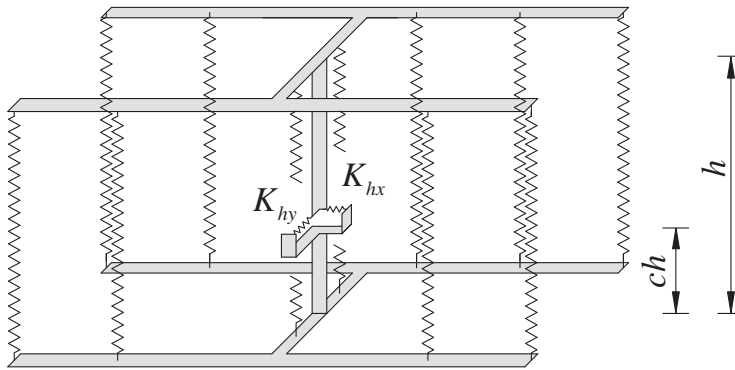


FIGURE 14 3-D MVLEM.

reflected in MVLEM than in TVLEM for the use of more vertical elements. The numerical investigation by Vulcano *et al.* [1988] showed the MVLEM is more accurate than the TVLEM in the simulation of the 1/3-scale RC wall test conducted by Vallenat *et al.* [1979]. Nevertheless, one of the shortcomings of the MVLEM is that the hysteretic rules of the modified ASME and OOH depend on somewhat arbitrary parameters based on engineering judgment. In addition, the nonlinear shear deformation is not adequately described, especially for the case of high shear stresses [Colotti, 1993]. It should be noted that although the axial and flexural interaction is incorporated, the flexure and shear interaction is still disregarded in the MVLEM (Fig. 13).

Despite the shortcomings mentioned above, the MVLEM obtains better response predictions for walls than the TVLEM. Because of its reasonable accuracy as well as simplicity, many modified models were further proposed based on the MVLEM. To reduce the complexity of the hysteretic model in MVLEM, Fischinger *et al.* [1990] simplified the force-deformation relations of horizontal and vertical springs. Fischinger *et al.* [1992] indicated that some of the parameters in the vertical spring hysteretic model needed further calibration. Fajfar and Fischinger [1990] used a stack of MVLEMs instead of a single MVLEM. Fischinger *et al.* [2004] expanded the 2-D MVLEM into Bi-directional (3-D) model to account for the nonlinear behavior of wall with T and H cross sections (Fig. 14).

4. 2-D Shear Panel Element Model (2-D SPEM)

Colotti [1993] added shear panel element into the conventional MVLEM. This modified model was intended to overcome the disadvantage of MVLEM in the poor descriptions

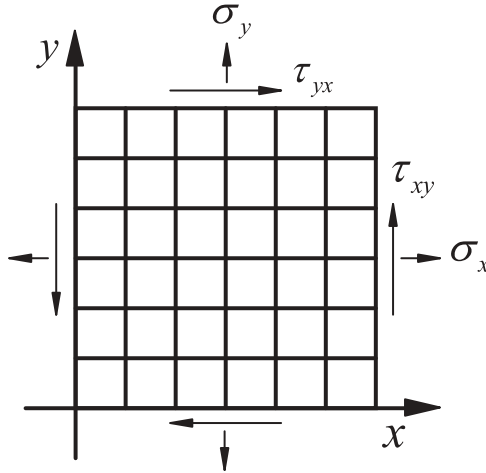


FIGURE 15 Stress state of shear panel element.

of the shear response as well as the shear-flexure interaction, especially under high shear stresses. The shear panel element is based on the theory developed by Vecchio and Collins [1986], also known as the modified compression-field theory (MCFT). In this wall model, the original horizontal spring in the MVLEM is replaced by a shear panel element, and other uniaxial vertical elements are included to represent the boundary elements. The panel element is assumed to contain an orthogonal grid of evenly distributed reinforcing bars which only resist axial stresses (neglecting dowel action) and subjected to the uniform axial stresses σ_x and σ_y as well as the uniform shear stress τ_{xy} (Fig. 15). Other assumptions include the perfect bonding between concrete and reinforcement, the coincidence of directions of principal strain and principal stress of concrete, and the rotating smeared cracks. For the shear panel model, the stress-strain relationship of concrete suggested by Vecchio and Collins [1986] is adopted while the curvilinear relationship of steel proposed by Menegotto and Pinto [1973] is employed.

To verify the reliability of the modified model, the numerical analysis by Colotti [1993] has been carried out on several squat structural walls tested by Vallenat *et al.* [1979] and Lefas *et al.* [1990]. The results exhibited an acceptable accuracy for walls subjected to monotonic loading. However, further improvement is still needed to simulate the fixed-end rotation at the wall-foundation interface, the rigid-body displacement of the foundation, and to extend this monotonic loading to a cyclic one. Orakcal and Wallace [2006] deemed that this model still showed discrepancies with experimental data since the predicted shear deformation was approximated 20% greater than the measured in some cases. Actually, this so-called modified MVLEM is more like a modified TVLEM, for the wall web is simulated by only one shear panel in which the deformation is average and the same along the X and Y directions; therefore, no flexural deformation of wall web is accounted for. As a result, this model retains the inability to incorporate SFI, and only captures the interaction between axial and shear responses.

A similar model was proposed by Milev [1996]. The interior 2-D nonlinear panel element was used to analyze the T-shaped walls tested by Kabeyasawa *et al.* [1995] subjected to both monotonic and reversed cyclic loading. This model is the extension of the previous TVLEM, by using one two-dimensional nonlinear panel element to substitute the original interior element with vertical, horizontal and rotational springs. The interior panel element

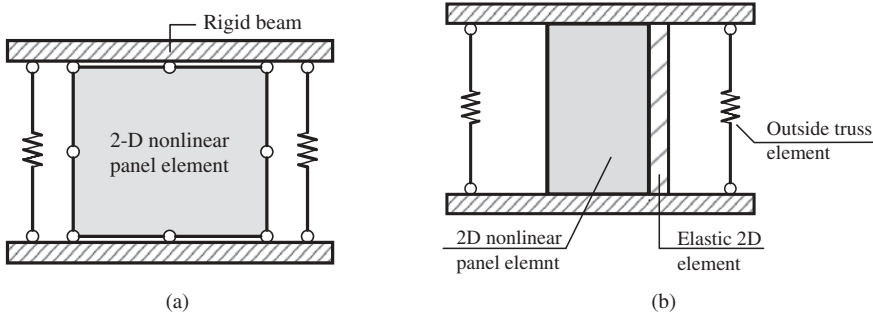


FIGURE 16 Model proposed by Milev.

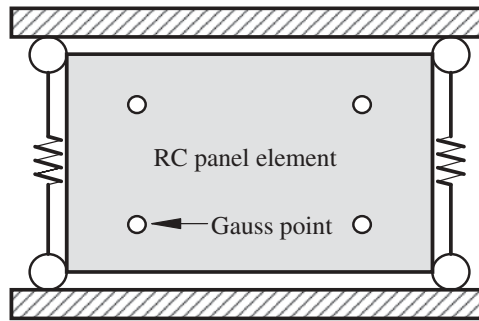


FIGURE 17 2-D SPEM proposed by Chen and Kabeyasawa.

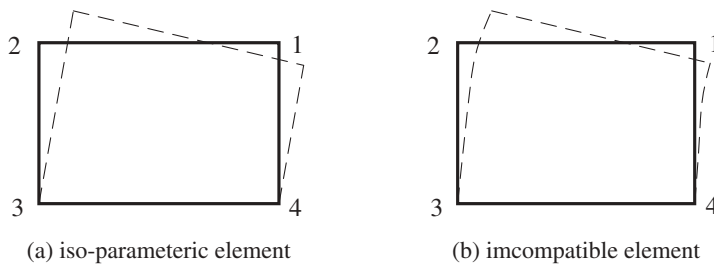


FIGURE 18 Two panel elements for wall web.

uses a quadrilateral serendipity element with 3-to-9-node iso-parametric shape functions and the 3 by 3 Gauss integration rule was applied (Fig. 16a). In analytical model, the flange of the T-shaped wall is modeled as elastic 2-D element (Fig. 16b). For concrete constitutive model, a modification introduced by Stevens *et al.* [1991] based on the MCFT was used. It was concluded that more refined model for flange of the shear wall should be proposed and problems of accuracy exist under reversed cyclic loading, particularly under high shear stresses.

Chen and Kabeyasawa [2000] also attempted to use similar 2-D nonlinear panel element to model the nonlinear behavior of the structural wall (Fig. 17). They adopted two different panel elements to represent the wall web. One is an iso-parametric element and the other is an incompatible element. The iso-parametric element (Fig. 18a) is a 4-node panel under biaxial loading and cannot obtain satisfactory result of flexural deformation. By adding a shape function for interior deformation of element $N_5 = 1 - \eta^2$, the flexural

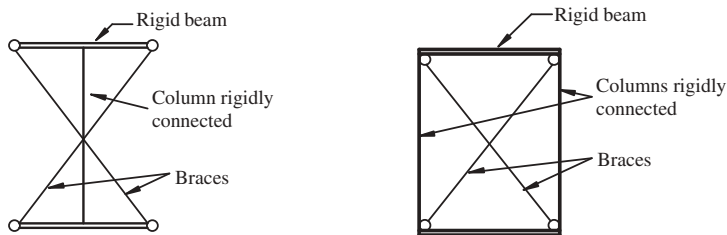
deformation was introduced in the incompatible element (Fig. 18b). Unloading and reloading stiffness of the conventional axial spring model for boundary elements of walls were modified. For the panel element model, tension stiffening model of concrete proposed by Izumo *et al.* [1989] and MCFT stress softening model for compression were adopted. Comparison between predicted and measured results of T-shaped structural walls tested by Kabeyasawa *et al.* [1995] was conducted, showing that for flexural yielding walls, shear deformation was overestimated by the iso-parametric panel element and the incompatible element provided a better correlation with the experimental results. For walls in shear failure, similar results were obtained from the two different panel elements. This model was also used to analyze a full-scale six-story RC frame-wall structure and good results were yielded [Chen *et al.*, 2007].

In fact, the latter two models are a combination of finite element (FE) panel element for wall web and the macroscopic element for boundary elements, resulting in significantly increased computational costs compared with the conventional TVLEM and MVLEM. Compared with the VLEM, the 2-D SPEM has rarely been used in simulating the isolated wall piers or complicated high-rise structures. Although the flexure and shear interaction is incorporated to some extent, these models still cannot function well when describing the responses of the structural wall under high shear stresses.

5. Equivalent Truss Model (ETM)

One of the simplest ETMs was developed based on the wide-column models. It consists of a column element, two rigid horizontal beams at the top and bottom and two hinge-ended diagonal braces connected to the ends of the rigid beams (Fig. 19a). Smith and Girgis [1984] called this model the braced wide column analogy and applied it to analyze a 15-story core wall for elevator. The wall segment bending stiffness is resisted by the column element, and shear stiffness is provided by the column element as well as the horizontal components of braces' stiffness. The vertical components of braces' stiffness and axial stiffness of the column element provide the wall axial stiffness. In another analogous frame model proposed by Smith and Girgis [1984] (Fig. 19b), the central column element was omitted and replaced by two vertical columns on the edge. This braced frame analogy can be divided into symmetric and asymmetric forms, allowing the adjacent columns of adjacent models to rotate independently in full-width models. In spite of the simplicity of those truss models, the limitation in inelastic analysis is obvious.

Hiraishi [1984] developed a measuring strategy that can be used in experiments to accurately measure the flexural and shear deformation of a structural wall, respectively. Based on the experimental results that clearly showed the shear and flexural deformation



(a) braced wide-column analogy model (b) braced frame analogy model

FIGURE 19 Equivalent truss model proposed by Smith and Girgis.

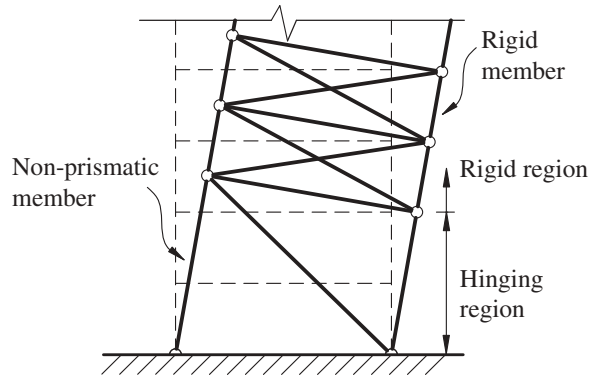


FIGURE 20 ETM proposed by Hiraishi and Kawashima.

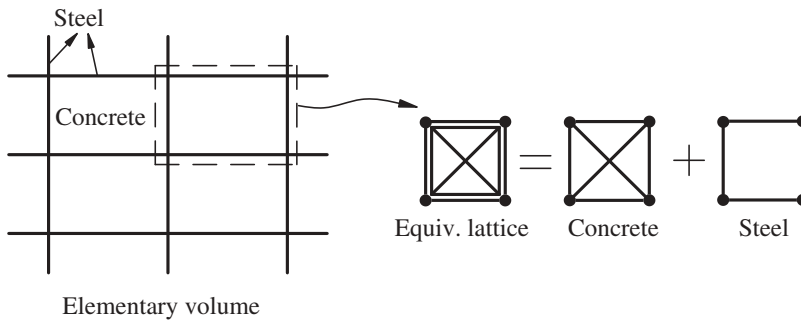


FIGURE 21 Equivalent lattice of an elementary volume.

of walls, an ETM with a non-prismatic truss member was also introduced by Hiraishi and Kawashima [1988], as illustrated by the deflected condition of the proposed ETM (solid lines) from its original condition (dashed lines) subjected to a left-to-right loading direction (Fig. 20). The length of the non-prismatic truss member was corresponding to the estimated hinging region of the structural wall. The area of the cross section of the non-prismatic truss member was determined taking into account the stress distribution along the height of the boundary element in tension. However, this truss model is only applicable to monotonic loading instead of cyclic loading, largely due to the difficulties in defining the structural topology and the hysteretic behavior of the truss elements.

Mazars *et al.* [2002] developed the equivalent reinforced concrete (ERC) model to predict the behavior of structural walls, where a structural wall is represented by concrete and reinforcement lattice meshes based on the grid of horizontal and vertical reinforcing steel, as depicted in Fig. 21. It was pointed out that when the stress field is homogeneous, the macroscopic model with largely reduced number of meshes can be used instead of the equivalent lattice, as shown in Fig. 22. The simulation yielded reasonable agreement with the experiment results, even for squat walls. However, the angle θ of the diagonal concrete is a critical parameter especially when analyzing the lightly reinforced wall and it is very challenging to select an appropriate value, depending on factors such as the horizontal and vertical reinforcement ratios, loading, and boundary condition, etc.

Park and Eom [2007] also adopted truss model to analyze RC members including squat walls and slender walls. The model consists of longitudinal, transverse, and diagonal truss

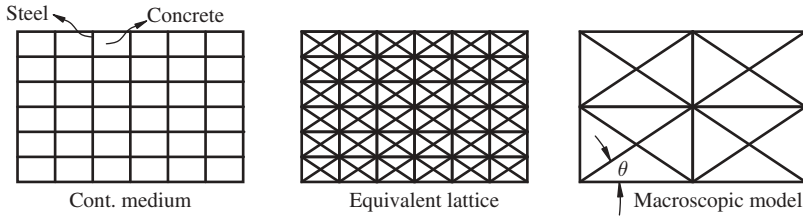


FIGURE 22 Equivalent lattice for walls.

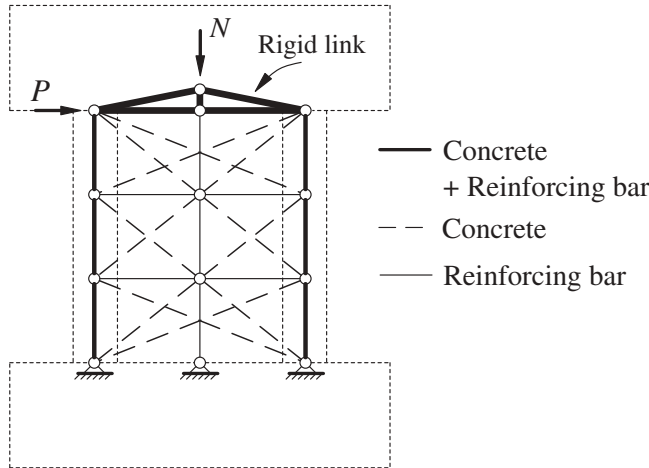
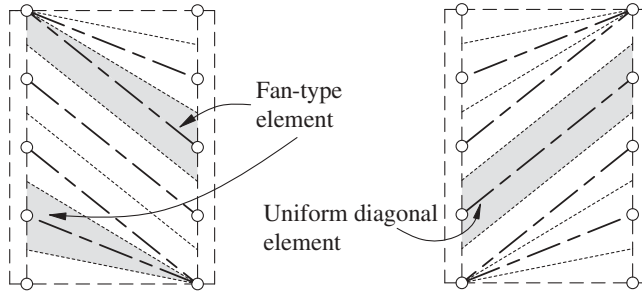
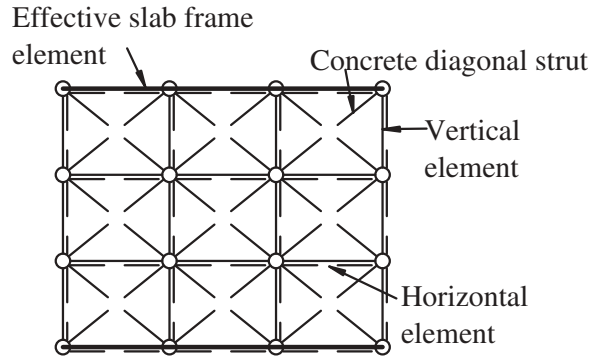
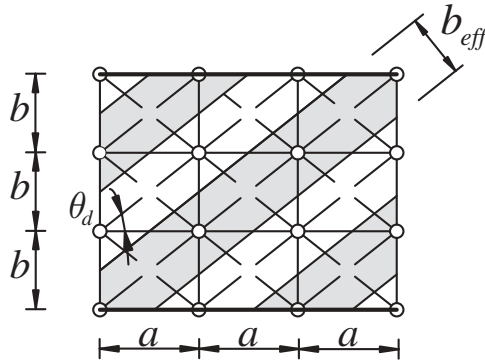


FIGURE 23 Truss model developed by Park and Eom.

elements, with each truss element consisting of concrete, reinforcing bar, or both. As illustrated in Fig. 23, the longitudinal truss elements (bold solid lines) at boundary elements of the wall include both concrete and reinforcing bars. The interior horizontal and vertical truss elements (solid lines) only involve reinforcing bars, disregarding the contribution of the compression zone of the intact concrete to the shear resistance. Two different forms of diagonal truss elements (dashed lines) were developed, i.e., the fan-type diagonal elements in the disturbed regions and the uniform diagonal elements in the Bernoulli region, as depicted in Fig. 24. It is obvious that the parameter θ , defining the angle of the diagonal truss elements, plays a critical role in the modeling and has significant influence on the analysis results. The comparison between experimental and computed results indicated that this model can well simulate the behavior of walls that failed due to web concrete crushing. Walls that failed due to the fracture or buckling of the longitudinal reinforcing bars cannot be satisfactorily simulated.

Panagiotou *et al.* [2012] developed the truss model shown in Fig. 25. This model consists of effective slab frame element, concrete diagonal strut, and vertical and horizontal elements. All the diagonal struts are parallel with a fixed inclination angle θ_d , as shown in Fig. 26. Panagiotou *et al.* suggested $\theta_d = 45^\circ$. The effective width of the concrete diagonal strut, b_{eff} , can be expressed in terms of the geometric relation between the mesh sizes a , b and θ_d . The vertical and horizontal elements include the concrete and reinforcing steel. Lu *et al.* [2014] further extended the 2-D truss model to 3-D to analyze the non-planar RC structural walls with T- and C-shaped cross sections. The two major modifications to the 2-D model were the inclination angle of the concrete diagonal strut and the concrete material

**FIGURE 24** Diagonal truss elements.**FIGURE 25** Truss model developed by Panagiotou *et al.* [2012].**FIGURE 26** Meshing and geometry of truss model developed by Panagiotou *et al.* [2012]

model. It turned out that the 3-D truss model could obtain more accurate results for walls with diagonal shear failure modes. In the 3-D model, the determination of θ_d for isolated walls with regular uniform shape over the height can be calculated by the follow equation:

$$\theta_d = \tan^{-1} \left(\frac{V_{\max}}{f_{y,t} \rho_t t_w d} \right) \leq 65^\circ, \quad (7)$$

where V_{\max} is the maximum resisted shear force; t_w is the wall thickness; d is the distance between the outer vertical lines in the direction of loading; $f_{y,t}$ is the yield strength of reinforcing steel; and ρ_t is the transverse reinforcement ratio.

For the walls with irregular cross sections, the strut-and-tie model can be used to determine the value of θ_d . In the 2-D model, both vertical and horizontal concrete elements incorporate the tension stiffening effect of concrete material; nonetheless, it was observed that before reaching the flexural strength, the distributed shear cracking has occurred. Therefore, in the 3-D truss model, the horizontal concrete element is assumed with no tensile strength.

6. Fiber-Based Model

Extensive attention has been paid to the fiber model during past several decades (e.g., Mahasuverachai and Powell, 1982; Kaba and Mahin, 1983; Mari, 1984; Spacone *et al.*, 1996). The fiber model was regarded as a simplified finite element method by some researchers (e.g., Kaba and Mahin, 1983) and a complicated form of multi-axial spring macroscopic model (e.g., Lai *et al.*, 1984); thus, it is difficult to classify the fiber model, especial when the MVLEM was combined with the fiber model to analyze the structural walls. The fiber model idealizes a section by breaking it into a number of discrete fibers. For each fiber, a uniaxial hysteretic model is assigned along with the kinematic and equilibrium requirements to obtain the mechanical behavior of the entire section. It is widely accepted that the fiber model can well describe the interaction between axial load and flexural moment; however, due to the assumption that plane sections remain plane, shear and bond-slip responses are generally neglected. Two main different approaches with different finite element formulations are adopted in general. One is the displacement-based element (e.g., Helleland and Scordelis, 1981; Mari, 1984), while the other is the force-based element (e.g., Spacone *et al.*, 1996; Neuenhofer and Filippou, 1997). The shortcoming of the displacement-based element is that a fine mesh of element is vital, due to the assumption of a linear distribution of the curvature along the element, which is not true where high inelastic deformation exists. Contrary to displacement-based element, the denser mesh is not necessary for a force-based element to improve its accuracy of inelastic response, since the use of force interpolation functions exactly satisfy the equilibrium. However, the force-based element requires more computational efforts than the displacement-based one [Alemedar and White, 2005; Fragiadakis and Papadrakakis, 2008]. Recently, Pugh *et al.* [2015] validated the force-based fiber model using data obtained from 21 slender wall experiments, and a generalized constitutive model was proposed for better predictions.

In 2006, the shake-table test of a 7-story RC load-bearing wall prototype building was conducted at the University of California at San Diego [Panagiotou and Restrepo, 2011; Panagiotou *et al.*, 2011]. Martinelli and Filippou [2009] adopted the 2-D fiber element to simulate this shake-table test. In each story, two fiber elements each with three integration points were used for simplicity, one at the centerline of the wall web and another at the centerline of the flange wall. Very good agreement was obtained in the displacement and deformation time histories. However, Martinelli and Filippou also pointed out that an underestimation of internal forces in the wall web since the 2-D model cannot capture the additional forces generated by the deformation of the slotted connector and the slab, and suggested a 3-D fiber model for a better prediction. Similar analysis has also been carried out by Schiotanus and Maffei [2008], and they concluded that slabs with columns increased the overturning resistance and the shear demand of the wall. Thus, it is important to consider the effect of gravity framing system on the structural walls.

Some fiber models for the analysis of the nonlinear behavior of structural walls are fundamentally based on the concept of MVLEM. Orakcal *et al.* [2004] adopted the macroscopic fiber-based model and implemented it with up-to-date cyclic constitutive relationships for concrete and reinforcement instead of the conventional hysteretic models of AESM and OOH. Tested specimens with rectangular and T-shaped cross sections [Thomsen and Wallace, 1995, 2004] were simulated to assess the accuracy of the proposed fiber-based model. Good correlations were obtained, in terms of the lateral load capacity, displacement profile, average rotations, etc. Nevertheless, this model under estimates the peak compressive strains and Wallace [2007] indicated that one of the reason is that this new model shared the general problems existing in fiber model that shear and flexural responses were uncoupled. In addition, for T-shaped walls, since the nonlinear tensile strain distribution along the flange cannot be captured, the lateral load capacity and inelastic rotations were overestimated while inelastic lateral displacements were underestimated. Jalali and Dashiti [2010] compared this fiber-based model with finite element model, and came to conclusion that taking the computational efforts, sensitivity of mesh sizes, and the accuracy of global behavior (e.g., overall wall lateral load vs. top displacement) into account, this fiber-based macro model exhibited its advantages, though same observation was obtained that the maximum compressive strain was substantially underestimated at critical section. Based on the Orakcal's fiber-based model, Bao and Kunnath [2010] even provided a modified model which assumed the unsupported second story wall as an upper triangular region modeled by a diagonal spring to simplify the modelling of progressive collapse of frame-wall structures.

Petrangeli *et al.* [1999] proposed a new fiber model to describe the shear mechanism for each concrete fiber of the cross section. Adopting this idea, fiber-based macro models to capture the interaction between shear, flexural, and axial behavior have been introduced [Massone *et al.*, 2006, 2009; Jiang and Kurama, 2010]. Massone *et al.* [2006] modified the Orakcal's model [2004] by assigning a shear spring for each uniaxial element, so that each uniaxial element was treated as a RC panel element (Fig. 27). The rotating-angle softened-truss model [Pang and Hsu, 1995] was used herein to represent the constitutive panel behavior, while MCFT can also be adopted as an alternative method. Unlike the 2-D SPEM proposed by Colotti [1993], this fiber-based model considered the flexure deformation of the central panel. By assigning shear spring for each vertical element (or fiber), the axial and shear responses were coupled. Since the multiple vertical elements can

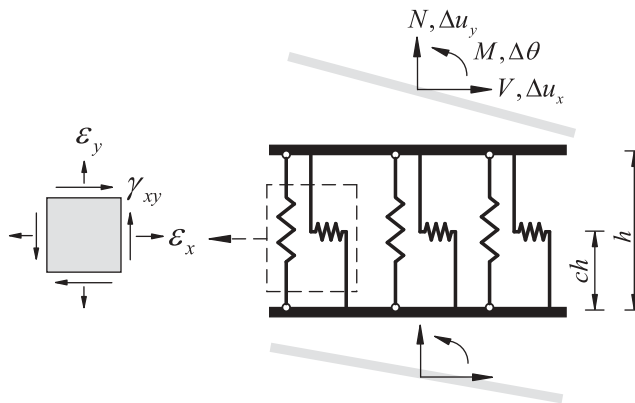


FIGURE 27 Coupled model element [Massone *et al.*, 2006].

account for the flexural response of structural walls, the flexural and shear interaction is incorporated. Results of monotonic analysis for slender shear walls showed that the coupling of nonlinear shear and flexural responses was captured, but the flexural deformation was overestimated and the shear deformation underestimated. For a shear-dominated wall, the lateral load capacities predicted using Massone's coupled model and common flexural fiber model were very different, showing the necessity to consider the SFI [Massone *et al.*, 2006]. Whyte and Stojadinovic [2013] adopted Massone's coupled model to analyze the squat walls and proved that this model can well predict the behavior of structural walls up to the peak strength, while it was also pointed out that the prediction of the post-peak behavior of different failure modes needed further research.

Based on the fiber model introduced by Orakcal *et al.* [2004], Kolozvari *et al.* [2015] proposed a SFI-MVLEM (Fig. 28), which can capture the shear-flexure interaction by replacing each original uniaxial element with a RC panel element subjected to membrane actions (Fig. 29). Although this model is very similar to the modified fiber-based MVLEM introduced by Massone *et al.* [2006], the SFI-MVLEM can perform well in simulating cyclic responses while the previous one can only capture monotonic responses. Behavior of the RC panel element in the SFI-MVLEM adopted the extended fixed-strut-angle-model (FSAM) proposed by Orakcal *et al.* [2012] and incorporated the shear aggregate interlock and dowel action. However, squat walls with aspect ratios less than 1.0 cannot be satisfactorily modeled by the SFI-MVLEM due to the zero resultant horizontal stress assumption.

Fischinger *et al.* [2012] developed a fiber-based MVLEM that can model the shear transfer mechanism across the cracks corresponding to the diagonal tension failure. In this model, as depicted in Fig. 30, the inelastic shear behavior and shear-flexure interaction are accounted for. Each vertical element (strip) contains a horizontal spring (HS) consisting of three parallel components (Fig. 31). These three components—HSA, HSD, and HSS—represent the interlock of aggregate in the crack, dowel effect of vertical reinforcement, and the axial resistance of shear reinforcement respectively. Compared with shaking table test results of a 1:3 scaled 5-story H-shaped coupled wall, this new model captured the

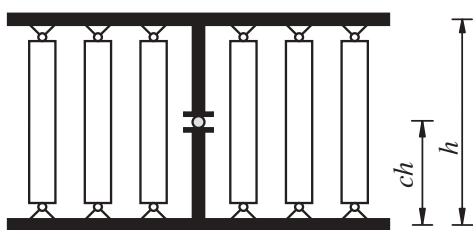


FIGURE 28 SFI-MVLEM proposed by Kolozvari *et al.* [2015].

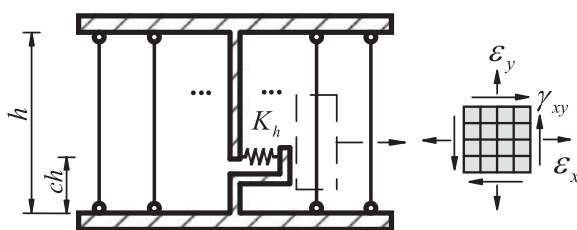


FIGURE 29 RC panel replacement in SFI-MVLEM.

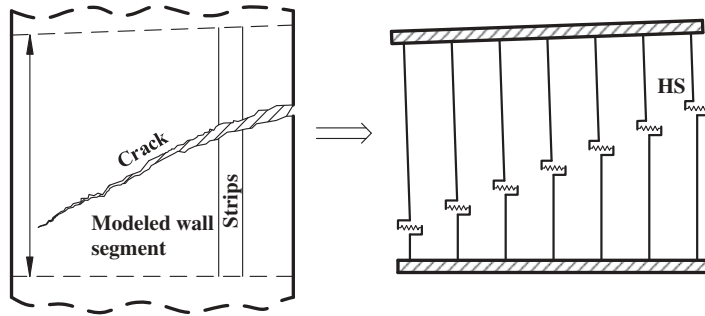


FIGURE 30 Fiber-based MVLEM developed by Fischinger *et al.* [2012].

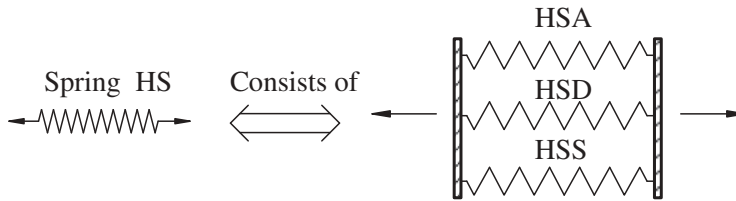


FIGURE 31 Three components of horizontal spring.

global responses quite well. The HSA spring successfully described the deterioration of the aggregate interlock in one direction and the other two (i.e. HSD and HSS) were subsequently activated. However, the interlock of aggregate was highly dependent on the cracks' width and the constitutive relations for each individual spring were semi-empirical.

It is noteworthy that in contrast with the heated research of rectangular and T-shaped walls, fewer studies concerned about the nonlinear behavior of core structural walls with U-shaped and channel-shaped sections [Beyer *et al.*, 2008a; Lowes *et al.*, 2013]. Based on the wide-column model of early stage [MacLeod and Hosny, 1977; Kwan, 1991], Beyer *et al.* [2008b] combined the fiber model with wide-column model to simulate the nonlinear behavior of U-shaped section RC walls. Different from the common fiber model, Beyer's wide-column model needs the subdivision of the U-shaped section into planar sections, and carefully choose the properties and vertical spacing of the horizontal links. This model has been recommended in SeismoStruct [2015].

The fiber-based macro models of RC structural walls have been widely applied both in practical engineering and computational research platform such as aforementioned SeismoStruct [2015]. In addition, in Perform-3D, a nonlinear structural analysis program distributed by CSI [2011], users can analyze structural walls by adopting fiber models with a general shear modulus. In OpenSees [2015], the fiber-based models developed by Massone *et al.* [2009] and Kolozvari *et al.* [2015] have been incorporated. One should notice that, however, most of fiber-based models cannot satisfactorily model the diagonal failures of RC walls, as indicated by Lu *et al.* [2014].

7. Concluding Remarks

From the early OVLEMs to the recently developed ones, the macroscopic modeling approach has been significantly progressed with the increasing capability of capturing complicated response characteristics of structural walls under earthquake ground motions. Important behaviors such as the migration of neutral axis and rocking of the wall can be

considered by most of macro models reviewed in this article. In addition, the fulfilling of even more challenging modeling objectives in terms of the SFI and the nonlinear performance of squat walls has been noticeably improved. Although this article does not elaborate the development of hysteretic model in detail, it is crucial to the success of the simulations when adopt macro models; therefore, more refined hysteretic models and material constitute relationship corresponding to different macro models are still needed. By thoroughly reviewing the four groups of macroscopic models, the following conclusion remarks can be drawn.

1. The VLEMs can be regarded as a solid basis for the development of most of other macro models, such as the 2-D SPEMs and fiber-based models. The VLEMs are simple, easy to implement and accurate enough to simulate the flexural behavior of walls. The main disadvantages of the VLEMs are the poor simulation of the SFI and the empirical hysteretic rules of model elements.
2. The 2-D SPEMs attempt to acquire more accurate results for less slender walls under high shear stresses, as well as to incorporate the SFI, by introducing shear panel elements. Some of the 2-D SPEMs combine the FE element and macro model elements; however, the simulation for walls under high shear stresses are still less than satisfactory.
3. The ETMs are suitable to be used in analyzing structural walls of complicated geometric configurations such as walls with T-shaped and H-shaped cross sections, openings and low aspect ratios. The modeling of the SFI is much improved in the ETMs compared with those previously developed macro models. In addition, the diagonal tension failure, which is a great challenge to other macro models, can be very well considered by the ETMs.
4. The fiber-based model has become one of the most accepted macro models nowadays, which has been incorporated into several wide spread analysis programs such as the Perform-3D, OpenSees, and SeismoStruct. Adopting material constitute relationship, the fiber-based model avoids some empirical parameters for hysteretic models. Modified by assigning a shear spring to each vertical fiber in fiber-based models, the axial and shear responses are coupled and the flexural response can be accounted for; consequently, the flexure and shear interaction is considered. However, as for more detailed simulation, especially when diagonal failure occurs, the fiber-based model still has limitation to obtain accurate results.

Suggestions for further research on the macroscopic models of structural walls are as follows.

1. The macro modeling issues of structural walls with special configurations need to be addressed, including the walls with irregular cross sections (U-shaped, channel-shaped, and T-shaped section), openings and the walls with special reinforcement details such as embedded steel elements.
2. The macro modeling techniques that can capture the significant 3-D effects need further research. Examples of such walls include those with substantial load transfer and/or deformation compatibility relations with adjoining structural elements such as slabs, intersecting beams and walls.
3. The fiber-based macro models have shown a good balance between analysis accuracy and computational cost and have been incorporated into several computer programs. However, further studies are necessary for this type of macro models to enhance their simulation capabilities for squat walls exhibiting complicated shear behavior.

Acronyms

AESM:	two-axial-element-in-series model
ASHM:	axial-stiffness hysteresis model
ETM:	equivalent truss model
FSAM:	fixed-strut-angle model
HS:	horizontal spring
HSA:	horizontal spring for aggregate interlock
HSD:	horizontal spring for dowel effect
HSS:	horizontal spring for shear (horizontal) reinforcement
MCFT:	modified compression-field theory
MVLEM:	multiple-vertical-line-element model
OOHH:	origin-oriented hysteresis model
OVLEM:	one-vertical-line-element model
SFI:	shear-flexure interaction
SFI-MVLEM:	shear-flexure interaction multiple-vertical-line element model
SPEM:	shear panel element model
TVLEM:	three-vertical-line-element model
VLEM:	vertical-line-element model

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