Linear Regression

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1 Discrete to Continuous Labels

From classification to regression

1.1 Task

Given $X \in \mathcal{X}$, predict $Y \in \mathcal{Y}$, Construct prediction rule $f : \mathcal{X} \to \mathcal{Y}$

1.2 Performance Measure

- Quantifies knowledge gained.
- Measure of closeness between true label Y and prediction f(X)
  - 0/1 lose:$loss(Y, f(X)) = 1_{f(X) \neq Y}$. Risk: probability of error
  - square loss: $loss(Y, f(X)) = (f(X) - Y)^2$. Risk: mean square error
- How well does the predictor perform on average?

$$Risk\ R(f) = \mathbb{E}[loss(Y, f(X))],\ (X, Y) \sim P_{XY}$$

1.3 Bayes Optimal Rule

- ideal goal: Construct prediction rule $f^* : \mathcal{X} \to \mathcal{Y}$

$$f^* = \arg\min_f E_{XY}[loss(Y, f(X))]$$

  (Bayes optimal rule)

- Best possible performance:

$$\forall f,\ R(f^*) \leq R(f)$$

  (Bayes Risk)

Problem: P_{XY} is unknown.
Solution: Training data provides a glimpse of P_{XY}

$$\text{(observed) } \{(X_i, Y_i)\} \sim_{i.i.d} P_{XY} \text{ unknown}$$

2 Macine Learning Algortihm

- Model based approach: use data to learn a model for P_{XY}
- Model-free approach: use data to learn mapping directly

2.1 Empirical Risk Minimization (model-free)

- Optimal predictor:

$$f^* = \arg\min_f \mathbb{E}[(f(X) - Y)^2]$$

- Empirical Minimizer:

$$\hat{f}_n = \arg\min_{f \in \mathcal{F}} \frac{1}{n} \sum_{i=1}^{n} (f(X) - Y)^2$$

$\mathcal{F}$ is the class of predictors:

- Linear
- Polynomial
- Nonlinear

3 Linear Regression

$$f(\vec{X}) = \sum_{i=0}^{p} \beta_0 X^i = \vec{X}^T \vec{\beta},\ where\ X^0 = 1,\ \vec{\beta} = [\beta_0, \ldots, \beta_p]^T$$

$$\hat{\vec{\beta}} = \arg\min_{\vec{\beta}} (A^T \vec{\beta} - \vec{Y})^T (A^T \vec{\beta} - \vec{Y}),\ where\ A = [\vec{X}_1, \ldots, \vec{X}_n]$$

$$J(\beta) = (A^T \vec{\beta} - \vec{Y})^T (A^T \vec{\beta} - \vec{Y})$$

$$\begin{aligned} \frac{\partial J(\vec{\beta})}{\partial \vec{\beta}} &= \frac{\partial (A^T \vec{\beta} - \vec{Y})^T (A^T \vec{\beta} - \vec{Y})}{\partial \vec{\beta}} \\ &= \frac{\partial \vec{\beta}}{den} \end{aligned}$$