Homework 1
MLE, MAP estimates; Linear and Logistic Regression

CMU 10-701: Machine Learning (Spring 2017)

OUT: Jan 31
DUE: Feb 10, 11:59 PM
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Part A: Multiple Choice Questions

1. For each case listed below, what type of machine learning problem does it belong to?
   (a) Advertisement selection system, which can predict the probability whether a customer will click on an ad or not based on the search history.
   Answer: B. Supervised learning: Regression
   A task, with ads click statistics and search history as input data, outputs the prediction of continuous probability of clicking an ad.

   (b) U.S post offices use a system to automatically recognize handwriting on the envelope.
   Answer: A. Supervised learning: Classification
   A task, with handwriting samples and their labels as input data, outputs the prediction of discrete numbers/letters of a handwriting on the envelope.

   (c) Reduce dimensionality using principal components analysis (PCA).
   Answer: C. Unsupervised learning
   A task, without training data as input, outputs a description of reduced dimensionality.

   (d) Trading companies try to predict future stock market based on current market conditions.
   Answer:

   - A. Supervised learning: Classification
     A task, with current market conditions as input, outputs the prediction of discrete stock market conditions, say bull or bear market.
   - B. Supervised learning: Regression
     A task, with current market conditions as input, outputs the prediction of continuous stock market conditions, say stock price.

   (e) Repair a digital image that has been partially damaged.
   Answer:

   - A. Supervised learning: Classification
     A task, with digital images database as input, outputs the prediction of discrete pixel value in the damaged zone.
   - B. Supervised learning: Regression
     A task, with digital images database as input, outputs the prediction of continuous parameters of a color distribution model to form a discrete patch to cover the damaged zone.
   - C. Unsupervised learning
     A task, without training data as input, outputs a description of a damaged pixel according to its surrounding pixel values, e.g. interpolation or extrapolation.

   Type of machine learning problem:

A. Supervised learning: Classification

B. Supervised learning: Regression

C. Unsupervised learning

2. For four statements below, which one is wrong?

   A. In maximum a posterior (MAP) estimate, data overwhelms the prior if we have enough data.

   B. There are no parameters in non-parametirc models.

   C. $P(X \cap Y \cap Z) = P(Z|X \cap Y)P(Y|X)P(X)$.

   D. Compared with parametric models, non-parameter models are flexible, since they don't make strong assumptions.

   Answer: B. There are no parameters in non-parametirc models. is wrong.
   Non-parametric model still needs parameters to describe the model, but the number of model's parameters is not fixed and will grow with the data size. The non-parametric only means that there is weak assumption on the model's type defined by a fixed number of parameters.

3. There are about 12% people in U.S. having breast cancer during their lifetime. One patient has a positive result for the medical test. Suppose the sensitivity of this test is 90%, meaning the test will be positive with probability 0.9 if one really has cancer. The false positive is likely to be 2%. Then what is the probability this patient actually having cancer based on Bayes Theorem?
   A. 90% B. 86% C. 12% D. 43%
   Answer: B. 86%

   - $P(C = 1) = 0.12$
   - $P(T = 1|C = 1) = 0.90$
   - $P(T = 1|C = 0) = 0.02$

$$\begin{aligned} P(C=1|T=1) &= \tfrac{P(T=1|C=1)P(C=1)}{P(T=1|C=1)P(C=1)+P(T=1|C=0)P(C=0)} \\ &= \tfrac{0.90\times0.12}{0.90\times0.12+0.02\times0.88} \\ &= 0.86 \end{aligned}$$

4. What is the most suitable error function for gradient **descent** using logistic regression?

   A. The negative log-likelihood function

   B. The number of mistakes

   C. The squared error

   D. The log-likelihood function

   Answer: A. The negative log-likelihood function
   The negative log-likelihood function of a logistic regression is a convex function.

Part B, Problem 1: Bias-Variance Decomposition

Consider a p-dimensional vector $\vec{x} \in \mathbb{R}^p$ drawn from a Gaussian distribution with an identity covariance matrix $\Sigma = I_p$ and an unknown mean $\vec{\mu}$, i.e. $\vec{x} \sim \mathcal{N}(\vec{\mu}, I_p)$. Our goal is to evaluate the effectiveness of an estimator $\hat{\vec{\mu}} = f(\vec{x})$ of the mean from only a single sample (i.e. $n = 1$) by measuring its mean squared error $\mathbb{E}[||\hat{\vec{\mu}} - \vec{\mu}||^2]$, where $|| \cdot ||^2$ is the squared Euclidean norm and the expectation is taken over the data generating distribution.

Note that for any estimator $\hat{\vec{\theta}}$ of a parameter vector $\vec{\theta}$, its mean squared error can be decomposed as:

$$\mathbb{E}[||\hat{\vec{\theta}} - \vec{\theta}||^2] = ||Bias[\hat{\vec{\theta}}]||^2 + trace(Var[\hat{\vec{\theta}}])$$

where,

$$Bias[\hat{\vec{\theta}}] = \mathbb{E}[\hat{\vec{\theta}}] - \vec{\theta} \; and \; Var[\hat{\vec{\theta}}]_{i,i} = Var[\hat{\theta}_i] = \mathbb{E}[(\hat{\theta}_i - \mathbb{E}[\hat{\theta}_j])^2]$$

1. Derive the maximum likelihood estimator:

$$\hat{\vec{\mu}}_{MLE} = \arg\max_{\vec{\mu}} P(\vec{x}|\vec{\mu})$$

Answer:
$\because$ We estimate the mean $\vec{\mu}$ from only a single sample $\vec{x}_1 \sim \mathcal{N}(\vec{\mu}, I_p)$
$\therefore$ The likelihood function is

$$L(\vec{\mu}) = P(\vec{x}_1|\vec{\mu}) = \frac{1}{(2\pi)^{p/2}|I_p|^{1/2}} \exp(-\frac{1}{2}(\vec{x}_1 - \vec{\mu})^T I_p^{-1}(\vec{x}_1 - \vec{\mu}))$$

$\therefore$

$$\hat{\vec{\mu}}_{MLE} = \arg\max_{\vec{\mu}} L(\vec{\mu}) = \vec{x}_1$$

What is its mean squared error?
Answer:
$\because$ The MSE of $\hat{\vec{\mu}}_{MLE}$ is

$$\begin{aligned}\mathbb{E}[||\hat{\vec{\mu}}_{MLE} - \vec{\mu}||^2] &= ||Bias[\hat{\vec{\mu}}_{MLE}]||^2 + trace(Var[\hat{\vec{\mu}}_{MLE}]) \\ &= ||\mathbb{E}[\hat{\vec{\mu}}_{MLE}] - \vec{\mu}||^2 + \textstyle\sum_{i=1}^{p} \mathbb{E}[(\hat{\mu}_{MLE_i} - \mathbb{E}[\hat{\mu}_{MLE_i}])^2]\end{aligned}$$

$\because$ $\hat{\vec{\mu}}_{MLE} = \vec{x}_1$
$\therefore$ $\mathbb{E}[\hat{\vec{\mu}}_{MLE}] = \mathbb{E}[\vec{x}_1] = \vec{\mu}$
$\therefore$

$$\mathbb{E}[||\hat{\vec{\mu}}_{MLE} - \vec{\mu}||^2] = \sum_{i=1}^{p} \mathbb{E}[(\vec{x}_{1_i} - \vec{\mu}_i)^2]$$

$\because$ $\vec{x} \sim \mathcal{N}(\vec{\mu}, I_p)$
$\therefore$

$$MSE = \mathbb{E}[||\hat{\vec{\mu}}_{MLE} - \vec{\mu}||^2] = p$$

2. Derive the ℓ_2-regularized maximum likelihood estimator:

$$\hat{\vec{\mu}}_{RMLE} = \arg\max_{\vec{\mu}} \log P(\vec{x}|\vec{\mu}) - \lambda||\vec{\mu}||^2$$

Answer:
$\because$ We estimate the mean $\vec{\mu}$ from only a single sample $\vec{x}_1 \sim \mathcal{N}(\vec{\mu}, I_p)$
$\therefore$ The ℓ_2-regularized log-likelihood function is

$$L(\vec{\mu}) = C - \frac{1}{2}(\vec{x}_1 - \vec{\mu})^T(\vec{x}_1 - \vec{\mu}) - \lambda\vec{\mu}^T\vec{\mu} = C - \frac{1}{2}\vec{x}_1^T\vec{x}_1 + \vec{x}_1^T\vec{\mu} - (\frac{1}{2} + \lambda)\vec{\mu}^T\vec{\mu}$$

$\therefore$

$$\begin{aligned}
\left.\frac{\partial L(\vec{\mu})}{\partial \vec{\mu}}\right|_{\vec{\mu}_{RMLE}} &= \vec{0} \\
(\vec{x}_1 - (1+2\lambda)\vec{\mu})|_{\vec{\mu}_{RMLE}} &= \vec{0} \\
\vec{\mu}_{RMLE} &= \tfrac{1}{1+2\lambda}\vec{x}_1
\end{aligned}$$

What is its mean squared error?

Answer:

From question 1, we know that

$$\mathbb{E}[||\hat{\vec{\mu}}_{RMLE} - \vec{\mu}||^2] = ||\mathbb{E}[\hat{\vec{\mu}}_{RMLE}] - \vec{\mu}||^2 + \sum_{i=1}^{p} \mathbb{E}[(\hat{\mu}_{RMLE_i} - \mathbb{E}[\hat{\mu}_{RMLE_i}])^2]$$

$\because \vec{\mu}_{RMLE} = \frac{1}{1+2\lambda}\vec{x}_1$

$\therefore \mathbb{E}[\hat{\vec{\mu}}_{RMLE}] = \mathbb{E}[\frac{1}{1+2\lambda}\vec{x}_1] = \frac{1}{1+2\lambda}\vec{\mu}$

$\therefore$

$$\mathbb{E}[||\hat{\vec{\mu}}_{RMLE} - \vec{\mu}||^2] = (\frac{2\lambda}{1+2\lambda})^2||\vec{\mu}||^2 + (\frac{1}{1+2\lambda})^2 \sum_{i=1}^{p} \mathbb{E}[(\vec{x}_{1_i} - \vec{\mu}_i)^2]$$

$\because \vec{x} \sim \mathcal{N}(\vec{\mu}, I_p)$

$\therefore$

$$MSE = \mathbb{E}[||\hat{\vec{\mu}}_{RMLE} - \vec{\mu}||^2] = (\frac{2\lambda}{1+2\lambda})^2||\vec{\mu}||^2 + (\frac{1}{1+2\lambda})^2 p$$

3. Consider an estimator of the form $\hat{\vec{\mu}}_{SCALE} = c\vec{x}$ where $c \in \mathbb{R}$ is a constant scaling factor. Find the value c^* that minimizes its mean squared error:

$$c^* = \arg\min_c \mathbb{E}[||c\vec{x} - \vec{\mu}||^2]$$

Answer:

From question 2, if we assume $c = \frac{1}{1+2\lambda}$, we can easily get the objective function for $\hat{\vec{\mu}}_{SCALE} = c\vec{x}$:

$$J_{MSE}(c) = (c-1)^2||\vec{\mu}||^2 + c^2 p$$

$\therefore$

$$\begin{aligned}
\left.\frac{dJ_{MSE}(c)}{dc}\right|_{c^*} &= 0 \\
((||\vec{\mu}||^2 + p)c - ||\vec{\mu}||^2)\Big|_{c^*} &= 0 \\
c^* &= \tfrac{||\vec{\mu}||^2}{||\vec{\mu}||^2 + p}
\end{aligned}$$

What is the corresponding minimum mean squared error?

Answer:

If $c^* = \frac{||\vec{\mu}||^2}{||\vec{\mu}||^2+p}$, then

$$MSE^* = J_{MSE}(c^*) = (\frac{p}{||\vec{\mu}||^2 + p})^2||\vec{\mu}||^2 + (\frac{||\vec{\mu}||^2}{||\vec{\mu}||^2+p})^2 p = \frac{||\vec{\mu}||^2 p}{||\vec{\mu}||^2 + p}$$

4. Consider the James-Stein estimator:

$$\hat{\vec{\mu}}_{JS} = \left(1 - \frac{p-2}{||\vec{x}||^2}\right)\vec{x}$$

Note that $\hat{\vec{\mu}}_{JS}$ can be written as $\vec{x} - g(\vec{x})$ where $g(\vec{x}) = \frac{p-2}{||\vec{x}||^2}\vec{x}$. This allows us to separate the mean squared error into three parts:

$$\begin{aligned}
\mathbb{E}[||\hat{\vec{\mu}}_{JS} - \vec{\mu}||^2] &= \mathbb{E}[||\vec{x} - g(\vec{x}) - \vec{\mu}||^2] \\
&= \mathbb{E}[\vec{x}^T\vec{x} - 2\vec{x}^T\vec{\mu} + \vec{\mu}^T\vec{\mu} + g(\vec{x}^T g(\vec{x}) - 2\vec{x}^T g(\vec{x}) + 2\vec{\mu}^T g(\vec{x})] \\
&= \mathbb{E}[||\vec{x} - \vec{\mu}||^2] + \mathbb{E}[||g(\vec{x})||^2] - 2\mathbb{E}[(\vec{x} - \vec{\mu})^T g(\vec{x})]
\end{aligned}$$

Furthermore, from Stein's lemma, we know that:

$$\mathbb{E}[(\vec{x}-\vec{\mu})^T g(\vec{x})] = \mathbb{E}\left[\sum_{j=1}^{p} \frac{\partial}{\partial x_j} g_j(\vec{x})\right]$$

- Find $\mathbb{E}[||\vec{x}-\vec{\mu}||^2]$.
  Answer:

$$\begin{aligned}\mathbb{E}[||\vec{x}-\vec{\mu}||^2] &= \mathbb{E}[\textstyle\sum_{i=1}^{p}(x_i-\mu_i)^2] \\ &= \textstyle\sum_{i=1}^{p}\mathbb{E}[(x_i-\mu_i)^2]\end{aligned}$$

  $\because \vec{x} \sim \mathcal{N}(\vec{\mu}, I_p)$
  $\therefore$

$$\mathbb{E}[||\vec{x}-\vec{\mu}||^2] = p$$

- Find $\mathbb{E}[||g(\vec{x})||^2]$. (Hint: your answer will include $\mathbb{E}[||\vec{x}||^{-2}]$)
  Answer:
  $\because g(\vec{x}) = \frac{p-2}{||\vec{x}||^2}\vec{x}$
  $\therefore$

$$\mathbb{E}[||g(\vec{x})||^2] = (p-2)^2\mathbb{E}[\frac{\vec{x}^T\vec{x}}{(||\vec{x}||^2)^2}] = (p-2)^2\mathbb{E}[||\vec{x}||^{-2}]$$

- Show that:

$$\frac{\partial}{\partial x_j} g_j(\vec{x}) = (p-2)\frac{||\vec{x}||^2 - 2x_j^2}{||\vec{x}||^4}$$

  where x_j is the jth element of x and $g_j(\vec{x})$ is the jth element of $g(\vec{x})$.
  Answer:
  $\because g_j(\vec{x}) = \frac{p-2}{||\vec{x}||^2}x_j$
  $\therefore$

$$\begin{aligned}\tfrac{\partial}{\partial x_j} g_j(\vec{x}) &= \tfrac{\partial}{\partial x_j}(\tfrac{p-2}{||\vec{x}||^2}x_j) \\ &= (p-2)(\tfrac{\partial(\sum_{i=1}^{p}x_i^2)^{-1}}{\partial x_j}x_j + \tfrac{1}{||\vec{x}||^2}) \\ &= (p-2)(\tfrac{-2x_j^2}{(\sum_{j=1}^{p}x_j^2)^{-2}} + \tfrac{1}{||\vec{x}||^2}) \\ &= (p-2)\tfrac{||\vec{x}||^2-2x_j^2}{||\vec{x}||^4}\end{aligned}$$

- What is the resulting mean squred error. (Hint: your answer will include $\mathbb{E}[||\vec{x}||^{-2}]$)
  Answer:
  $\because$
  - $\mathbb{E}[||\vec{x}-\vec{\mu}||^2] = p$
  - $\mathbb{E}[||g(\vec{x})||^2] = (p-2)^2\mathbb{E}[||\vec{x}||^{-2}]$
  - $\frac{\partial}{\partial x_j} g_j(\vec{x}) = (p-2)\frac{||\vec{x}||^2-2x_j^2}{||\vec{x}||^4}$

  $\therefore$

$$\begin{aligned}\mathbb{E}[||\hat{\vec{\mu}}_{JS}-\vec{\mu}||^2] &= \mathbb{E}[||\vec{x}-\vec{\mu}||^2] + \mathbb{E}[||g(\vec{x})||^2] - 2\mathbb{E}\left[\textstyle\sum_{j=1}^{p}\tfrac{\partial}{\partial x_j}g_j(\vec{x})\right] \\ &= p + (p-2)^2\mathbb{E}[||\vec{x}||^{-2}] - 2(p-2)\mathbb{E}[\tfrac{p||\vec{x}||^2-2\sum_{j=1}^{p}x_j^2}{||\vec{x}||^4}] \\ &= p + (p-2)^2\mathbb{E}[||\vec{x}||^{-2}] - 2(p-2)^2\mathbb{E}[||\vec{x}||^{-2}] \\ &= p - (p-2)^2\mathbb{E}[||\vec{x}||^{-2}]\end{aligned}$$

5. Qualitatively compare these estimators, noting any similarities between them. How does regularization affect an estimator's bias and variance? Which estimator would you choose to approximate $\vec{\mu}$ from

real data about which you have no prior knowledge? How does the data dimensionality p affect your answer?
Answer:

- Similarities:
  - For $\hat{\vec{\mu}}_{RMLE}$, if $\lambda = 0$, $\hat{\vec{\mu}}_{RMLE} = \hat{\vec{\mu}}_{MLE} = \vec{x}_1$
  - For $\hat{\vec{\mu}}_{SCALE}$, if $c = 1$, $\hat{\vec{\mu}}_{SCALE} = \hat{\vec{\mu}}_{MLE} = \vec{x}_1$
  - For $\hat{\vec{\mu}}_{JS}$, if $p = 2$, $\hat{\vec{\mu}}_{JS} = \hat{\vec{\mu}}_{MLE} = \vec{x}_1$
- The regularization increases the bias, but decreases the variance.
- If I have no prior knowledge, I will choose MLE, because its MSE is not related with the prior knowledge of $\vec{\mu}$ and thus is predictable.
- The increase of dimensionality p will increase the MSE except for the James-Stein estimator.
  $\because \mathbb{E}[||\vec{x}||^{-2}] \geq 0$
  $\therefore MSE(p) = -\mathbb{E}[||\vec{x}||^{-2}]p^2 + (4\mathbb{E}[||\vec{x}||^{-2}] + 1)p - 4\mathbb{E}[||\vec{x}||^{-2}]$ must have maximum value.
  $\therefore$ If the dimensionality p is very large, we can choose James-Stein estimator to constrain its MSE level.

Part B, Problem 2: Linear Regression

Suppose we observe N data pairs $\{(x_i, y_i)\}_{i=1}^N$, where y_i is generated by the following rule:

$$y_i = \vec{x}_i^T\vec{\beta} + \epsilon_i$$

where $\vec{x}_i, \vec{\beta} \in \mathbb{R}^d$, and ϵ_i is an i.i.d random noise drawn from the Gaussian Distribution:

$$\epsilon_i \sim \mathcal{N}(0, \sigma^2)$$

with a known constant σ. We further denote $\vec{Y} = [y_1, y_2, \ldots, y_N]^T$ and $X = [\vec{x}_1, \vec{x}_2, ..., \vec{x}_N]^T$.

Now, we are interested in estimating $\vec{\beta}$ from the observed data.

1. Derive the likelihood function $\mathcal{L}(\vec{\beta})$
   Answer:
   $\because y_i = \vec{x}_i^T\vec{\beta} + \epsilon_i$ and $\epsilon_i \sim \mathcal{N}(0, \sigma^2)$
   $\therefore y_i \sim \mathcal{N}(\vec{x}_i^T\vec{\beta}, \sigma^2)$
   $\therefore$
   $$\begin{aligned} \mathcal{L}(\vec{\beta}) &= \textstyle\prod_{i=1}^N P(y_i|\vec{x}_i, \vec{\beta}) \\ &= \textstyle\prod_{i=1}^N (\frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{(y_i - \vec{x}_i^T\vec{\beta})^2}{2\sigma^2})) \end{aligned}$$

2. Show that the MLE estimator $\hat{\vec{\beta}}_{MLE}$ of $\vec{\beta}$ is equivalent to the solution of the following linear regression problem:
   $$\min_{\vec{\beta}} \frac{1}{2}||\vec{Y} - X\vec{\beta}||_2^2 \tag{1}$$

   Answer:
   $\because$ We can derive MLE estimator $\hat{\vec{\beta}}_{MLE}$ via the log likelihood:
   $$\begin{aligned} \hat{\vec{\beta}}_{MLE} &= \arg\max_{\vec{\beta}} \log\mathcal{L}(\vec{\beta}) \\ &= \arg\max_{\vec{\beta}} \textstyle\sum_{i=1}^N -\frac{1}{2\sigma^2}(y_i - \vec{x}^T\vec{\beta})^2 \\ &= \arg\min_{\vec{\beta}} \frac{1}{2}\textstyle\sum_{i=1}^N (y_i - \vec{x}^T\vec{\beta})^2 \\ &= \arg\min_{\vec{\beta}} \frac{1}{2}||\vec{Y} - X\vec{\beta}||_2^2 \end{aligned}$$

   $\therefore$ The MLE estimator is equivalent to the solution of the following linear regression problem:
   $$J^*(\vec{\beta}) = \min_{\vec{\beta}} \frac{1}{2}||\vec{Y} - X\vec{\beta}||_2^2$$

3. Now we suppose $\vec{\beta}$ is not a deterministic parameter, but a random variable having a Gaussian prior distribution:
   $$p(\vec{\beta}) \sim \mathcal{N}(\vec{0}, \frac{\sigma^2}{2\lambda}I_d)$$
   where I is a $d \times d$ identity matrix and $\lambda > 0$ is a known parameter. Show that the MAP estimation $\hat{\vec{\beta}}_{MAP}$ of $\vec{\beta}$ is equivalent to the solution of the following ridge regression problem:
   $$\min_{\vec{\beta}} \frac{1}{2}||\vec{Y} - X\vec{\beta}||_2^2 + \lambda||\vec{\beta}||_2^2 \tag{2}$$

   Answer:
   $\because p(\vec{\beta}) \sim \mathcal{N}(\vec{0}, \frac{\sigma^2}{2\lambda}I_d)$
   $\therefore$
   $$P(\vec{\beta}) = \frac{1}{(2\pi)^{d/2}|\frac{\sigma^2}{2\lambda}I_d|^{1/2}}\exp(-\frac{1}{2}\vec{\beta}^T(\frac{\sigma^2}{2\lambda}I_d)^{-1}\vec{\beta}) = \frac{1}{(\frac{\pi}{\lambda})^{d/2}\sigma^d}\exp(-\frac{\lambda}{\sigma^2}\vec{\beta}^T\vec{\beta})$$

$\because$ the posteriori distribution $P(\vec{\beta}|y_i, \vec{x}_i) \propto P(y_i, |\vec{x}_i, \vec{\beta})P(\vec{\beta})$
$\therefore$ the MAP estimator $\hat{\vec{\beta}}_{MAP}$ can be derived from

$$\begin{aligned}
\hat{\vec{\beta}}_{MAP} &= \arg\max_{\vec{\beta}} \textstyle\prod_{i=1}^{N} P(y_i, |\vec{x}_i, \vec{\beta})P(\vec{\beta}) \\
&= \arg\max_{\vec{\beta}} \textstyle\sum_{i=1}^{N}(-\frac{1}{2\sigma^2}(y_i - \vec{x}^T\vec{\beta})^2) - \frac{\lambda}{\sigma^2}\vec{\beta}^T\vec{\beta} \\
&= \arg\min_{\vec{\beta}} \textstyle\frac{1}{2}\sum_{i=1}^{N}(y_i - \vec{x}^T\vec{\beta})^2 + \lambda\vec{\beta}^T\vec{\beta} \\
&= \arg\min_{\vec{\beta}} \textstyle\frac{1}{2}||\vec{Y} - X\vec{\beta}||_2^2 + \lambda||\vec{\beta}||_2^2
\end{aligned}$$

$\therefore$ The MAP estimator is equivalent to the solution of the following ridge regression problem:

$$J^*(\vec{\beta}) = \min_{\vec{\beta}} \frac{1}{2}||\vec{Y} - X\vec{\beta}||_2^2 + \lambda||\vec{\beta}||_2^2$$

4. Refer to the closed form solutions of (1) and (2) in the lecture slides, what might be an issue of $\hat{\vec{\beta}}_{MLE}$ if $d >> N$? How can $\hat{\vec{\beta}}_{MAP}$ possibly address it?
Answer:
From the lecture, we know the closed form solutions of (1) and (2) as below:

- $\hat{\vec{\beta}}_{MLE} = (X^TX)^{-1}X^TY$
- $\hat{\vec{\beta}}_{MAP} = (X^TX + \lambda I)^{-1}X^TY$

$\because X$ is a $N \times d$ matrix.
$\therefore X^TX$ is a $d \times d$ matrix.
$\because rank(X^TX) \leq rank(X) \leq min(d, N) = N << d$
$\therefore X^TX$ is not invertible.
$\therefore \hat{\vec{\beta}}_{MLE}$ is not feasible.
$\because (X^TX + \lambda I)$ is a positive definite matrix, if $\lambda > 0$
$\therefore (X^TX + \lambda I)$ is invertible.
$\therefore \hat{\vec{\beta}}_{MAP}$ is feasible.

Part B, Problem 3: MLE, MAP and Logistic Regression

We learnt about Maximum Likelihood estimation in class. For a fixed set of data and underlying statistical model, the method of maximum likelihood selects the set of values of the model parameters that maximizes the likelihood function.

In this problem, we will look at two different ways of estimating parameters in a probability distribution. Suppose we observe n i.i.d. random variables $X_1, ..., X_n$, drawn from a distribution with parameter θ. That is, for each X_i and a natural number k,

$$P(X_i = k) = (1-\theta)^k \theta$$

Given some observed values of X_1 to X_n , we want to estimate the value of θ.

3.1 Maximum Likelihood Estimation

The first kind of estimator for θ we will consider is the Maximum Likelihood Estimator (MLE). The probability of observing given data is called the likelihood of the data, and the function that gives the likelihood for a given parameter $\hat{\theta}$ (which may or may not be equal to the true parameter θ) is called the likelihood function, written as $L(\hat{\theta})$. When we use MLE, we estimate θ by choosing the $\hat{\theta}$ that maximizes the likelihood.

$$\hat{\theta}_{MLE} = \arg\max_{\hat{\theta}} L(\hat{\theta})$$

It is often convenient to deal with the log-likelihood ($\ell(\hat{\theta}) = \log L(\hat{\theta})$) instead, and since log is an increasing function, the argmax also applies in the log space:

$$\hat{\theta}_{MLE} = \arg\max_{\hat{\theta}} \ell(\hat{\theta})$$

1. Given a dataset $\mathcal{D}$, containing observations $\{X_1 = k_1, X_2 = k_2, \ldots, X_n = k_n\}$, write an expression for $\ell(\hat{\theta})$ as a function of $\mathcal{D}$ and $\hat{\theta}$. How does the order of the variables affect the function?
   Answer:
   $\because P(X_i = k_i|\theta) = (1-\theta)^{k_i}\theta$
   $\therefore$
   $$\begin{aligned} \ell(\hat{\theta}) &= \prod_{i=1}^n P(X_i = k_i|\hat{\theta}) \\ &= (1-\theta)^{\sum_{i=1}^n k_i}\theta^n \end{aligned}$$

2. Derive an expression for the maximum likelihood estimate.
   Answer:
   Assume $K = \sum_{i=1}^n k_i$
   $$\begin{aligned} \hat{\theta}_{MLE} &= \arg\max_{\hat{\theta}} \ell(\hat{\theta}) \\ \frac{d\ell(\hat{\theta})}{d\hat{\theta}}\Big|_{\hat{\theta}_{MLE}} &= 0 \\ ((1-\theta)^{K-1}\theta^{n-1}(n-(n+K)\theta))\Big|_{\hat{\theta}_{MLE}} &= 0 \\ \hat{\theta}_{MLE} &= \frac{n}{n+K} = \frac{n}{n+\sum_{i=1}^n k_i} \end{aligned}$$

3.2 Maximum a Posteriori Estimation

Now we assume that we have some prior knowledge about the true parameter θ. We express it by treating θ itself as a random variable and defining a prior probability distribution over it. Precisely, we suppose that the data $X_1, \ldots, X_n$ are drawn as follows:

- θ is drawn from the prior probability distribution

- Then $X_1, \ldots, X_n$ are drawn independently from a Geometric distribution with θ as the parameter.

Now both X_i and θ are random variables, and they have a joint probability distribution. We now estimate θ as follows

$$\hat{\theta}_{MAP} = \arg\max_{\hat{\theta}} P(\theta = \hat{\theta}|X_1, \ldots, X_n)$$

This is called Maximum a Posteriori (MAP) estimation. Using Bayes rule, we can rewrite the posterior probability as follows.

$$P(\theta = \hat{\theta}|X_i, \ldots, X_n) = \frac{P(X_i, \ldots, X_n|\theta = \hat{\theta})P(\theta = \hat{\theta})}{P(X_1, \ldots, X_n)}$$

Applying this to the MAP estimate, we get the following expression. Notice that we can ignore the denominator since it is not a function of $\hat{\theta}$

$$\begin{aligned} \hat{\theta}_{MAP} &= \arg\max_{\hat{\theta}} P(X_1, \ldots, X_n|\theta = \hat{\theta})P(\theta = \hat{\theta}) \\ &= \arg\max_{\hat{\theta}} L(\hat{\theta})P(\theta = \hat{\theta}) \\ &= \arg\max_{\hat{\theta}}(\ell(\hat{\theta}) + \log P(\theta = \hat{\theta})) \end{aligned}$$

Thus, the MAP estimator maximizes the sum of the log-likelihood and the log-probability of the prior distribution on θ. WHen the prior is a continuous distribution with density function p, we have

$$\hat{\theta}_{MAP} = \arg\max_{\hat{\theta}}(\ell(\hat{\theta}) + \log p(\hat{\theta}))$$

For this problem, we will use the Beta distribution (a popular choice when the data dstribution is Geometric or Bernoulli) as the prior, and the density function is given by

$$p(\hat{\theta}) = \frac{\hat{\theta}^{\alpha-1}(1-\hat{\theta})^{\beta-1}}{B(\alpha, \beta)}$$

where $B(\alpha, \beta)$ is the beta function.

4. Derive a close form expression for the maximum a posteriori estimate. (hint: If x^* maximizes f, $f'(x^*) = 0$).
   Answer:
   $\because P(X_i = k_i|\theta) = (1-\theta)^{k_i}\theta$ and $p(\hat{\theta}) = \frac{\hat{\theta}^{\alpha-1}(1-\hat{\theta})^{\beta-1}}{B(\alpha,\beta)}$
   $\therefore$ The MAP estimate of $\hat{\theta}$ can be derived by (assume $K = \sum_{i=1}^{n} k_i$):

$$\begin{aligned} \hat{\theta}_{MAP} &= \arg\max_{\hat{\theta}} \prod_{i=1}^{n} p(X_i = k_i|\hat{\theta})p(\hat{\theta}) \\ &= \arg\max_{\hat{\theta}}(1-\hat{\theta})^K \hat{\theta}^n p(\hat{\theta}) \\ &= \arg\max_{\hat{\theta}} K \ln(1-\hat{\theta}) + n \ln \hat{\theta} + \ln p(\hat{\theta}) \\ &= \arg\max_{\hat{\theta}} K \ln(1-\hat{\theta}) + n \ln \hat{\theta} + (\alpha - 1)\ln \hat{\theta} + (\beta - 1)\ln(1-\hat{\theta}) \\ &= \arg\max_{\hat{\theta}}(n + \alpha - 1)\ln \hat{\theta} + (K + \beta - 1)\ln(1-\hat{\theta}) \end{aligned}$$

   Assume the objective function $J(\hat{\theta}) = (n + \alpha - 1)\ln \hat{\theta} + (K + \beta - 1)\ln(1-\hat{\theta})$, then

$$\begin{aligned} \left.\frac{dJ(\hat{\theta})}{d\hat{\theta}}\right|_{\hat{\theta}_{MAP}} &= 0 \\ \left.\left(\frac{n+\alpha-1}{\hat{\theta}} - \frac{K+\beta-1}{1-\hat{\theta}}\right)\right|_{\hat{\theta}_{MAP}} &= 0 \\ \hat{\theta}_{MAP} &= \frac{n+\alpha-1}{K+n+\alpha+\beta-2} \end{aligned}$$

5. Is the bias of Maximum Likelihood Estimate (MLE) typically greater than or equal to the bias of Maximum A Posteriori (MAP) estimate? (Explain your answer in a sentence)
   Answer:
   No, it depends on the difference between the prior distribution and real distribution of parameters.

6. What can you say about the value of Maximum Likelihood Estimate (MLE) as compared to the value of Maximum A Posteriori (MAP) estimate with a uniform prior? Why?
Answer:
If the prior is a uniform distribution, then with such a weak priori, the MLE is same with MAP. Because the priori has no effect on the MAP estimation, and only the shared likelihood between both estimators will determine the estimation value.

3.3 Logistic Regression

In class, we wil learn about MLE of parametrers in logistic regression. For a given data $\vec{x} \in \mathbb{R}^p$, the probability of Y being 1 in logistic regression is

$$P(Y = 1|\vec{X} = \vec{x}) = \frac{\exp(w_0 + \vec{x}^T\vec{w})}{1 + \exp(w_0 + \vec{x}^T\vec{w})} \tag{3}$$

where w_0 and $\vec{w} = (w_1, w_2, \ldots, w_p)^T$ are model parameters. In this problem, we consider the maximum a posteriori setting, where we put a Gaussian prior on the parameters:

$$w_i \sim \mathcal{N}(\mu, 1),\ for\ i = 0, 1, 2, \ldots, p.$$

7. Choose a conjugate prior for Gaussian on μ (choose any higher parameters as you want to ease the computation). Assuming you are given a dataset with n training examples and p features, write down a formula for the conditional log posterior likelihood of the training data in terms of the class labels $y^{(i)}$, the features $x_1^{(i)}, \ldots, x_p^{(i)}$, and the parameters $w_0, w_1, \ldots, w_p$, where the superscript (i) denotes the sample index. This will be your objective function for gradient ascent.
Answer:
Choose $\mu \sim \mathcal{N}(\tilde{\mu}, \sigma^2)$ as a conjugate prior for Gaussian on μ, then the conditional log posterior likelihood of the training data is (assume $\vec{\mathcal{Y}} = [y^{(1)}, y^{(2)}, \ldots, y^{(n)}]$, and $\mathcal{X} = [\vec{x}^{(1)}, \vec{x}^{(2)}, \ldots, \vec{x}^{(n)}]^T$):

$$\begin{aligned} f(w_0, \vec{w}, \mu) &= \ln(P(\vec{\mathcal{Y}}|\mathcal{X}, w_0, \vec{w}, \mu)P(w_0, \vec{w}|\mu)P(\mu)) \\ &= \textstyle\sum_{i=1}^n \ln(P(y^{(i)}|\vec{x}^{(i)}, w_0, \vec{w}, \mu)) + \sum_{j=0}^p \ln(P(w_j|\mu)) + \ln(P(\mu)) \\ &= \textstyle\sum_{i=1}^n (y^{(i)}(w_0 + \sum_{k=1}^p w_k x_k^{(i)}) - \ln(1 + \exp(w_0 + \sum_{k=1}^p w_k x_k^{(i)}))) \\ &\quad \textstyle -\frac{1}{2}\sum_{j=0}^p (w_j - \mu)^2 - \frac{1}{2\sigma^2}(\mu - \tilde{\mu})^2 + C \end{aligned}$$

8. Compute the partial derivative of the objective with respect to w_0, to an arbitrary w_i, and μ, i.e. derive $\partial f/\partial w_0$, $\partial f/\partial w_i$, $\partial f/\partial \mu$ where f is the objective that you provided above. Use (3) to simplify the formula. What is the MAP estimation of μ given w_0 and $\vec{w}$
Answer:

- $\partial f/\partial w_0$:

$$\begin{aligned} \partial f/\partial w_0 &= \textstyle\sum_{i=1}^n (y^{(i)} - \frac{\exp(w_0 + \sum_{k=1}^p w_k x_k^{(i)})}{1+\exp(w_0 + \sum_{k=1}^p w_k x_k^{(i)})}) \\ &= \textstyle\sum_{i=1}^n (y^{(i)} - P(Y^{(i)} = 1|\vec{x}^{(i)}, w_0, \vec{w})) \end{aligned}$$

- $\partial f/\partial w_i$:

$$\begin{aligned} \partial f/\partial w_i &= \textstyle\sum_{j=1}^n (y^{(j)} x_i^{(j)} - \frac{x_i^{(j)} \exp(w_0 + \sum_{k=1}^p w_k x_k^{(i)})}{1+\exp(w_0 + \sum_{k=1}^p w_k x_k^{(j)})}) \\ &= \textstyle\sum_{j=1}^n x_i^{(j)} (y^{(j)} - P(Y^{(j)} = 1|\vec{x}^{(j)}, w_0, \vec{w})) \end{aligned}$$

- $\partial f/\partial \mu$:

$$\begin{aligned} \partial f/\partial \mu &= \textstyle\sum_{j=0}^p (w_j - \mu) - \frac{1}{\sigma^2}(\mu - \tilde{\mu}) \\ \partial f/\partial \mu\big|_{\hat{\mu}_{MAP}} &= 0 \\ \textstyle\sum_{j=0}^p (w_j - \mu) - \frac{1}{\sigma^2}(\mu - \tilde{\mu})\Big|_{\hat{\mu}_{MAP}} &= 0 \\ \hat{\mu}_{MAP} &= \frac{\sum_{j=0}^p w_j + \tilde{\mu}/\sigma^2}{p+1+1/\sigma^2} \end{aligned}$$

Part C: Programming Exercise

Exploring The Effect of Priors in Batting Average Estimation

In this problem, you will explore how prior knowledge can effect your estimates of batting averages.

Dataset

In this problem, we have generated data for 5000 fictional baseball players. The data is divided into 3 parts – '*pre_season.txt*', '*mid_season.txt*', and '*end_season.txt*'. Each of these files has 3 columns: the *id* for the player (an integer), the number of *at_bats* for the player (an at-bat is an opportunity to get a hit), and the number of *hits* the player got during those at-bats. The data files can be loaded using the provided *load_data* function in *hw1_baseball.py.* The batting average for a player can be computed by dividing the number of hits by the number of *at_bats*.

Maximum Likelihood Estimator

Assume for the momen that you only have access to the data in '*mid_season.txt*'. Midway through the season, you would like to estimate the end of season batting averages for all 5000 players. Write a function to compute the maximum likelihood estimate of the batting average for all 5000 players. Make sure to turn in your code.

Maximum a Posteriori Estimator

Unsatisfied with the MLE estimates, you decide that you would like to use the pre-season statistics of the players as a prior on what their in-season batting average will be. Write a function to compute the maximum a posteriori estimate of the batting average for all 5000 players. Briefly describe how you choose to incorporate prior information. Make sure to turn in your code.

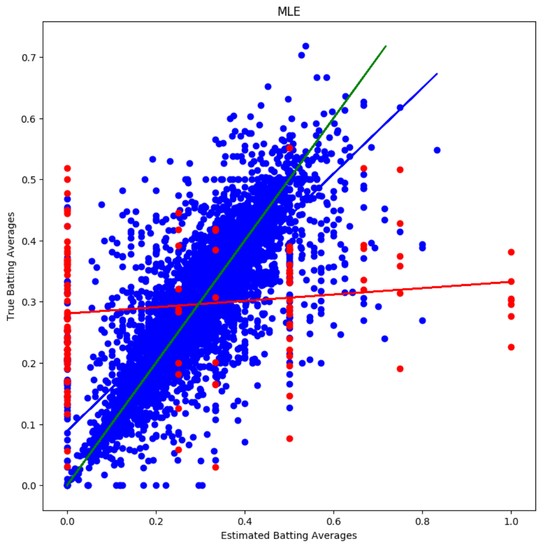
Visualize Your Estimates

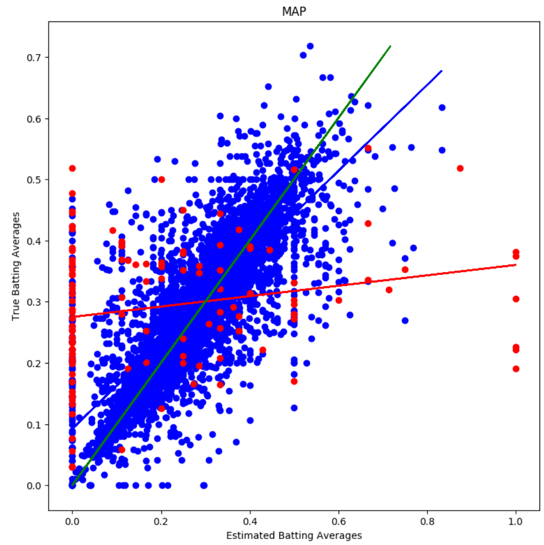
Compute the actual batting averages from '*end_season.txt*' (do not include statistics from the other files in these actual averages) and compare your estimates of the batting average to these estimates. Use the provided visualize function in *hw1_baseball.py* to visualize and compare your MLE and MAP estimators. Make sure to turn in your visualizations.

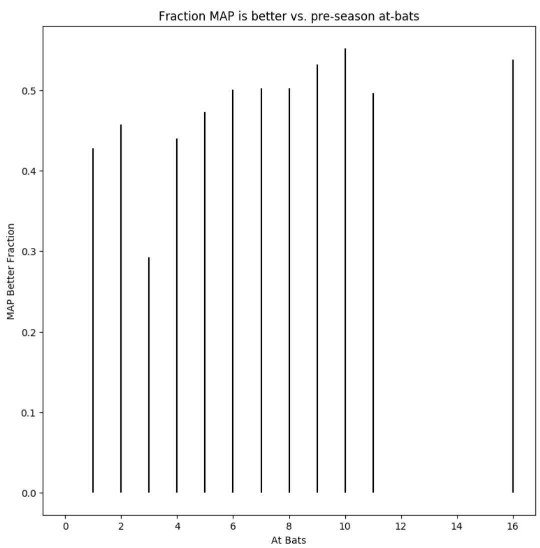
- Does the MLE estimator appear to fail in certain cases? Why?
  Answer:
  Yes, most of the MLE estimators with less than 5 at_bats appear to fail. Because the number of sample is too few to represent the true batting average.
- Does the MAP estimator appear to fail in certain cases? Why?
  Answer:
  Yes, most of the MAP estimators underestimate the true batting average. Because the prior from pre-season where players may not play their best does not match the true prior of batting average during the second half season.
- What conclusions do you draw from this experiment?
  Answer:

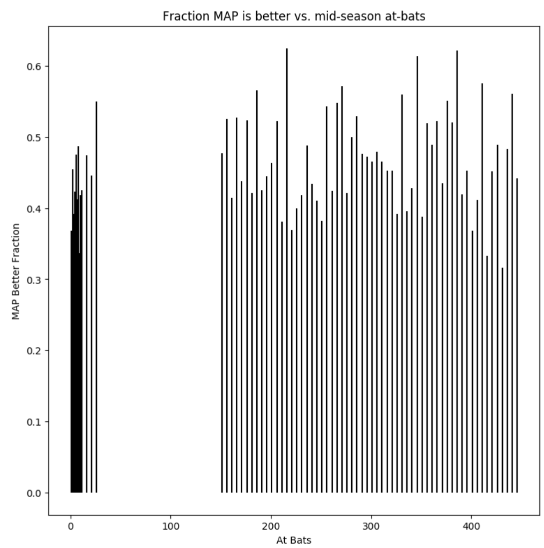
  - If we need to use MLE or MAP, we should collect more data to shrink its variance.

– If we need to use MAP, we should use the right prior which fulfills the same conditions of targets.









Logistic Regression on Movie Review Dataset

In this problem, you will explore logistic regression to classify movie reviews into two classes - positive & negative. The dataset to be used in IMDB Large Movie Review dataset (Maas et. al, ACL 2011). The datafiles are present in the link shared above.

Details about dataset

The dataset comprises of two folders: '*train*' and '*test*', and each of these in turn contain two subfolders - *pos* & *neg*. Each file in these subfolders is a unique review. In total, we have 25K training reviews (12.5K positive, and remaining 12.5K negative). The test folder too has 12.5K positive and 12.5K negative reviews. For our task, we will use bag of word representation.

Exercises

For this exercise, we will directly use Logistic Regression library from *sklearn.linear_models*. We will experiment with different values of $C \in \{0.001, 0.01, 0.1, 1, 10, 100\}$. Here, C is the inverse of regularization constant. We will also closely study the learnt parameter/weight/coefficient vector.

- Plot train and test accuracy for varying values of C. First plot should contain both train and test accuracy vs C with $L2$ regularizer (penalty) and the second plot should employ $L1$ regularizer (penalty). What do you observe in the two plots? Which value of C is optimum in these two cases?
  Answer:
  Fig.1
- While using $L2$ regularizer, and different values of C, plot the $L2$ norm of weight vector vs C. What do you observe?
  Answer:
  Fig.1
- While using $L1$ regularizer, and different values of C, plot the $L1$ norm of weight vector vs C. What do you observe?
  Answer:
  Fig.1
- Study how sparsity (i.e. percentage of zero elemenets in a vector) of the parameter vector changes with different values of C. In one plot, depict two curves – one for $L1$ regularizer and the other one for $L2$ regularizer. Jot down your observations.
  Answer:
  Fig.2

Now we will try to visualize the basis of the classification! One way to do so is to look at the weight vector and analyze the top (least) K values.

- While using $L2$ regularizer, and the optimum value of C (with respect to test accuracy), which 5 words correspond to the largest weight indices in the learnt weight vector? Which 5 words correspond to the least weight indices in the learnt parameter vector?
  Answer:
  - Largest weight indexed words: perfect, favorite, wonderful, loved, excellent
  - Least weight indexed words: worst, waste, awful, boring, poor
- While using $L2$ regularizer, and the optimum value of C(with respect to test accuracy), which review is predicted positive with highest probability? Similary, which review is predicted negative with highest certainty?

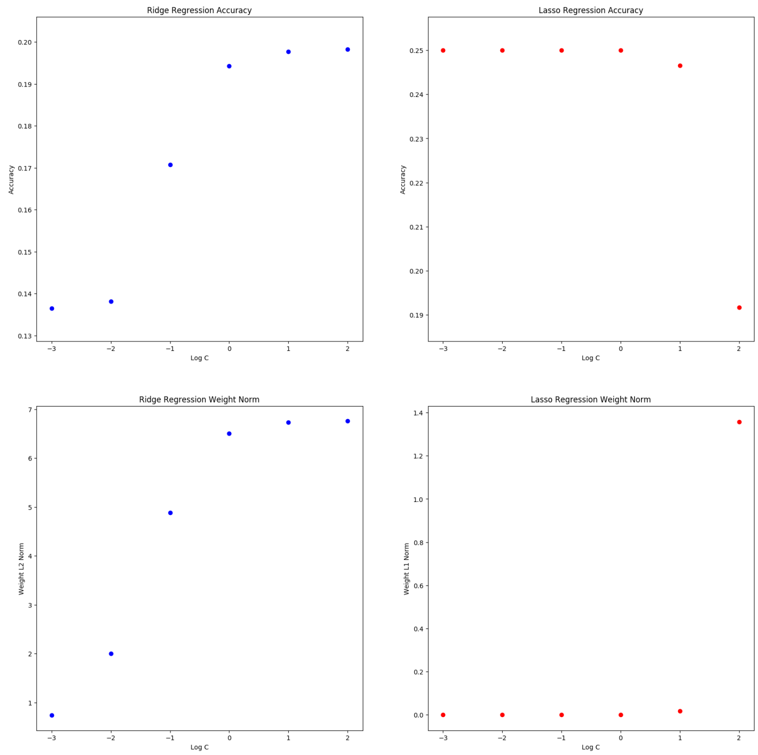


Figure 1: Q1,2,3

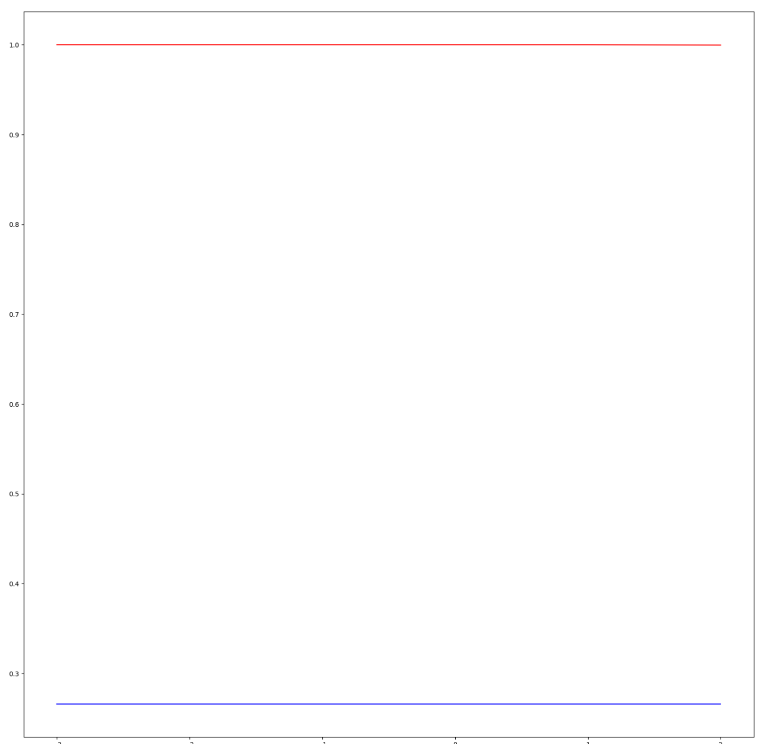


Figure 2: Q4

– ”Tony Hawk’s Pro Skater 2x isn’t much different at all from the previous games excluding Tony Hawk 3 The only thing new that is featured in Tony Hawk’s Pro Skater 2x is the new selection of levels and tweaked out graphics Tony Hawk’s Pro Skater 2x offers a new career mode and that is the 2x career The 2x career is basically Tony Hawk 1 career because there is only about five challenges per level If you missed Tony Hawk 1 and 2 I suggest that you buy Tony Hawk’s Pro Skater 2x but if you have played the first two games you should still try this one Overall there really isn’t anything new but it is still very fun to go through the game Hopefully this review benefits your needs Graphics 7 out of 10 Overall the clean visuals isn’t really one of Tony Hawk’s Pro Skater 2x’s main characteristics The atmosphere has been changed around a lot from Tony Hawk 1 and 2 and the character models look a little bit improved When you look back to Tony Hawk’s Pro Skater 1 and 2 on the old PS1 the thought that those old graphics are ugly run through your head In Tony Hawk’s Pro Skater 2x the graphics are rendered A LOT better The character models are no longer filled with jaggys the textures are more smooth but not to the farthest extent Tony Hawk’s Pro Skater 2x’s visuals do not compare to Tony Hawk 3’s graphics but Activision probably didn’t want to make Tony Hawk’s Pro Skater 2x have extraordinary graphics Overall the graphics deserve an average score of 7 because they did not put the full power of the Xbox to use in here Graphics are nice and clean that’s all I have to say Sound 8 out of 10 The sound effects don’t deliver much to the imagination but the skateboards popping off of the ground sound great The main reason why I gave the sound factor a rating was because you are not obligated to listen to the below average Tony Hawk soundtrack because there is a custom soundtrack feature The sound effects sound a lot better than the sounds in Tony Hawk 1 and 2 mainly because it is more clearer and just the fact that everything sounds great One of the main reasons why I bought this game is because of the custom soundtrack The grind sound effects still sound the same as the first two games did just a little tweaked out One of the major problems of the sound factor is the fact that if the song is over it will NOT proceed to the next track the song that you have just listened to will just play over again I don’t like the in-game soundtrack but like I said you are not obligated to listen to it Controls 10 out of 10 The controls are the best part of Tony Hawk’s Pro Skater 2x The control set-up is marvelously comfortable and easy to get used to Back in the Playstation days people thought that the controls were the best ever but it looks like 2x has done a better job with the Xbox control Surprisingly it is very easy to use the control stick to execute tricks Activision has done great work with Tony Hawk’s Pro Skater 2x’s controls They have made the Xbox controller the best for Tony Hawk games You will not be disappointed with the control style and that is a guarantee Game play 10 out of 10 Excluding the fact that Tony Hawk’s Pro Skater 2x is basically Tony Hawk 1 and 2 put together the game play is still unbelievably fun The game play factor has been changed around a bit This time you get A LOT more air than in the first two games and it is a lot easier to perform tricks In Tony Hawk’s Pro Skater 2x each character has three career modes consisting of Tony Hawk 1 career Tony Hawk 2 career and the 2x career Tony Hawk 1 career is rather easy because in the first game you get NOTHING for air The Tony Hawk 2 career delivers the same amount of difficulty as the playstation version did The only amount of difficulty that applies to the 2x career is finding out where all items are but after you’ve done that 2x career is no hard at all In the 2x career there is a total of 3 levels and the first two levels consist of finding the secret tapes collecting S-K-A-T-E and doing whatever else is required for that particular level The third level out of the three is the competition level where you have to get a certain amount of points to get the gold In the first two levels the secret tapes and collecting the letters S-K-A-T-E are featured in both of them Overall Tony Hawk’s Pro Skater 2x still maintains the old Tony Hawk’s Pro Skater vibe Story - Fun factor 10 out of 10 Tony Hawk’s Pro Skater 2x is by far the most funnest game on Xbox today I have played Tony Hawk’s Pro Skater 1 and 2 and back then I didn’t like them but for some reason Tony Hawk’s Pro Skater 2x is really fun There really isn’t much to say except that Tony Hawk’s Pro Skater 2x is by far the best game on Xbox today One problem is that if you’ve already gone through the game once you will play it a couple more times but it will be repetitive Replay value 10 out of 10 Tony Hawk’s Pro Skater 2x delivers a high amount of replay value There is a lot of cheats to unlock and a lot of character videos Overall Tony Hawk’s Pro Skater 2x has lots of replay value mainly because it is so fun Best feature You are not obligated to listen to the crappy in-game soundtrack

Worst feature The custom soundtrack is a bit messed up Final Statement Lots of people have complained in the past that they didn't like Tony Hawk's Pro Skater 2x because there is nothing new but they should stop complaining because your getting a lot of game for 50 00 Graphics 7 out of 10 Sound 8 out of 10 Control 10 out of 10 Game play 10 out of 10 Story N A Fun factor 10 out of 10 Overall score 9 out of 10"

– Plankton or Creatures from the Abyss as I'm positive it's more commonly known as filmed under as the title Creatures from the Abyss appears over a moving image in the same font type as the rest of the credits starts with five 20 something kids Mike Clay Rogers his girlfriend Margaret Sharon Twomey sisters Julie Ann Wolf Dorothy Loren DePalm an annoying idiot named Bobby Michael Bon whom decide to all fit into a small rubber boat head out to sea don't ask why as I don't know Oh the complete idiot Bobby left the petrol behind never thought to tell anyone so it comes as no great surprise that they end up stranded out at sea without any petrol for the motor to make matters worse they become trapped in a thunder storm discover a dead body floating in the water Shortly after their luck seems to change when they come across a yacht potential safety in a flash everyone boards the yacht begin to explore First of all they find a scientific lab with various fish specimens computer equipment then down below they find fully furnished luxurious cabins They find a chemist Deran Sarafian who appears mad can't talk They eat fish from the fridge which makes Dorothy puke up green vomit beetles slugs They learn that these fish are living fossil's 1000's of years old have been contaminated by toxic waste dumped in the sea that they fly mutate bite are generally unpleasant to be around I really can't be bothered to go on with this plot outline so I won't here's what I think This Italian production was produced directed by Massimiliano Cerchi under the pseudonym Al Passeri I'd hide under a different name if I made a film this bad too I think Plankton is quite simply one of the worst films ever there are so many things wrong with this film it's difficult to know where to start First the script by Richard Baumann is total crap it makes no sense whatsoever is so slow dull it was torture for me to sit through Why would five people just simply set sail for the middle of the ocean on a rubber dinghy barely big enough to fit them all in What were they planning on doing exactly Why do we often get point-of-view shots from these fish creatures but they seem to be totally invisible to the characters as they are never shown on screen even though they are right next to a character how do these fish get around the boat as there is no water for them to swim in People's actions reactions to things are all wrong they constantly split up they make bizarre decisions that simply don't make any sense in the situation they find themselves in some of the dialogue is as awful as anything I've heard I could go on all day about all the plot holes ridiculous goings on but I'll run out of space if I do The fish creatures themselves look awful a mixture of rubbish rubber puppets some really bad stop motion animation at the end the scenes where they interact with the human cast also look terrible with some bad super imposition I have heard a lot of comments saying that Plankton is gory don't make me laugh Forget it there is virtually no blood or gore in Plankton whatsoever there are a couple of slimy scenes when Bobby transforms into a fish monster while having sex with Julie but it's pretty brief he doesn't kill her he just sort of drips slime on her grows a couple of tentacles a fish head comes out of his mouth Later on Julie's vagina starts to drip some dark slime but that's it we never get to actually see what happens to her or what the slime is Dorothy has a fish creature come out of her back off screen control her but again we never get to see what happens to her while Margaret commits suicide a very brief shot of a plastic harpoon stuck to her forehead Easily the grossest scene is when Dorothy pukes up that green stuff with what looks like beetles slugs in it That's it only one person actually dies on screen for the most part Plankton is quite tame as exciting as watching paint dry I nearly fell asleep it's so boring I can't see how anybody can like this total crap I just can't The acting is awful the dubbing is awful the characters are awful I hated all of them Tecnically Plankton is predictably crap as well with an estimated budget of only 250 000 all I can say is where did the money go The sets are monotonous dull with one lab a few cabins the special effect's are bottom of the barrel stuff including the most fake looking exploding boat ever the cinematography is bland the music sucks there is zero atmosphere or tension as a whole Plankton like it's name sake is as low in the food chain as it could possibly be I hate Plankton it's awful in every single aspect of it's overlong

86 minute duration Do yourself a favour avoid this one at all costs unless your either a masochist or insomniac