1.7 PRIMARY SENSORS

Primary sensors obtain signals that can be transduced from the physical quantities measured. From the point of view of information domains (Figure 1.2), we regard them as elements with input and output in the same domain unlike sensors which work upon the output of primary sensors to pass to the electric domain. We classify primary sensors here according to the detected input quantity. We do not consider those devices that have output directly in electric form. In [8] there is more extensive information about the primary sensors discussed in this section.

1.7.1 Temperature Sensors: Bimetals

A bimetal is a sensor formed by two metals having different thermal expansion coefficients that are firmly joined, for example, by welding and that are exposed to the same temperature. Whenever the temperature changes, the piece changes its shape and curves to form a uniform circular arc.

In the notation of Figure 1.11, the radius of curvature r, when changing from a temperature T_1 to another T_2, is given by [2]:

$$r = \frac{t[3(1 + m)^2 + (1 + mn)(m^2 + 1/mn)]}{6(\alpha_A - \alpha_B)(T_2 - T_1)(1 + m)^2} \tag{1.25}$$

where

t = the total thickness of the piece
n = the ratio between moduli of elasticity = E_B/E_A
m = the ratio between thicknesses = t_B/t_A
α_A, α_B = the thermal expansion coefficients

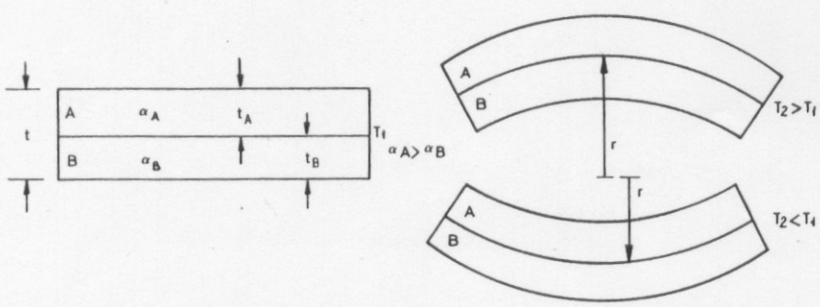

FIGURE 1.11 Bimetal. Dimensions and curvature have been exaggerated in order to better illustrate the working principle. (From E. O. Doebelin, *Measurement Systems Application and Design,* 4th ed., © 1990. Reprinted by permission of McGraw-Hill, New York.)

If use is made of metals having similar moduli of elasticity and thicknesses ($m \cong 1$, $n \cong 1$), which is the usual practice, equation (1.25) reduces to

$$r \cong \frac{2t}{3(\alpha_A - \alpha_B)(T_2 - T_1)} \tag{1.26}$$

Therefore the radius of curvature is inversely proportional to the difference between temperatures. A position or displacement sensor would yield a corresponding electric signal. Alternatively, the force exerted by a total or partially bonded or clamped element can be measured.

In practice, thicknesses from 10 μm to 3 mm are common. To achieve a high sensitivity, it is desirable to have $\alpha_B < 0$, but given that there is not any useful metal having that property, use is made of invar (an alloy of steel and nickel) that shows $\alpha = 1.7 \times 10^{-6}$/°C. As metal A, brass and other proprietary alloys are used.

These devices are used in the range from −75°C to +540°C, and mostly from 0°C to +300°C. They are manufactured in forms such as cantilever, spiral, helix, and diaphragm, and the force or displacement they undergo is measured. They are also used to directly open or close contacts (thermostats, on–off controls) and as thermal protection elements in electric circuit interrupters. In electric circuit interrupters the current flows along the element that experiences a heating by Joule effect until it reaches a temperature high enough to exert a mechanical force on a trigger device that opens the circuit and interrupts the current flow.

Other nonmeasurement applications are the thermal compensation of temperature-sensitive devices and fire alarms. Their response is slow because of their high mass.

1.7.2 Pressure Sensors

Pressure measurement in liquids or gases is common, particularly in process control. Pressure is defined as the force per unit area. To measure a pressure, it is either compared with a known force or its effect on an elastic element is measured (deflection measurement). Table 1.5 shows some of the methods available.

In liquid column manometers such as the U-tube in Figure 1.12, second-order effects are disregarded in comparing the pressure to be measured with a reference pressure; the result is a difference h of liquid level

$$h = \frac{P - P_{\text{ref}}}{\rho g} \tag{1.27}$$

where ρ is the density of the liquid and g is the acceleration of gravity. A level sensor (photoelectric, float, etc.) yields an electric output signal.

TABLE 1.5 Some Common Methods to Measure Pressure in its Normal Range

Liquid Column + Level Detection			
Elastic element	Bourdon tube + Displacement measurement		Potentiometer LVDT Inductive sensor Digital encoder
	Diaphragm + Deformation measurement	Central deformation	Potentiometer LVDT Inductive sensor Unbonded strain gages Cantilever and strain gage Vibrating wire
		Global deformation	Variable reluctance Capacitive Piezoelectric
		Local deformation	Bonded gages Deposited gages Diffused gages

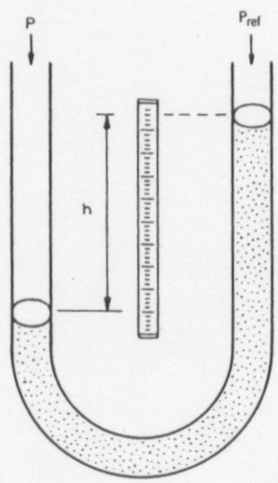

FIGURE 1.12 Liquid-column U-tube manometer. The liquid must be compatible with the fluid for which pressure is to be measured, and the tube must be able to withstand the mechanical stress. (From E. O. Doebelin, *Measurement Systems Application and Design*, 4th ed., © 1990. Reprinted by permission of McGraw-Hill, New York.)

A pressure applied to an elastic element deforms it until the internal stress balances the applied pressure. Depending on the material and its geometry, the resulting displacement or deformation may be small or large, which determines which sensors can be applied (Table 1.5). The usual devices utilize the Bourdon tube or the clamped or bonded diaphragm.

The Bourdon tube was developed by Eugene Bourdon in 1849. It consists of a metallic tube with a noncircular cross section obtained by deforming a tube having a circular cross section. A pressure difference applied across the wall causes the tube to tend to recover its original circular section. Figure 1.13 shows that if one of the tube ends is closed and the other one is firmly held, that tendency to recover the original shape displaces the free end. This displacement is not linear along its entire range but is linear enough in short ranges. The configurations that offer the greater displacements have the drawbacks of their greater compliance and length that results in a small frequency passband. Displacement sensors are used to obtain an electric output signal.

A diaphragm is a flexible circular plate consisting of a taut membrane or a clamped sheet that strains under the action of the pressure difference to be measured. The transduction is then made by detecting the displacement of the central part of the diaphragm, its global deformation, or the local strain (in this case using strain gages; Section 2.2).

For a thin plate with thickness t and radius R experiencing a pressure difference P between both sides, if the maximal deformation z is less than one-third of the thickness, we have [2]

$$P = \frac{16Et^4}{3R^4(1 - \nu^2)}\left[\frac{z}{t} + 0.488\left(\frac{z}{t}\right)^3\right] \tag{1.28}$$

where E is Young's modulus and ν the Poisson's ratio for the plate material.

If piezoresistive sensors are to be applied, it is necessary to know the mechanical stress at the different points across the plate. At all points at a distance r from the center, the radial stress is

$$s_r = \frac{3PR^2\nu}{8t^2}\left[\left(\frac{1}{\nu} + 1\right) - \left(\frac{3}{\nu} + 1\right)\left(\frac{r}{R}\right)^2\right] \tag{1.29}$$

FIGURE 1.13 Different shapes for Bourdon tubes. (From E. O. Doebelin, *Measurement Systems Application and Design*, 4th ed., © 1990. Reprinted by permission of McGraw-Hill, New York.)

The tangential stress is

$$s_t = \frac{3PR^2\nu}{8t^2}\left[\left(\frac{1}{\nu}+1\right)-\left(\frac{1}{\nu}+3\right)\left(\frac{r}{R}\right)^2\right] \tag{1.30}$$

Across the diaphragm there are tensions and compressions. This requires the placement of several strain gages as well as combinations of these gages in a measurement bridge to benefit from additive effects and provide temperature compensation (Section 3.4.4).

Some of the elastic materials used are beryllium-copper, stainless steel, nickel-copper alloys, and even silicon if the diaphragm is going to incorporate silicon strain gages.

If the resulting displacement in a single diaphragm is not large enough, capsules and bellows provide larger displacements. Figure 1.14 shows a capsule that consists of twin diaphragms joined by their external border and placed on opposite sides of the pressurized chamber. Figure 1.15 shows bellows, which are flexible chambers with axial elongation that undergo even larger deflections than capsules, even reaching in some cases 10% of their length. But both devices are vibration- and acceleration-sensitive and do not withstand high overpressures.

1.7.3 Flow Velocity and Flow-rate sensors

Flow is the movement of a fluid in a channel or in open or closed conduits. The flow rate is the quantity of matter, in volume or weight, that flows in a unit time. Flow-rate measurements are present in all energy and mass transport processes to control or monitor those processes and for metering purposes, as happens with water, gas, gasoline, or crude oil among others.

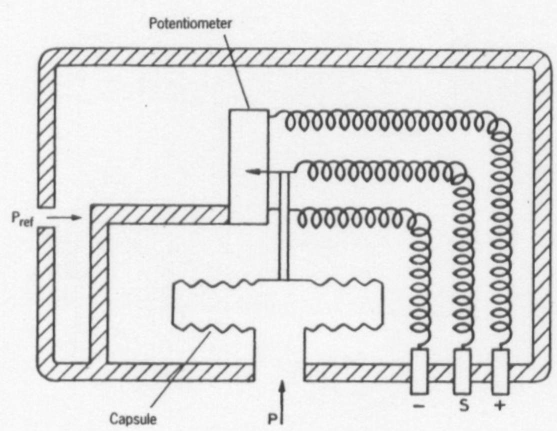

FIGURE 1.14 Capsule bellows for pressure measurement. (Courtesy of Vernitron)

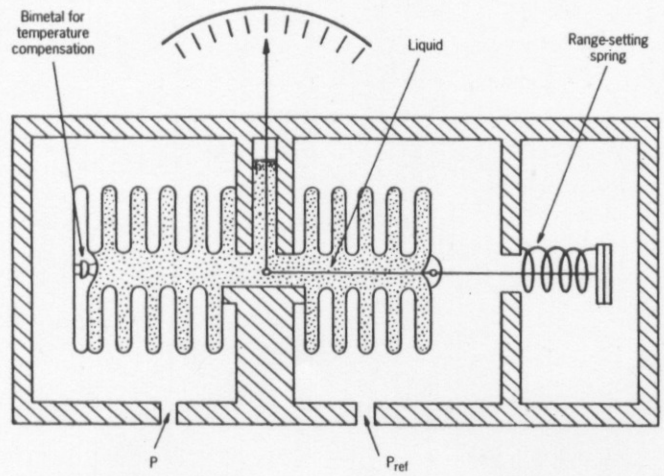

FIGURE 1.15 Bellows for pressure measurement incorporating a temperature compensation based on a bimetal.

Most flowmeters use indirect measurement methods, in particular, the sensing of the drop in pressure caused by introducing a resistive element in the conduit in which we wish to measure the flow rate. To understand these and other flow-rate measurement methods, a brief review of basic fluid flow theory is necessary.

A viscous or laminar flow is that of a fluid flowing along a straight smooth-walled and uniform transverse section conduit, where all particles have a trajectory parallel to the conduit walls and move in the same direction. In a turbulent flow, in contrast, some of the fluid particles have longitudinal and transverse velocity components, thus resulting in whirls.

Bernoulli's theorem applies to an incompressible fluid experiencing only gravity as internal force (i.e., without friction), flowing in stationary movement and with no heat entering or leaving it. Any change in velocity produces an opposite change in pressure that equals the change of kinetic energy per unit of volume added to the change due to any difference in level. That is,

$$p + \rho g h + \frac{\rho v^2}{2} = \text{constant} \tag{1.31}$$

where

p = the static pressure
ρ = the fluid density (incompressible)
g = the acceleration of gravity
h = the height with respect to a reference level

v = the fluid velocity at the point considered
$\rho v^2/2$ = the dynamic pressure

A simple device that can be described by means of (1.31) is the Pitot tube used to measure the velocity of a fluid at a point. If a tube with a right angle is introduced into an open conduit where an incompressible and frictionless fluid flows in a given known direction, and its open end is oriented against the flow (Figure 1.16), the liquid enters into the tube and rises until the pressure exerted by the fluid column balances the force produced by the impacting velocity on the open end. Since in front of the opening the velocity is zero, flow lines distribute around the end creating a stagnation point. It holds therefore that

$$\frac{v^2}{2g} = \frac{p_1}{\rho g} = \frac{p_2}{\rho g} = h_0 + h \tag{1.32}$$

Also in an open conduit we have $p_1 = \rho g h_0$, thus giving

$$v^2 = 2gh \tag{1.33}$$

In a closed conduit we must measure both the static pressure and the total or stagnation pressure p_t using a Pitot tube. The arrangement of Figure 1.17 permits us to make both measurements at the same time. From (1.31) we can derive the following relationship between the fluid velocity and the difference between both pressures,

$$v^2 = \frac{2(p_t - p)}{\rho} \tag{1.34}$$

The Pitot tube is commonly used to measure air velocity in avionics.

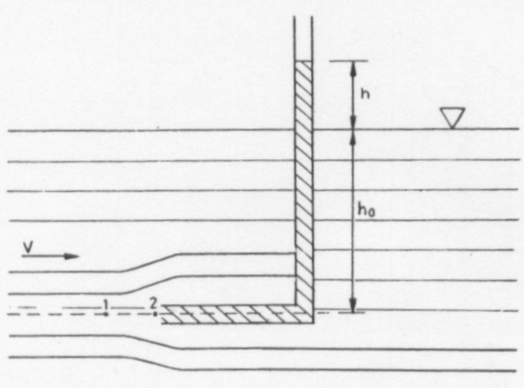

FIGURE 1.16 Pitot tube in an open conduit.

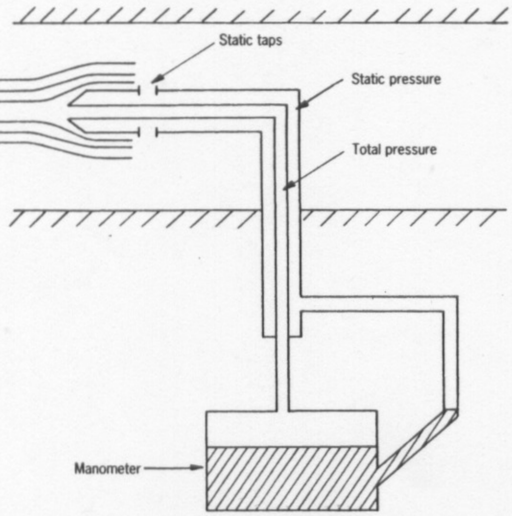

FIGURE 1.17 Pitot tube in a closed conduit. The manometer measures the difference between total pressure and static pressure and can be of an electronic type. (From E. O. Doebelin, *Measurement Systems Application and Design,* 4th ed., © 1990. Reprinted by permission of McGraw-Hill, New York.)

Obstruction flowmeters are the most frequently used type. We can use equation (1.31) to understand how they work. A restriction having constant cross section forms the obstruction to the flow. Across it there is a drop in pressure, as we will show later. This way the measurement of flow reduces to a measurement of pressure difference.

If we introduce in a closed conduit a plate having a hole, the fluid vein contracts changing from a cross section A_1 (that of the conduit) to a cross section A_2 (that of the hole) (Figure 1.18). This change results in a corresponding change of velocity. Because of the principle of mass conservation, we have

$$Q = A_1 v_1 = A_2 v_2 \tag{1.35}$$

At the same time, by applying Bernoulli's theorem, we have

$$p_1 + \rho g h_1 + \frac{\rho v_1^2}{2} = p_2 + \rho g h_2 + \frac{\rho v_2^2}{2} \tag{1.36}$$

If $h_1 = h_2$, these two equations yield

$$v_2^2 = \frac{2(p_1 - p_2)/\rho}{1 - (A_2/A_1)^2} \tag{1.37}$$

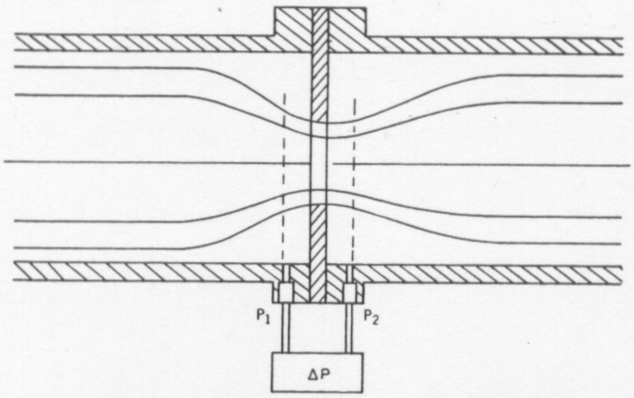

FIGURE 1.18 Obstruction plate for flow measurement based upon a differential pressure sensor.

The theoretical flow rate is $Q = A_2v_2$. The real flow rate, however, is somewhat lower, and it is determined by experimentally calculating a correction coefficient, called a *discharge factor* C_d that depends on A_1, A_2, and other parameters. Its value is of the order of 0.6. We have then

$$Q_r = C_d Q \tag{1.38}$$

Some of the principal shortcomings of this method are the loss in pressure it causes and that it is difficult to measure fluctuating flows accurately unless the differential pressure sensor is fast enough, including the effects of the hydraulic connections. Nozzles and Venturi tubes are based on the same principles, but their internal shapes are not so blunt, thus reducing the loss of pressure (C_d can be even 0.97).

The application of Bernoulli's theorem and the principle of mass conservation to volume flow-rate measurement can also be done in a reciprocal way. That is, it is possible to make the fluid pass section variable and keep the difference in pressure between both sides of the obstruction constant. The measured flow rate is then related to the area of the pass section.

Figure 1.19 shows a rotameter, which is an average flow-rate indicator based on this method. It consists of a uniform conic section tube and a grooved float inside it that is dragged by the fluid to a height determined by its weight and the flow. The fluid—gas or liquid—flows upward. When the flow increases, the float rises thus allowing an increased pass section and keeping the pressure difference between both ends constant. The displacement of the float is a measure of the fluid flow rate. For pressures lower than 3.5 kPa and nonopaque liquids, the tube can be of glass and include the scale to read the position of the float. For higher pressures and flows the tube must be of metal and the position of the float is detected by magnetic means. It is also possible to apply the null-measurement method by using a magnetic

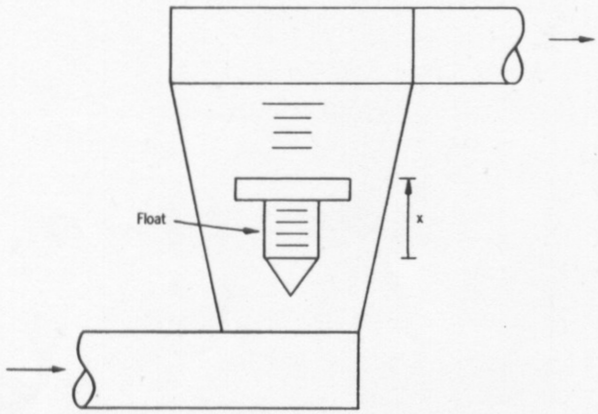

FIGURE 1.19 Variable area flowmeter: rotameter.

float and a solenoid outside the tube. The float position is measured by a photoelectric detector. The flow is determined from the amplitude of the current supplied to the solenoid in order to reposition the float at zero [11].

Turbine flowmeters are vaned wheels placed inside a moving fluid that makes them turn at a speed proportional to fluid velocity when it is high enough. The turning velocity is detected by a magnetic pickup.

In open channel flows and also in nonfilled conduits, the flow measurement methods used are different from those described for totally filled conduits. One of them is based on flumes.

Figure 1.20 shows that a flume is a narrow gorge in the top part of a wall perpendicular to the flow direction that produces a backward stagnation, the liquid flowing then through the gorge. With this method part of the kinetic energy of the fluid is converted into potential energy, and the fluid rises to a height relative to the lower point of the gorge that depends on the flow rate. If the gorge is rectangular, for example,

$$Q = K \cdot L \cdot H^{3/2} \tag{1.39}$$

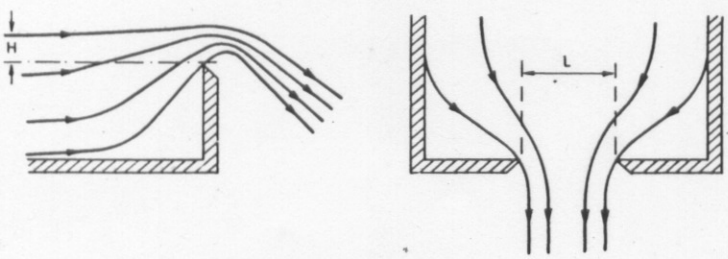

FIGURE 1.20 Flume with rectangular gorge.

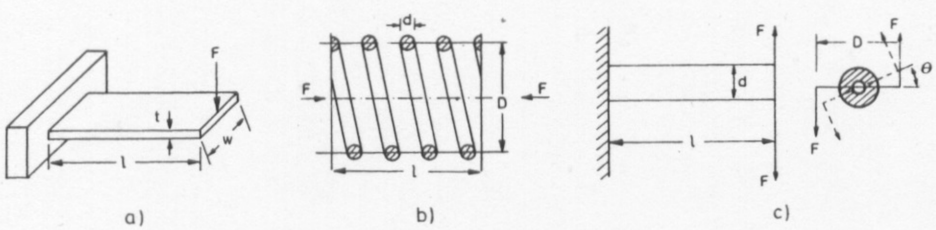

FIGURE 1.21 Different springs with linear and angular deflection.

where Q is the flow, H the height raised by the fluid, L is the flume width, and K is a constant. H can be measured using a fluid level sensor.

1.7.4 Force and Torque Sensors

A method to measure force (or torque) is to compare it with a known force, as is done on scales. Another method consists of measuring the effect of the force on an elastic element, called a *load cell*. In electric load cells that effect is a deformation or a displacement. In hydraulic and pneumatic load cells it is an increase in the pressure of, respectively, a liquid or a gas.

When a mechanical force is applied to a nonmobile elastic element, it strains until the strain-generated stresses balance those due to the applied force. The result is a change in the dimensions of the element that is proportional to the applied force, if the shape is appropriate.

Figure 1.21 shows three suitable arrangements. The corresponding equations are those in Table 1.6. In [9] additional shapes and their corresponding equations are given.

TABLE 1.6 Deflection x or θ and Stress s_M or τ_M for the Elastic Elements Shown in Figure 1.21

	Deflection	Maximal Stress
Cantilever	$x = \dfrac{4Fl^3}{Ewe^3} = \dfrac{2\sigma l^2}{3Ee}$	$s_M = \dfrac{6Fl}{e^2w} = \dfrac{3Eex}{2l^2}$
Helical	$x = \dfrac{8FnD^3}{Gd^4} = \dfrac{\pi nD^2}{Gdk_1}$	$\tau_M = \dfrac{8k_1DF}{\pi d^3} = \dfrac{Gdxk_1}{\pi nD^2}$
Torsion bar	$\theta = \dfrac{32FDl}{\pi d^4G} = \dfrac{2\pi l}{dG}$	$\tau_M = \dfrac{16FD}{\pi d^3} = \dfrac{dG\theta}{2l}$

Source: From H. K. P. Neubert, ***Instrument Transducers***, © 1975. Reprinted by permission of Oxford University Press, Fair Law, N.J.

Note: All quantities are in SI units (lengths in meters, forces in newtons, angles in radians). E = longitudinal modulus of elasticity (Young), G = modulus of rigidity (torsion elasticity modulus), k_1 = stress factor (function of D/d) (valued from 1.1 to 1.6), n = number of turns.