Multiprocessor Scheduling - I

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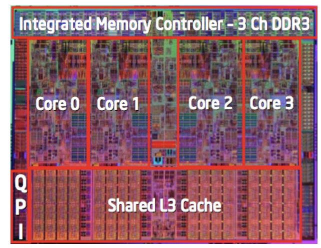
Lecture #13

Outline

- Multi-core Trends
- Multiprocessor Scheduling Approaches
- Global Scheduling
- Anomalies in multiprocessor scheduling

System Trends

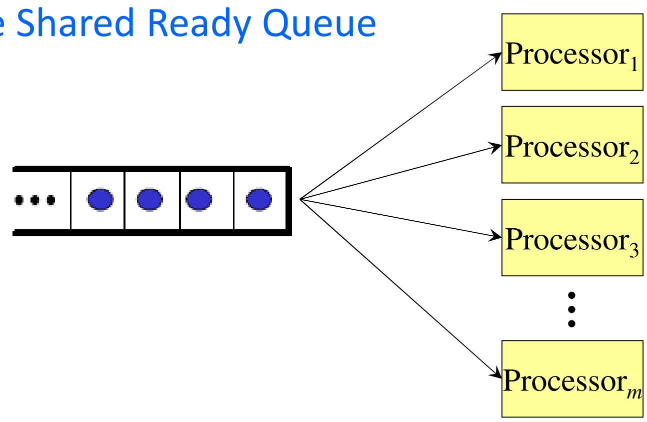
- **Processor trends**
  - Intel Xeon processor E5-2600 offers up to 22 cores
  - Intel Knights Landing: 72 cores!
  - Trend - numerous cores / chip
  - Major driving forces
    - Cooling constraints
    - Fundamental limits on clock speeds



How do we schedule real-time tasks on multicore processors (or multiprocessors)?

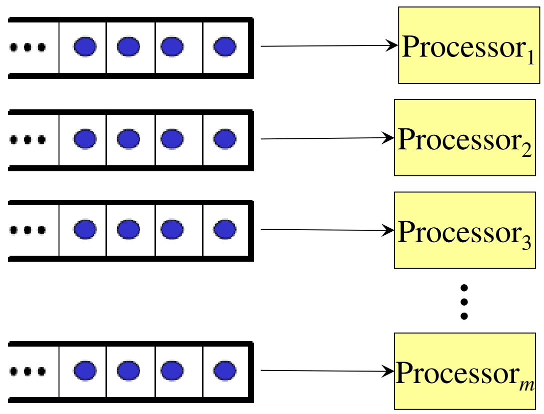
Global Scheduling for Multiprocessors

Single Shared Ready Queue



Partitioned Scheduling for Multiprocessors

One Ready Queue Per Processor



Global Multiprocessor Scheduling Characteristics

- All ready tasks are kept in a common (global) queue
- When selected for execution, a task can be dispatched to an arbitrary processor, even after being preempted
- Task execution is assumed to be "greedy":
  - If higher-priority tasks occupy all processors, a lower-priority task cannot execute until the execution of a higher-priority task is complete.

Global Scheduling

Advantages:

- Supported by most multiprocessor operating systems
  - Windows NT, Solaris, Linux, ...
- Effective utilization of processing resources
  - Unused processor time can easily be reclaimed

Disadvantages:

- Weak theoretical framework
  - Few results from the uni-processor case can be used
- Poor resource utilization for hard timing constraints
  - No more than 50% resource utilization can be guaranteed
- Suffers from several scheduling anomalies
  - Sensitive to period adjustments

Global Scheduling

Complexity of schedulability analysis for global scheduling: (Leung & Whitehead, 1982)

- *The problem of deciding if a task set is schedulable on m processors with respect to global scheduling is NP-complete in the strong sense.*

Consequence:

- There can only exist a pseudo-polynomial time algorithm for
  - (i) finding an optimal static priority assignment, or
  - (ii) feasibility testing
- But not both at the same time (unless P = NP)!

Global Scheduling

- The root-cause in global scheduling: (Liu, 1969)

  Few of the results obtained for a single processor generalize directly to the multiple processor case; bringing in additional processors adds a new dimension to the scheduling problem. The simple fact that a task can use only one processor even when several processors are free at the same time adds a surprising amount of difficulty to the scheduling of multiple processors.

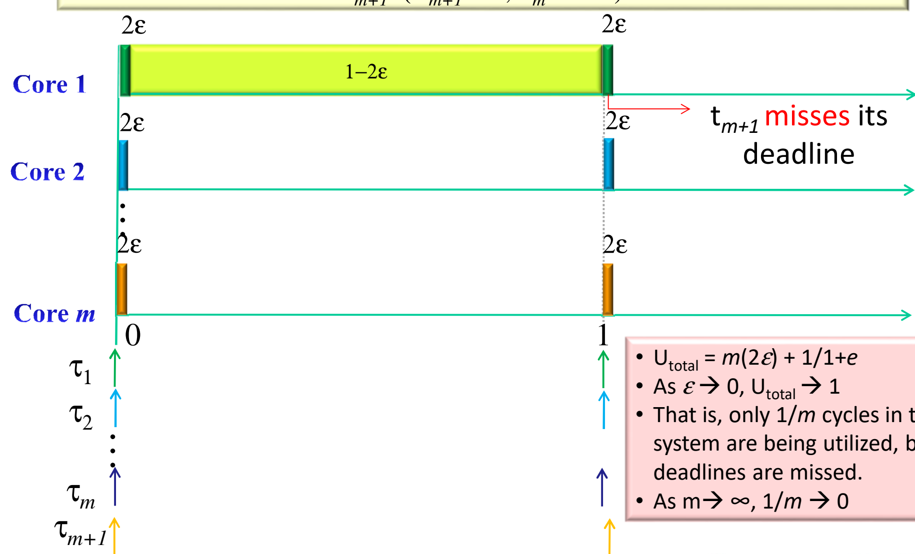
Weak Theoretical Framework

Underlying causes:

- Dhall's effect:
  - With RM, DM and EDF, some low-utilization task sets can be unschedulable regardless of how many processors are used.
- Dependence on Relative Priority Ordering:
  - Changing the relative priority ordering among higher-priority tasks may affect the schedulability for a lower-priority task.
- Uncharacterized Critical Instant:
  - A critical instant does not always occur when a task arrives at the same time as all its higher-priority tasks.

Dhall Effect (Dhall & Liu, 1978)

τ_1: ($C_1 = 2\varepsilon$, $T_1 = 1$); τ_2: ($C_2 = 2\varepsilon$, $T_2 = 1$); …;τ_m: ($C_m = 2\varepsilon$, $T_m = 1$)
τ_{m+1}: ($C_{m+1} = 1$, $T_m = 1+\varepsilon$)



- $U_{total} = m(2\varepsilon) + 1/1+e$
- As $\varepsilon \rightarrow 0$, $U_{total} \rightarrow 1$
- That is, only $1/m$ cycles in the system are being utilized, but deadlines are missed.
- As $m \rightarrow \infty$, $1/m \rightarrow 0$

Dhall's Effect (2 of 2)

- Also applies to (greedy) RM, DM and EDF scheduling
- The lowest utilization of unschedulable task sets can be ar tter how many processo

$$U_{global} = m\frac{2\varepsilon}{1} + \frac{1}{1+\varepsilon} \rightarrow 1$$

when $\varepsilon \rightarrow 0$

Consequence:

- New multiprocessor priority-assignment schemes are needed!

Impact of Relative Priority Ordering

- The response time of a task depends on the relative priority ordering of the higher-priority tasks
- This property does not exist for a uniprocessor system
- This means that well-known uniprocessor methods for finding optimal priority assignments cannot be applied

Consequence:

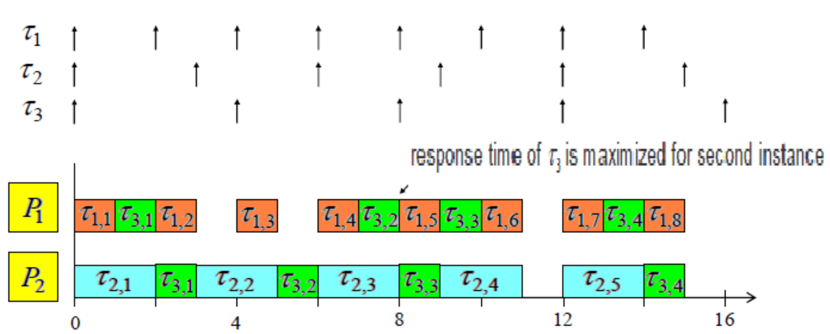
- New methods for constructing multiprocessor priority assignments are needed!

Uncharacterized Critical Instant

- Critical instant, the relative phasing at which a task has its maximum response time, cannot be easily characterized.

RM scheduling

$$\tau_1 = \{ C_1 = 1, T_1 = 2 \}$$
$$\tau_2 = \{ C_2 = 2, T_2 = 3 \}$$
$$\tau_3 = \{ C_3 = 2, T_3 = 4 \}$$



More on Critical Instant

- Critical Instant:
  - A critical instant does *not* always occur when a task arrives at the same time as all its higher-priority tasks.
  - Finding the critical instant is a very (NP-?) hard problem
  - **Note**: recall that knowledge about the critical instant is a fundamental property in uniprocessor feasibility tests.

Consequence:

- New methods for constructing effective multiprocessor feasibility tests are needed!

Underlying Causes

- Dhall's Effect:
  - With RM, DM and EDF, some low-utilization task sets can be unschedulable regardless of how many processors are used.
- Dependence on relative priority ordering:
  - Changing the relative priority ordering among higher-priority tasks may affect schedulability for a lower-priority task.
- Uncharacterized critical instant:
  - A critical instant does not always occur when a task arrives at the same time as all its higher-priority tasks.

New techniques for priority assignments and schedulability tests are needed!

Poor Resource Utilization

- The utilization guarantee bound for any static-priority multiprocessor scheduling algorithm cannot be higher than ½ of the capacity of the processors.
- This applies for all types of static-priority scheduling. That is, partitioned and global, greedy and p-fair scheduling.
- Hence, in the worst case, one cannot utilize more than half the processing capacity if hard timing constraints exist.

Scheduling Anomalies

Scheduling anomaly: A seemingly positive change in the system (reducing load or adding resources) causes a non-intuitive decrease in performance.

- Uniprocessor systems:
  - Anomalies only found for non-preemptive scheduling (Mok, 2000)
- Multiprocessor systems:
  - Richard's anomalies for non-preemptive scheduling
  - Execution-time-based anomalies for preemptive scheduling
  - Period-based anomalies for preemptive scheduling

Richard's Anomalies: (Graham, 1969)

- Assumptions:
  - Non-preemptive scheduling
  - Precedence constraints
  - Restricted migration (individual task instances cannot migrate)
  - Fixed execution times
- Task completion times may increase as a result of:
  - Changing the task priorities
  - Increasing the number of processors
  - Reducing task execution times
  - Weakening the precedence constraints
  - Having shared resources

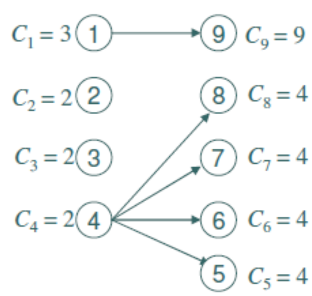
Execution-time-based anomalies: (Ha & Liu, 1994)

- Assumptions:
  - Preemptive scheduling
  - Independent tasks
  - Restricted migration (individual task instances cannot migrate)
  - Fixed execution times
- Task completion times may increase as a result of:
  - Reducing task execution times

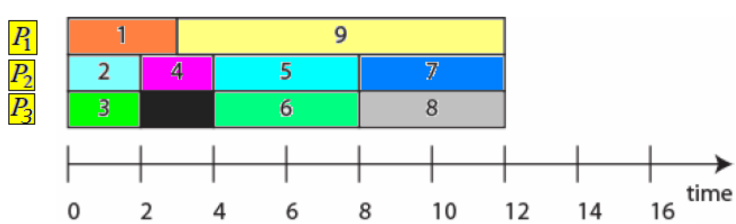
Period-based anomalies (Andersson & Jonsson, 2000)

- **Assumptions**:
  - Preemptive scheduling
  - Independent tasks
  - Full migration
  - Fixed execution times
- A task's completion time may increase as a result of:
  - Increasing the period of a higher-priority task
  - Increasing the period of the task itself
- **Note**: increasing the periods is commonly used to reduce the load in real-time systems!

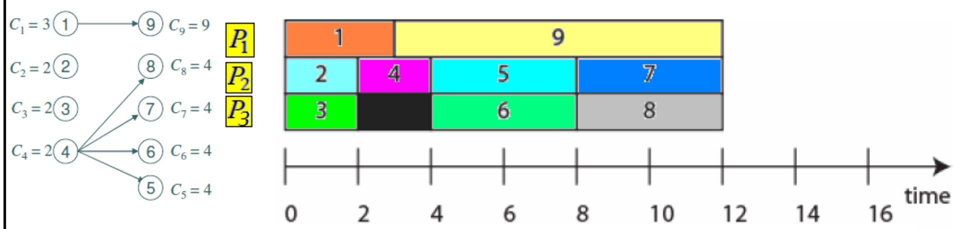
Precedence Constraints



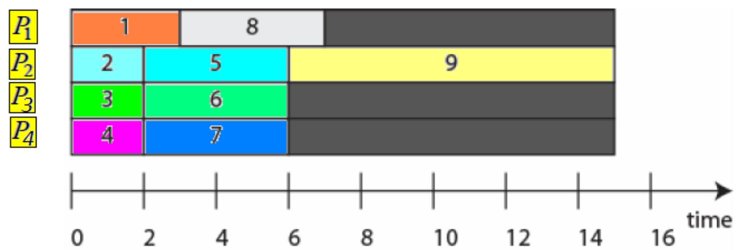
An edge in the ***precedence graph*** from job i to job j means that job j can execute only <u>after</u> job i has completed execution.



Anomaly with # of Processors



What happens if one more processor is introduced?



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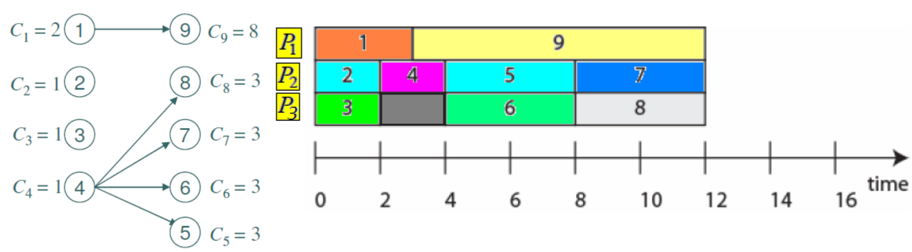
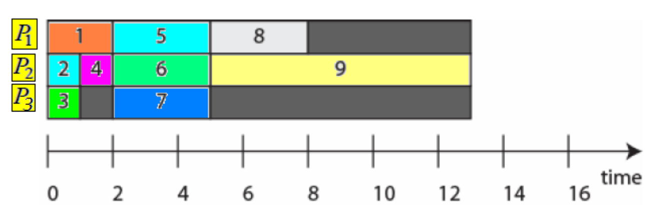
Anomaly with Reduction of Computation Time



What happens if all computation times are reduced by 1?



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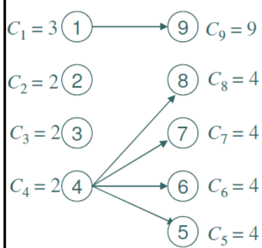
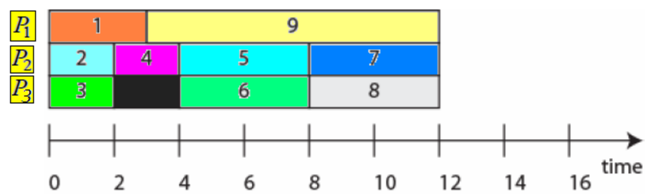
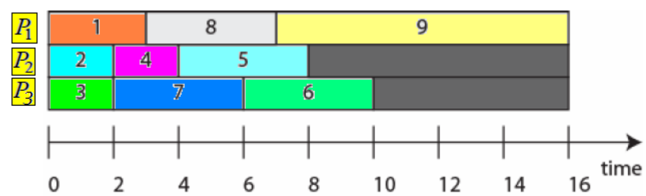
Anomaly with Relaxation of Precedence Constraints



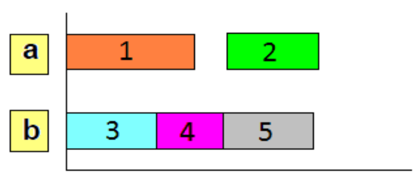


What happens if you remove the precedence constraints (4 → 8) and (4 → 7)?

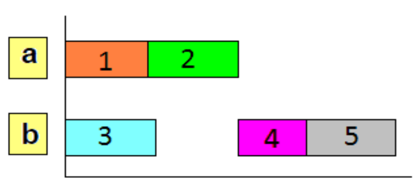


Anomaly with Resource Sharing

Jobs 2 and 4 share a resource: i.e. must execute mutually exclusively



What happens if you reduce C_1?



Global Scheduling

Some Older Basic Results in global scheduling:

- **Static priorities**:
  - The RM-US[m/(3m-2)] priority assignment scheme offers a way to circumvent Dhall's effect and a non-zero resource utilization guarantee bound of $m/(3m-2) \geq 33.3\%$.
- In 2003, Baker generalized the RM-US results to DM.
- **Dynamic priorities**:
  - In 2002, Srinivasan & Baruah proposed the EDF-US[m/(2m-1)] scheme with a corresponding non-zero resource utilization guarantee bound of $m/(2m-1) \geq 50\%$.
- Optimal multiprocessor scheduling:
  - Using *p*-fair scheduling and dynamic priorities, it is possible to achieve 100% resource utilization on a multiprocessor.
  - Impractical to use?

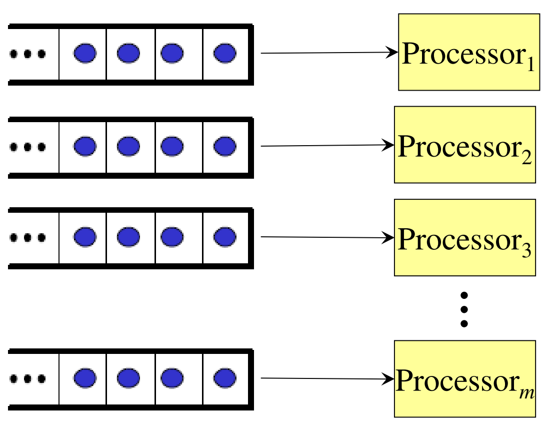
Conclusions

- Multi-core systems are a part of the future
- Global and partitioned scheduling techniques are two approaches to multiprocessor scheduling
- Timing behavior under task scheduling strategies can be brittle.
  - Small changes can have big (and unexpected) consequences.

PARTITIONED SCHEDULING

Partitioned Scheduling for Multiprocessors

One Ready Queue Per Processor



Partitioned Scheduling

- Given a set of tasks, how to assign each task to a queue (processor)?
- Notes:
  - Once assigned, a task only executes on the processor it is assigned to
  - Single-processor schedulability analysis can be carried out on each node
    - i.e. look at all the tasks that can be in the same ready queue
  - No anomalies to worry about
  - No Dhall's Effect
- Question:

The answer lies in the so-called Bin-Packing problem.

Bin-Packing

- The Bin-Packing Problem
  - *n* objects, each of size ≤ 1.0, must be packed into *m* identical "bins", each of which is of size 1.0 such that
    a. the sum of the objects allocated to any bin is ≤ 1.0
    b. the number of bins used is minimized.

Bin-Packing Heuristics

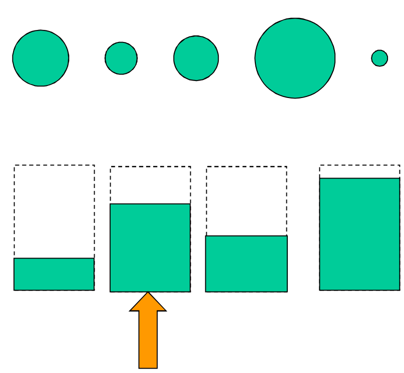
- First-Fit (FF)
- Best-Fit (BF)
- Next-Fit (NF)
- Worst-Fit (WF)
- First-Fit Decreasing (FFD)
- Best-Fit Decreasing (BFD)
- Worst-Fit Decreasing (WFD)

Questions to Ask:

- How many bins do you need?
  - What is the minimum number of bins you need?
  - What is the maximum number of bins you need?
- Can you find the optimum number of bins?

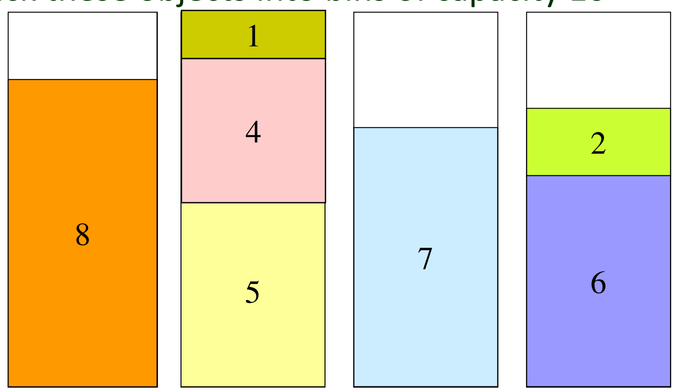
Next-Fit Heuristic

1. Fit next object into <u>next</u> bin that it can fit into.
   - If it does not fit into any bin, add a new bin to fit into.
2. If no object left, done. Go to Step 1.



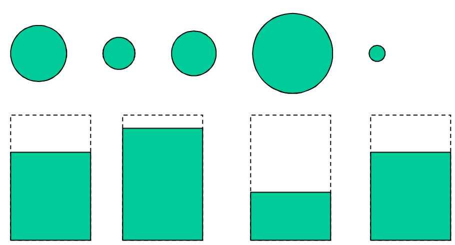
Next Fit Example

- Given a list of objects of size 8, 5, 7, 6, 2, 4, 1 in that order
- Pack these objects into bins of capacity 10



First-Fit (FF) Heuristic

1. Fit next object into the <u>first</u> bin that it can fit into
   - If it does not fit into any bin, add a new bin
2. If no object left, done. Else, go to Step 1.



First Fit Example

- Given a list of objects of size 8, 5, 7, 6, 2, 4, 1 in that order
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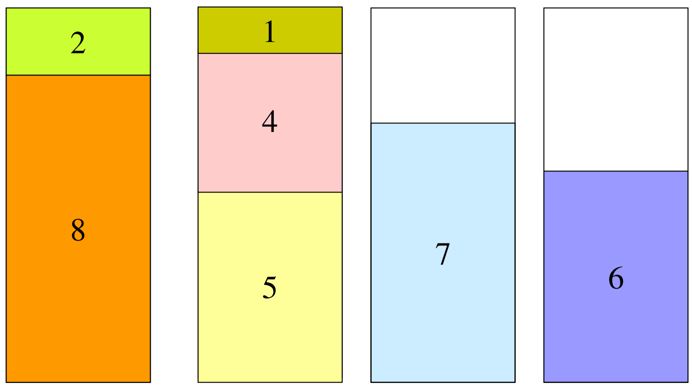
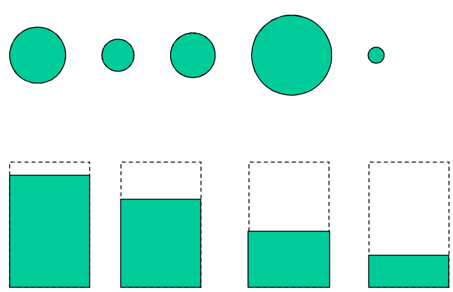


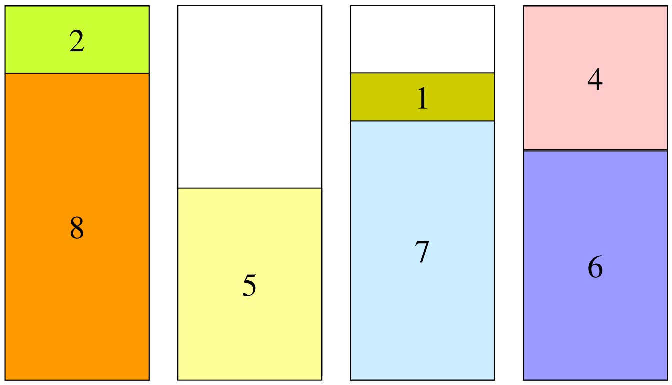
Best-Fit Heuristic

1. Sort the bins in <u>descending</u> order of consumed space
2. Fit next object into first sorted bin that it can fit into
   - If it does not fit into any bin, add a new bin to fit into
3. If no object left, done. Go to Step 1.



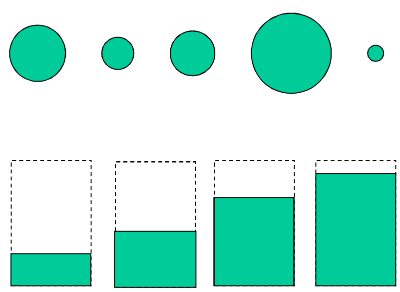
Best Fit Example

- Given a list of objects of size 8, 5, 7, 6, 2, 4, 1 in that order
- Pack these objects into bins of capacity 10



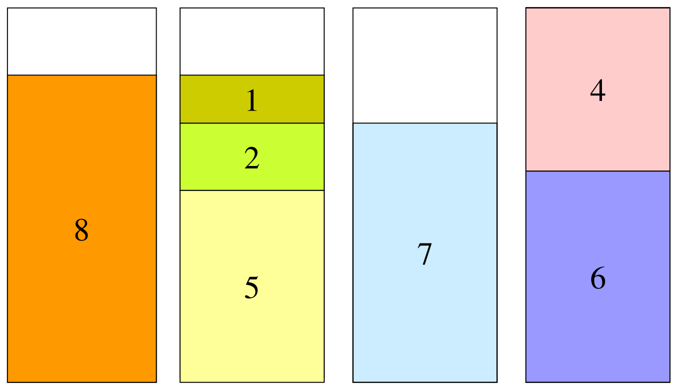
Worst-Fit Heuristic

1. Sort the bins in <u>ascending</u> order of consumed space
2. Fit next object into first sorted bin that it can fit into
   - If it does not fit into any bin, add a new bin to fit into
3. If no object left, done. Go to Step 1.



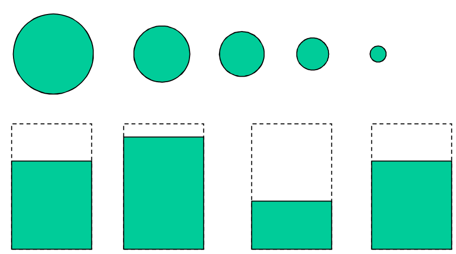
Worst Fit Example

- Given a list of objects of size 8, 5, 7, 6, 2, 4, 1 in that order
- Pack these objects into bins of capacity 10



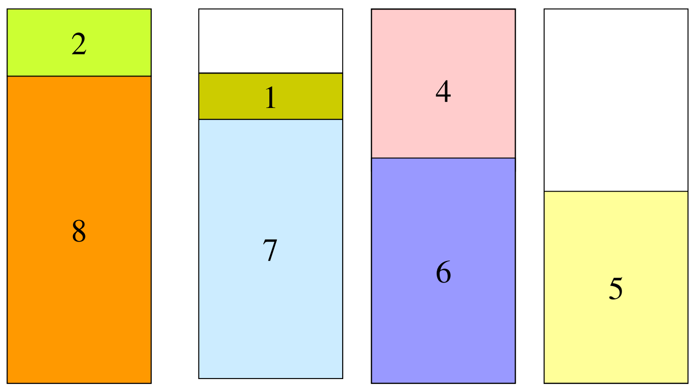
First-Fit Decreasing (FFD) Heuristic

1. Sort the objects in descending order of size
2. Fit first object into the first bin that it can fit into
   - If it does not fit into any bin, add a new bin
3. If no object left, done. Else, go to Step 2.



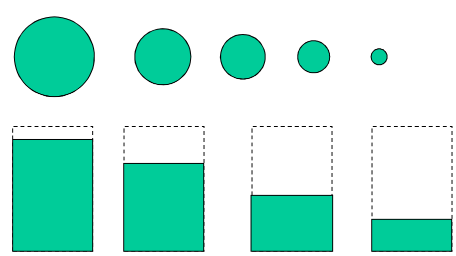
First Fit Decreasing Example

- Given a list of objects of size 8, 5, 7, 6, 2, 4, 1 in that order
- Pack these objects into bins of capacity 10



Best-Fit Decreasing (BFD) Heuristic

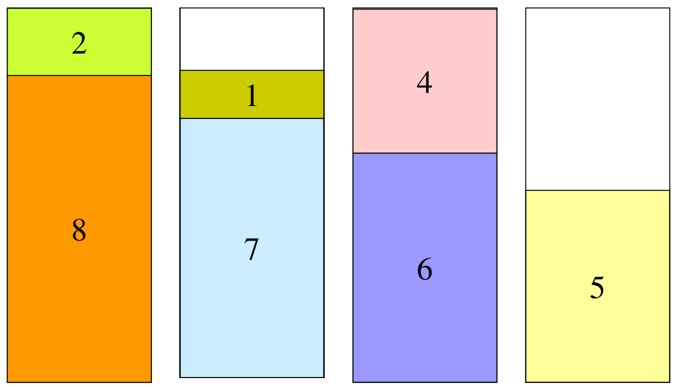
1. Sort the objects in descending order of size
2. Sort the bins in <u>descending</u> order of consumed space
3. Fit next object into first sorted bin that it can fit into
   - If it does not fit into any bin, add a new bin to fit into
4. If no object left, done.



"Tries to pack bins as much as possible with empty space in the 'last' bins"

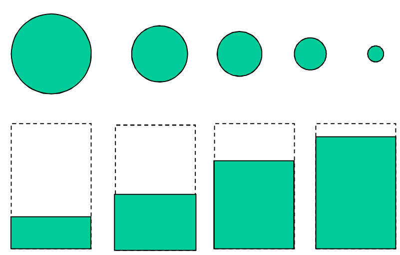
Best Fit Decreasing Example

- Given a list of objects of size 8, 5, 7, 6, 2, 4, 1 in that order
- Pack these objects into bins of capacity 10



Worst-Fit Decreasing (WFD) Heuristic

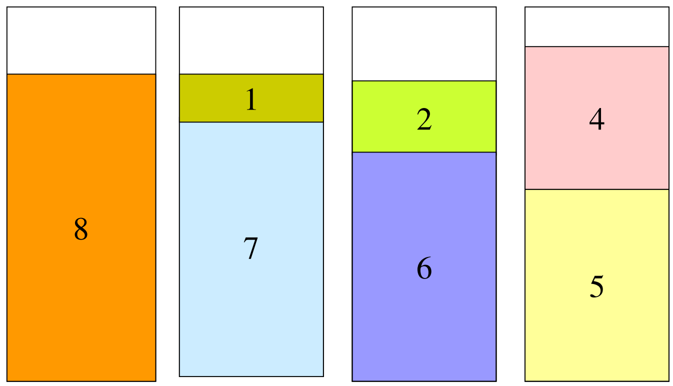
1. Sort the objects in descending order of size
2. Sort the bins in <u>ascending</u> order of consumed space
3. Fit next object into first sorted bin that it can fit into
   - If it does not fit into first bin, add a new bin to fit into
4. If no object left, done. Go to Step 2



"Balances load across available bins"

Worst Fit Decreasing Example

- Given a list of objects of size 8, 5, 7, 6, 2, 4, 1 in that order
- Pack these objects into bins of capacity 10



Complexity of Bin-Packing

- The bin-packing problem is known to be NP-complete:
  - It could take an exponential # of steps in the worst case to determine the optimal number of bins
- Very efficient near-optimal heuristics exist
  - *Best Fit Decreasing* and *First Fit Decreasing* heuristics use no more than **11/9 OPT + 2 bins** (where **OPT** is the number of bins given by the optimal solution).
    - Exception: there are simple cases which have worse performance, but can be detected and addressed as special cases.

The Bad Case of Bin Packing

- Given a list of objects of size 0.51, 0.51, 0.51, 0.51, 0.51
- Pack these objects into bins of capacity 1.0
- How many bins are required?
- With objects of size 0.50+ε, 0.50+ε, 0.50+ε, 0.50+ε, 0.50+ε,
  - What is the utilization of the system?
- Large objects can be a problem!
  - Think *rocks*…

The Good Case of Bin-Packing

- Consider a list of objects of size 0.01, 0.01, 0.01, 0.01, …. (1000 objects)
- Pack these objects into bins of size 1.0
- How many bins are required?
- Are small objects a problem?
  - Think sand…

Getting Back to Partitioned Scheduling

- Only having tasks with high utilization can lead to poor utilization of processors
  - We refer to these tasks as "heavyweight tasks"
- Only having tasks with low utilization leads to extremely good packing and high system utilization
  - We refer to these tasks as "lightweight tasks"
- What does having a hybrid of "lighweight" and "heavyweight" tasks mean??

Important Points to Remember

- When we perform bin-packing for partitioned scheduling, consider which scheduling policy is used
  - Under EDF, 100% schedulability is possible → bin-packing condition is straightforward (modulo overheads and worst-case execution time estimates).
  - Under RM, schedulability analysis must be carried out (exact or otherwise) → "empty space" test is less straightforward.
- All tasks are assumed to be independent, i.e. there is no resource sharing
  - In practice, this assumption is generally not true.

Conclusions

- Partitioned scheduling for multiprocessors requires the bin-packing problem to be solved
- Bin-packing is an NP-complete problem
- Many efficient but sub-optimal heuristics are available
  - First fit, next fit, best fit and worst fit
    - Usually applied to pack objects as they arrive
  - First-fit decreasing, next-fit decreasing, best-fit decreasing and worst-fit decreasing
    - Usually applied when all objects to be packed are readily available (and can be sorted before packing)