Uniprocessor Scheduling - II

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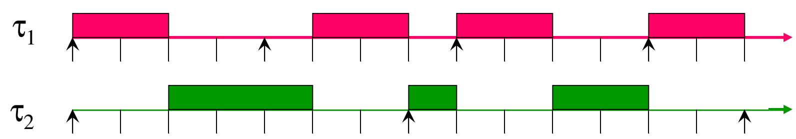
Lecture #4

Scheduling Policies

- CPU scheduling policy: a rule to select task to run next
  - cyclic executive
  - Rate-monotonic/deadline-monotonic
  - earliest deadline first
  - least laxity first
- Assume preemptive, priority scheduling of tasks
  - analyze effects of non-preemption later

Earliest Deadline First (EDF) Scheduling

- Can be used to schedule periodic tasks
- Uses dynamic priorities and preemptive scheduling
  - Higher priority to task with earlier deadline
- **Example**: 2-task set where each task is (C, T=D) → {(2, 4), (3, 7)}



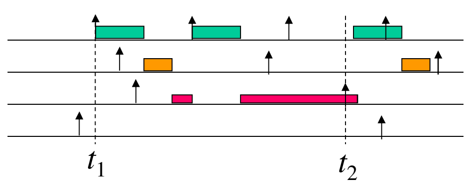
- Utilization U of task set $\{\tau_i\}$ for i = 1, …, n:

$$U_i = \frac{C_i}{T_i} \qquad\qquad U = \sum_{i=1}^{n} U_i = \sum_{i=1}^{n} \frac{C_i}{T_i}$$

EDF Schedulability Condition

Theorem: A task set is schedulable under EDF if and only if $U \le 1$.

Proof:



- Assume that "overflow" occurs at time t_2.
- Let t_1 be the latest time before t_2 such that
  - the processor is fully utilized in the interval $[t_1, t_2]$
  - only instances with deadlines before t_2 execute in $[t_1, t_2]$
- If such a t_1 cannot be found, then set $t_1 = 0$.
- Let C_d be the computational demand in $[t_1, t_2]$

$$C_d = \sum_{r_i \ge t_1, d_i \le t_2} \left\lfloor \frac{t_2 - t_1}{T_i} \right\rfloor * c_i \le \sum_{i=1}^{n} \frac{t_2 - t_1}{T_i} * c_i = (t_2 - t_1)U$$

- But an overflow implies that $C_d > (t_2 - t_1)$: a contradiction if $U \le 1$.

Points to Note

- If deadlines are shorter than periods, the necessary condition for schedulability under EDF is an open problem.
- If $U > 1$, which task will first miss its deadline is unpredictable.

Rate-Monotonic Scheduling (RMS)

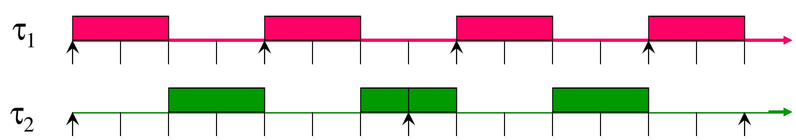
- The priorities of periodic tasks are based on their rates: a higher rate gets a higher priority.
- Theoretical basis
  - optimal fixed-priority scheduling policy (when deadlines are at the end of period)
  - analytic formulas to check schedulability
- Must distinguish between scheduling and analysis
  - Rate-Monotonic Scheduling (RMS) forms the basis for Rate-Monotonic Analysis (RMA)
  - However, we will consider later how to analyze systems in which rate-monotonic scheduling is *not* used
  - Any scheduling approach may be used, but all real-time systems should be analyzed for timing

Rate Monotonic Analysis (RMA)

- Rate-monotonic analysis is a set of mathematical techniques for analyzing sets of real-time tasksets.
- The original RMS theory applied only to independent, periodic tasks, but has been extended to address
  - priority inversion
  - task interactions
  - aperiodic tasks
- Focus is on RM**A**, not RM**S**

Rate-Monotonic Scheduling (RMS)

- Higher (fixed) priority to higher frequency task
  - i.e. higher priority to task with shorter period
- **Example**: 2-task set where each task is (C, T=D) → {(2, 4), (3, 7)}



Liu and Layland proved that RMS leads to a feasible schedule if $U \leq n\,(2^{1/n} - 1)$

- If $C_2 = 3.1$, then C_2 will miss its deadline although the utilization is (2/4)+(3.1/7) = 0.5+0.443=0.943, which is < 1.
- The above bound is sufficient for schedulability but not necessary
- RMS is an optimum fixed-priority assignment algorithm
  - if a taskset cannot be scheduled by RMS, all other fixed-priority schedules will also be unschedulable.

More on the RMS Bound

- The RMS bound is a sufficient condition, not a necessary condition
- Occurs only under mathematically extreme conditions
- In practice
  - Many tasksets are **harmonic**
    - In a harmonic taskset, every period is an integral multiple (or sub-multiple) of all other periods in the task set
  - For harmonic tasksets, RMS is schedulable if $U \leq 1$.
  - Or for nearly harmonic tasksets
    - The schedulable utilization is about 0.92
- Mathematically, if *C*'s and *T*'s are chosen randomly, the average schedulable task set utilization is 88%.
- Remaining utilization can still be used for other purposes
  - Background tasks, testing, etc.

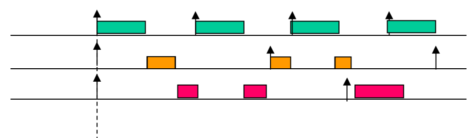
n	$n\,(2^{1/n} - 1)$
1	1
2	0.828
3	0.780
4	0.757
5	0.743
6	0.735
7	0.729
8	0.724
9	0.721
10	0.718
…	…
∞	$ln\ 2 = 0.693$

The Derivation of the "RMS" Least Upper Bound

- **Step 1**: Find the worst-case relative phasing between tasks
- **Step 2**:
  - Assume that the ratio of the largest to the smallest period (T_n / T_1) is no more than 2.
  - For *n* tasks, find U_{lub} such that if a task set has $U \leq U_{lub}$, then it can be feasibly scheduled by RMS.
- **Step 3**: Generalize the result for an arbitrary T_n / T_1 ratio.

The Worst-Case Relative Phasing

- **Critical instant**: the instant or relative phasing at which a task arrival encounters its worst-case response time
- **Critical zone**: the duration between the critical instant of a task instance and its completion



- For a periodic task set $\{(C, T)\}$, the critical zone for a task τ occurs when τ arrives simultaneously with *all* other higher priority tasks.
  - So, all tasks arrive together
- <u>A taskset is feasible if the first instance of a task arrives along with all other tasks and completes by its deadline</u>

The Least Upper Schedulable Bound for 2 Tasks

Goal: Among all the feasibly scheduled task sets $\{\tau_1, \tau_2\}$, we want to find for any given combination of periods $\{T_1\ T_2\}$, the maximum feasible schedulable utilization U_{ub}.

- Find the minimum value of U_{ub}, U_{lub}, for all possible combinations of T_i.

Method:

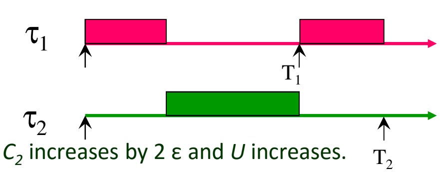
- Fix T_2 and consider all possible values of T_1, C_1 and C_2 (relative to T_2).
- For any T_1, assuming a fully utilized processor, the minimum U occurs when $C_1 = T_2 - T_1$

  Thus, $C_2 = T_1 - C_1$

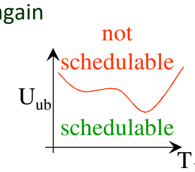
  $U = U_1 + U_2$

  $= (T_2 - T_1) / T_1 + (T_1 - T_2 + T_1) / T_2$

  $= T_2 / T_1 + 2\,T_1 / T_2 - 2$



- If $C_1 = T_2 - T_1 - \varepsilon$, then, for full utilization, C_2 increases by $2\,\varepsilon$ and U increases.
- If $C_1 = T_2 - T_1 + \varepsilon$, then, for full utilization, C_2 decreases by ε and U again increases (remember that $T_2 / T_1 < 2$).
- Next, differentiate U w.r.t. T_1 and equate the result to zero
  - Obtain $\dfrac{T_2}{T_1} = \sqrt{2}$



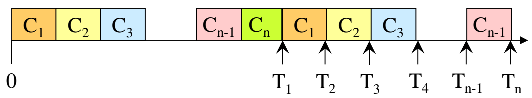
- Hence, among all possible values of T_1, the minimum utilization is

$$2(\sqrt{2} - 1)$$

Least Upper Schedulable Bound for n Tasks

- For given values of $T_1, \ldots, T_n$, the upper bound, U_{ub}, on the utilization of feasible sets is obtained when

  $C_1 = T_2 - T_1$
  $C_2 = T_3 - T_2$
  ….
  $C_{n-1} = T_n - T_{n-1}$
  $C_n = T_1 - (C_1 + \ldots + C_{n-1}) = 2\,T_1 - T_n$



- For the above values, $$U = \sum_{i=1}^{n-1} R_i + \frac{2}{R_1 R_2 \ldots R_{n-1}} - n$$

  where $R_i = T_{i+1} / T_i$ and $R_1 \ldots R_{n-1} = T_n / T_1$
- Differentiate U w.r.t. $R_1, \ldots, R_{n-1}$ and equate the result to zero to obtain

  $R_1 = \ldots = R_{n-1} = 2^{1/n}$
- Hence, among all the T's and C's, the minimum utilization for feasibility is given by $U = n\,(2^{1/n} - 1)$

Response Time (RT) Test

- Also, referred to as "Exact Schedulability Test" or "Completion Time Test"
- Can be used for computing response times with *any* fixed-priority preemptive scheduling scheme
- Let a_k^i **= the worst-case response time of task** τ_i. a_k^i of task τ_i may be computed by the following iterative formula:

$$a_{k+1}^i = C_i + \sum_{j=1}^{i-1} \left\lceil \frac{a_k^i}{T_j} \right\rceil C_j \qquad \text{where } a_0^i = \sum_{j=1}^{i} C_j$$

- Test terminates when $a_{k+1}^i = a_k^i$
- Task i is schedulable if its response time is before its deadline: $a_k^i \le D_i$
  - Stop test once current iteration yields a value of a_k^i beyond the deadline (else, you may never terminate).
- This test determines the schedulability of only **task τ_i**
- Repeat for other tasks as needed**:** i will change with the task

The 'square bracketish' thingies represent the 'integer ceiling' function, NOT brackets

Example: Testing for Schedulability

	C_i	T_i	U_i
Task τ_1:	40	100	0.4
Task τ_2:	40	150	0.267
Task τ_3:	100	350	0.286

- Utilization of first two tasks: 0.667 < LUB(2) = 0.828
  - The first two tasks are schedulable by UB test
- Utilization of all three tasks: 0.953 > LUB(3) = 0.779
  - UB test is inconclusive for the third task
  - Need to apply RT test

Applying RT Test

Use RT test to determine if τ_3 meets its first deadline: $\boxed{i = 3}$

$$a_0^3 = \sum_{j=1}^{3} C_j = C_1 + C_2 + C_3 = 40 + 40 + 100 = 180$$

$$a_1^3 = C_i + \sum_{j=1}^{i-1} \left\lceil \frac{a_0^3}{T_j} \right\rceil C_j = C_3 + \sum_{j=1}^{2} \left\lceil \frac{a_0^3}{T_j} \right\rceil C_j$$

$$= 100 + \left\lceil \frac{180}{100} \right\rceil (40) + \left\lceil \frac{180}{150} \right\rceil (40) = 100 + 80 + 80 = 260$$

Applying the RT Test (contd.)

$$a_2^3 = C_3 + \sum_{j=1}^{2} \left\lceil \frac{a_1^3}{T_j} \right\rceil C_j = 100 + \left\lceil \frac{260}{100} \right\rceil (40) + \left\lceil \frac{260}{150} \right\rceil (40) = 300$$

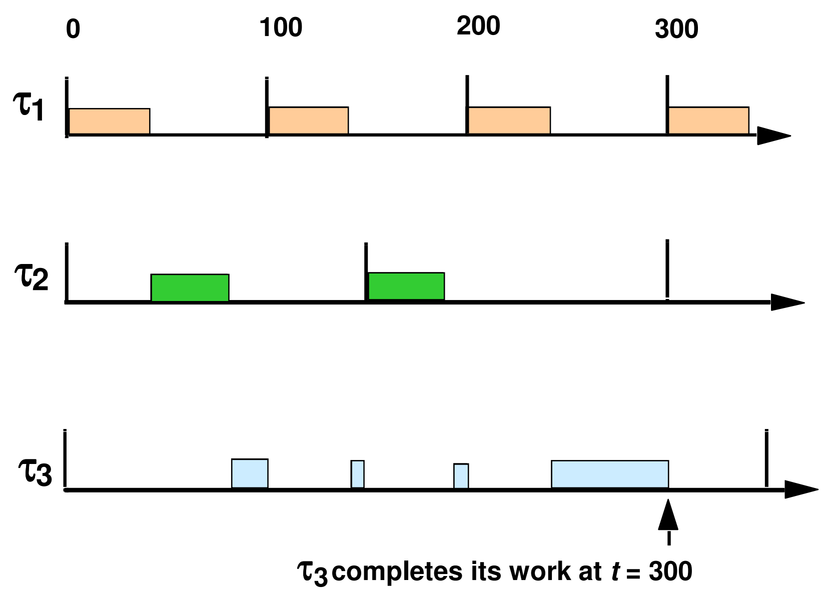
$$a_3^3 = C_3 + \sum_{j=1}^{2} \left\lceil \frac{a_2^3}{T_j} \right\rceil C_j = 100 + \left\lceil \frac{300}{100} \right\rceil (40) + \left\lceil \frac{300}{150} \right\rceil (40) = 300$$

$a_3 = a_2 = 300$ **Done!**

Task τ_3 is schedulable using the RT test

$$\boxed{a_3 = 300 < D = T = 350}$$

Timeline for Example



Exercise: Applying RT Test

Task τ_1: $C_1 = 1$ $T_1 = 4$

Task τ_2: $C_2 = 2$ $T_2 = 6$

Task τ_3: $C_3 = 2$ $T_3 = 10$

a) Apply the LUB test

b) Draw timeline

c) Apply RT test

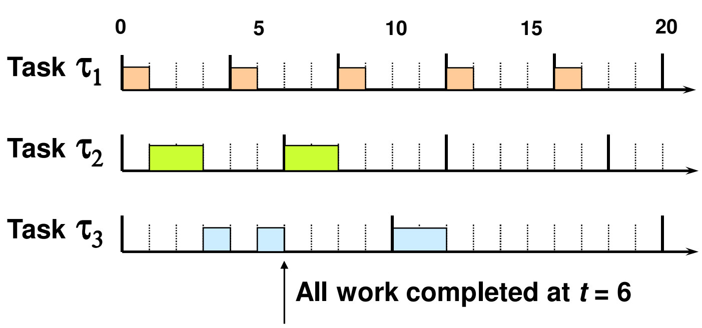
Solution: Applying RT Test

a) UB test

τ_1 and τ_2 OK -- no change from previous exercise

.25 + .34 + .20 = .79 > .779 ==> Test inconclusive for τ_3

b) RT test and timeline



Solution: Applying RT Test (cont.)

c) RT test

$$a_0^3 = \sum_{j=1}^{3} C_j = C_1 + C_2 + C_3 = 1 + 2 + 2 = 5$$

$$a_1^3 = C_3 + \sum_{j=1}^{2} \left\lceil \frac{a_0^3}{T_j} \right\rceil C_j = 2 + \left\lceil \frac{5}{4} \right\rceil 1 + \left\lceil \frac{5}{6} \right\rceil 2 = 2 + 2 + 2 = 6$$

$$a_2^3 = C_3 + \sum_{j=1}^{2} \left\lceil \frac{a_1^3}{T_j} \right\rceil C_j = 2 + \left\lceil \frac{6}{4} \right\rceil 1 + \left\lceil \frac{6}{6} \right\rceil 2 = 2 + 2 + 2 = 6$$

Done

Why Are Deadlines Missed?

- For a given task, consider
  - **preemption**: time waiting for higher priority tasks
  - **execution**: time to do its own work
  - **blocking**: time delayed by lower priority tasks
- The task is schedulable if the sum of its preemption, execution, and blocking is less than (or equal to) its deadline.
- **Focus**: identify the biggest hits among the three and reduce, as needed, to achieve schedulability

Summary

- Dynamic-priority EDF scheduling can schedule periodic task sets with total utilization ≤ 100%
- Under fixed-priority Rate-Monotonic Scheduling (RMS),
  – Pathological tasksets with about 70% utilization could miss deadlines
    - Nearly impossible to find in practice
  – Least-Upper Bound tests are simple but conservative
    - Also referred to as a "utilization bound"
  – The Response time test is more exact but needs more calculations.
    - Easily automated
  – Harmonic tasksets can be scheduled up to 100%
  – Nearly harmonic tasksets (found in practice very often) are scheduled upwards of 92%
  – Randomly chosen tasksets have an average schedulable utilization of 88%
- RMS priority assignments are used widely in practice.