Embedded Systems

11. Architecture Synthesis

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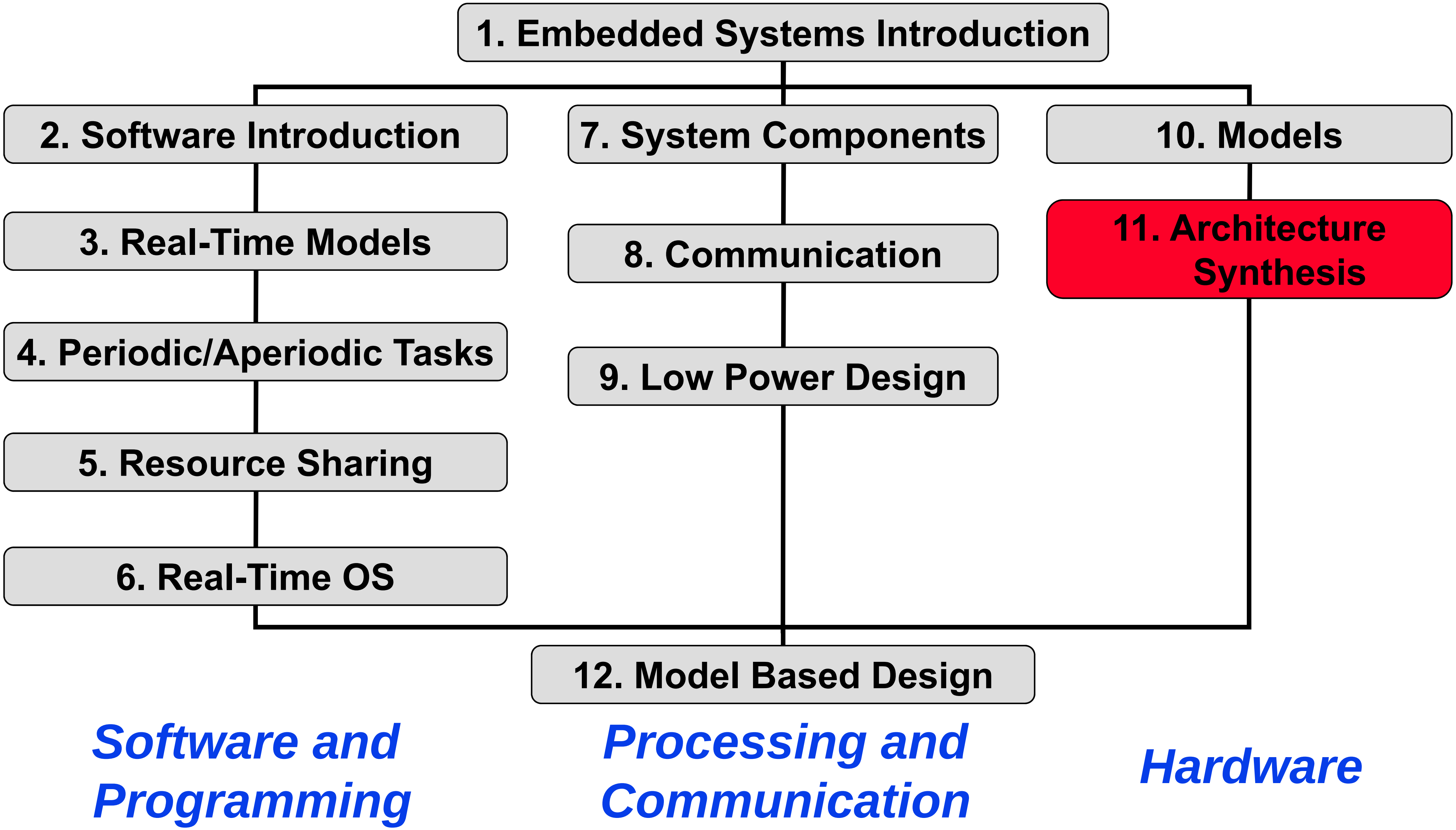
Contents of Course



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- Dynamic Voltage Scaling

Architecture Synthesis

Determine a hardware architecture that efficiently executes a given algorithm.

- ***Major tasks*** of architecture synthesis:
  - ***allocation*** (determine the necessary hardware resources)
  - ***scheduling*** (determine the timing of individual operations)
  - ***binding*** (determine relation between individual operations of the algorithm and hardware resources)
- ***Classification*** of synthesis algorithms:
  - ***heuristics*** or ***exact methods***
- Synthesis methods can often be applied ***independently of granularity*** of algorithms, e.g. whether operation is a whole complex task or a single operation.

Models

- ***Sequence graph*** $G_S = (V_S, E_S)$
  where V_S denotes the operations of the algorithm and E_S the dependence relations.
- ***Resource graph*** $G_R = (V_R, E_R),\ V_R = V_S \cup V_T$
  where V_T denote the resource types of the architecture and G_R is a bipartite graph. An edge $(v_s, v_t) \in E_R$ represents the availability of a resource type v_t for an operation v_s.
- ***Cost function*** $c : V_T \rightarrow \mathbf{Z}$
- ***Execution times*** $w : E_R \rightarrow \mathbf{Z}^{\geq 0}$
  are assigned to each edge $(v_s, v_t) \in E_R$
  and denote the execution time of operation $v_s \in V_S$
  on resource type $v_t \in V_T$.

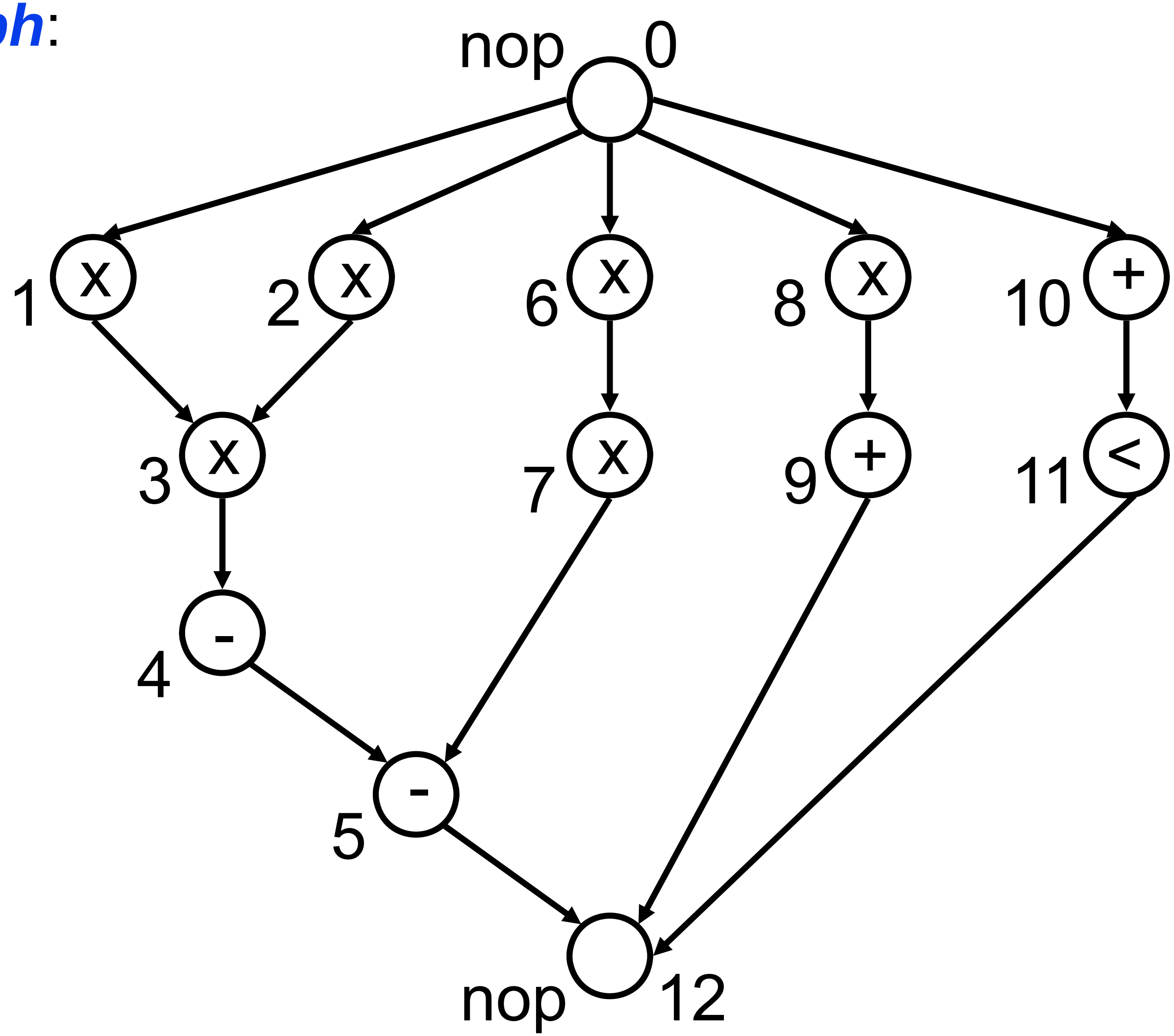
Models

- ***Example*** sequence graph:
  - Given ***algorithm*** (differential equation):

```
int diffeq(int x, int y, int u, int dx, int a) {
  int x1, u1, y1;
  while ( x < a ) {
    x1 = x + dx;
    u1 = u - (3 * x * u * dx) - (3 * y * dx);
    y1 = y + u * dx;
    x = x1;
    u = u1;
    y = y1;
  }
  return y;
}
```

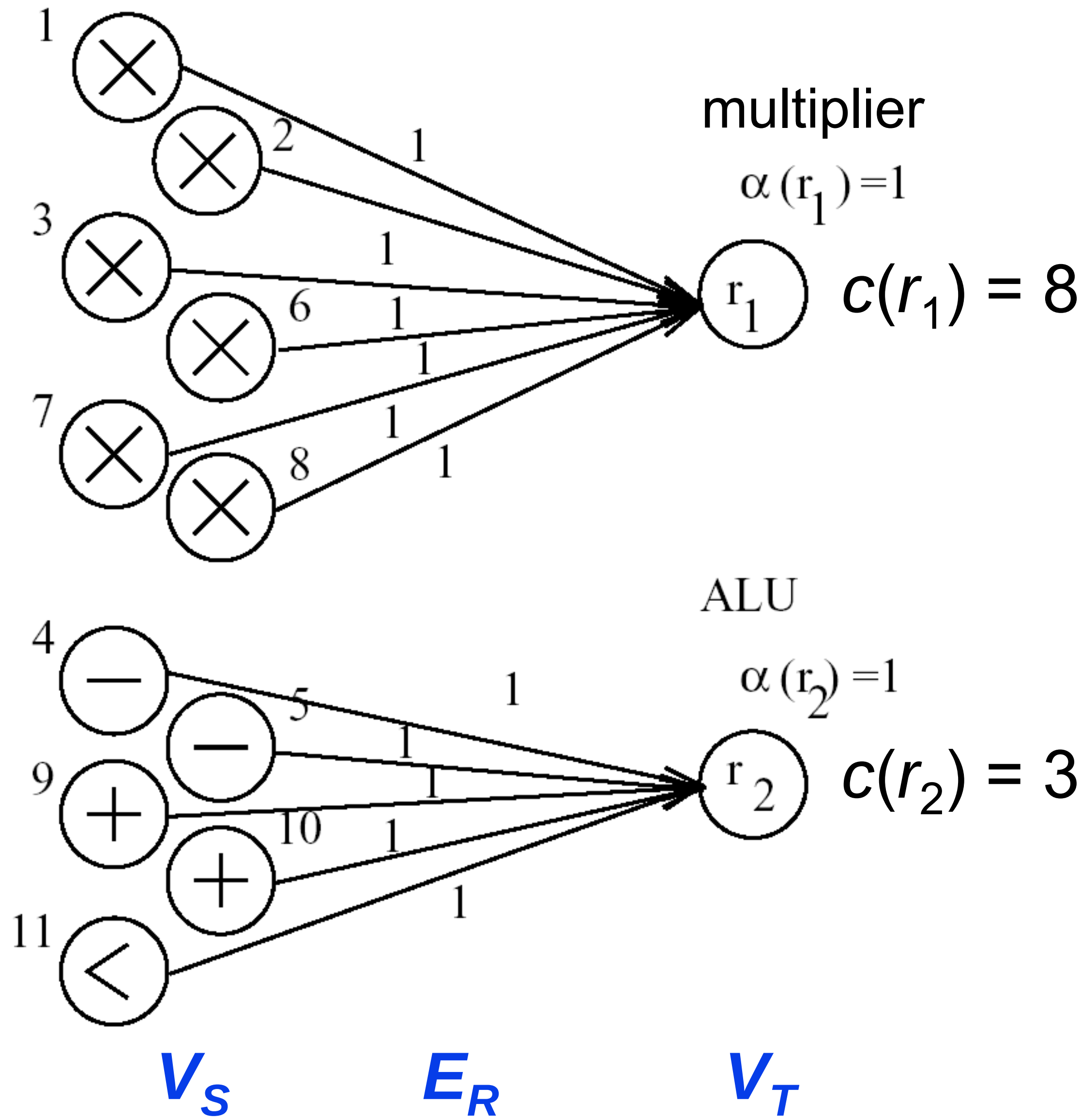
Models

- ***Sequence graph***:



Models

- ***Resource graph***:



Allocation and Binding

An allocation is a function $\alpha : V_T \rightarrow \mathbf{Z}^{\geq 0}$ that assigns to each resource type $v_t \in V_T$ the number $\alpha(v_t)$ of available instances.

A binding is defined by functions $\beta : V_S \rightarrow V_T$ and $\gamma : V_S \rightarrow \mathbf{Z}^{>0}$. Here, $\beta(v_s) = v_t$ and $\gamma(v_s) = r$ denote that operation $v_s \in V_S$ is implemented on the rth instance of resource type $v_t \in V_T$.

Scheduling

A schedule is a function $\tau : V_S \to \mathbf{Z}^{>0}$ that determines the starting times of operations. A schedule is feasible if the conditions

$$\tau(v_j) - \tau(v_i) \geq w(v_i) \quad \forall (v_i, v_j) \in E_S$$

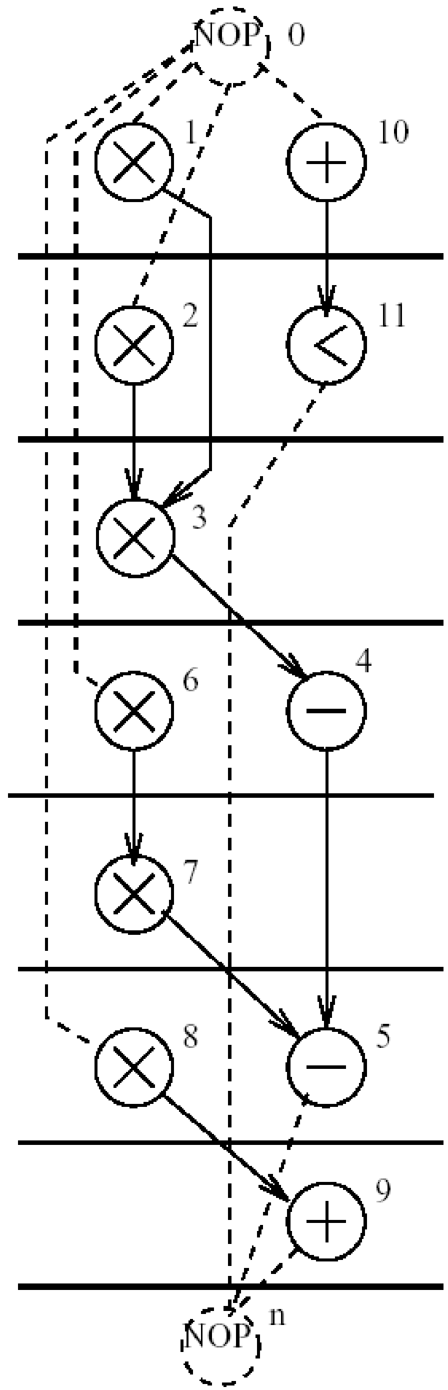
are satisfied. $w(v_i) = w(v_i, \beta(v_i))$ denotes the execution time of operation v_i.

The latency L of a schedule is the time difference between start node v_0 and end node v_n: $L = \tau(v_n) - \tau(v_0)$.

Scheduling

- ***Example***:

  $L = \tau(v_{12}) - \tau(v_0) = 7$



$\tau(v_0) = 1$

$\tau(v_1) = \tau(v_{10}) = 1$

$\tau(v_2) = \tau(v_{11}) = 2$

$\tau(v_3) = 3$

$\tau(v_6) = \tau(v_4) = 4$

$\tau(v_7) = 5$

$\tau(v_8) = \tau(v_5) = 6$

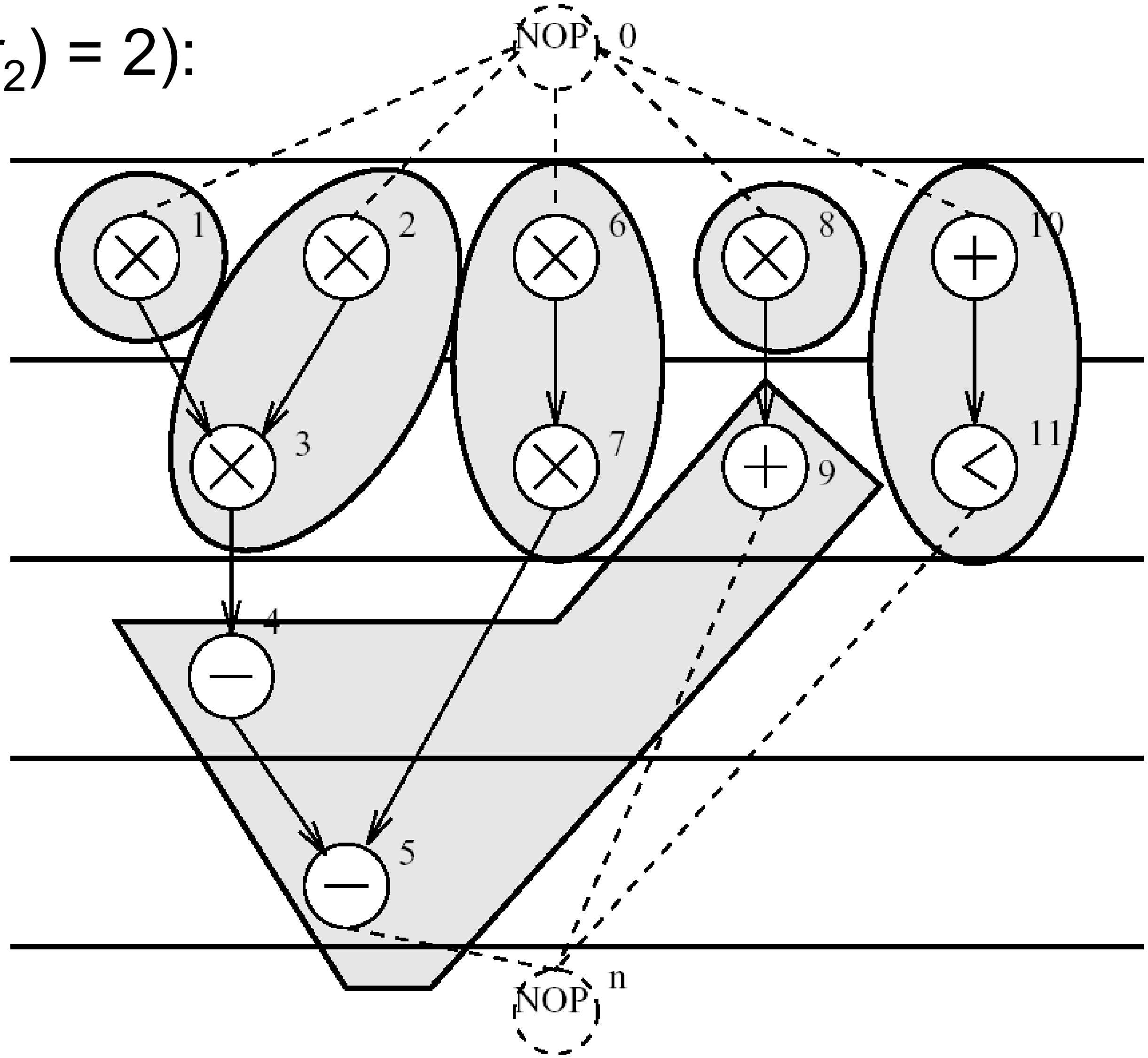
$\tau(v_9) = 7$

$\tau(v_{12}) = 8$

Binding

- Example ($\alpha(r_1) = 4$, $\alpha(r_2) = 2$):

$$\beta(v_1) = r_1, \gamma(v_1) = 1,$$
$$\beta(v_2) = r_1, \gamma(v_2) = 2,$$
$$\beta(v_3) = r_1, \gamma(v_3) = 2,$$
$$\beta(v_4) = r_2, \gamma(v_4) = 1,$$
$$\beta(v_5) = r_2, \gamma(v_5) = 1,$$
$$\beta(v_6) = r_1, \gamma(v_6) = 3,$$
$$\beta(v_7) = r_1, \gamma(v_7) = 3,$$
$$\beta(v_8) = r_1, \gamma(v_8) = 4,$$
$$\beta(v_9) = r_2, \gamma(v_9) = 1,$$
$$\beta(v_{10}) = r_2, \gamma(v_{10}) = 2,$$
$$\beta(v_{11}) = r_2, \gamma(v_{11}) = 2$$



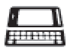
Multiobjective Optimization

- Architecture Synthesis is an optimization problem with more than one objective:
  - Latency of the algorithm that is implemented
  - Hardware cost (memory, communication, computing units, control)
  - Power and energy consumption

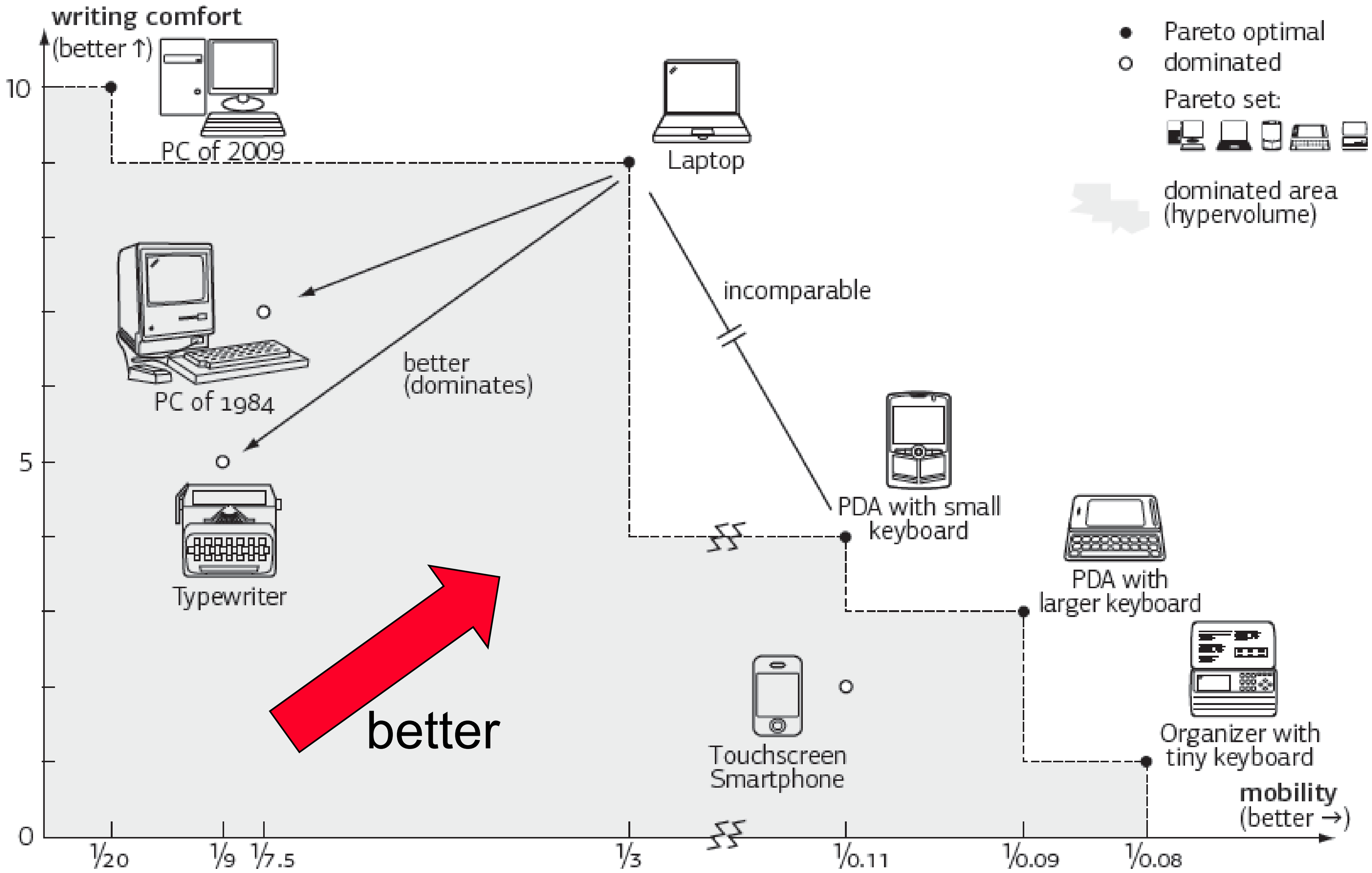
- Optimization problems with several objectives are called "multiobjective optimization problems".

Multiobjective Optimization

- Let us suppose, we would like to select a typewriting device. Criteria are
  - mobility (related to weight)
  - comfort (related to keyboard size and performance)

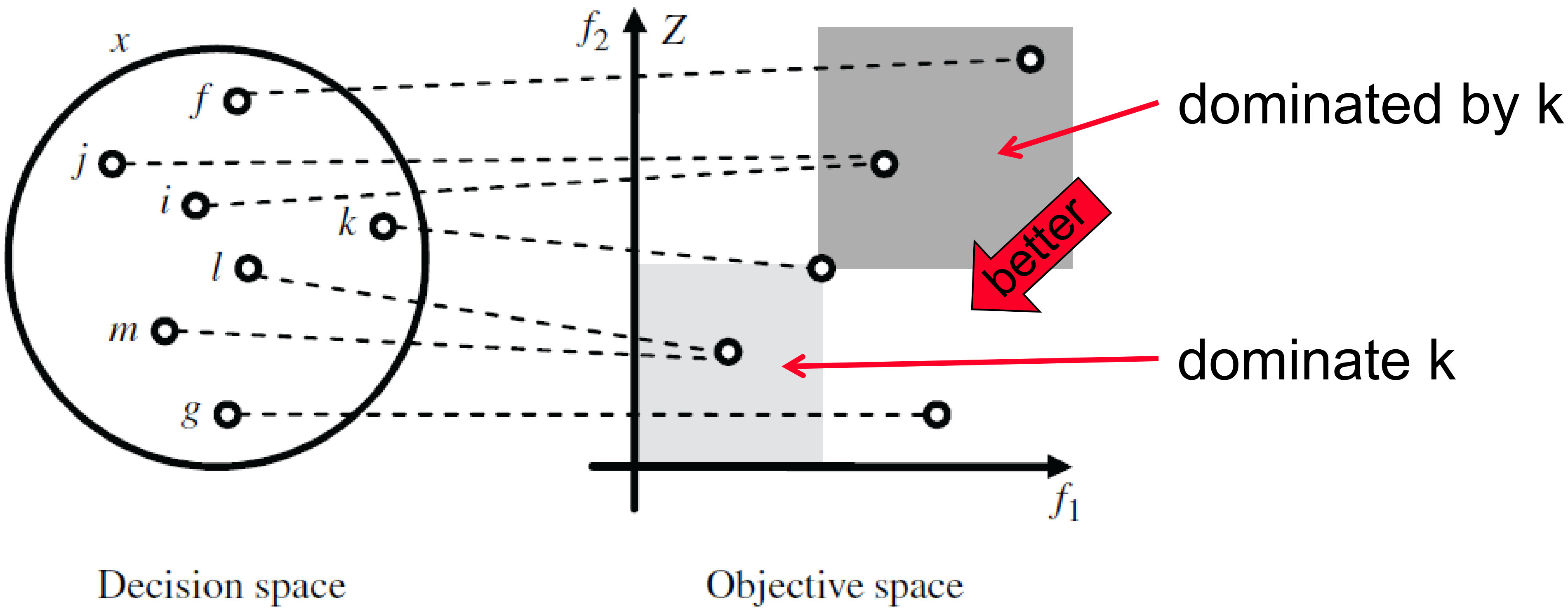
Icon	Device	weight (kg)	comfort rating
	PC of 2009	20.00	10
	PC of 1984	7.50	7
	Laptop	3.00	9
	Typewriter	9.00	5
	Touchscreen Smartphone	0.11	2
	PDA with large keyboard	0.09	3
	PDA with small keyboard	0.11	4
	Organizer with tiny keyboard	0.08	1

Multiobjective Optimization



Pareto-Dominance

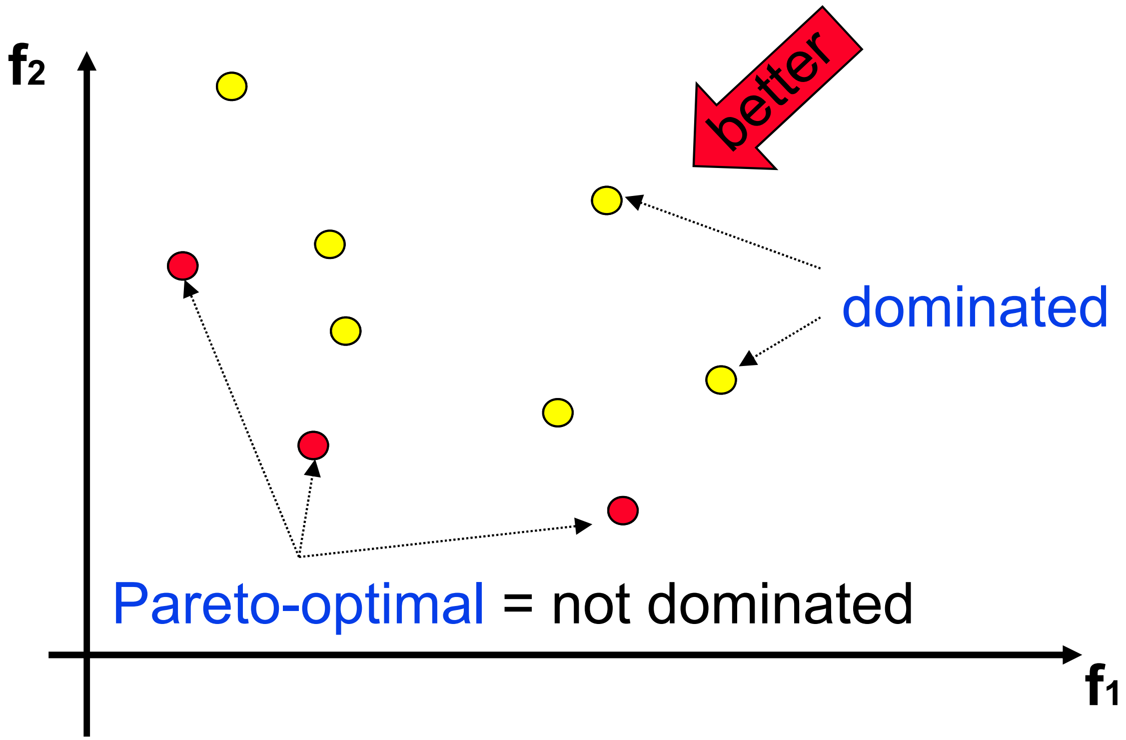
Definition : *A solution $a \in \mathrm{X}$ weakly Pareto-dominates a solution $b \in \mathrm{X}$, denoted as $a \preceq b$, if it is as least as good in all objectives, i.e., $f_i(a) \leq f_i(b)$ for all $1 \leq i \leq n$. Solution a is better then b, denoted as $a \prec b$, iff $(a \preceq b) \wedge (b \npreceq a)$.*



Pareto-optimal Set

- A solution is named ***Pareto-optimal***, if it is not Pareto-dominated by any other solution in X.
- The set of all Pareto-optimal solutions is denoted as the Pareto-optimal set and its image in objective space as the ***Pareto-optimal front***.

objective space Z:



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Scheduling Algorithms

- *Classification*
  - ***unlimited resources***: no constraints in terms of the available resources are defined.
  - ***limited resources***: constrains are given in terms of the number a types of available resources.

  - ***iterative algorithms***: an initial solution to the architecture synthesis is improved step by step.
  - ***constructive algorithms***: the synthesis problem is solved in one step.
  - ***transformative algorithms***: the initial problem formulation is converted into a (classical) optimization problem.

Scheduling Without Resource Constraints

- The scheduling method can be used
  - as a ***preparatory step*** for the general synthesis problem
  - to ***determine bounds*** on feasible schedules in the general case
  - if there is a ***dedicated resource*** for each operation.

Given is a sequence graph $G_S = (V_S, E_S)$ and a resource graph $G_R = (V_R, E_R)$. Then the latency minimization without resource constraints is defined as

$$L = \min\{\tau(v_n) : \tau(v_j) - \tau(v_i) \geq w(v_i) \;\; \forall (v_i, v_j) \in E_S\}$$

Contents

The ASAP Algorithm

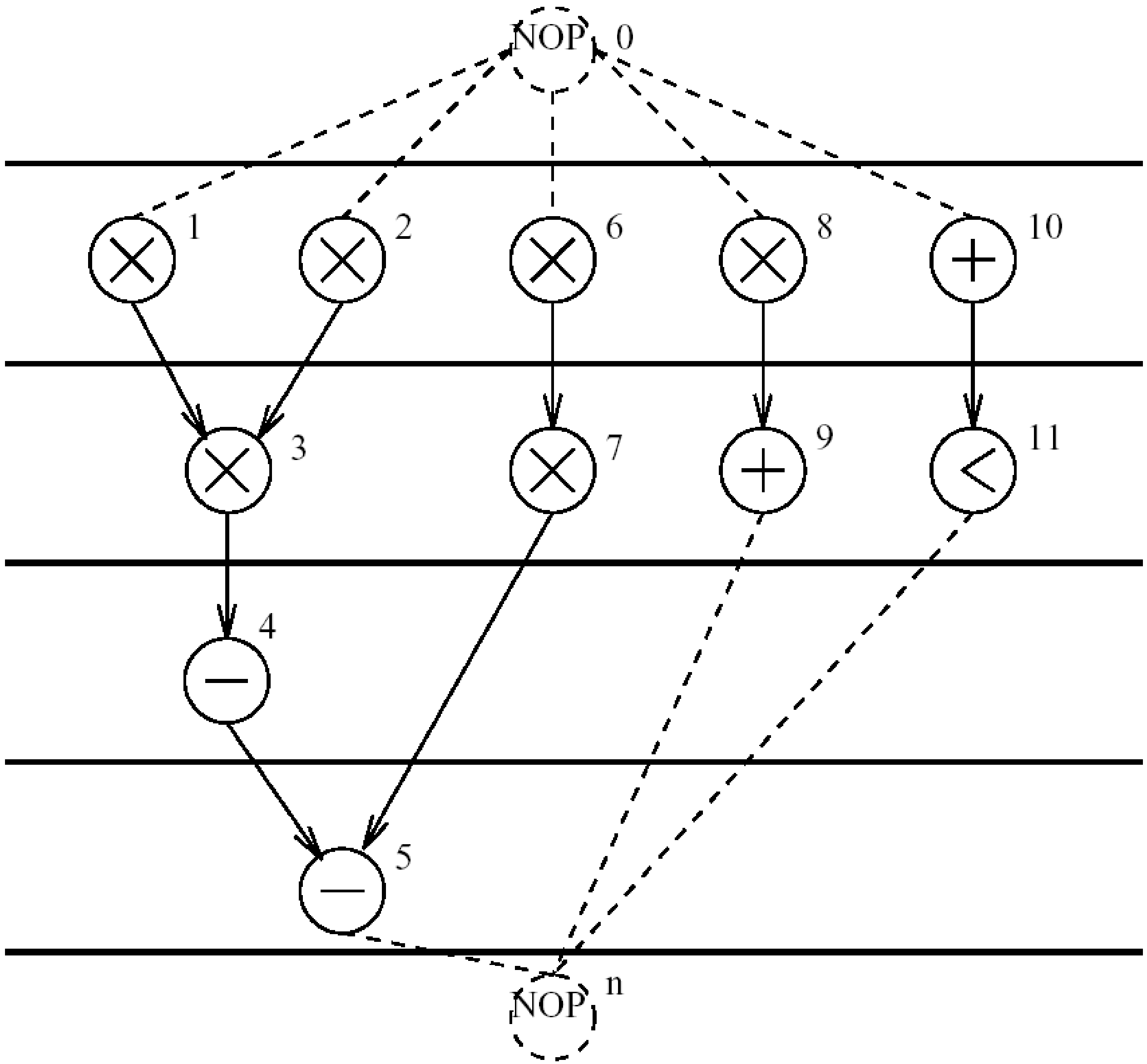
- *ASAP = As Soon As Possible*

```
ASAP(G_S(V_S, E_S), w) {
    τ(v_0) = 1;
    REPEAT {
        Determine v_i whose predec. are planed;
        τ(v_i) = max{τ(v_j) + w(v_j) ∀(v_j, v_i) ∈ E_S}
    } UNTIL (v_n is planned);
    RETURN (τ);
}
```

The ASAP Algorithm

- ***Example:***

  $w(v_i) = 1$



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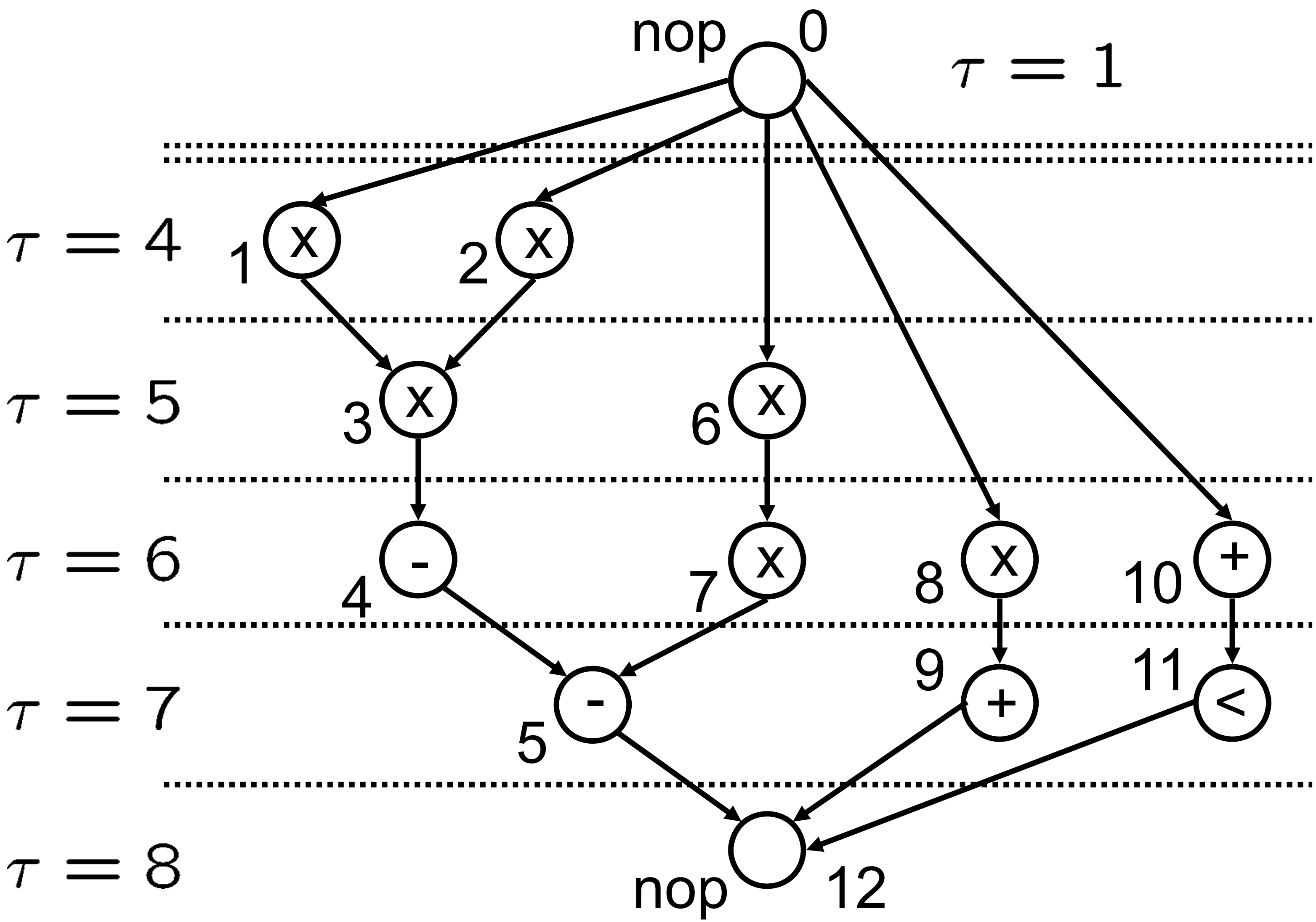
The ALAP Algorithm

- ***ALAP = As Late As Possible***

```
ALAP(G_S(V_S, E_S), w, L_max) {
    τ(v_n) = L_max + 1;
    REPEAT {
        Determine v_i whose succ. are planed;
        τ(v_i) = min{τ(v_j) ∀(v_i, v_j) ∈ E_S} − w(v_i)
    } UNTIL (v_0 is planned);
    RETURN (τ);
}
```

The ALAP Algorithm

- ***Example:***
  $L_{max} = 7$
  $w(v_i) = 1$



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Scheduling with Timing Constraints

- Different classes of timing constraints:
  - ***deadline*** (latest finishing times of operations), for example

$$\tau(v_2) + w(v_2) \leq 5$$

  - ***release times*** (earliest starting times of operations), for example

$$\tau(v_3) \geq 4$$

  - ***relative constraints*** (differences between starting times of a pair of operations), for example

$$\tau(v_6) - \tau(v_7) \geq 4$$
$$\tau(v_4) - \tau(v_1) \leq 2$$

Scheduling with Timing Constraints

- We will model all timing constraints using ***relative constraints***. Deadlines and release times are defined relative to the start node v_0.
- Minimum, maximum and equality constraints can be converted into each other:
  - ***Minimum constraint***:

$$\tau(v_j) \geq \tau(v_i) + l_{ij} \longrightarrow \tau(v_j) - \tau(v_i) \geq l_{ij}$$

  - ***Maximum constraint***:

$$\tau(v_j) \leq \tau(v_i) + l_{ij} \longrightarrow \tau(v_i) - \tau(v_j) \geq -l_{ij}$$

  - ***Equality constraint***:

$$\tau(v_j) = \tau(v_i) + l_{ij} \longrightarrow \begin{array}{l} \tau(v_j) - \tau(v_i) \leq l_{ij} \wedge \\ \tau(v_j) - \tau(v_i) \geq l_{ij} \end{array}$$

Weighted Constraint Graph

- Timing constraints can be represented in form of a weighted constraint graph:

A weighted constraint graph $G_C = (V_C, E_C, d)$ related to a sequence graph $G_S = (V_S, E_S)$ contains nodes $V_C = V_S$ and a weighted edge for each timing constraint. An edge $(v_i, v_j) \in E_C$ with weight $d(v_i, v_j)$ denotes the constraint $\tau(v_j) - \tau(v_i) \geq d(v_i, v_j)$.

Weighted Constraint Graph

- In order to represent a ***feasible schedule***, we have one edge corresponding to each precedence constraint with

$$d(v_i, v_j) = w(v_i)$$

where $w(v_i)$ denotes the execution time of v_i.

- A consistent assignment of starting times $\tau(v_i)$ to all operations can be done by solving a ***single source longest path*** problem.
- A possible algorithm (***Bellman-Ford***) has complexity $O(|V_C|\,|E_C|)$:

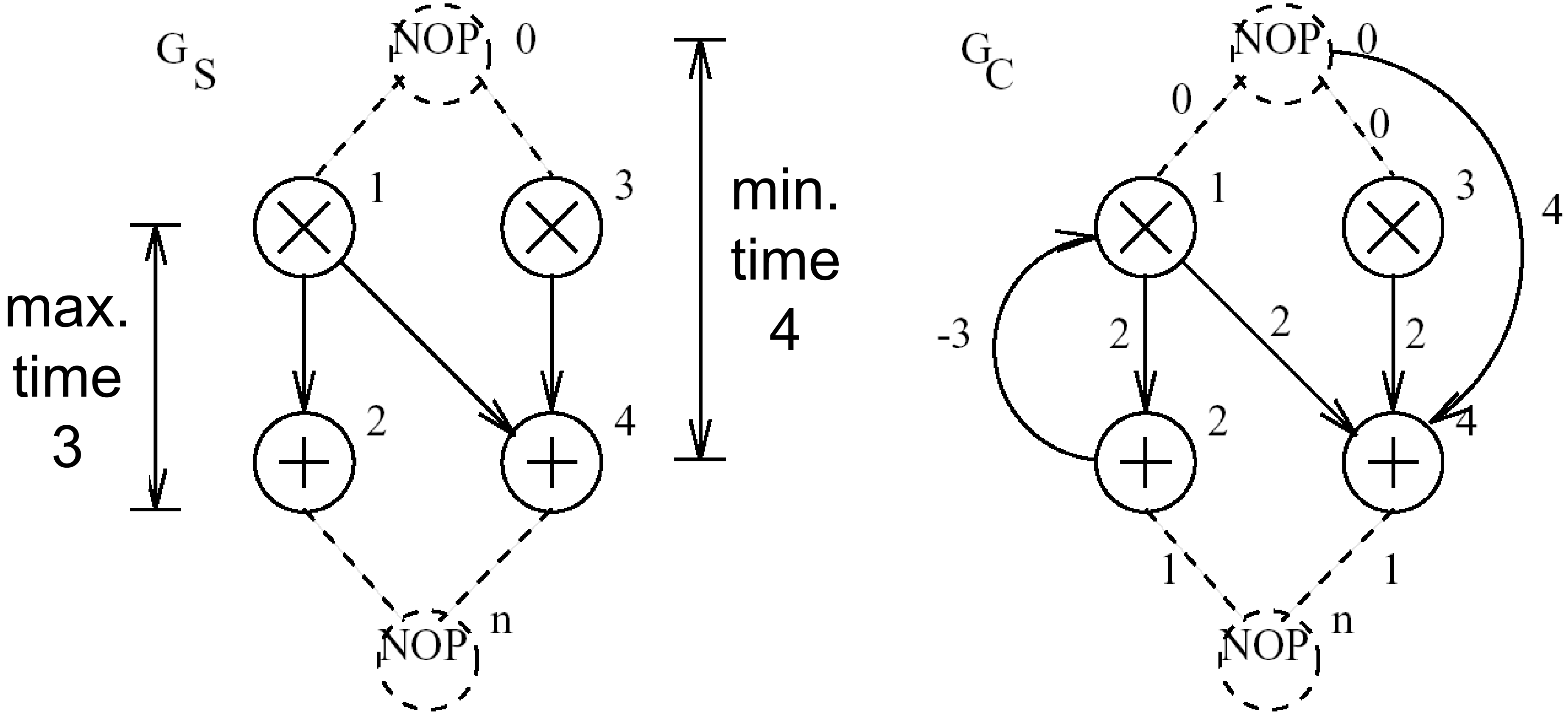
  Iteratively set $\tau(v_j) := \max\{\tau(v_j), \tau(v_i)+d(v_i, v_j) : (v_i, v_j) \in E_C\}$ for all $v_j \in V_C$ starting from $\tau(v_i) = -\infty$ for $v_i \in V_C \setminus \{v_0\}$ and $\tau(v_0) = 1$.

Weighted Constraint Graph

- ***Example***: $w(v_1) = w(v_3) = 2 \quad w(v_2) = w(v_4) = 1$

  $\tau(v_0) = \tau(v_1) = \tau(v_3) = 1,\ \tau(v_2) = 3,$

  $\tau(v_4) = 5,\ \tau(v_n) = 6,\ L = \tau(v_n) - \tau(v_0) = 5$



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Scheduling With Resource Constraints

Given is a sequence graph $G_S = (V_S, E_S)$, a resource graph $G_R = (V_R, E_R)$ and an associated allocation α and binding β.

Then the minimal latency is defined as

$$
\begin{aligned}
L = \min\{\tau(v_n) : \\
& (\tau(v_j) - \tau(v_i) \geq w(v_i, \beta(v_i)) \; \forall (v_i, v_j) \in E_S) \;\wedge \\
& (|\{v_s : \beta(v_s) = v_t \;\wedge\; \tau(v_s) \leq t < \tau(v_s) + w(v_s, v_t)\}| \leq \alpha(v_t) \\
& \forall v_t \in V_T, \forall 1 \leq t \leq L_{max})\}
\end{aligned}
$$

where L_{max} denotes an upper bound on the latency.

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List Scheduling

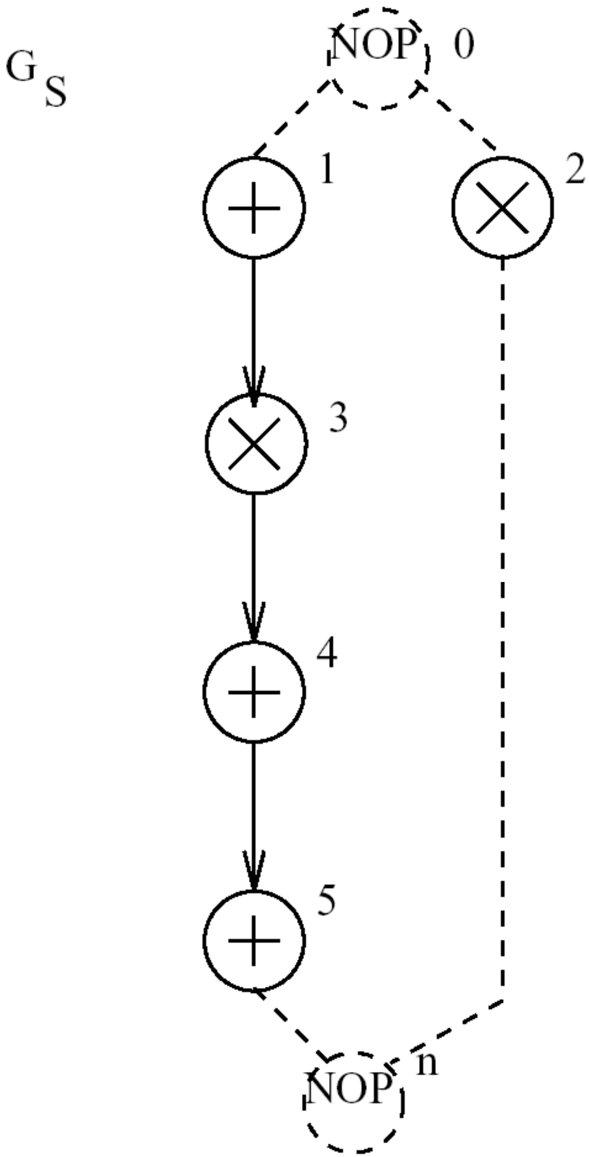
```
LIST(G_S(V_S, E_S), G_R(V_R, E_R), α, β, priorities){
  t = 1;
  REPEAT {
    FORALL v_k ∈ V_T {
      determine candidates to be scheduled U_k;
      determine running operations T_k;
      choose S_k ⊆ U_k with maximal priority
        and |S_k| + |T_k| ≤ α(v_k);
      τ(v_i) = t  ∀v_i ∈ S_k;   }
    t = t + 1;
  } UNTIL (v_n planned)
  RETURN (τ);   }
```

List Scheduling

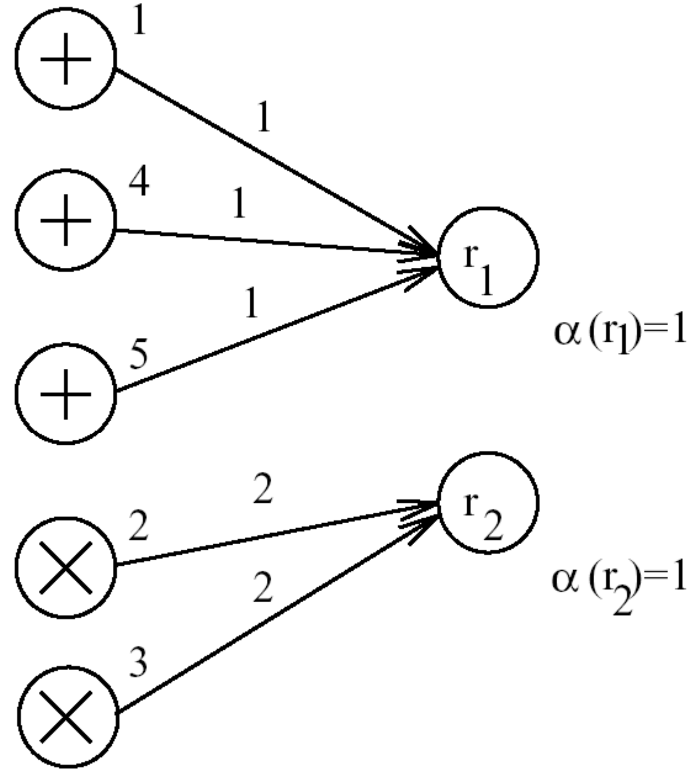
- One of the most ***widely used*** algorithms for scheduling under resource constraints.
- ***Principles***:
  - To each operation there is a ***priority*** assigned which denotes the urgency of being scheduled. This ***priority is static***, i.e. determined before the List Scheduling.
  - The algorithm schedules ***one time step after the other***.
  - U_k denotes the set of operations that (a) are mapped onto resource v_k and whose predecessors finished.
  - T_k denotes the currently running operations mapped to resource v_k .

List Scheduling

- *Example*:

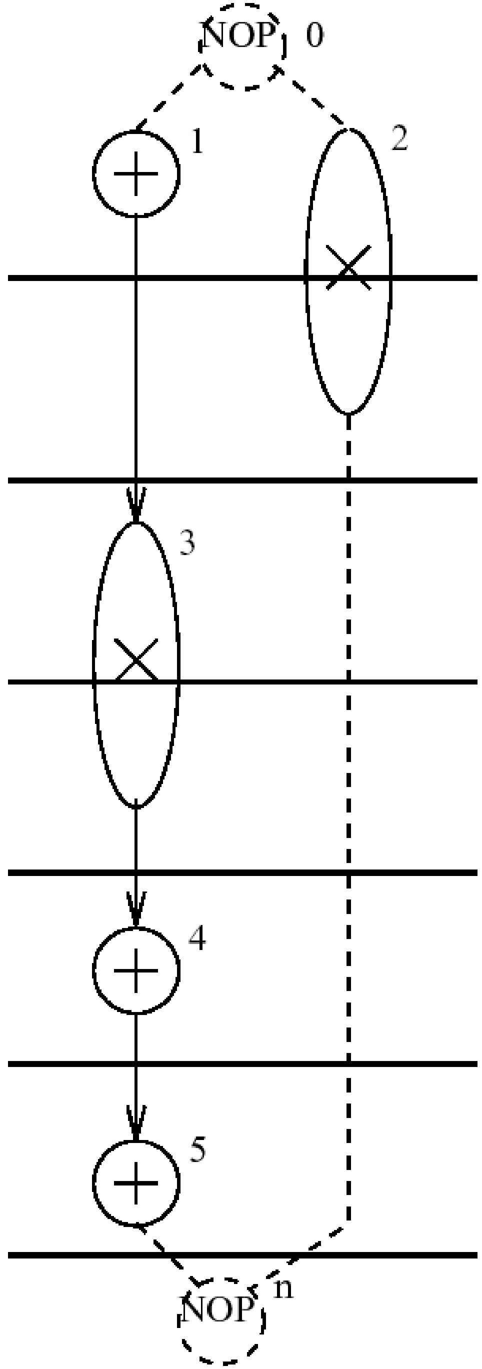


b) G_R



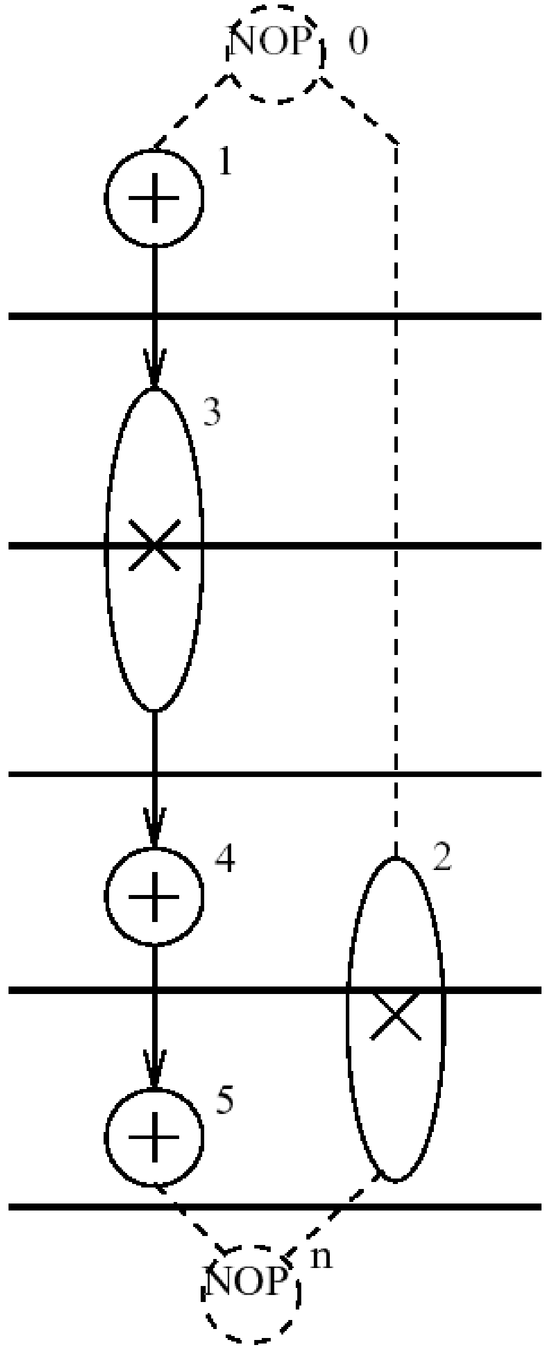
List Scheduling

- Solution via ***list scheduling***:
  - In the example, the solution is independent of priority.
  - Because of the ***greedy*** principle, all resources are directly occupied.
  - List scheduling is a ***heuristic algorithm***.
    In this example, it does not yield the minimal latency!



List Scheduling

- Solution via an ***optimal method***:
  - ***Latency is smaller*** than with list scheduling.
  - An example of an optimal algorithm is the transformation into an ***integer linear program***.



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Integer Linear Programming

- ***Principle***:

Synthesis Problem

↓ transformation into ILP

Integer Linear Program (ILP)

↓ optimization of ILP

Solution of ILP

↓ back interpretation

Solution of Synthesis Problem

Integer Linear Program

- Yields ***optimal solution*** to synthesis problems as it is based on an exact mathematical description of the problem.
- Solves ***scheduling, binding and allocation*** simultaneously.
- ***Standard optimization*** approaches (and software) are available to solve integer linear programs:
  - in addition to linear programs (linear constraints, linear objective function) some variables are forced to be integers.
  - much more complex than solving linear program
  - efficient methods are based on (a) branch and bound methods and (b) determining additional hyperplanes (cuts).

Integer Linear Program

- Many ***variants*** exist, depending on available information, constraints and objectives, e.g. minimize latency, minimize resources, minimize memory. Just an example is given here!!
- For the following example, we use the ***assumptions***:
  - The ***binding is determined*** already, i.e. every operation v_i has a unique execution time $w(v_i)$.
  - We have determined the ***earliest and latest starting times*** of operations v_i as l_i and h_i, respectively. To this end, we can use the ASAP and ALAP algorithms that have been introduced earlier. The maximal latency L_{max} is chosen such that a feasible solution to the problem exists.

Integer Linear Program

minimize: $\tau(v_n) - \tau(v_0)$

subject to $x_{i,t} \in \{0,1\} \quad \forall v_i \in V_S \;\; \forall t : l_i \leq t \leq h_i \qquad (1)$

$$\sum_{t=l_i}^{h_i} x_{i,t} = 1 \quad \forall v_i \in V_S \qquad (2)$$

$$\sum_{t=l_i}^{h_i} t \cdot x_{i,t} = \tau(v_i) \quad \forall v_i \in V_S \qquad (3)$$

$$\tau(v_j) - \tau(v_i) \geq w(v_i) \quad \forall (v_i, v_j) \in E_S \qquad (4)$$

$$\sum_{\forall i:(v_i,v_k)\in E_R} \; \sum_{p'=\max\{0,t-h_i\}}^{\min\{w(v_i)-1,t-l_i\}} x_{i,t-p'} \leq \alpha(v_k)$$

$$\forall v_k \in V_T \;\; \forall t : 1 \leq t \leq \max\{h_i : v_i \in V_S\} \qquad (5)$$

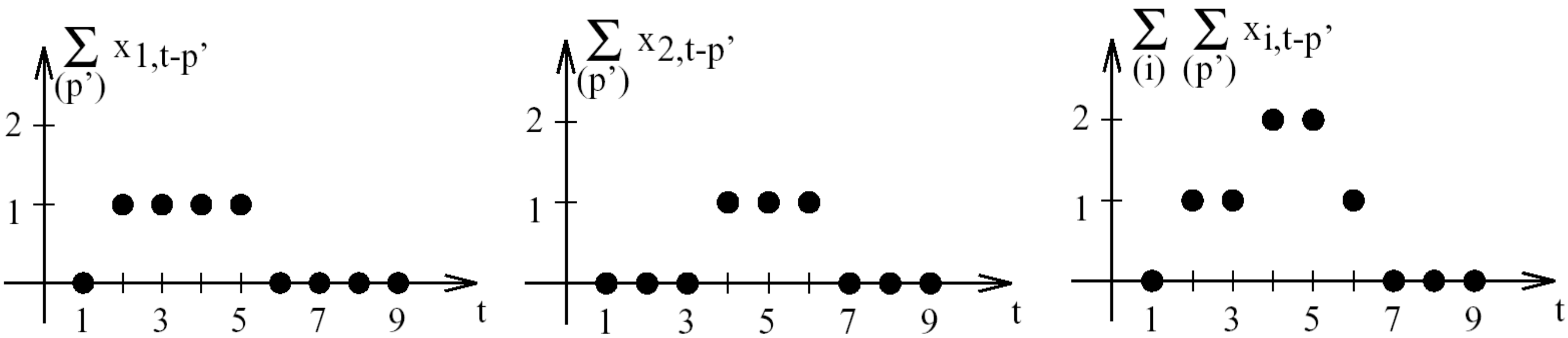
Integer Linear Program

- ***Explanations***:
  - (1) declares variables x to be binary .
  - (2) makes sure that exactly one variable $x_{i,t}$ for all t has the value 1, all others are 0.
  - (3) determines the relation between variables *x* and starting times of operations τ. In particular, if $x_{i,t} = 1$ then the operation v_i starts at time *t*, i.e. $\tau(v_i) = t$.
  - (4) guarantees, that all precedence constraints are satisfied.
  - (5) makes sure, that the resource constraints are not violated. For all resource types $v_k \in V_T$ and for all time instances t it is guaranteed that the number of active operations does not increase the number of available resource instances.

Integer Linear Program

- ***Explanations***:
  - (5) The first sum selects all operations that are mapped onto resource type v_k. The second sum considers all time instances where operation v_i is occupying resource type v_k :

$$\sum_{p'=0}^{w(v_i)-1} x_{i,t-p'} = \begin{cases} 1 & : \quad \forall t : \tau(v_i) \leq t \leq \tau(v_i) + w(v_i) - 1 \\ 0 & : \quad \text{sonst} \end{cases}$$



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Iterative Algorithms

- ***Iterative algorithms*** consist of a set of indexed equations that are evaluated for all values of an index variable *l*:

$$x_i[l] = \mathbf{F}_i[\ldots, x_j[l - d_{ji}], \ldots] \quad \forall l \;\; \forall i \in I$$

  Here, x_i denote a set of indexed variables, $\boldsymbol{F}_i$ denote arbitrary functions and d_{ji} are constant index displacements.
- Examples of well known representations are ***signal flow graphs*** (as used in signal and image processing and automatic control), ***marked graphs*** and special forms of ***loops***.

Iterative Algorithms

- ***Several representations*** of the same iterative algorithm:
  - One ***indexed equation*** with constant index dependencies:

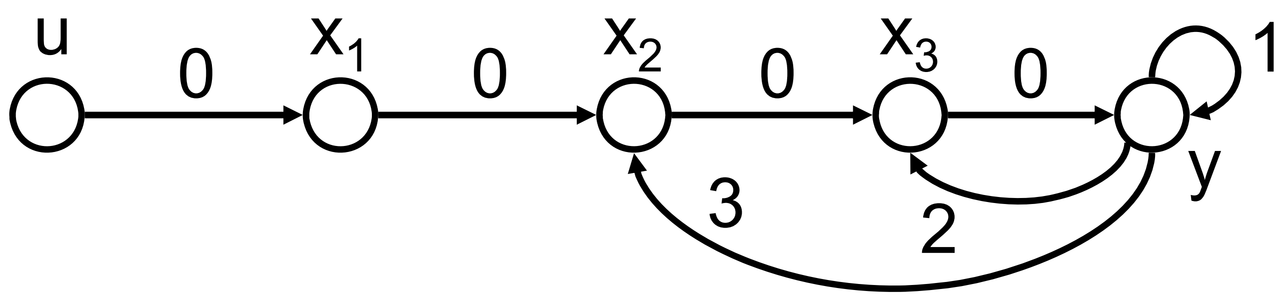
$$y[l] = au[l] + by[l-1] + cy[l-2] + dy[l-3] \qquad \forall l$$

  - Equivalent set of indexed equations:

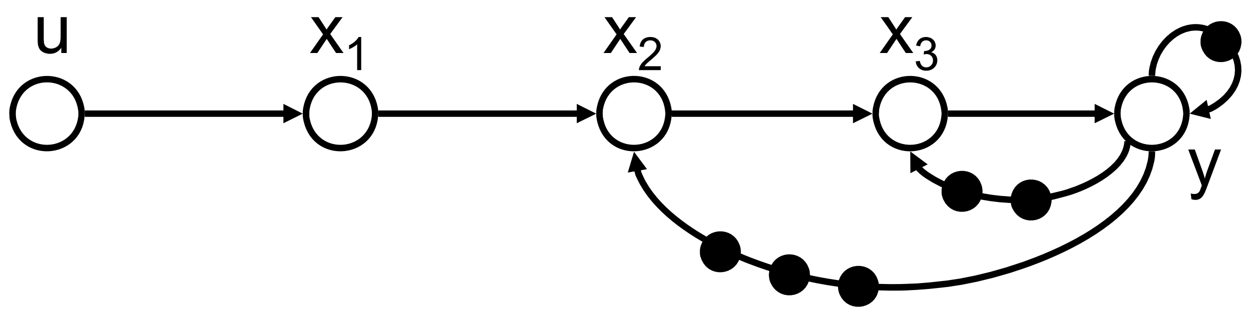
$$\begin{aligned} x_1[l] &= au[l] & \forall l \\ x_2[l] &= x_1[l] + dy[l-3] & \forall l \\ x_3[l] &= x_2[l] + cy[l-2] & \forall l \\ y[l] &= x_3[l] + by[l-1] & \forall l \end{aligned}$$

Iterative Algorithms

- ***Extended sequence graph*** $G_S = (V_S, E_S, d)$: To each edge $(v_i, v_j) \in E_S$ there is associated the ***index displacement*** d_{ij}. An edge $(v_i, v_j) \in E_S$ denotes that the variable corresponding to v_j depends on variable corresponding to v_i with displacement d_{ij}.

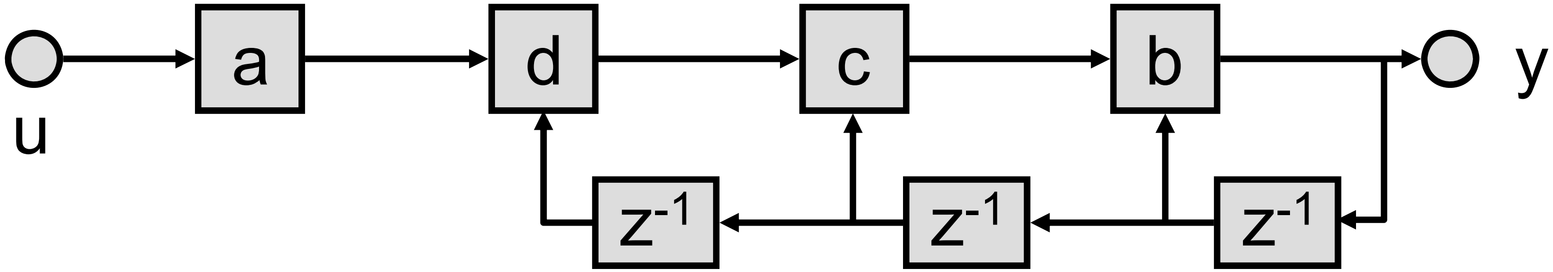


- Equivalent ***marked graph***:



Iterative Algorithms

- Equivalent ***signal flow graph***:



- Equivalent ***loop program***:

```
while(true) {
        t1 = read(u);
        t5 = a*t1 + d*t2 + c*t3 + b*t4;
        t2 = t3;
        t3 = t4;
        t4 = t5;
        write(y, t5);}
```

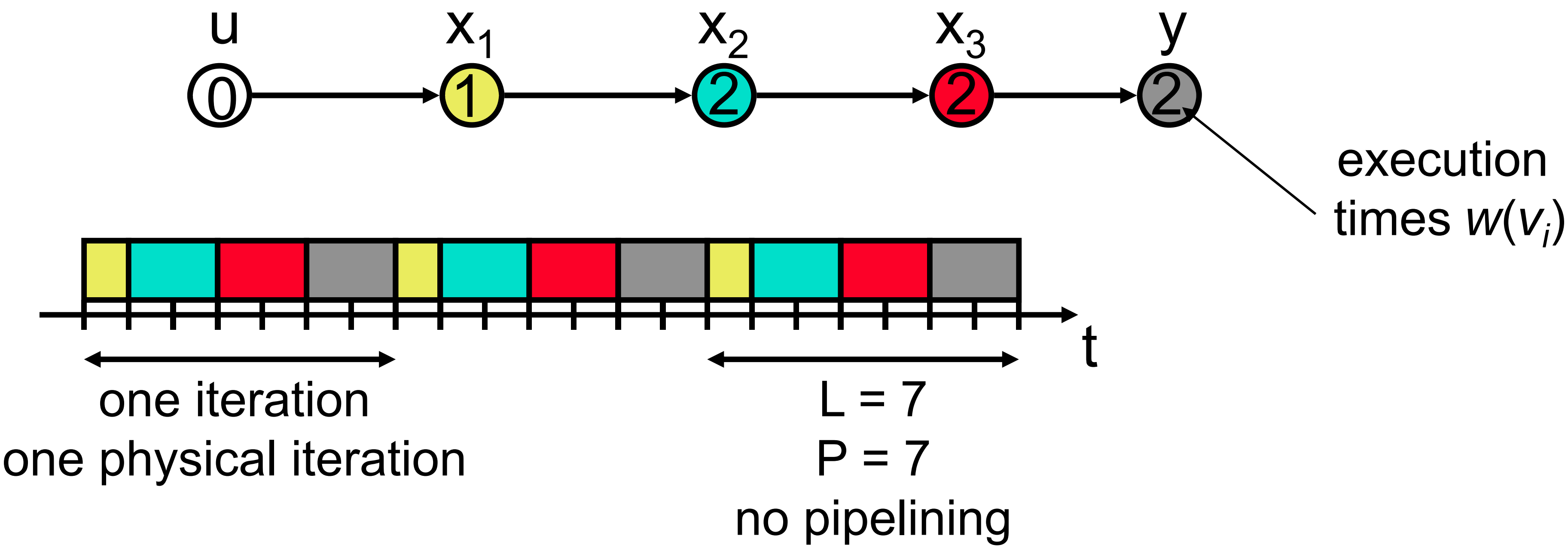
Iterative Algorithms

- An ***iteration*** is the set of all operations necessary to compute all variables $x_i[l]$ for a fixed index l.
- The ***iteration interval*** P is the time distance between two successive iterations of an iterative algorithm. $1/P$ denotes the ***throughput*** of the implementation.
- The ***latency*** L is the maximal time distance between the starting and the finishing times of operations belonging to one iteration.
- In a pipelined implementation (***functional pipelining***), there exist time instances where the operations of different iterations l are executed simultaneously.
- In case of ***loop folding***, starting and finishing times of an operation are in different physical iterations.

Iterative Algorithms

- ***Implementation principles***
  - A ***simple possibility***, the edges with $d_{ij} > 0$ are removed from the extended sequence graph. The resulting simple sequence graph is implemented using ***standard methods***.

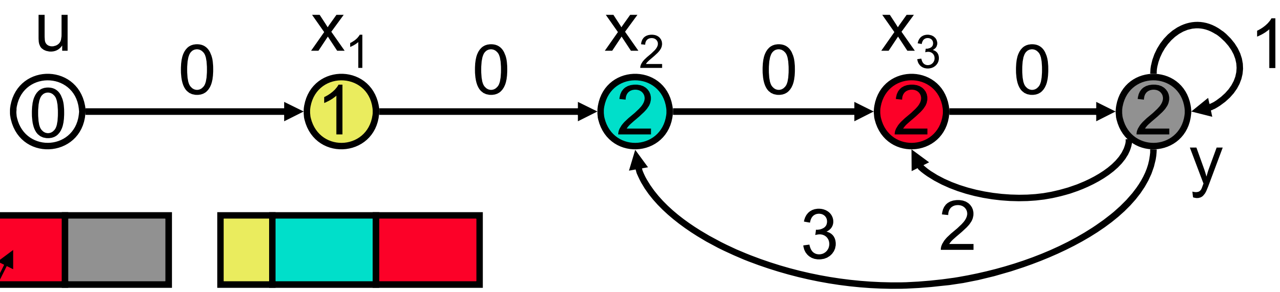
Example with unlimited resources:

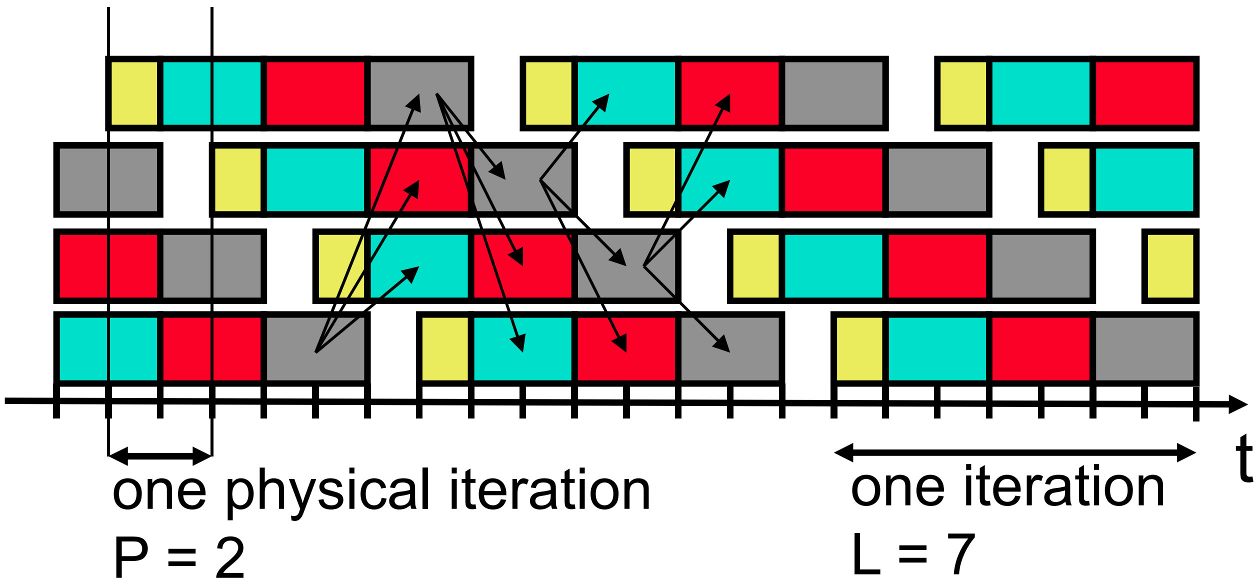


Iterative Algorithms

- ***Implementation principles***
  - Using ***functional pipelining***: Successive iterations overlap and a higher throughput (1/P) is obtained.

    Example with unlimited resources (note data dependencies across iterations!)





- 4 resources
- functional pipelining
- loop folding

Iterative Algorithms

- Solving the synthesis problem using ***integer linear programming***:
  - Starting point is the ILP formulation given for simple sequence graphs.
  - Now, we use the ***extended sequence graph*** (including displacements d_{ij}.
  - ***ASAP*** and ***ALAP*** scheduling for upper and lower bounds h_i and l_i use only edges with $d_{ij} = 0$ (remove dependencies across iterations).
  - We suppose, that a suitable ***iteration interval*** P is chosen beforehand. If it is too small, no feasible solution to the ILP exists and P needs to be increased.

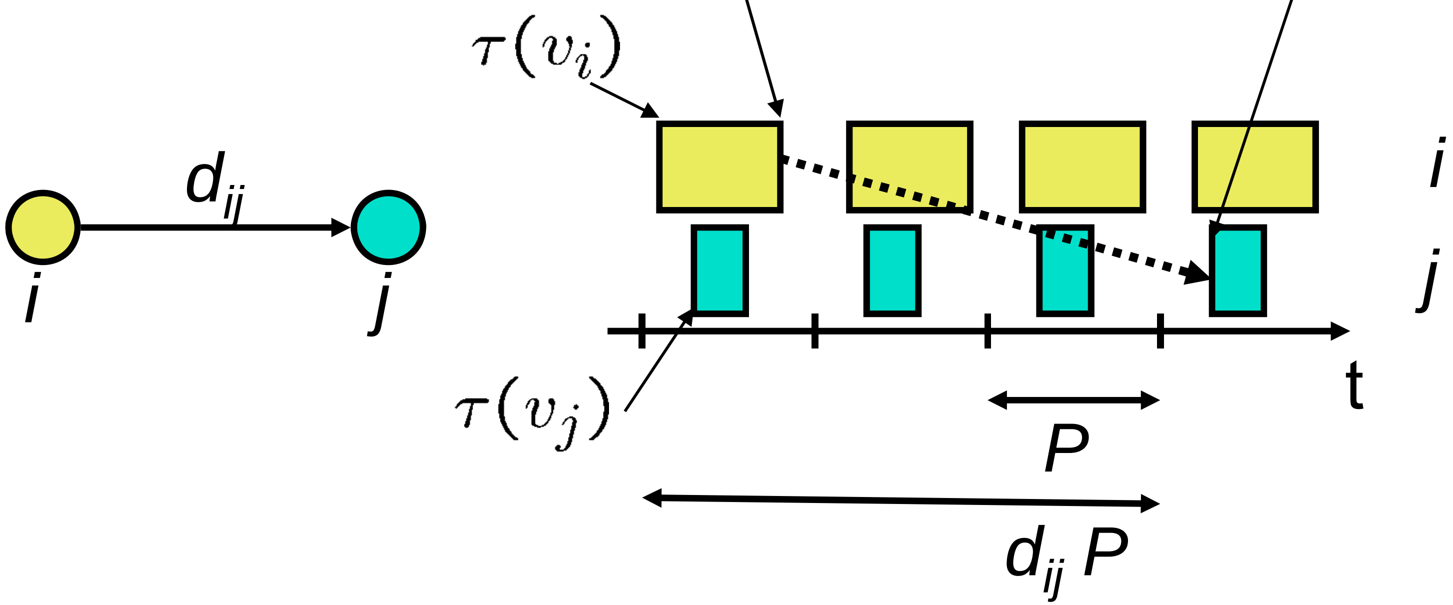
Iterative Algorithms

- Eqn.(4) is replaced by:

$$\tau(v_j) - \tau(v_i) \geq w(v_i) - d_{ij} \cdot P \qquad \forall (v_i, v_j) \in E_S$$

Proof of correctness:

$$\tau(v_i) + w(v_i) \leq \tau(v_j) + d_{ij} \cdot P$$



Iterative Algorithms

- Eqn. (5) is replaced by

$$\sum_{\forall i:(v_i,v_k)\in E_R} \sum_{p'=0}^{w(v_i)-1} \sum_{\forall p: l_i \le t-p'+p\cdot P \le h_i} x_{i,t-p'+p\cdot P} \le \alpha(v_k)$$

$$\forall 1 \le t \le P,\ \forall v_k \in V_T$$

Sketch of ***Proof***: An operation v_i starting at $\tau(v_i)$ uses the corresponding resource at time steps *t* with

$$t = \tau(v_i) + p' - p \cdot P$$

$$\forall p', p : 0 \le p' < w(v_i) \wedge l_i \le t - p' + p \cdot P \le h_i$$

Therefore, we obtain

$$\sum_{p'=0}^{w(v_i)-1} \sum_{\forall p: l_i \le t-p'+p\cdot P \le h_i} x_{i,t-p'+p\cdot P}$$

Contents

- Models
- Scheduling without resource constraints
  - ASAP
  - ALAP
  - Timing Constraints
- Scheduling with resource constraints
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  - Integer Linear Programming
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- ***Dynamic Voltage Scaling***

Dynamic Voltage Scaling

- If we transform the DVS problem into an integer linear program optimization: we can ***optimize the energy*** in case of ***dynamic voltage scaling***.
- As an ***example***, let us model a set of tasks with dependency constraints.
  - We suppose that a task $v_i \in V_S$ can use one of the execution times $w_k(v_i)$ $\forall$ $k \in K$ and corresponding energy $e_k(v_i)$. There are $|K|$ different voltage levels.
  - We suppose that there are ***deadlines*** $d(v_i)$ for each operation v_i.
  - We suppose that there are no resource constraints, i.e. all tasks can be executed in parallel.

Dynamic Voltage Scaling

minimize: $\sum_{k \in K} \sum_{v_i \in V_S} y_{ik} \cdot e_k(v_i)$

subject to:

$$y_{ik} \in \{0, 1\} \quad \forall v_i \in V_S, k \in K \qquad (1)$$

$$\sum_{k \in K} y_{ik} = 1 \quad \forall v_i \in V_S \qquad (2)$$

$$\tau(v_j) - \tau(v_i) \geq \sum_{k \in K} y_{ik} \cdot w_k(v_i) \quad \forall (v_i, v_j) \in E_S \qquad (3)$$

$$\tau(v_i) + \sum_{k \in K} y_{ik} \cdot w_k(v_i) \leq d(v_i) \quad \forall v_i \in V_S \qquad (4)$$

Dynamic Voltage Scaling

- ***Explanations***:
  - The objective functions just sums up all individual energies of operations.
  - Eqn. (1) makes decision variables y_{ik} binary.
  - Eqn. (2) guarantees that exactly one implementation (voltage) $k \in K$ is chosen for each operation v_i .
  - Eqn. (3) implements the precedence constraints, where the actual execution time is selected from the set of all available ones.
  - Eqn. (4) guarantees deadlines.