

Name: SOLUTIONS (Bonus: 1 point)

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## 18-648: Embedded Real-Time Systems

### Quiz #1 Solutions

Fall 2016

60 minutes

#### Instructions

1. Show all relevant work.
2. Partial credit may be given for some questions.
3. The use of a calculator is allowed.
4. The time limit will be *strictly* enforced.
5. Be efficient. Come back to questions you are not sure of later.
6. Watch the screen for any clarifications.
7. This is a **CLOSED-BOOK/CLOSED-NOTES** quiz.

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#### For Graders' Use Only

Bonus: \_\_\_\_\_ / 2

1. \_\_\_\_\_ / 22

2. \_\_\_\_\_ / 24

3. \_\_\_\_\_ / 22

4. \_\_\_\_\_ / 20

5. \_\_\_\_\_ / 12

**TOTAL.** \_\_\_\_\_ / 100

## Question 1. True/False and Multiple Choices.

### (a) True/False Questions (18 points)

- False** Using the Priority Ceiling Protocol (PCP) can lead to mutual deadlocks.
- True** It takes a taskset with one shared resource and at least three or more tasks to exhibit unbounded priority inversion.
- False** Consider a fixed-priority scheduler which assigns random (fixed) priority values to periodic real-time tasks. There might be priority assignments on some task set for which this scheduler would find a feasible schedule but the rate-monotonic scheduler will not.
- False** Earliest deadline first (EDF) is a static priority preemptive scheduling scheme.
- True** There are periodic task sets that can be scheduled by EDF but not RMS.
- False** There are periodic task sets that can be scheduled by RMS but not EDF.
- False** With a priority-based preemptive scheduler, a ready task scheduled to run will run until its completion, after which the highest-priority ready task on the ready queue will be run.
- False** Each thread in a process has its own data segment.
- True** A taskset with three periodic tasks that have periods 8, 16 and 48 time-units respectively is considered harmonic.
- True** Context-switching which consumes a non-zero amount of time can be accounted for in schedulability analysis.

**(b) Circle one or more answers for the questions below. (4 points)**

I. Which best describes the EDF scheduler used by real-time tasks:

- a. Static preemptive
- b. Static non-preemptive
- ☒ c. **Dynamic preemptive**
- d. Dynamic non-preemptive

II. Which of the following protocols do not provide deadlock avoidance (assuming tasks do not suspend within critical regions):

- ☒ a. **Basic Priority Inheritance Protocol**
- b. Highest Locker Priority Protocol
- c. Priority Ceiling Protocol
- d. Non-Preemption Protocol

**Question 2. Will it schedule? (24 points)**

For the following task sets, determine if the set listed is schedulable using Rate-Monotonic Scheduling. Each task  $\tau_i$  is characterized by  $\{C_i, T_i\}$ , its worst-case execution time and period.. The relative deadline of each task is the same as its period. If a task set is not schedulable, please indicate which task misses its deadline. The RMS bound is  $n(2^{1/n} - 1)$ . For  $n = 2$ ,  $U_b = 0.828$ . For  $n = 3$ ,  $U_b = 0.78$ . Use the response-time test if appropriate. The response-time test for task  $i$  is given by:

$$a_{k+1}^i = C_i + \sum_{j=1}^{i-1} \left\lceil \frac{a_k^i}{T_j} \right\rceil C_j \quad \text{where } a_0^i = \sum_{j=1}^i C_j$$

Test terminates when  $a_{k+1}^i = a_k^i$

a) *Hint:* Remember to use RMS.

$$\tau_1 = \{64, 160\}$$

$$\tau_2 = \{10, 80\}$$

$$\tau_3 = \{8, 32\}$$

Since RMS being used, let's re-arrange the tasks in RMS priority order.

$$\tau'_1 = \{8, 32\} \quad \rightarrow U'_1 = \frac{8}{32} = 0.25$$

$$\tau'_2 = \{10, 80\} \quad \rightarrow U'_2 = 10/80 = 0.125$$

$$\tau'_3 = \{64, 160\} \quad \rightarrow U'_3 = 64 / 160 = 0.40$$

$$U_{\text{total}} = 0.25 + 0.125 + 0.40 = 0.775$$

Since  $U_{\text{bound}}(3) = 0.78$ , we can immediately declare that **all 3 tasks are schedulable** under RMS.

b) *Hint*: Remember to use RMS.

$$\tau_1 = \{3, 12\}$$

$$\tau_2 = \{6, 20\}$$

$$\tau_3 = \{6, 15\}$$

Since RMS being used, let's re-arrange the tasks in RMS priority order.

$$\tau'_1 = \{3, 12\} \rightarrow U'_1 = \frac{3}{12} = 0.25$$

$$\tau'_2 = \{6, 15\} \rightarrow U'_2 = 6/15 = 0.4$$

$$\tau'_3 = \{6, 20\} \rightarrow U'_3 = 6 / 20 = 0.30$$

$$U_{\text{total}} = 0.25 + 0.4 + 0.30 = 0.95$$

Since  $U_{\text{bound}}(3) = 0.78 < U_{\text{total}}$ , the schedulability bound cannot be used to determine the schedulability of the entire taskset. We therefore have to analyze the tasks in descending priority order.

Task  $\tau'_1$ : the task is schedulable since  $U'_1 = 0.25 \leq U_{\text{bound}}(1) = 1.0$ .

Task  $\tau'_2$ : we need to check if  $U'_1 + U'_2 = 0.25 + 0.4 \leq U_{\text{bound}}(2) = 0.828$ .

Yes, the task is schedulable.

Task  $\tau'_3$ : when we check if  $U'_1 + U'_2 + U'_3 = 0.25 + 0.4 + 0.3 \leq U_{\text{bound}}(3) = 0.78$ , we find that the test is not passed. So, we perform the exact completion time for  $\tau'_3$ .

$$a_{k+1}^i = C_i + \sum_{j=1}^{i-1} \left\lceil \frac{a_k^i}{T_j} \right\rceil C_j \quad \text{where } a_0^i = \sum_{j=1}^i C_j \quad \boxed{i = 3}$$

Test terminates when  $a_{k+1}^i = a_k^i$

$$a^3_0 = C'_1 + C'_2 + C'_3 = 3 + 6 + 6 = 15$$

$$a^3_1 = C'_3 + \text{ceiling}(15/12)*3 + \text{ceiling}(15/15)*6 = 6 + 2*3 + 1*6 = 18$$

$$a^3_2 = C'_3 + \text{ceiling}(18/12)*3 + \text{ceiling}(18/15)*6 = 6 + 2*3 + 2*6 = 24 > T'_3$$

Since  $a^3_2 = 24$ , which is already greater than 20 (the deadline of  $\tau'_3$ ), we can stop. Task  $\tau'_3$  is **not** schedulable.

### Question 3. Sharing is caring (22 points)

Suppose each periodic real-time task is represented by  $(C, T=D)$ .  
Consider the 4-task set  $\tau_1: \{4, 12\}$ ,  $\tau_2: \{3, 16\}$ ,  $\tau_3: \{3, 20\}$ ,  $\tau_4: \{3, 24\}$ .

The tasks arrive in following manner:

1.  $\tau_4$  at time 0
2.  $\tau_3$  at time 2
3.  $\tau_2$  at time 4
4.  $\tau_1$  at time 6

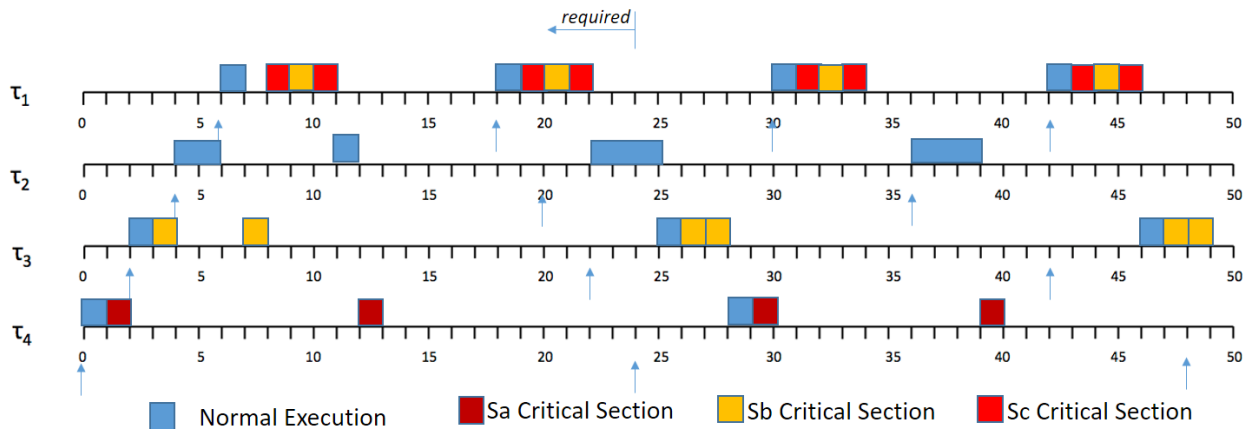
And they execute in the following manner:

1.  $\tau_4$  executes normally for 1 time unit, locks mutex  $S_a$ , executes for 2 time units and finally unlocks mutex  $S_a$ . (...P( $S_a$ )...V( $S_a$ ))
  2.  $\tau_3$  executes normally for 1 sec, locks mutex  $S_b$ , executes for 2 time units and finally unlocks mutex  $S_b$ . (...P( $S_b$ )...V( $S_b$ ))
  3.  $\tau_2$  execute normally for 3 time units. (...)
  4.  $\tau_1$  executes normally for 1 time unit, locks mutex  $S_c$ , executes for 1 more time unit, locks mutex  $S_b$  executes for 1 time unit, unlocks mutex  $S_b$ , executes for 1 time unit and finally unlocks mutex  $S_c$ . (...P( $S_c$ )...P( $S_b$ )...V( $S_b$ )...V( $S_c$ ))
- a) Draw the timeline under RMS using Priority Ceiling Protocol. Please mark all the critical sections.

Priority ceiling( $S_a$ ) = priority( $\tau_4$ )

Priority ceiling( $S_b$ ) = priority( $\tau_1$ )

Priority ceiling( $S_c$ ) = priority( $\tau_1$ )



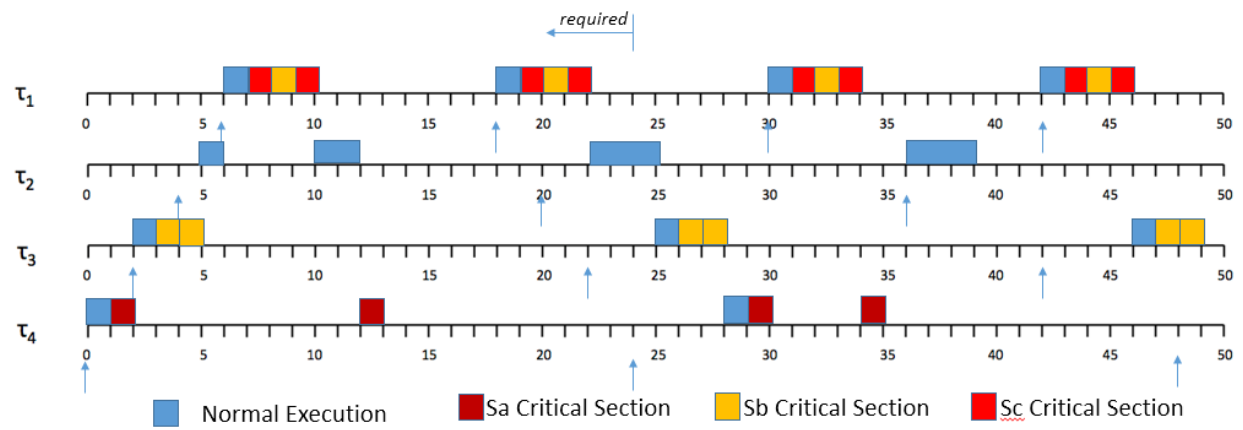
Note: Task  $\tau_1$  is blocked by  $\tau_3$  when trying to lock  $S_c$ , Task  $\tau_3$  which had locked  $S_b$  executes at  $\tau_1$ 's priority at time 7 until  $S_b$  is released at time 8.

- b) Draw the timeline under RMS using Highest Locker Priority. Please mark all the critical sections.

Priority ceiling( $S_a$ ) = priority( $\tau_4$ ) → also the highest locker priority

Priority ceiling( $S_b$ ) = priority( $\tau_1$ ) → also the highest locker priority

Priority ceiling( $S_c$ ) = priority( $\tau_1$ ) → also the highest locker priority



*Note:* Task  $\tau_3$  executes at the priority of  $\tau_1$  as soon as it locks Sb and therefore cannot be preempted by  $\tau_1$  or  $\tau_2$  until Sb is released at time 5.

#### Question 4. An overloaded question (20 points)

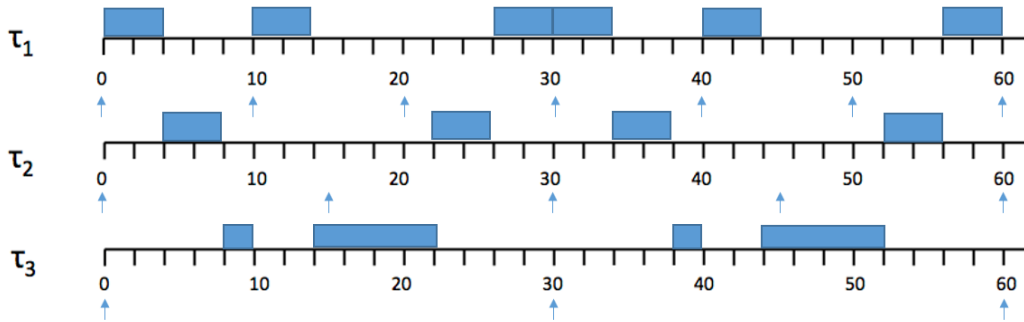
There are 3 real-time tasks in the system. Each task  $\tau_i$  is characterized by  $\{C_i, T_i\}$ , its worst-case execution time and period parameters. The relative deadline of each task is the same as its period.

$$\tau_1 = \{4, 10\} \rightarrow U_1 = 0.4$$

$$\tau_2 = \{4, 15\} \rightarrow U_2 = 0.2667$$

$$\tau_3 = \{10, 30\} \rightarrow U_3 = 0.3333 \rightarrow U_{\text{total}} = 0.4 + 0.2667 + 0.3333 = 1.0$$

a) Determine if this given task set is schedulable or not using the Earliest Deadline First scheduling algorithm. Fill in the timeline to verify your answer. (Draw the time period from 0-60)



b) Determine if this given task set is schedulable or not using the Rate-Monotonic scheduling algorithm.

*Note:* The taskset is *not* harmonic since  $T_1 = 10$  and  $T_2 = 15$  are not integral multiples (or sub-multiples) of one another.

Since  $U_{\text{total}} > U_{\text{bound}}(3) = 0.78$ , we cannot use the schedulability bound test or the harmonic utilization bound to determine the schedulability of the entire taskset.

Since  $U_1 + U_2 = 0.6667 < U_{\text{bound}}(2) = 0.828$ , tasks  $\tau_1$  and  $\tau_2$  are schedulable.

For task  $\tau_3$ , the exact completion time test must be run. We therefore have,

$$a^3_0 = C_1 + C_2 + C_3 = 4 + 4 + 10 = 18$$

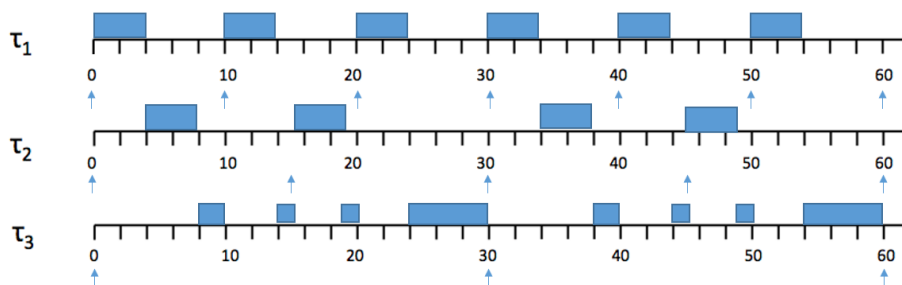
$$a^3_1 = C_3 + \text{ceiling}(18/10) \cdot 4 + \text{ceiling}(18/15) \cdot 4 = 10 + 2 \cdot 4 + 2 \cdot 4 = 26$$

$$a^3_2 = C_3 + \text{ceiling}(26/10) \cdot 4 + \text{ceiling}(26/15) \cdot 4 = 10 + 3 \cdot 4 + 2 \cdot 4 = 30$$

$$a^3_3 = C_3 + \text{ceiling}(30/10) \cdot 4 + \text{ceiling}(30/15) \cdot 4 = 10 + 3 \cdot 4 + 2 \cdot 4 = 30$$

Since  $a^3_2 = a^3_3$ , we can stop. The completion time of  $\tau_3$  is  $30 \leq$  its deadline of 30. Hence, task  $\tau_3$  and the entire taskset is schedulable!

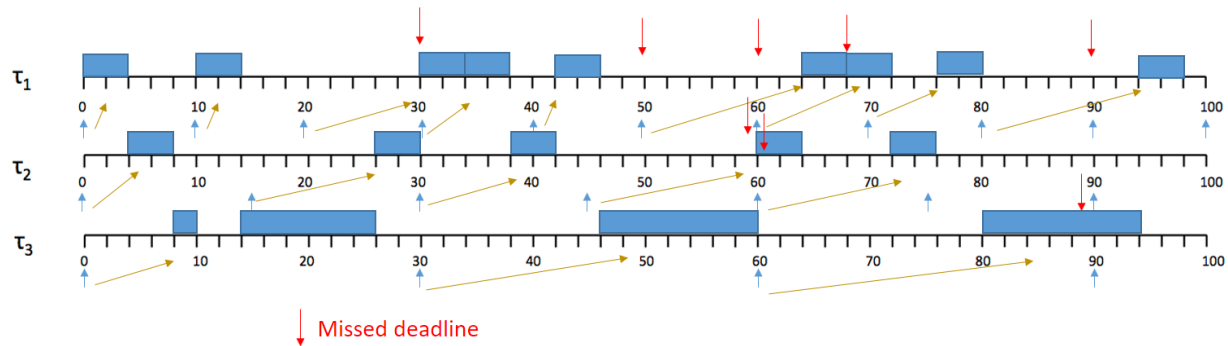
*Comment:* The timeline looks as follows.





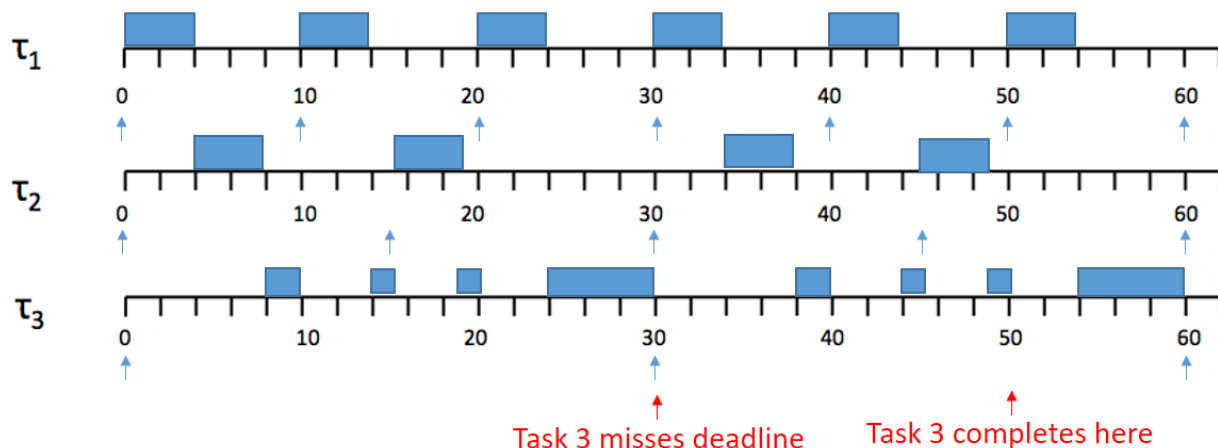
Now assume  $\tau_3$ 's worst-case execution time increases from 10 to 14.

c) Determine if the new task set is schedulable using the Earliest Deadline First scheduling algorithm. If the task set cannot be scheduled, indicate which tasks will miss their deadlines. Draw the timeline for this given task set to verify your answer. (Draw the time period from 0-90). If there is a tie, break when applicable in favor of the task that arrived earlier (else break the tie arbitrarily). If a task misses a deadline, it continues to be eligible to execute at the priority of its absolute deadline.



**Note:** All tasks miss their deadlines and the overloaded situation becomes worse over time. In other words, dynamic priority scheduling algorithms are 'unstable' under overloaded conditions.

d) Determine if the new task set from (c) is schedulable using the Rate-Monotonic scheduling algorithm. If the task set cannot be scheduled, figure out which task will miss the deadline. Draw the timeline for this given task set to verify your answer. (Draw the time period from 0-90)



**Note:** The lowest priority task  $\tau_3$  will miss its deadlines but tasks  $\tau_1$  and  $\tau_2$  will meet all their future deadlines. This is referred to as the stability of RMS (fixed-priority scheduling) under overloaded conditions.

**Question 5. Time to block. (12 points)**

A real-time task set has 3 tasks  $\tau_1$ ,  $\tau_2$  and  $\tau_3$  in decreasing order of (fixed) priority. They share some logical resources like critical sections and global variables that are protected using mutexes.

- Task  $\tau_1$  accesses a mutex  $M_1$  for 5 ms.
- Task  $\tau_2$  accesses two mutexes  $M_2$  and  $M_3$  for 7 ms and 5 ms respectively.
- Task  $\tau_3$  accesses three mutexes  $M_1$ ,  $M_2$  and  $M_4$  for 8 ms, 12 ms and 12 ms respectively.

Note: There are no nested mutex accesses of any kind: one mutex is locked and then released, *before* another mutex is locked.

Please fill in the table below.

Blocking Term	Under Basic PIP	Under PCP	Under HLP	Under NPP
$B_1$	8	8	8	12
$B_2$	12	12	12	12
$B_3$	0	0	0	0

**Note:** The lowest-priority task experiences 0 blocking time since there is no lower-priority task to cause priority inversion under all these protocols (unless the OS refuses to schedule it even when the processor is idle – which would be a ridiculous OS and scheduling policy).

First, compute the priority ceilings of the mutexes.

Priority ceiling of  $M_1$  = priority of  $\tau_1$ .

Priority ceiling of  $M_2$  = priority of  $\tau_2$

Priority ceiling of  $M_3$  = priority of  $\tau_2$

Priority ceiling of  $M_4$  = priority of  $\tau_3$

**Under Basic PIP:** A task  $\tau$  can encounter priority inversion from one or more lower-priority tasks that use mutexes with priority ceilings greater than or equal to the priority of task  $\tau$ . But each mutex can cause at most one critical section of priority inversion, and one task can also cause at most one priority inversion. Therefore,

- task  $\tau_1$  can only be blocked by the critical section of task  $\tau_3$  locking  $M_1$ . Hence,  $B_1 = 8$ .
- task  $\tau_2$  can be blocked by the critical sections of task  $\tau_3$  locking  $M_1$  and  $M_2$ . However, both of them cannot cause priority inversion – only one can (because when task  $\tau_2$  arrives, task  $\tau_3$  could have locked either  $M_1$  or  $M_2$  but not both). So, we pick  $\max(8, 12)$ .

**Under PCP:** A task  $\tau$  can encounter priority inversion from no more than one lower-priority task that uses mutexes with priority ceilings greater than or equal to the priority of task  $\tau$ . Therefore,

- task  $\tau_1$  can only be blocked by the critical section of task  $\tau_3$  locking  $M_1$ . Hence,  $B_1 = 8$ .
- task  $\tau_2$  can be blocked by one of the critical sections of task  $\tau_3$  locking  $M_1$  or  $M_2$ . So, we pick  $\max(8, 12)$ .

**Under HLP:** Same as PCP when tasks do not suspend voluntarily within critical sections. That is, a task  $\tau$  can encounter priority inversion from no more than one lower-priority task that uses mutexes with priority ceilings greater than or equal to the priority of task  $\tau$ . The blocking terms are the same as PCP. (This is why HLP is a very good approximation of PCP. Their property differences come in only when tasks can suspend voluntarily inside a critical section – that's bad practice anyway).

**Under NPP:** Under the non-preemption protocol, a task starts executing at the highest system priority as soon as it locks a mutex. In other words, it acts like the HLP except that all mutexes are assigned a priority ceiling = the highest priority in the system. Hence, a task  $\tau$  can encounter priority inversion from no more than one lower-priority task, but any task that locks a mutex can cause priority inversion. Hence,

$$B_1 = \max(7, 5, 8, 12, 12) = 12$$

$$B_2 = \max(8, 12, 12) = 12$$