Lagrange multipliers are used to solve constrained optimization problems. If we have a function $f(x, y)$ and we want to optimize (maximize or minimize) to find the maximum/minimum value. Here we are not allow to consider all (x, y) while looking for this value. Instead, the (x, y) you can consider are constrained to lie on some curve or surface

Typical, Lagrange multipliers problem can be stated as follows:

Minimize (or maximize) $w = f(x, y, z)$ constrained by $g(x, y, z) = c$.

Lagrange multipliers solution: Local minima (or maxima) must occur at a critical point. This is a point where $\bigtriangledown f = \lambda \bigtriangledown g$, and $g(x, y, z) = c$.

Example Find the point on (called P) on the line 2x + y = 1 such that the rectangle formed by P and the origin O has maximum area

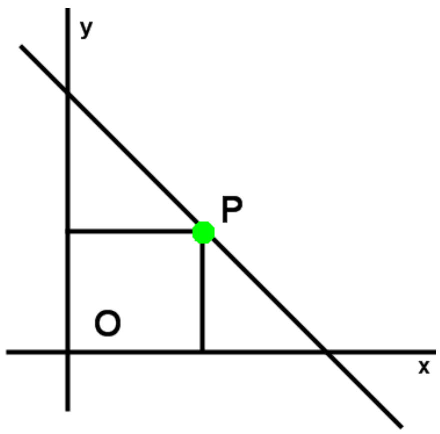

Figure 1:

Answer

We need to solve: g(x, y) = x + 2y = 1 and the constraint that maximize f(x, y) = xy (the area)

The gradients are: $\bigtriangledown g =< 1, 2 >$ and $\bigtriangledown f =< y, x >$

Lagrange multipliers: λ

We have 3 equation for 3 variables as follows:

$y = \lambda$

$x = 2\lambda$

$x + 2y = 1$

The solution is: $4\lambda = 1$

Thus, $y = \frac{1}{4}$ and $x = \frac{1}{2}$

We know this is a maximum because the maximum occurs either at a critical point or on the boundary. In this case, the boundary points are on the axes at (1,0) and (0,1/2), which gives a rectangle with area = 0.