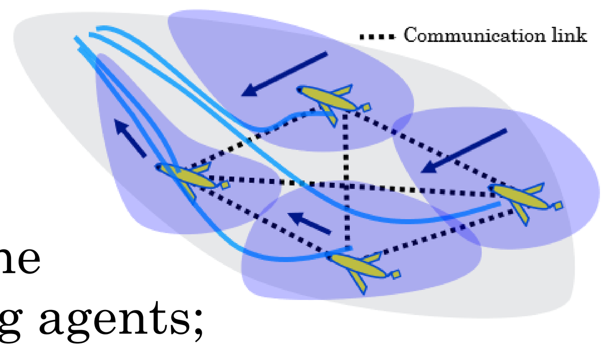
Flow Field Partition for Underwater Vehicle Path Planning

Mengxue Hou, Georgia Institute of Technology

Objective

To develop a compressed representation of the spatial and temporal variation of the ocean flow field that facilitate fast path planning.

Motivation

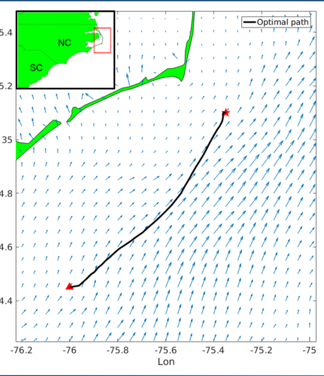
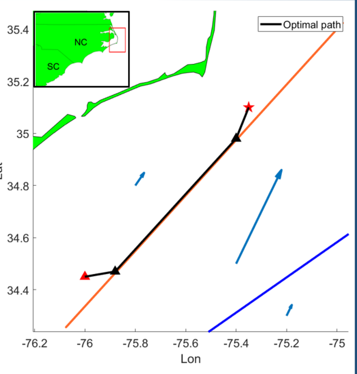


Sharing field estimation data among agents will facilitate planning performance of the fleet.

- Constrained communication capacity limits the amount of information that can be shared among agents;
- Computation cost of AUV path planning increases in the case of complicated flow map.

Novelty and Contribution

Grid-based flow map	Feature map
Spatial variation	Partitions of uniform flow speed
Temporal variation	Temporal variation of the partitioned flow

		Map size	Planning cost
Grid-based map Grid based path planning, e.g. Level Set Method		Large	High
Feature map Method of Evolving Junctions		Small	Low

Relating flow partition to Koopman Operator Theory

Evolution of state $\mathbf{z}_{k+1} = \mathbf{f}(\mathbf{z}_k)$ → Observable $\mathbf{u}(\mathbf{z}, \mathbf{x}, t)$ → Evolution of observables $\mathcal{K}\mathbf{u}(\mathbf{z}_k, \mathbf{x}) = \mathbf{u}(\mathbf{z}_{k+1}, \mathbf{x})$

Finite dimension nonlinear dynamics — Infinite dimension linear dynamics

Define $\phi_j(\mathbf{x}) = \mathbb{I}_{\mathbf{x}\in R_j}$,

$$\mathbf{u}(\mathbf{z}_k, \mathbf{x}) = \sum_{j=1}^{\infty} \bar{\mathbf{u}}_j(\mathbf{z}_k)\phi_j(\mathbf{x})$$

$$\mathcal{K}\mathbf{u}(\mathbf{z}_k, \mathbf{x}) = \sum_{j=1}^{\infty} \bar{\mathbf{u}}_j(\mathbf{z}_{k+1})\phi_j(\mathbf{x})$$

Flow partition can be a novel data driven method to compute the Koopman modes and Koopman eigenfunctions of the flow field.

Representing Spatial Variation of the Flow Field

$$\mathbf{y}(t) = [\underbrace{\mathbf{x}}_{\text{Data point position}}; \underbrace{\mathbf{F}(\mathbf{x}, t)}_{\text{Flow speed}}]$$

$$\mathrm{dist}^2(\mathbf{y}, \mathbf{y}') = (\mathbf{y} - \mathbf{y}')^T \mathbf{Q}(\mathbf{y} - \mathbf{y}')$$

$$\min J = \sum_{\alpha=1}^{k} \sum_{\mathbf{y}\in R_\alpha} \sum_{t\in T} \mathrm{dist}^2(\mathbf{y}(t), \mu_\alpha)$$

Partitioned regions — Centroid of data points in α^{th} region

⬇

$$\min \sum_{\alpha=1}^{k} \sum_{\mathbf{y}\in R_\alpha} \mathrm{dist}^2(\bar{\mathbf{y}}, \mu_\alpha)$$

Difference between time-averaged flow obs. and centroid of data points in α^{th} region

Representing Temporal Variation of the Flow Field

$$\min_{\Theta_\alpha} J_\alpha = \sum_{\mathbf{x}\in R_\alpha} \sum_{t\in T} ||\mathbf{f}_\alpha(\Theta_\alpha, t) - \mathbf{F}(\mathbf{x}, t)||^2$$

Time series model containing a set of unknown parameters Θ_α

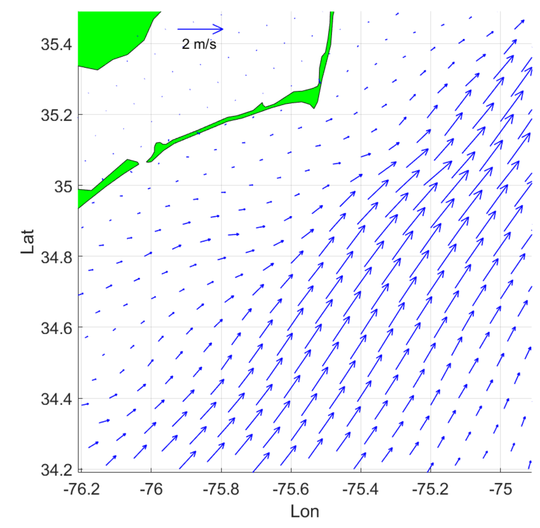
⬇

$$\min_{\Theta_\alpha} \sum_{t\in T} ||\mathbf{f}_\alpha(\Theta_\alpha, t) - \phi_\alpha(t)||^2$$

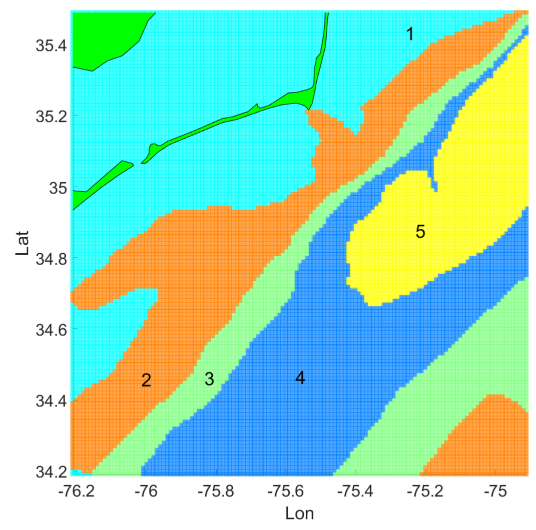
Difference between spatial-averaged flow and time-series model in α^{th} region

[1] Haoyan Zhai, Mengxue Hou, Fumin Zhang, and Haomin Zhou, "Method of evolving junction on optimal path planning in flow fields," in preparation for submission. Preprint available at http://arxiv.org/abs/1904.11554

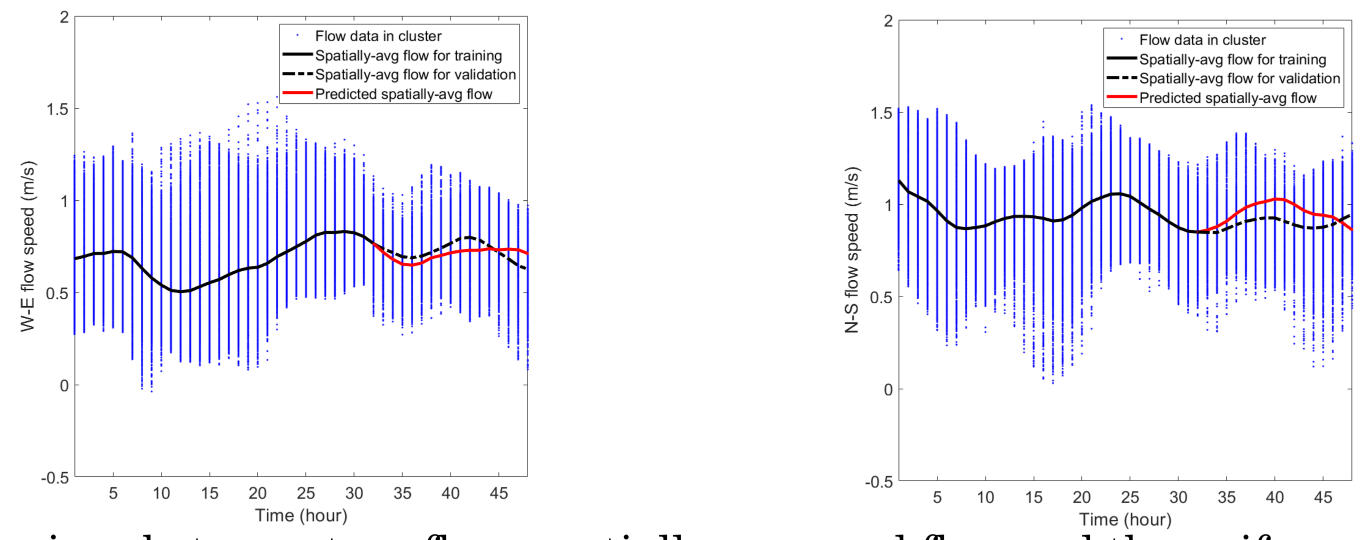
Partition of Ocean Surface Flow Field



48 hrs time-averaged flow 5/27, 00:00– 5/29, 00:00 UTC, 2017 at Cape Hatteras, NC



Partitioned flow field

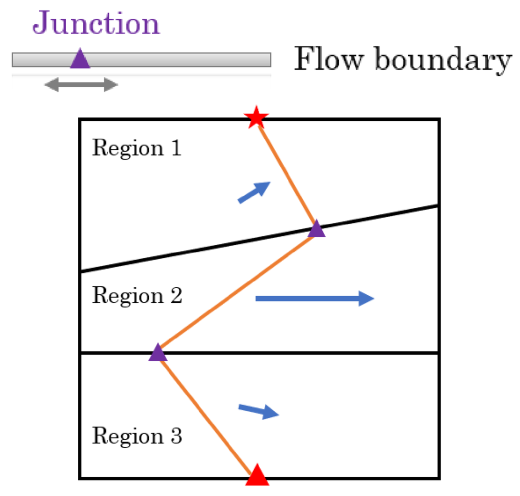


Comparison between true flow, spatially averaged flow and the uniform flow predicted by ARIMA model in region 4.

Original flow field ~ 5×10^5 data points — Partitioned flow field ~ 10^3 data points

Method of Evolving Junctions

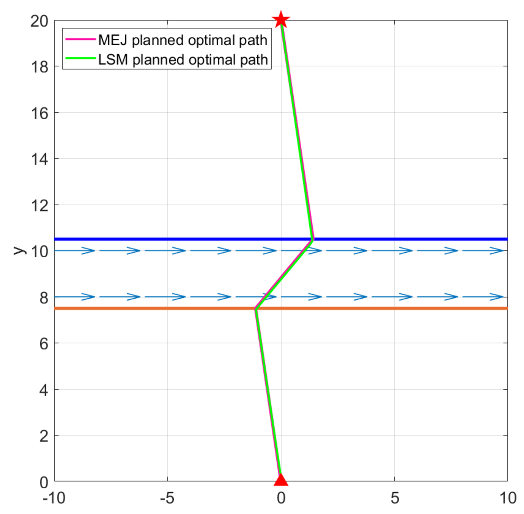
Infinite dimensional path planning ➡ Finite dimension optimization on junction positions



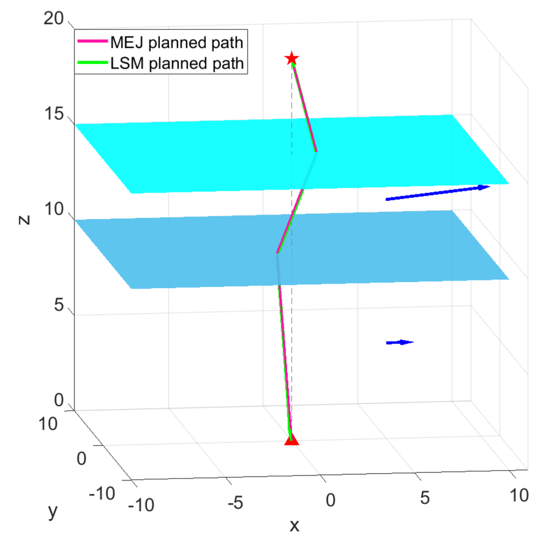
Features:

- Low computation cost
- Guaranteed global optimality
- Applicable to various cost functions

Time-optimal path planning



Comparison between MEJ and LSM planned time optimal path in 2D jet flow

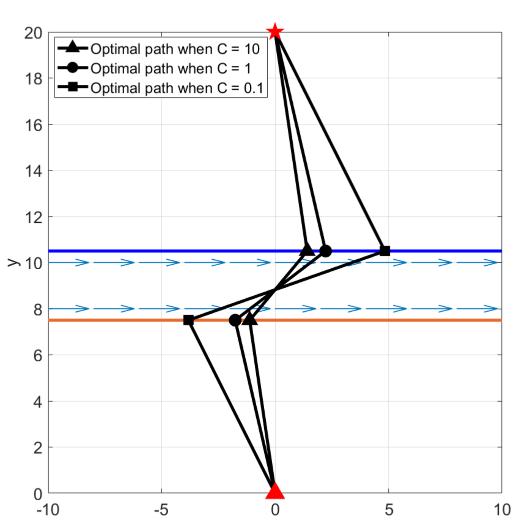


Comparison between MEJ and LSM planned time optimal path in 3D jet flow

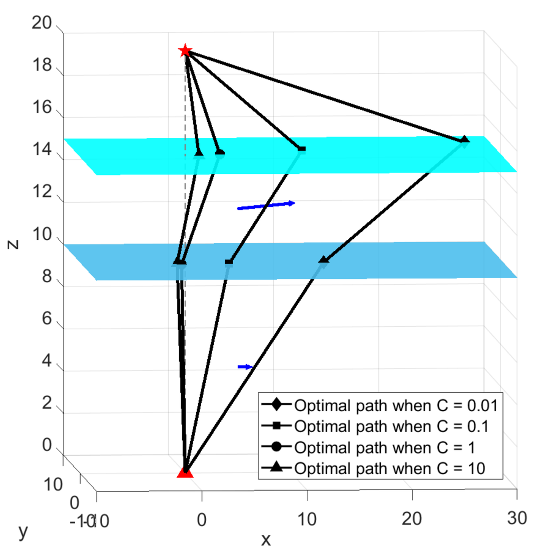
Computation cost comparison between path planning using MEJ and LSM

	MEJ	LSM
2D jet flow	0.110 secs	10.328 secs
3D jet flow	0.570 secs	10125.2 secs

Energy-optimal path planning



MEJ planned energy optimal path in 2D jet flow



MEJ planned energy optimal path in 3D jet flow

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Co-Authors

Haoyan Zhai, Haomin Zhou, and Fumin Zhang
Georgia Institute of Technology

Georgia Tech Systems Research