Broadcast Channel (BC)

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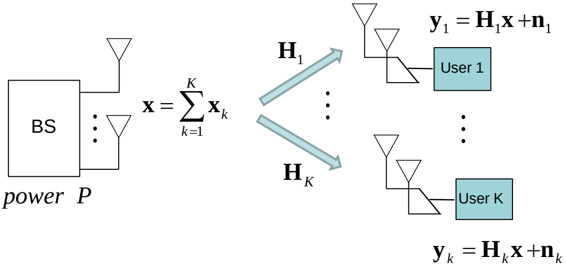
Contents

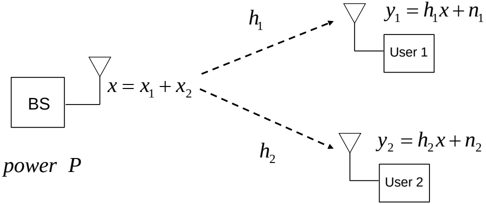
Broadcast channel

- It consists of a transmitter, with total power P, sending to K different receivers over the same time slot and frequency band
- Downlink channel in cellular systems



Capacity region (SISO case)

- Let us first consider the 2-user case
- The transmitter (BS) and the two receivers have a single antenna



- The total power, P, is split between the 2 users: $P = P_1 + P_2$,
- The noises are Gaussian with the same variance: $n_1 \sim CN(0, \sigma^2)$ and $n_2 \sim CN(0, \sigma^2)$ (total noise power WN_0)
- The channels do not change (AWGN channels)
- We define the channel power gain as $g_k = |h_k|^2$

Optimal scheme: Superposition coding

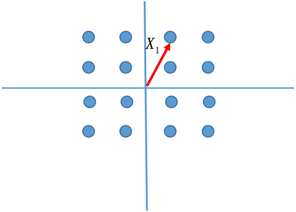
Let's assume that user 1 has a better channel than user 2:
$g_1 > g_2$,

- The signal transmitted by the BS is the linear superposition of the signals of the two users: $x = x_1 + x_2$
- Each user should use a different codebook
- The total power P is split between the two users

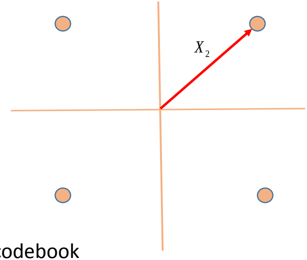
$$P = P_1 + P_2$$

  for each possible split we attain a pair of rates (R_1, R_2)
- Decoding procedure:
  - User 2 receives $y_2 = h_2 x_1 + h_2 x_2 + n_2$ and decodes x_2 considering x_1 as interference
  - User 1 receives $y_1 = h_1 x_1 + h_1 x_2 + n_1$ and does the following:
    1. First, it decodes x_2 and cancel the interference
    2. Then, it decodes x_1 without interference

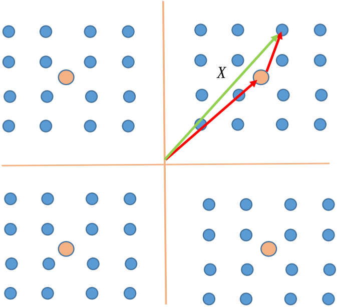
Codebook User 1



Codebook User 2



Superposition codebook

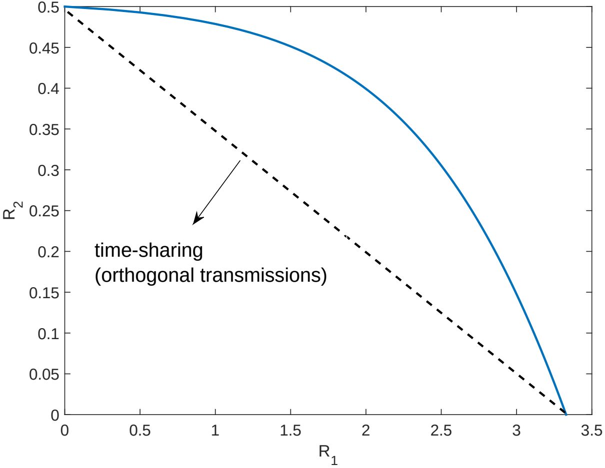


The capacity region for the 2-user Gaussian BC with total power P and $g_1 > g_2$ is

$$\begin{aligned} R_1 &\leq \log\left(1+\frac{\alpha P g_1}{\sigma^2}\right) \\ R_2 &\leq \log\left(1+\frac{(1-\alpha)P g_2}{\alpha P g_2+\sigma^2}\right) \end{aligned}$$

where $0 \leq \alpha \leq 1$, $P_1 = \alpha P$, $P_2 = (1-\alpha)P$

Example: $P = 10$, $g_1/\sigma^2 = 10$, $g_2/\sigma^2 = 0.1$



Example

- $P = 10$ mW; channel gains: $|h_1|^2 = g_1 = 10$ and $|h_2|^2 = g_2 = 1$
- System bandwidth: $W = 100$ kHz
- $N_0 = 10^{-8}$ W/Hz
- Suppose we apply the optimal strategy: **superposition coding at the Tx and successive cancellation at the Rx**

Problem: If user 1 has a rate requirement of 300 kbps, find the rate that user 2 can achieve

The K-user case

- For K users, the channels in the SISO case for each user can be ordered $\rightarrow$ **degraded BC**
- Let's assume $g_1 > g_2 > \cdots > g_K$; then, user j can decode and cancel interference coming from users $k = 1, 2, \ldots j-1$
- The capacity region becomes

$$C = \bigcup_{P_k : \sum_k P_k = P} \left\{ (R_1, \ldots, R_k) : R_k = \log\left(1 + \frac{P_k g_k}{\sigma^2 + \sum_{j<k} P_j g_j}\right) \right\}$$

- The optimal strategy is the same: **superposition coding at the Tx and successive cancellation at the Rx**

Sum-rate capacity

- A point of interest is the **sum-rate capacity**, i.e., the maximum sum of rates $\sum_{k=1}^{K} R_k$ where the maximum is taken over all rate vectors $(R_1, \ldots, R_K)$ in the BC capacity region

$$C_{SR} = \max_{(R_1,\ldots,R_k)\in C} \sum_{k=1}^{K} R_k$$

- It does not take into account fairness among users, but it is easier to characterize
- The sum-rate capacity is a point on the boundary of the capacity region: Pareto optimal
- The sum-rate capacity is achieved by assigning all power P to the user with the best channel, and therefore the sum-rate capacity is given by

$$C_{SR} = \log\left(1 + \frac{P g_{\max}}{\sigma^2}\right)$$

Example

Let's consider again the previous example (2-user case)

- $P = 10$ mW; channel gains: $|h_1|^2 = g_1 = 10$ and $|h_2|^2 = g_2 = 1$
- System bandwidth: $W = 100$ kHz
- $\sigma^2 = 10^{-3}$ W

The sum-rate capacity is achieved when only user 1 transmits with power P, i.e.,

$$C_{SR} = W \log\left(1 + \frac{Pg_1}{\sigma^2}\right) = 6.644 \cdot 10^5 \text{ bits/s}$$

Notice that to achieve this point we do not need superposition coding

BC capacity in fading channels

Again, there are two notions of capacity in BC fading channels

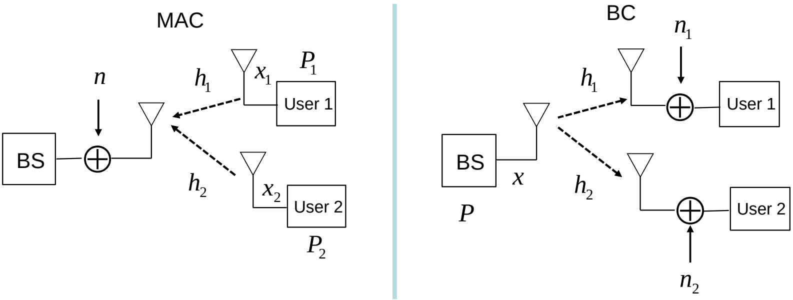
- Fast fading: we can code over multiple channel realizations $\rightarrow$ **Ergodic capacity**
- Slow fading: we code over a single channel realization $\rightarrow$ **Outage capacity**
- Also, the results would depend on whether the Tx (BS) knows all fading states (CSIT) or not
- To achieve the capacity of the BC we need CSIT (otherwise we cannot apply superposition coding), therefore existing results for fading channels focus on the CSIT case

Sum-rate capacity for the SISO-BC with fading channels and CSIT

- At each channel realization, the BS transmits only the user with the best channel
- The optimal power is assigned via waterfilling

Uplink-downlink duality

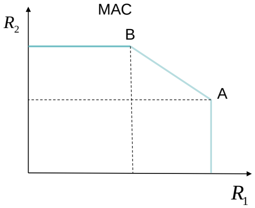
The uplink/downlink channels appear to be quite similar

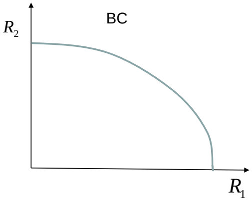


But there also some important differences...

Can you find them?

- In the MAC we have only one additive noise term, in the BC we have different noise terms for each user
- The BC has a single power constraint, whereas the MAC has different power constraints associated to each user
- In the BC the signal and the interference travel through the same channel, in the MAC the signal and the interference travel through different channels $\rightarrow$ near-far effect
- The capacity regions and the optimal TX-RX schemes for the MAC and BC look quite different





Duality conditions

Under the following conditions there is a duality between the MAC and BC that can be exploited, for instance, for capacity analysis as well as for the design of uplink downlink transmission strategies

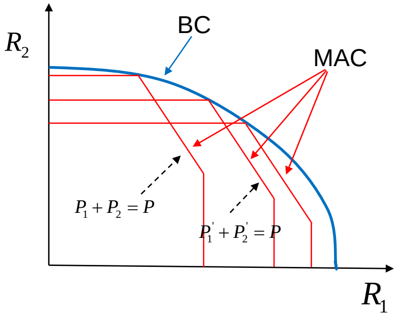
- The channels, h_k, are the same for the uplink and downlink channels
- Each channel in the downlink/uplink has the same noise statistics: for instance, all noises are $CN(0, \sigma^2)$
- The power constraint P on the downlink equals the sum of per-user power constraints in the MAC

$$P = \sum_{k=1}^{K} P_k$$

- Under these duality conditions the capacity region of a BC with power P is the capacity region of a MAC with a power constraint $P = \sum_{k=1}^{K} P_k$
- For the 2-user case, we have

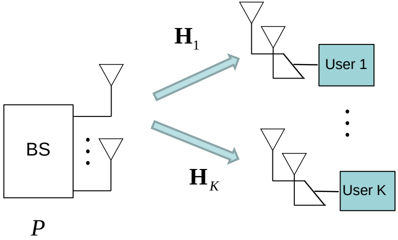
$$C_{BC} = \bigcup_{P_1+P_2=P} C_{MAC}(P_1, P_2)$$

where for each pair (P_1, P_2) the capacity region of the MAC is a pentagon



Capacity region of the BC-MIMO

- Let us consider the K-user case. The channel from the BS to the k-th receiver is an $M_k \times N$ matrix: $\mathbf{H}_k$, $(k = 1, \ldots, K)$



- The BS transmits a signal $\mathbf{x}$ with covariance matrix $\mathbf{Q} = E\left[\mathbf{x}\mathbf{x}^H\right]$, with $\mathrm{tr}(\mathbf{Q}) \leq P$
- The signal $\mathbf{x}$ encodes the information intended for all users
- The noise is Gaussian: $\mathbf{n} \sim CN(0, \mathbf{I})$ (Notice that $\sigma^2 = 1$)
- The channels do not change over time

- The capacity in the MIMO-BC channel is based on the notion of **Dirty Paper Coding (DPC)**: a technique that allows to "presubtract" interference at the Tx side without increasing the transmit power
- The MAC-BC duality can be applied here to greatly simplify the calculation of the MIMO-BC capacity region
- For instance, for the 2-user case

$$C_{BC} = \bigcup_{\mathrm{tr}(\mathbf{Q}_1)+\mathrm{tr}(\mathbf{Q}_2)=P} C_{MAC}(\mathbf{Q}_1, \mathbf{Q}_2)$$

- For each pair (P_1, P_2) the optimal transmit covariance matrices for the MAC can be easily achieved (convex problem or iterative waterfilling), and the capacity region is again a pentagon
- The capacity of the BC is the union of all these pentagons

Linear precoding

- The optimal Tx scheme for the MIMO-BC (Dirty Paper Coding) is a nonlinear encoding technique, which is difficult to apply in practice
- From a practical standpoint, is preferable to use **linear precoding** techniques: the transmitted signal for user k is

$$\mathbf{x}_k = \mathbf{W}_k \mathbf{s}_k,$$

  where $\mathbf{s}_k$ is the vector containing M_k symbols, and $\mathbf{W}_k$ is an $N \times M_k$ precoding matrix
- The signal transmitted by the BS is

$$\mathbf{x} = \sum_{k=1}^{K} \mathbf{W}_k \mathbf{s}_k$$

- The signal received by the k-th user is

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{W}_k \mathbf{s}_k + \mathbf{H}_k \left(\sum_{l=1, l \neq k}^{K} \mathbf{W}_l \mathbf{s}_l \right) + \mathbf{n}_k \qquad (1)$$

- The problem consists on designing the precoding matrices $\mathbf{W}_k$ ($k = 1, \ldots, K$) according to some criterion/goal
  - (Pre)canceling the interference (**Block Diagonalization**)
  - Minimize the transmit power subject to individual SINR constraints
  - Maximize the minimum SINR achieved by the worst user subject to a total power constraint

and many more...

Block Diagonalization

- The BS has N antennas
- Each user has M_k antennas and receives M_k symbols
- Let's assume that $N = \sum_{k=1}^{K} M_k$

The idea of block-diagonalization[1] is to choose the linear precoding matrices $\mathbf{W}_k$ such that the interference form other users is completely eliminated

$$\mathbf{H}_k \mathbf{W}_l = \mathbf{0}, \qquad \forall l \neq k$$

Using these matrices, the interference term in Eq. (1) is canceled and the signal received by user k becomes

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{W}_k \mathbf{s}_k + \mathbf{n}_k$$

[1] Q. H. Spencer, A. L. Swindlehurst, M. Haardt "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels", *IEEE Trans. on Signal Proc.*, vol. 52, n02, pp. 461-471, Feb. 2004.

How can we find these precoding matrices?

- Let us define

$$\overline{\mathbf{H}}_k = \left[\mathbf{H}_1^T, \cdots, \mathbf{H}_{k-1}^T, \mathbf{H}_{k+1}^T, \cdots, \mathbf{H}_K^T\right]^T$$

- What are the dimensions of $\overline{\mathbf{H}}_k$? $\sum_{l=1, l\neq k}^{K} M_l \times N$
- For instance, if all users have the same number of antennas M, $\overline{\mathbf{H}}_k$ is an $M(K-1) \times N$ matrix
- Any suitable $\mathbf{W}_k$ must belong to the null space of $\overline{\mathbf{H}}_k$ [2]
- Remember that we have assumed $N = \sum_{k=1}^{K} M_k$ and therefore $\overline{\mathbf{H}}_k$ always has an M_k-dimensional right null space

[2] In Matlab null(A) returns an orthogonal basis for the null space of A.

Conclusions

- The capacity achieving scheme for the BC-SISO is superposition encoding at the Tx and successive cancelation at the Rx
- The capacity achieving scheme for the BC-MIMO is dirty paper coding at the Tx and MMSE decoding at the Rx
- Under some conditions there is a MAC-BC duality that turns out to be very helpful for capacity calculations as well as in many design algorithms for the BC
- In practice, for the BC we use linear precoding techniques
- Block diagonalization is a typical example