For the $\Sigma_c \to \Lambda_c^+ \pi^-$ resonant states in the $\Lambda_b^0 \to \Lambda_c^+ p\bar{p}\pi^-$ decays.

1 The decay amplitude

According to PDG, the decay amplitude can be written as

$$\mathscr{M} \propto \frac{1}{M_0^2 - m^2 - iM_0\Gamma(m)} q^L F_L(q, q_0), \tag{1}$$

where M_0 is the PDG mass of the resonant state, m is the $\Lambda_c^+\pi^-$ invariant mass, $\Gamma(m)$ is the mass-dependent width, q is the momentum of the Λ_c^+ baryon in the Σ_c rest frame, L is the orbit angular momentum, $F_L(q, q_0)$ is a phenomenological form factor, and is choosen to be a Blatt-Weisskopf form in our analysis. The mass-dependent width can also be found in the "Resonance" chapter of PDG, namely,

$$\Gamma(m) = \Gamma_0 \times \left(\frac{q}{q_0}\right)^{2L+1} \frac{M_0}{m} B_L(q, q_0, d)^2. \tag{2}$$

The decay amplitude gives the probability amplitude in the phase space,

$$d\Gamma = \frac{2\pi}{M} |M|^2 \, d\Phi_n(P; p_1, ..., p_n), \tag{3}$$

where M is the invariant mass of the primary particle, which decays to n final-state particles, Φ_n is the phase space, P stands for the four-momentum of the primary particle, and p_i is the four-momentum of the i_{th} final-state particle.

Now we need to propagate the probability density function in four-body phase space to the probability density function of the $\Lambda_c^+\pi^-$ invariant mass.

2 PDF of invariant mass

In the decay $\Lambda_b^0 \to \Lambda_c^+ p\bar{p}\pi^-$, we label Λ_c^+ as particle 1, π^- as particle 2, $\bar{p}$ as particle 3, and p as particle 4.

The defination of the n body phase space element is given by

$$d\Phi_n(P; p_1, ..., p_n) = \delta^4(P - \sum_{i=1}^{n} p_i) \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 2E_i}. \tag{4}$$

Using this definition, we can get

$$
\begin{aligned}
& d\Phi_3(q; p_1, p_2, p_3) \times d\Phi_2(P; q, p_4)(2\pi)^3 dq^2 \\
& = \delta^4(q - \sum_{i=1}^{3} p_i)\delta^4(P - q - q_4)(2\pi)^3 \prod_{i=1}^{4} \frac{d^3 p_i}{(2\pi)^3 2E_i} dq^2 \frac{d^3 q}{(2\pi)^3 2E_q} \\
& = \delta^4(q - \sum_{i=1}^{4} p_i)\delta^4(P - q - q_4)(2\pi)^3 \prod_{i=1}^{4} \frac{d^3 p_i}{(2\pi)^3 2E_i} \\
& = d\Phi_4(P; p_1, p_2, p_3, p_4)
\end{aligned}
$$

where q is the sum of four-momentum of particle 1, 2 and 3, and the Jacobi determinant is used for the transformation from $dq^2 d^3q$ to d^4q.