$\nabla f = \frac{d}{dx}\left(\frac{F(x)}{g(x)}\right)$

(1)

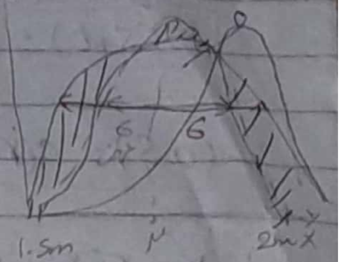
Task 26 - Mathematics for Machine Learning

Linear Algebra

هو دراسة المتجهات و مساحات المتجهات ورسم الخرائط بينها

Introduction to linear algebra

المتجهات



Motivation:

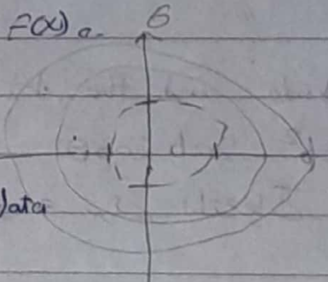
$2a + 3b = 8$

$10a + 1b = 13$

$$\underbrace{\begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix}}_{\text{معامل}} \begin{bmatrix} a \\ b \end{bmatrix} = \underbrace{\begin{bmatrix} 8 \\ 13 \end{bmatrix}}_{\text{Cost}}$$

Ex: If $f(x) = \frac{1}{6\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2 6^2}\right)$

$F(x)$ بوجد 6 - متوسط تغيرات



The Problem of fitting a function to some data

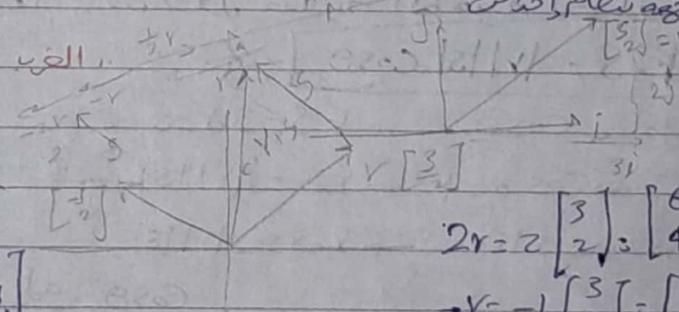
Operations with vectors

جمع

1. addition

$s + r = r + s$

* التعبير عن المتجه بنظام إحداثي



2. Multiplication الضرب العددي

If $s = \begin{bmatrix} -1 \\ 2 \end{bmatrix} = -1i, 2j$

$r = \begin{bmatrix} 3 \\ 2 \end{bmatrix} = 3i, 2j$

$s + r = \begin{bmatrix} -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

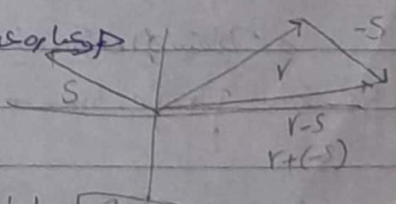
$2r = 2\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$

$-r = -1\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$

3. Associative "الترابط": $(r+s)+t = r+(s+t)$

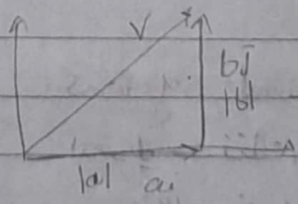
4. Subtraction الطرح: هو عبارة عن جمع سالب المتجه على المتجه الآخر

$r + (-r) = \begin{bmatrix} 3 \\ 2 \end{bmatrix} + \begin{bmatrix} -3 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

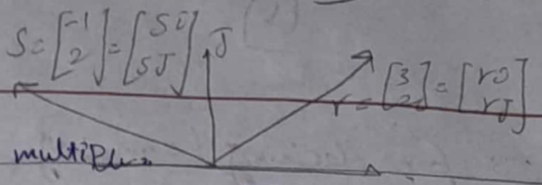


length of the vector

$r = ai + bj = \begin{bmatrix} a \\ b \end{bmatrix}$



length $|r| = \sqrt{a^2 + b^2}$



Dot Product "Scalar multiple"

$r.S = r_i S_i + r_J S_J = 3*-1 + 2*2 = \boxed{1}$

$\boxed{S.r = r.S}$ تبادلي "Commutative" وهنا ضاف العرض ليه الفتا ان

انه "Distrubation" توزيعي

$\boxed{r.(S+t) = r.S + r.t}$

If $r=\begin{bmatrix}r_1\\r_2\\ \vdots\\r_n\end{bmatrix}$ $S=\begin{bmatrix}S_1\\S_2\\ \vdots\\S_n\end{bmatrix}$, $t=\begin{bmatrix}t_1\\t_2\\ \vdots\\t_n\end{bmatrix}$

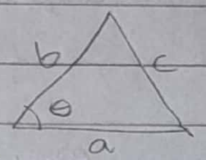
"associative"

$\boxed{r.(aS) = a(r.S)}$ over Scalar multiple → $r.(S+t) = r_1.(S_1+t_1) + r_2.(S_2+t_2) - - + r_n.(S_n+t_n)$

→ $r.r = r_i r_i + r_J r_J = r_i^2 + r_J^2 = |r|^2$ → $\boxed{r.r = |r|^2}$

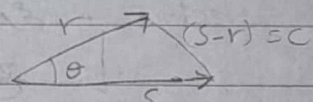
- The cosine rule and the dot product

$c^2 = a^2 + b^2 - 2ab\cos\theta$



تقوم بالتعامل معاها على مستوى المتجهات

$|r-s|^2 = |r|^2 + |s|^2 - 2|r||s|\cos\theta$



↳ $(r-s).(r-s) = r.r - S.r - r.S - S.-S = \underline{r.r} + S.S - 2(S.r)$

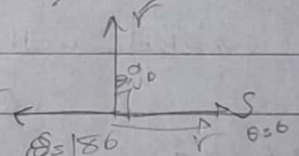
من خلال مقارنة الطرفين نجد ان $|r|^2 + |S|^2 - 2(S.r)$

$\boxed{r.S = |r||s|\cos\theta}$ ##

→ If $\theta = 90$ → $\cos\theta = 0$ → $r.S = S.r = 0$

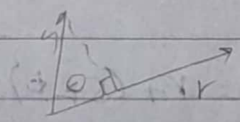
→ If $\theta = 0$ → $\cos 0 = 1$ → $r.S = |r||S|$

→ " $\theta = 180$ → $\cos 180 = -1$ → $r.S = -|r||S|$



$\cos\theta = \frac{adj}{hyp} = \frac{adj}{|S|}$

- vector Projection



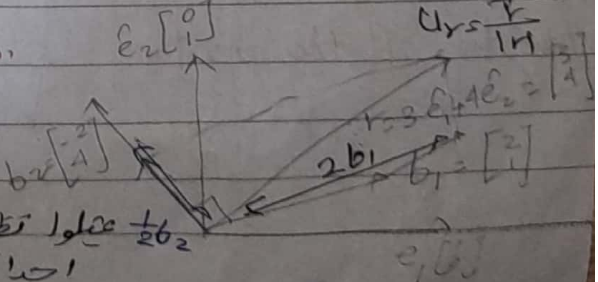
$r.S = |r|\underbrace{|S|\cos\theta}_{adj}$ → Projection of S on r

في حالة S, r متعامدين تكون الإسقاط = 0

Scalar Projection → $\frac{r.S}{|r|} = |S|\cos\theta$ → vector Project $= \frac{r.S}{|r|}.U_r$ → unit vector on r direction

$U_r = \frac{r}{|r|}$

→ changing Basics "Co-ordinate systems"



لتحويل من نظام e لنظام b يجب ان b_1, b_2 يكونوا متعامدين علشان اتأكد من تعامدهم

$\boxed{b_1 . b_2 = 0}$

↳ $b_1 . b_2 = 0*1 + 1*0 = 0$ → $b_1 \perp b_2$ → عملوا نظام احداثي

vector Projection of r_e in b_1

↳ $\frac{r_e . b_1}{|b_1|^2} = \frac{6+4}{(\sqrt{5})^2} = \frac{10}{5} = 2$ → b_2

3

$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Vectors , Basis , applications

Basis of as is a set of n vectors that

1. are not linear combinations of each other , linearly independent.
2. span the space
3. The space is then n-dimensional

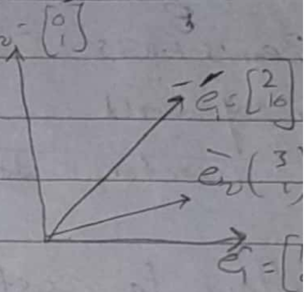
Introduction to Matrices

$2a + 3b = 8$

$10a + 1b = 13$ $\rightarrow \begin{pmatrix} 2 & 3 \\ 10 & 1 \end{pmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{pmatrix} 8 \\ 13 \end{pmatrix}$

$x \rightarrow \begin{pmatrix} 2 & 3 \\ 10 & 1 \end{pmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{pmatrix} 2 \\ 10 \end{pmatrix}$ $\quad \leftarrow \begin{pmatrix} 2a & 3b \\ 10a & 1b \end{pmatrix} = \begin{pmatrix} 8 \\ 13 \end{pmatrix}$

$y \rightarrow \begin{pmatrix} 2 & 3 \\ 10 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ $\quad \underbrace{\begin{pmatrix} 2 & 3 \\ 10 & 1 \end{pmatrix}}_{A} \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{r} = \underbrace{\begin{bmatrix} 8 \\ 13 \end{bmatrix}}_{r'}$

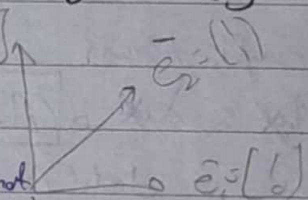


$A(nr) = nr'$

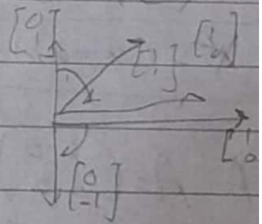
$A(r+s) = (r' + s) = Ar + As$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

Matrix I $\rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$



Matrices: Composition or Combination of Matrix transformat.



inverses (Solving the apples and banana Problem)

$\underbrace{\begin{pmatrix} 2 & 3 \\ 10 & 1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} a \\ b \end{pmatrix}}_{r} = \underbrace{\begin{pmatrix} 8 \\ 13 \end{pmatrix}}_{S}$ $\rightarrow \boxed{\underbrace{A^{-1}}_{\text{inverse}} A = I}$

$\rightarrow \underbrace{(A^{-1}A)}_{I} r = A^{-1}s \rightarrow \boxed{Ir = r = A^{-1}s}$

ex:
$\begin{matrix} 1. \\ 2. \\ 3. \end{matrix} \begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 15 \\ 21 \\ 13 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 15 \\ 6 \\ -2 \end{pmatrix}$

يطرح الصف الاول من الصف الثاني والثالث

→ Going from gaussian Elimination to finding the inverse Matrix

محدد معكوس

Determinants and inverses

عند ضرب المصفوفات بتقرب لصف من عمود

- Summation convention and the symmetry of the dot product

- Transformations in a changed basis

$$\left(\begin{pmatrix}3\\1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}\right)\begin{pmatrix}x\\y\end{pmatrix} \cdot \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\1 & 1\end{pmatrix}$$

B — Bear's basis in my world

R — 45°

vector rotated in my basis

* orthogonal matrices متعامدة صور المصفوفة

$A^T_{ij} = A_{ji}$

$\rightarrow \boxed{A^TA = I}$ $\rightarrow |A| = \pm 1$

$\rightarrow \boxed{A^T = A^{-1}}$

- How to construct an orthonormal basis — gram-schmidt process

v_3, v_1, v_2, e_1, $e_1 = \frac{v_1}{|v_1|}$

Eigenproblems — Eigenvectors $A = \begin{pmatrix}a & b\\c & d\end{pmatrix}$

$Ax = \lambda x \rightarrow \boxed{(A - \lambda I)x = 0}$

$\det(A - \lambda I) = 0 \rightarrow \det\left(\begin{pmatrix}a & b\\c & d\end{pmatrix} - \begin{pmatrix}\lambda & 0\\0 & \lambda\end{pmatrix}\right) = 0$

$\boxed{\lambda^2 - (a+d)\lambda + ad - bc = 0}$

If $A = \begin{pmatrix}1 & 0\\0 & 2\end{pmatrix} \rightarrow \det\begin{pmatrix}1-\lambda & 0\\0 & 2-\lambda\end{pmatrix} = (1-\lambda)(2-\lambda) = 0$

$\rightarrow \lambda = 1 \rightarrow x = \begin{pmatrix}t\\0\end{pmatrix}$

$\rightarrow \lambda = 2 \rightarrow x = \begin{pmatrix}0\\t\end{pmatrix}$

If $\lambda = 1$ $(A - \lambda I)x = 0$

$\begin{pmatrix}1-1 & 0\\0 & 2-1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}0\\x_2\end{pmatrix} = 0$

If $\lambda = 2$

$\begin{pmatrix}-1 & 0\\0 & 0\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}-1 & 0\\0 & 0\end{pmatrix} = \begin{pmatrix}-x_1\\0\end{pmatrix} = 0$

$$\begin{array}{ccc} v & \xrightarrow{T^n} & T^n v \\ \big\downarrow C^{-1} & & \big\uparrow C \\ [v]_E & \xrightarrow{D^n} & [T^n v]_E \end{array}$$

$C = \begin{pmatrix}x_1 & x_2 & x_3\\ \vdots & \vdots & \vdots\end{pmatrix}$ $D = \begin{pmatrix}\lambda_1 & 0 & 0\\0 & \lambda_2 & 0\\0 & 0 & \lambda_3\end{pmatrix}$ $T^n = \begin{pmatrix}a^n & 0 & 0\\0 & b^n & 0\\0 & 0 & c^n\end{pmatrix}$

$T = CDC^{-1} \rightarrow T^2 = CDC^{-1}CDC^{-1} = CD^2C^{-1} \rightarrow T^n = CD^nC^{-1}$

PageRank

$r_A = \sum_{j=1}^{n} L_{A,j} r_j \rightarrow r = Lr$