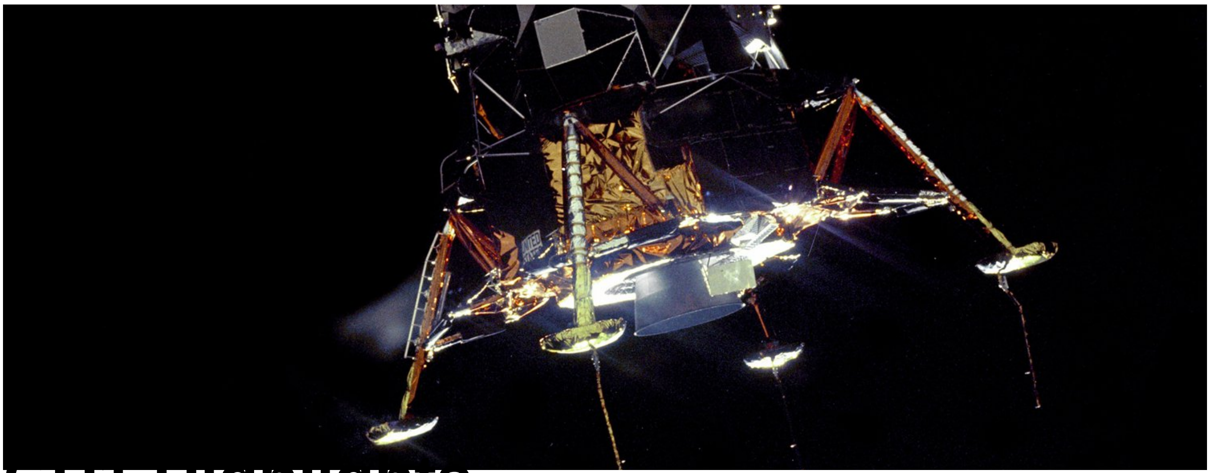

CH1 Highlights

Operator	Symbol	English Expressions
Negation	¬	"not", "it is not the case that"
Conjunction (And)	∧	"and", "both...and"
Disjunction (Or)	∨	"or", "either...or", "at least one of"
Exclusive disjunction (XOR)	⊕	"exclusive or", "either...or but not both"
Implication (If-then)	→	"if...then", "implies", "only if"
Biconditional (If and only if)	↔	"if and only if", "iff", "is equivalent to", “p is necessary and sufficient for q” “if p then q, and conversely” “p iff q.” “p exactly when q.”

We use letters to denote propositional variables (sentential variables), that is, variables that represent propositions, just as letters are used to denote numerical variables. The conventional letters used for propositional variables are p, q, r, s, ….

> The truth value of a proposition is true, denoted by T, if it is a true proposition, and the truth value of a proposition is false, denoted by F if it is a false proposition.
>
> Propositions that cannot be expressed in terms of simpler propositions are called **atomic propositions.**
>
> Many mathematical statements are constructed by combining one or more propositions. **New propositions, called compound propositions, are formed from existing propositions using logical operators**

Different ways to express the conditional statement "p → q":

Expression	Meaning
if p, then q	p implies q
if p, q	p only if q
p is sufficient for q	a sufficient condition for q is p

Expression	Meaning
q if p	q whenever p
q when p	q is necessary for p
a necessary condition for p is q	q follows from p
q unless ¬p	q provided that p

“q whenever p” is one of the ways to express the conditional statement p → q

1. The converse of "p → q" is "q → p."
2. The **contrapositive** of "p → q" is "¬q → ¬p."
3. The inverse of "p → q" is "¬p → ¬q."

Of these three statements, only the **contrapositive** is always equivalent to the original statement

"p → q." This means that the **contrapositive** has the same truth value as the original statement, regardless of the truth values of the variables p and q.

The converse and the inverse are not always equivalent to the original statement. In fact, the converse and the inverse are only equivalent to the original statement in certain cases. One of the most common logical errors is to assume that the converse or the inverse of a conditional statement is equivalent to the original statement. **The original statement and its contrapositive are equivalent, but the converse and inverse are not equivalent to the original statement.**

Construct the truth table of the compound proposition (p ∨ ¬q) → (p ∧ q).

p	q	¬q	p ∨ ¬q	p ∧ q	(p ∨ ¬q) → (p ∧ q)
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

The **"precedence of logical operators"** refers to the order in which logical operations are performed in a compound proposition.

First, the negation operator (¬) is always performed first, before any other logical operators.

Second, the conjunction operator (∧)

Third, the disjunction operator (∨)

Fourth, The conditional operator (→) and the biconditional operator (↔) have lower precedence than conjunctions and disjunctions.

▼ Find the bitwise OR, bitwise AND, and bitwise XOR of the bit strings 01 1011 0110 and 11 0001 1101.

Answer

11 1011 1111 bitwise OR

01 0001 0100 bitwise AND
10 1010 1011 bitwise XOR

- A compound proposition that is always true, no matter what the truth values of the propositional variables that occur in it, is called a **tautology**.
- A compound proposition that is always false is called a **contradiction**.
- A compound proposition that is neither a tautology nor a contradiction is called a **contingency.**
- Compound propositions that have the same truth values in all possible cases are called **logical equivalents.**
- The compound propositions p and q are called logically equivalent if p ↔ q is a **tautology**.
  The notation **p ≡ q** denotes that p and q are **logically equivalent**.

p	¬p	p ∨ ¬p	p ∧ ¬p
T	F	T	F
F	T	T	F

And here are the examples of a tautology and a contradiction:

Propositional Formula	Truth Value
p ∨ ¬p	T
p ∧ ¬p	F

And here are De Morgan's Laws:

Propositional Formula	Equivalent Propositional Formula
¬(p ∧ q)	¬p ∨ ¬q
¬(p ∨ q)	¬p ∧ ¬q