FI6022 - Financial Engineering

Structured Investment Products

(40%)

Niall O'Donnell – 15179087
Eoghan O'Shea – 12160296
Akhil Menon – 19008414
Amrit Chachadi – 19023626

MSc Computational Finance

UNIVERSITY OF LIMERICK
OLLSCOIL LUIMNIGH

Project Objectives:

Structured investment products such as Autocallable Notes and the Ladder Forwards have gathered significant popularity over recent years, primarily through market participants pursuing a tailored risk-return product. The motivation behind this growth originates from the objective of increasing yields or enhanced coupons in exchange for acquiring additional downside risk. Ultimately, the attraction to investors lies in a product that does not bear the full volatility and principal loss exposures of the equities market.

The low interest and low-yield conditions that encapsulated the financial industry since 2003 led to a surge in contingent income securities in search of high yields. Fabozzi (2005) states that newly issued autocallable structured sector was the quickest expanding market of the US investment-grade fixed income market. The market for new issuance autocallable securitized products from 2007-2010 was estimated to be $40 billion dollars in the US market according to Deng et al (2009). Notwithstanding their growth, risk management of autocallable products remains a serious issue (Kim and Lim 2018). A significant concern associated with these products remains that they are complex, and the risks are not appropriately clarified or appreciated. The enticement of an enhanced coupon tends to outweigh the understanding and clarification of the risks associated. For investors exploring alternate investment strategies and searching for increased yield, these products tend not to produce higher returns, due to the fact that the risks associated are not entirely measured by either the investor or indeed the issuer (McCann 2006).

The aim of this paper is to provide a complete report, analysing the pricing and return characteristics of two popular structured products and to acquire an extensive and intuitive comprehension of structured investment products and their financial engineering. This will be applied to Autocallable Notes and the Ladder Structured Forward. To accomplish this the both investment products will be financially engineered, commencing from January 2008 and expiring in December 2012 with the S&P 500 Index acting as the underlying asset. Subsequently, a transparent and complete examination into the risk-return trade-off will be undertaken.

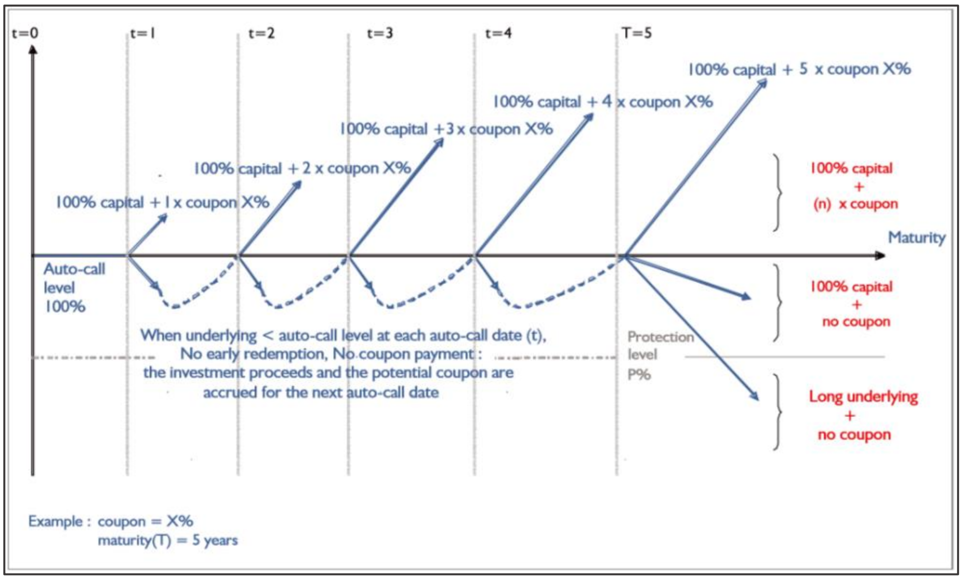
Section 1

1.1 An Introduction to Autocallable Notes

Autocallable contingent income securities/notes are products where the investor receives a superior coupon if certain conditions hold, in return for an increased exposure of the investor's capital, when compared to a plain vanilla instrument. It is comparable to the reverse convertible in relation to the selling of an at-the-money (ATM) put option to enhance the coupon element. However, the autocallable may reach an automatic expiration prior to the listed maturity date which is dissimilar to the reverse convertible. The enhanced coupon may be received on condition if the price of the underlying asset breaches a certain level, known as the auto-call barrier.

This auto-call feature causes the product to be automatically redeemed, on a prespecified date, if the price performance of an underlying asset reaches or goes above the auto-call barrier. The investor will receive 100% face value as well as the enhanced coupon, and the note will subsequently expire. If conditions are such that the underlying asset fails to reach or surpass the auto-call barrier at any of the prespecified dates throughout the investment, the investor will not receive a coupon, but will receive 100% face value at the final maturity date. If the lower barrier is penetrated throughout the product lifecycle, substantial deficits are conceivable contingent on the index level at future redemption dates. The lower level protection barrier is used as a protection mechanism within the autocallable. From Figure 1.1 it is clear that the auto-call barrier is 100%, and the auto-call will automatically occur if the referenced index tends upwards, above its first level, on any of the 5 auto-call dates.

Figure 1.1



1.2 Creating the Autocallable | S&P 500

Commencing on the 01/01/08 and terminating on the 12/31/12, a 5-year autocallable was constructed using the DLIB-AC function on Bloomberg, with the following features:

• Auto-Call Barrier:	110%	• Active on Last Trading Day of Year
• Contingent Coupon:	10%	• (1 *x* 10%) where (n *x* C) and n = 1,2,3,4,5
• Early Redemption:	100%	• If Auto-Call Barrier Breached.
• Lower Protection Barrier:	55%	• Continuously Active

Below, Figure 1.2 illustrates the final deal, with a price of 98.49 quoted on the 12/30/2012.

Figure 1.2: Autocallable Note Product

1.3 Performance | Backtesting the Autocallable | S&P 500

Overall Performance

Figure 1.3 below shows the 5-year price performance of the S&P 500. At conception of the product, the S&P 500 price was listed at 1447.16. At expiration, the price was held at 1426.19. With both the 110% and 55% barrier overlaid, it also illustrates the performance of the autocallable note over the 5-year lifespan. As can be seen, the S&P 500 failed to breach the 110% autocall barrier on any of the anniversary dates. Therefore, the buyer of the autocallable received no enhanced coupons. The Long Knockout Put with Strike 0% and Barrier 110% (Component 2) was never activated and therefore did not provide the investor with a guaranteed 100% face value at maturity. The Long Knockout Put with Strike 0% and Barrier 0% (Component 2) implanted for the final year of the product provided the investor with a 100% rebate. Given the product survives the final year, a 100% rebate is paid out, highlighting the importance of this leg of the deal in protecting the investor.

The lower protection barrier at 55% was breached on 11/20/2008, activating the Short Knock-In Put. Therefore, the investor is exposed to extreme downside risk as they shall now receive the performance of the index at maturity. Following lows of below $700 in March 2009, the investor is fortunate as the index rallies significantly, ultimately finishing at $1429.19 on 12/31/2012 or at 98.69% of its initial value. Although the sold Knock-In Put – the biggest source of risk in the product – expires in the money, the loss to the investor is not significant.

The table below (Figure 1.4) illustrates the (%) of the initial value the index was at each anniversary date. This provides an investor with an idea of how much the index was required to gain in order to secure the enhanced coupon and full redemption. Had the normalized value of the index breached 110 on any of the anniversary dates, the investor would have received the 100% face value, in addition to any accrued coupons. On the following page, a detailed illustration of the S&P 500's performance over this period relative to the 55% and 100% investment barriers will be provided (Figure 1.5). This will grant investors a comprehensive insight into the performance of the autocallable note over the period.

Figure 1.3 – Price Performance of the S&P 500 2008-2012

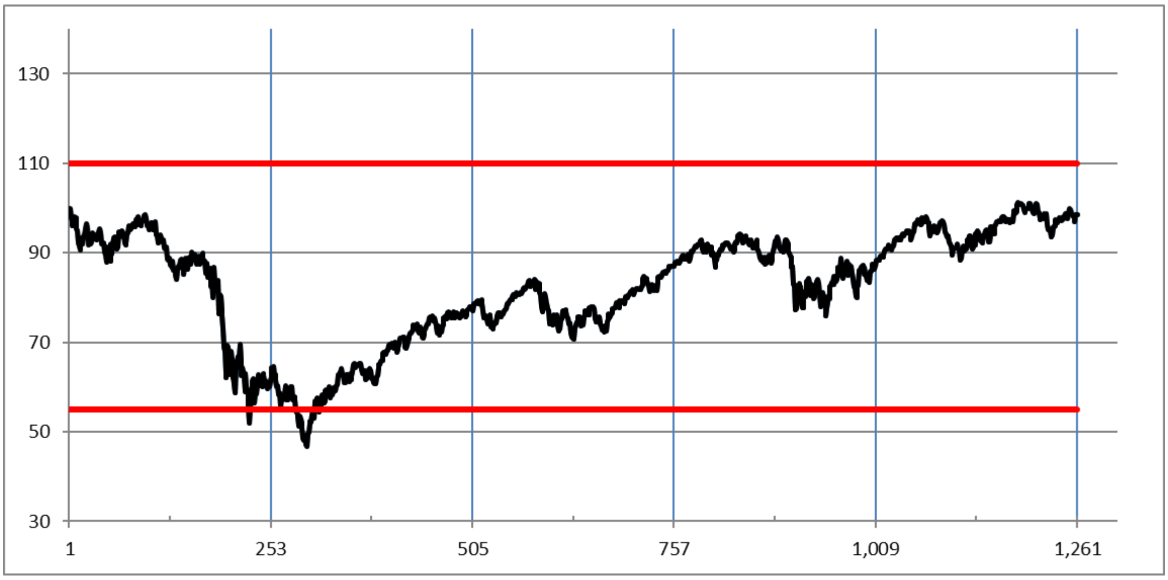


Figure 1.4 – S&P 500 Performance Relative to Starting Value

Year	Date	Normalized Price
Initial Price	01/01/2008	**100**
Year-1	12/31/2008	**62.4**
Year-2	12/31/2009	**77.1**
Year-3	12/31/2010	**86.9**
Year-4	12/30/2011	**86.9**
Year-5	12/31/2012	**98.6**

Figure 1.5 | Autocallable Note & SPX Performance | 2008-2012

Ultimately, the investor in the autocallable product having received no enhanced coupons, receives a pay-off equal to the price difference of the index at maturity versus at conception, for a total capital loss of $20.97, shown below. The profit and loss will be analysed in more detail, and backtested in Section 1.4 below, through the creation of a replicating portfolio.

$$\boxed{S_t - S_0}$$

$$\$1426.19 - \$1447.16$$

- Received 100% Rebate due to Knocked-Out Long Put | Strike = 0% | Barrier = 0%

- Liable due to the Knocked-In Short Put | Strike = 100% | Barrier = 55%

=

$$-\$20.97$$

1.4 Replicating the Autocallable | Price Verification

In order to assess whether the Autocallable Note has been priced correctly in Bloomberg, a replicating portfolio has been developed in Microsoft Excel. By financially engineering the autocallable note using its constituent parts, it is possible to investigate whether price discrepancies are present within particular 'legs' of the product. In total, the Autocallable Note consists of three components, each with a combination of both knock-in and knock-out options. Each component has been described in detail below.

Component 1:

Long 5Y Knockout Puts: | Strike = 0% | Barrier = 110%

This component is constructed by combining five knockout put options, with each barrier commencing on the anniversary date of its respective year and each option expiring at the product's expiry on the 12/31/12. This ensures the coupon accrues each year the product survives.

The role of this component allows investors to earn a contingent coupon if the index price surpasses the 110% strike on a discrete anniversary date. The 110% barrier is active only on these anniversary dates. If the price falls short on that date, there will be no coupon or early redemption and the coupon will accrue (n *x* C) to the next anniversary date. If the price exceeds the barrier, the product is 'auto-called', and accumulated coupons up to that point are paid out.

Component 2:

Long 5Y Knockout Put: Strike = 0% | Discrete Barrier = 110%

This option, with a discrete barrier active on the anniversary dates of years 1,2,3,4 – and expiring thereafter – works synchronously with Component 1 to provide the early redemption value of 100%, alongside the contingent coupons up to that point. If the index price surpasses 110% of the initial price on any anniversary date up to and including year 4, 100% of the principal is returned to the investor in addition to the contingent coupons supplied in Component 1. The product would be subsequently terminated.

Long 5Y Knockout Put: Strike = 0% | Barrier = 0%

If, at the end of Year 4, the value falls short of the 110% barrier, the product survives until Year 5. This option – with a 0% barrier active only on the final anniversary date – ensures the return of the full principal amount to the investor only if the value of the index fall below the 110% barrier but stays above the 55% protection barrier. Component 3 addresses the scenario when the value of the index falls below the 55% protection barrier.

Component 3:

Short 5Y Knock-In Put: Strike = 100% | Continuous Barrier = 55%

This option is the primary source of both the risk and reward of the investor. This option ensures the enhanced coupon to the investor. However, it also ensures that if the underlying falls below the 55% barrier, the investor receives the performance of the index. This short option is also a way for the option maker to recoup costs.

The continuous barrier means that if the index breaches the 55% and ends the period worth 60% of its initial value, the investor will receive $60 for every $100 invested [$\frac{S_t}{S_0} * 100$] as the Knock-In Put would finish 40% ITM.

In the following section, the replicating portfolio constructed above will be compared to the structured product created in Bloomberg. First, the quoted price from Bloomberg will be verified by pricing the replicating portfolio with identical specifications. Secondly, the replicating portfolio will then be backtested with alternative barrier levels, to assess how this affects the pricing of the structured investment product.

1.5 Comparative Analysis & Benchmarking

Price Verification | Profit and Loss:

To verify the quoted price from Bloomberg, a replicating portfolio was constricted in excel consisting of the three components described above. Daily price data was first sourced from Bloomberg and normalized to the first data point to ensure accuracy. Using a notional amount of $100 and maintaining the 110%/55% auto-call/protection barriers, an almost identical pay-off structure was observed. A price of $98.49 was observed in Bloomberg compared with $98.55 in the replicating portfolio in Excel, amounting to a discrepancy of just $0.06. (Figure 1.6)

The P&L of the replicating portfolio can also be observed in (Figure 1.7), indicating no redeemable coupons, and no early redemption of the autocallable note. The final payoff of minus 1.45% is observed, computed from the difference between the 100% final redemption and the final price of 98.55 (98.55% of the initial starting price).

$$100\% - Max\left[100\% - \frac{98.55}{100} * 100\%, 0\right]$$

$$= 1.45\%$$

Figure 1.7 | P&L of Replicating Autocallable Portfolio

Year	1	2	3	4	5		
Row No.	257	509	761	1015	1265		
n	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	Min{S_t}	Max{S_t}
S(t)	**62.4**	**77.1**	**86.9**	**86.9**	**98.6**	46.75	101.3
C(t)	10%	20%	30%	40%	50%		
Coupons	-	-	-	-	-		
Early Redemption	-	-	-	-			
Final Redemption					**100.00**		
					- 1.45		
Total Payment	-	-	-	-	**98.55**		
- 100.0	-	-	-	-	**98.55**		

Autocall Barrier	110%
Lower Protection Barrier	55%
Knock-In Put Conventional Strike	100%
Y5 Knock-Out Barrier	0%

Figure 1.6 | Price Verification of Autocallable Note Quote Price

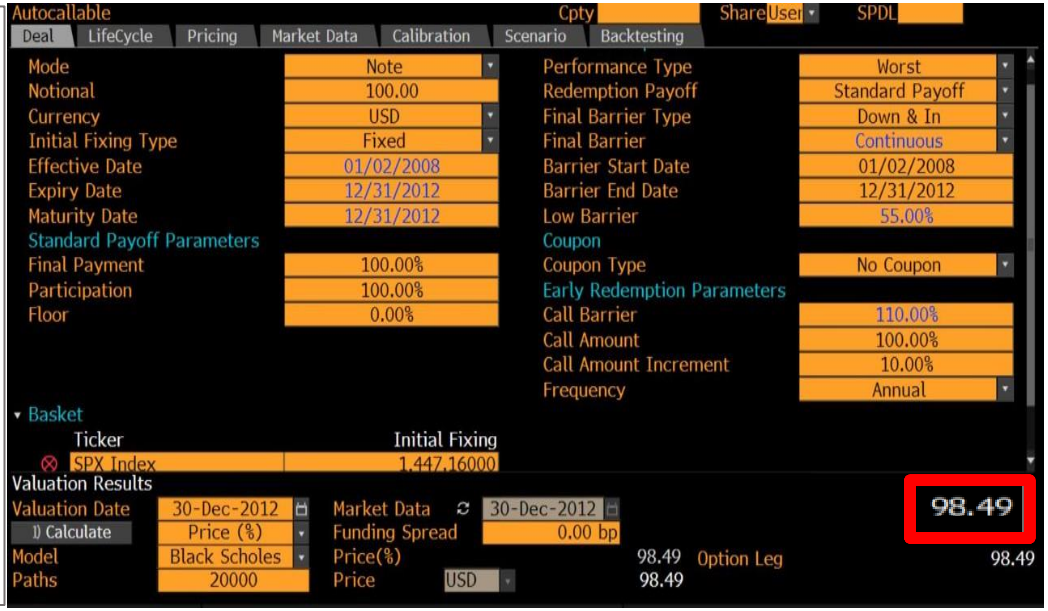
Replica Portfolio Excel

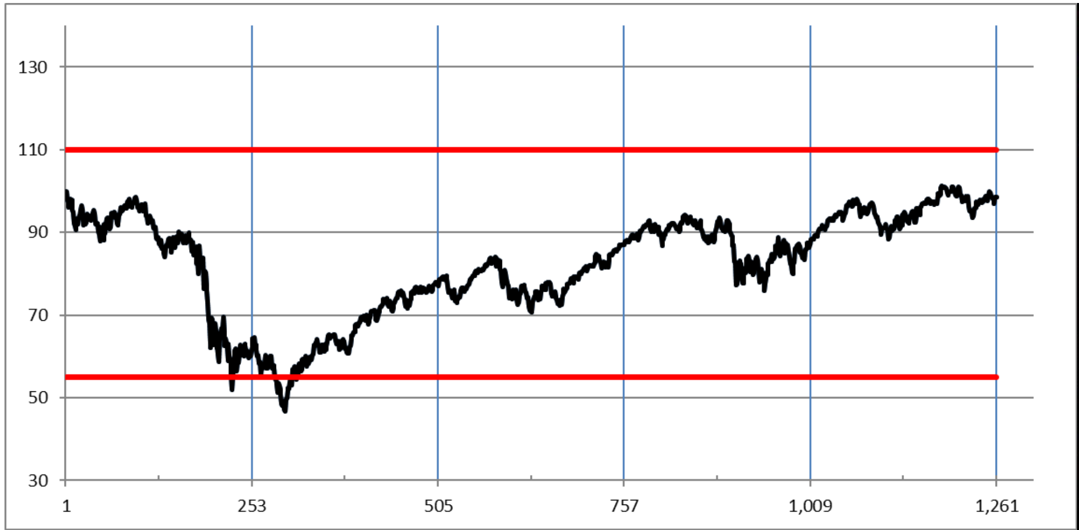
Year	1	2	3	4	5		
Row No.	257	509	761	1015	1265		
n	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	Min{S_t}	Max{S_t}
S(t)	**62.4**	**77.1**	**86.9**	**86.9**	**98.6**	46.75	101.3
C(t)	10%	20%	30%	40%	50%		
Coupons	-	-	-	-	-		
Early Redemption	-	-	-	-			
Final Redemption					**100.00**		
				-	**1.45**		
Total Payment	-	-	-	-	**98.55**		
- 100.0	-	-	-	-	98.55		

Autocall Barrier	110%
Lower Protection Barrier	55%
Knock-In Put Conventional Strike	100%
Y5 Knock-Out Barrier	0%

$\boxed{98.55}$

Bloomberg Product Quote





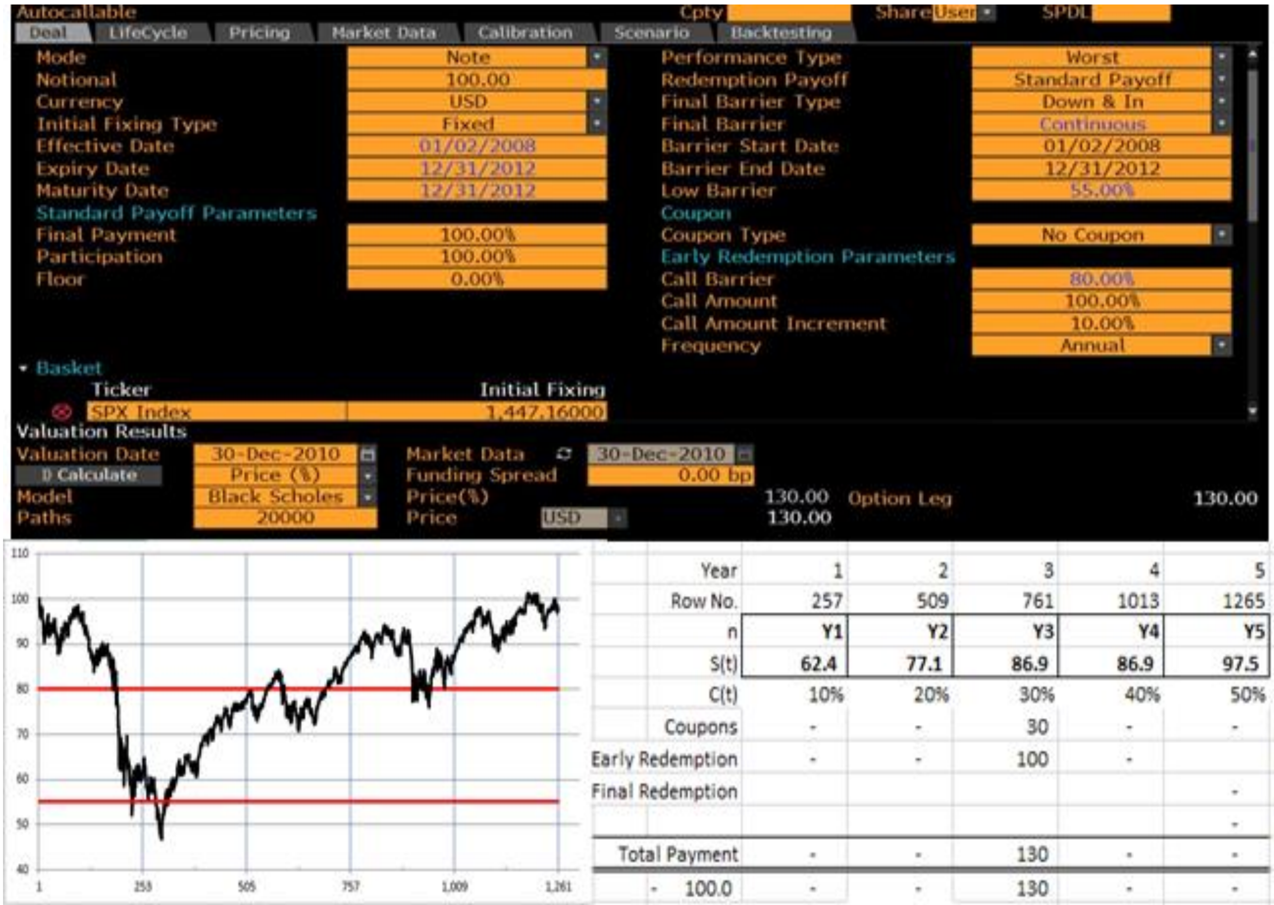
Secondly, an examination of different barrier levels will be undertaken which will allow for an evaluation of optimal market scenarios, whereby the auto-call barrier was breached, and the protection barrier was not breached. Firstly, a 90% auto-call barrier with the protection barrier set to 40% was assessed. The impact of this change in protection barrier and auto-call barrier results in the underlying asset breaching the auto-call barrier being breached only at the year of maturity (Figure 1.8), resulting in the investor receiving enhanced coupons and a final redemption of 100 FV. The autocallable note was subsequently priced high, at 150.

Figure 1.8 Alternative Barriers & Bloomberg Price

Year	1	2	3	4	5
Row No.	257	509	761	1013	1265
n	Y1	Y2	Y3	Y4	Y5
S(t)	62.4	77.1	86.9	86.9	103.1
C(t)	10%	20%	30%	40%	50%
Coupons	-	-	-	-	50
Early Redemption	-	-	-	-	
Final Redemption					100
					-
Total Payment	-	-	-	-	150.0
- 100.0	-	-	-	-	150.0

On the following page, the subsequent scenario comprises of an auto-call barrier at 80% and the protection barrier at 55%. This will highlight the premature redemption component of the product. The resulting consequence of the change in barrier level will be the early redemption of the product in the 3rd year. This is a positive situation for an investor allowing for 100% of the face value to be redeemed, in conjunction with the coupons that would have been received over the 3 years. The auto call barrier would be surpassed by the long KO put on the 3rd specified anniversary date. (Figure 1.9)

Figure 1.9 Alternative Barriers & Bloomberg Price



1.6 Risk-Return Evaluation

As noted in the previous sections, the downward protection barrier of 55% was breached shortly after the product commenced, thus exposing the investor to significant downside risk dependant on the index's path. This ultimately led to a capital loss of -1.45% to the investor, as the auto-call barrier of 110% was never breached. It is therefore prudent to analyse the autocallable note under various market conditions. By stress testing the profitability of the autocallable notes in conditions of varying returns and volatility, a more comprehensive profile of the risk and return characteristics of the investment product can be generated. To achieve this, three market conditions have been chosen.

- **Bear Market** - Approximately **-10% Return**
- **Flat Market** - Approximately ~**0% Return**
- **Bull Market** - Approximately **+10% Return**

Investment profiles such as those offered by (Citigroup 2007) have failed to inform investors of how the product performs in poor market conditions, instead choosing to focus on more stable and ideal market conditions. This has been criticised as irresponsible to investors, as it does not allow them to appreciate the full spectrum of risks involved in entering into such a contract.

A Monte Carlo Simulation methodology has been chosen to analyse the autocallable note over the three distinct market environments. One-year historical time periods for the S&P 500 have been selected – not necessarily a part of the 2008-2012 product period – to investigate the viability of this product for investors. Average drift and daily volatility data have been extracted from these time series to reflect the three market environments noted above. The Monte Carlo Simulations will then be run to investigate the probability of preferential outcomes for an investor in this autocallable note. As the data is based on real historical periods of the S&P 500, these scenarios reflect realistic scenarios which investors may encounter and therefore need to be aware of prior to purchasing investment products such as autocallable notes.

Bear Market Environment: (-10%)

To illustrate the risk and return profile during a bear market environment, the period from the 01/01/2000 to 12/31/2000 has been chosen. Over this time period, the S&P 500 experienced a one-year loss of 10.14%, making it an appropriate period for this analysis. This period and the evident bearish environment is graphed below in Figure 1.10

Figure 1.10 Bear Market Scenario

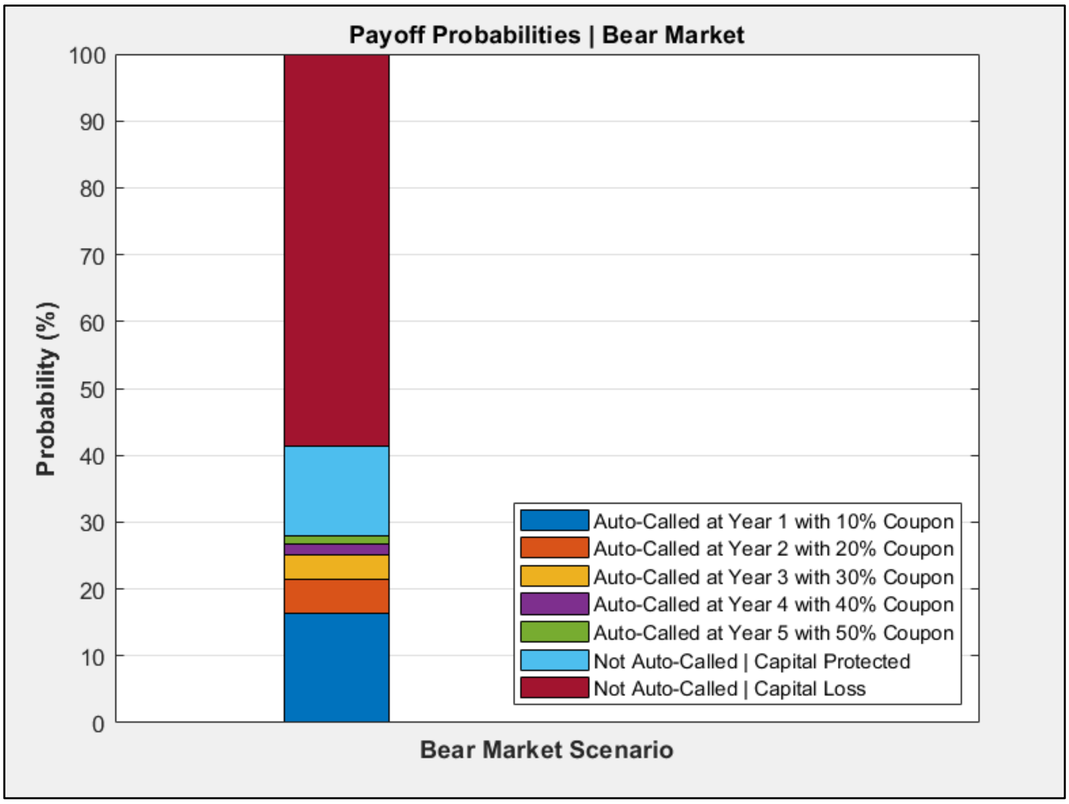
A Monte Carlo Simulation was used to analyse the autocallable note, using the time series characteristics derived from the above period. Firstly, daily prices were sourced from Bloomberg and were subsequently converted into returns in order to calculate the daily volatility. This daily volatility figure scaled for 252 trading days – calculated below – in addition to the average drift figure (-10.14%) was inputted into the Monte Carlo Simulation in order to simulate the probability of the option being autocalled over the time period. These probabilities have been illustrated in the table below and in Figure 1.11.

$$\sigma_{ANNUAL} = \sigma_{DAILY} * \sqrt{252}$$

$$\sigma_{ANNUAL} = 1.40\% * \sqrt{252}$$

$$\sigma_{ANNUAL} = 22.21\ \%$$

Figure 1.11 | Payoff Probabilities – Bear Market



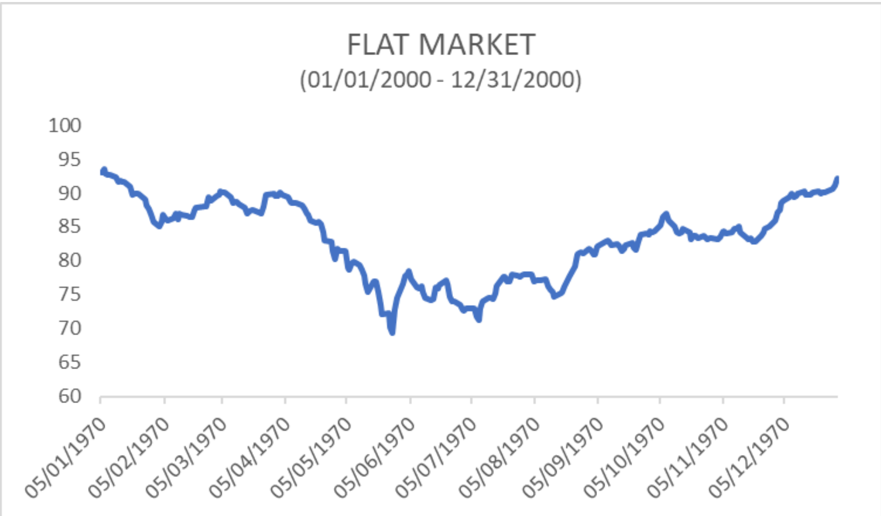
Bear Market Payoff Probabilities	
Auto-Called Year 1	**16.4** %
Auto-Called Year 2	**5.1** %
Auto-Called Year 3	**3.6** %
Auto-Called Year 4	**1.7** %
Auto-Called Year 5	**1.34** %
Not Auto-Called | Capital Protected	**13.4** %
Not Auto-Called | Capital Loss	**58.6** %

An interesting distribution of probabilities was observed for the bear market scenario. As can be seen, there is a significant probability (58.6%) that the autocallable note will not be auto-called, and the investor will incur a capital loss at maturity. A probability of 58.6% highlights the danger to investors in these poor market conditions. However, there is a 13.4% probability of the note returning a face value at maturity if it is not auto-called. As can be seen in the price plot above of the S&P (Figure 1.10), the market did in fact experience both upward jumps as well as downward jumps, ultimately finishing approximately 10.14% down end-of-year. Although 13.4% is not significantly high, it does illustrate how holders of autocallable notes may at times be fortunate during times of increased volatility. Despite this, the total probability of 72% for the product not being auto-called illustrates a significantly risky investment for potential investors.

Flat Market Environment: (0%)

To investigate the autocallable note's performance in a stable market environment, the period 01/01/1970 to 12/31/1970 was chosen. Over this time period, the S&P stayed approximately flat, realising a total gain of just 0.10% over the year. The index's flat performance has been graphed below in (Figure 1.12).

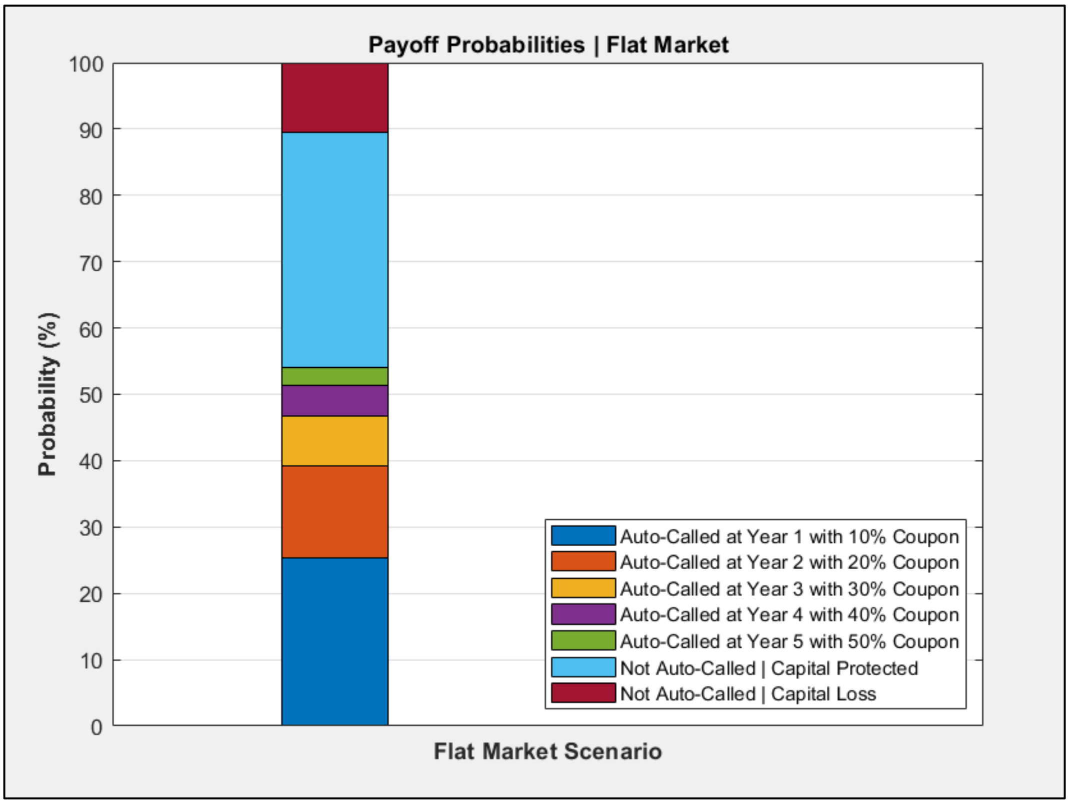
Figure 1.12 | Flat Market Scenario



Like before, an average drift of 0% was assumed and daily volatility – *calculated below* – was extracted from the dataset for use in the Monte Carlo Simulation. A higher volatility was observed as opposed to the bull market environment. As Mariathasan (2019) notes, low volatility is typically observed alongside period of high growth, so this was to be expected. Again, probabilistic estimates for the various autocallable outcomes were calculated to provide an insight into the level of risk associated with purchasing this structured product during a flat market environment (Figure 1.13)

$$\sigma_{ANNUAL} = \sigma_{DAILY} * \sqrt{252}$$
$$\sigma_{ANNUAL} = 0.96\% * \sqrt{252}$$
$$\sigma_{ANNUAL} = 15.20\ \%$$

Figure 1.13 | Payoff Probabilities – Flat Market



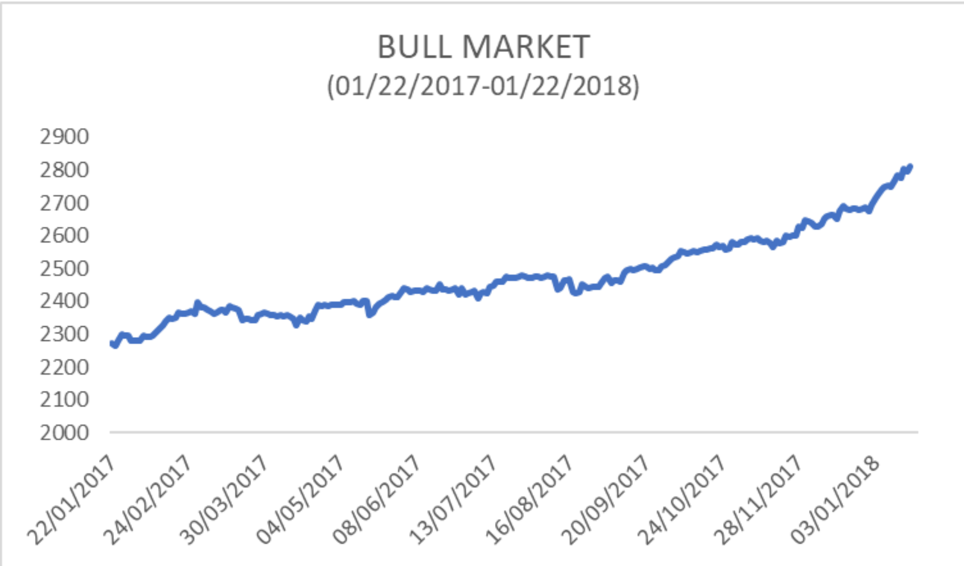
Flat Market Payoff Probabilities	
Auto-Called Year 1	**23.3** %
Auto-Called Year 2	**13.3** %
Auto-Called Year 3	**6.7** %
Auto-Called Year 4	**4.1** %
Auto-Called Year 5	**4.1** %
Not Auto-Called | Capital Protected	**37.6** %
Not Auto-Called | Capital Loss	**10.9** %

For a flat market scenario, there is a total probability of 51.5% that the autocallable note will be retired early over the 5-year window, with both a return of the full-face value and contingent coupons received by the investor. With this positive outlook, it must be noted that there is still a 10.9% that the investor will incur a total capital loss, with a total probability of 48.5% that the note will not be auto-called. The flat market scenario, like the bear market scenario also experiences upward and downward jumps before finishing the year with an approximate return of 0%. These jumps in a flat market again highlight an almost 50/50 probability of the autocallable note being auto-called. However, the 37.6% probability of the investor's capital being protected despite the note not being auto-called highlights a significantly reduced risk profile for a flat market scenario, relative to a bear market.

Bull Market Environment: (+10%)

To examine the autocallable's performance in a bull market scenario, the period of 01/22/2017 to 01/22/2018 has been selected. The bullish environment and positive returns of the index have been graphed below in Figure 1.14

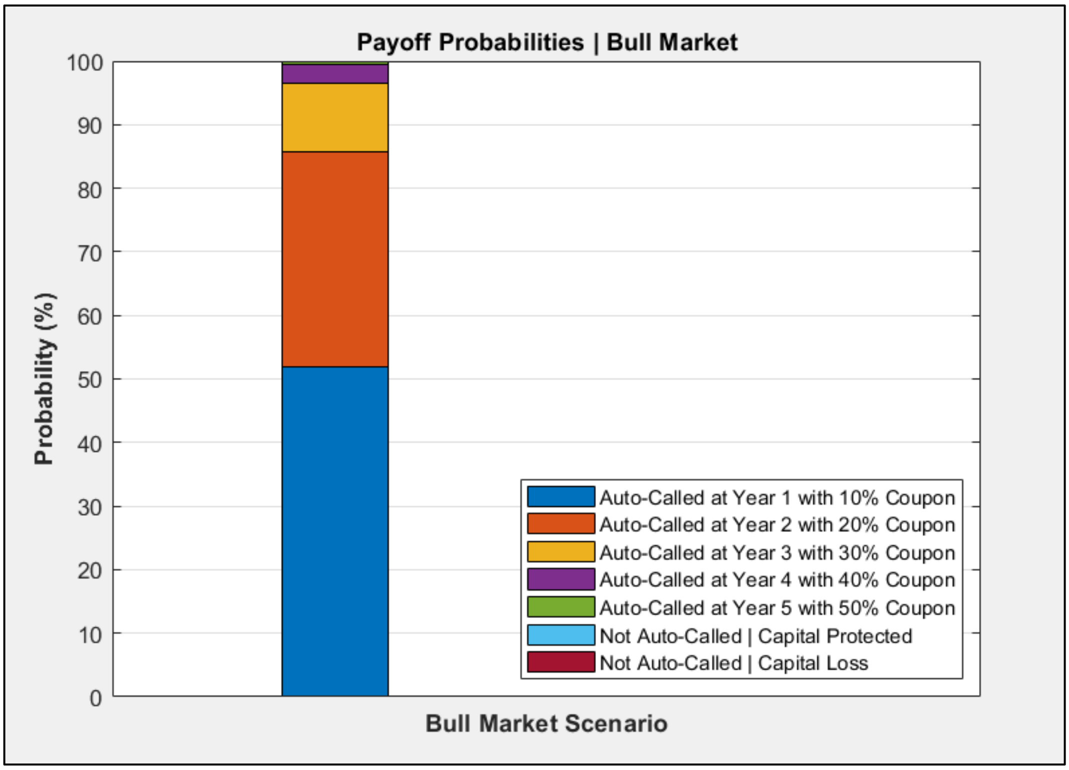
Figure 1.14 | Bull Market Scenario



The daily return standard deviation was once more extracted using the daily prices obtained from the Bloomberg Terminal. The daily return standard deviation was calculated to be 0.43%. This was then scaled to calculate the annual volatility which we calculated to be 6.79% shown below. With the use of the annual volatility figure calculated in conjunction with the 10% prescribed annual drift value, a Monte Carlo Simulation was once more undertaken to calculate the probability of each scenario for the 5-year autocallable product. Using a stacked bar chart, we then analysed the level of risks related to each of the outcomes, assigning seven of the autocallable outcomes a probabilistic value (Figure 1.15).

$$\boldsymbol{\sigma_{ANNUAL}} = \boldsymbol{\sigma_{DAILY}} * \sqrt{\mathbf{252}}$$
$$\boldsymbol{\sigma_{ANNUAL}} = \mathbf{0.43}\% * \sqrt{\mathbf{252}}$$
$$\boldsymbol{\sigma_{ANNUAL}} = \mathbf{6.79}\,\%$$

Figure 1.15 | Payoff Probabilities – Bull Market



Bull Market Payoff Probabilities	
Auto-Called Year 1	**51.9** %
Auto-Called Year 2	**33.8** %
Auto-Called Year 3	**10.8** %
Auto-Called Year 4	**3.0** %
Auto-Called Year 5	**0.3** %
Not Auto-Called | Capital Protected	**0.2** %
Not Auto-Called | Capital Loss	**0** %

The results of the bullish scenario with regard to the autocallable product can be seen in Figure 1.15 using a payoff probabilities graph. The most likely scenario in this instance is the autocallable product being auto-called in year 1, with a likelihood of 51.9%, including a 10% coupon and capital protection. The difference is obvious between the bullish scenario and the bearish scenario, with the capital loss in this scenario given to be 0%. The next most likely scenario, at 33.8%, would lead to the product being auto-called in year 2. This likelihood is illustrated in the plot of the S&P over the time period shown above, which trends predominantly upwards. Hidden among the enhanced coupons and extra returns, it is clear that the risks associated with this investment product lie beneath the lure of enhanced yields, with investors lacking the knowledge or understanding of the risks associated with the product over the various market environments.

1.7 Investment Summary | Conclusion

Autocallable notes offer investors a number of attractive features, including capital protection, early redemption, and the possibility of enhanced coupons. Whilst these features are easily marketable to potential investors, the performance of such a structured investment product must be stress-tested, as is abundantly clear from the above paper. An accurate presentation of the risk profile associated with investment products is not only financially responsible, but also ethically necessary.

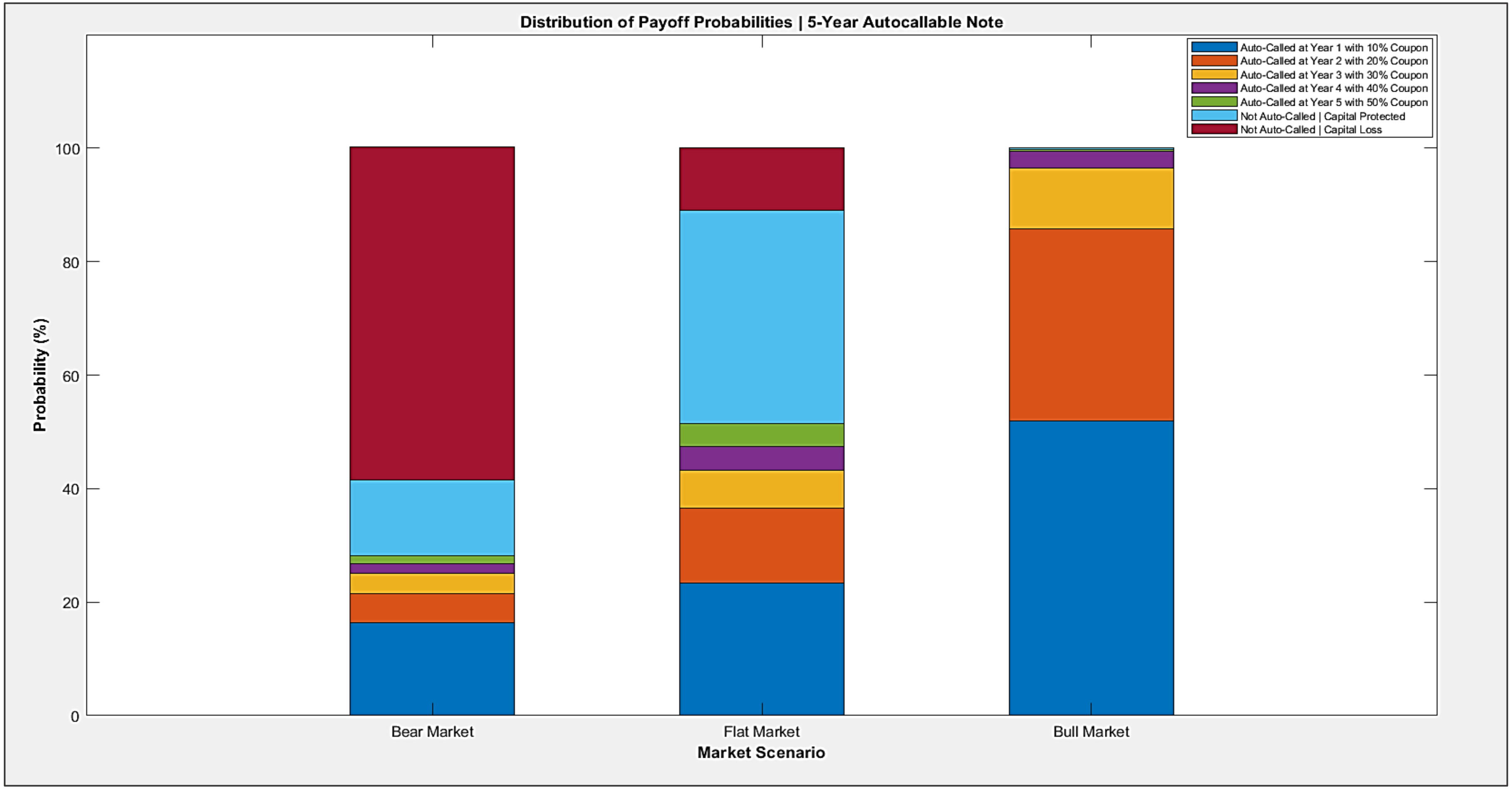
In this paper, the DLIB-AC function in Bloomberg was utilised to price the 5 year autocallable from 2008 and terminating in 2012, which was then compared to the real-time movements of the SPX. To underline the investment risks associated with autocallable products we utilised a GBM Monte-Carlo simulation to simulate the market environment scenario analysis of the autocallable product. Under both the bearish and flat scenario market environments, it was estimated that the autocallable note would not be 'auto-called'. However, under the flat market scenario there was a reduced risk, with the investor's capital remaining protected for a significant percent of the time. The bearish market environment led to the investor incurring a capital loss with a likelihood of 58.6%. The bullish market scenario had a cumulative likelihood of 99.7% that the autocallable note would be retired over the 5-year period. The 0% capital loss associated with the bullish scenario highlights the reward this structured product offers, but also the many risks that may accompany it.

This analysis was completed in conjunction within the specified 2008-2012 time period for the SPX Index, with a chosen 110% auto-call barrier and 55% lower protection barrier. Below, the distribution of payoff probabilities between the three market scenarios has been illustrated side-by-side (Figure 1.16). The significant discrepancy between market states illustrates a noteworthy risk profile for investors. Although the product performs well in a bull market environment – as most products will – there is a significant discrepancy between bull, flat and bear market scenarios. There is a 72% and 48.5% probability that the product will not be autocalled in bear and flat market scenarios, respectively. With this, there is a 58.6% and 10.9% probability of a capital loss being realised. These results must be considered by investors, as all market states were based on real historical periods of the S&P 500.

The 5-year autocallable note analysis spanning 2008-2012 presented a similar result. The contingent coupons never materialised over the period, as the index failed to breach the 110% autocallable barrier. Not only this, but less than a year after the period commenced the lower protection barrier of 55% was breached, due to the onset of the global financial crisis. Ultimately, an investor in this autocallable note would have realised a capital loss of 1.45%.

From this, it is concluded that a structured investment product such as this autocallable note is suitable for investors with a high-risk tolerance, and ideally, of a high net-worth. Investors will be required to withstand a potentially long waiting period before receiving a principal repayment. However, that is not a certainty, as capital preservation is not guaranteed. Additionally, there is no liquid secondary market for autocallable notes. Therefore, to safely to avail of the potentially high returns, investors must be willing to forego interest payments, dividends and accept the possibility of a substantial loss of principal and returns. (Merrill Lynch, 2018). In conclusion, the substantial risks associated with autocallable investment products must be relayed to potential investors. As such, investors should be of high net-worth and without the necessity for short-term liquidity, in order to tolerate the associated risks of autocallable structured investments.

Figure 1.16 | Distribution of Payoff Probabilities



Market Scenario	Bear	Flat	Bull
Auto-Called Year 1	**16.4** %	**23.3** %	**51.9** %
Auto-Called Year 2	**5.1** %	**13.3** %	**33.8** %
Auto-Called Year 3	**3.6** %	**6.7** %	**10.8** %
Auto-Called Year 4	**1.7** %	**4.1** %	**3.0** %
Auto-Called Year 5	**1.34** %	**4.1** %	**0.3** %
Not Auto-Called | Capital Protected	**13.4** %	**37.6** %	**0.2** %
Not Auto-Called | Capital Loss	**58.6** %	**10.9** %	**0** %

Section 2

2.1 Introduction: Ladder Forward Options

Ladder options, also known as lock-in options, are synthetic financial products that contain a combination of vanilla put and call options, along with a set of barrier options. These options are assigned an appropriate set of moneyness levels called as rung levels. Anytime the underlying asset breaches the fixed moneyness level or rung level before the maturity date, the ladder option locks in a profit, regardless of the path taken by the asset after breaching the rung level, even if it finishes out of the money (OTM) upon reaching maturity. Based on the characteristics of the option, these options can be referred to as path-dependent products. This product type is popular among investors due to the ability of the product to lock-in any upward movement exposure. Although this may make the product seem attractive, the price of these options increases when more rung levels are inputted. To combat this, a short ladder put position is often added along with long ladder call position in order to neutralise the increasing price effect.

LADDER PUT	**LADDER CALL**
The rung levels are assigned below the strike price	The rung levels are assigned above the strike price
The payoff is the difference between the lowest rung level breached and the strike price during the life of the option	The payoff is the difference between the highest rung level breached and the strike price during the life of the option
Ladder Put Payoff = max[max{K - S_T, K - R},0] where K = strike R = Rung level breached $S_{T=}$ Asset price at maturity	**Ladder Call Payoff = max[max{S_T - K, R - K},0]** where K = strike R = Rung level breached $S_{T=}$ Asset price at maturity

Figure 2.1 - SPX Prices from 01/09/2019 to 28/02/2020

Figure 2.1 illustrated the S&P 500 Index prices between October 2019 to February 2020. With the strike at 2900 and the rung level set at 3200, a Long ladder call position will lock-in a profit as the rung level was breached during the life of the option. If there were multiple rung levels set, a profit could have been locked-in for every rung level breached. It can be also noted that in case the price hits none of the rung levels that were set, the payoff of the product will be the same as that of a regular call option.

2.2 Constructing a Ladder Forward Option:

A ladder option allows the holder to lock-in an intrinsic value if the rung levels are breached during the life an option. In order to engineer a ladder forward option, we require a combination of vanilla calls, puts and knock out barrier options for this purpose. In the first part of this section we will discuss the engineering of a 5Y long Ladder Forward using an appropriate replicating portfolio of ladder call and ladder put options. We use the Bloomberg's OVME multi leg structuring tool for this purpose. We will then discuss the consistency of the price of the option by comparing it with ladder options with rebates and lookback options with limited rungs.

In order to engineer the Ladder option, we need to select the appropriate strike level. The conventional strike is the 100% ATM forward rate. This ATMF rate is derived using the concept of put-call parity. The pricing method of the forward price of a put and call option with same strike K and same time to maturity T is given by the Put-Call Parity. Considering two portfolios, one comprising a call option and cash and the other with a put option and an underlying stock. Both the portfolios are worth S_T at time T when $S_T > K$ and both the portfolios are worth K when $S_T < K$. Hence, we can say that both are worth max ($S_{T,}$ K) at expiration T. Since European options cannot be exercised prior to expiry and they have the same value at time T, they must have the same values at initiation otherwise there would be an arbitrage opportunity. Hence on initiation our two portfolios have the values of c and Ke^{-rT} in the first portfolio and p and S_0 in the second one. Hence as of time $T_{0,}$ the value of the portfolios are given by the below equation:

$$c + Ke^{-rT} = p + S0$$

For a dividend paying stock we could write the same equation as following, where q is the dividend yield

$$c + Ke^{-(r-q)t} = p + S_0 e^{-qt}$$

The options in the portfolio do not pay dividends but the underlying stocks in them do. Hence, we account for this while pricing the forwards. The equation can also be represented as

$$c - p = DF(F - K)$$

Long Call + Short Put = Discounted (Forward Price - Forward Strike)

Hence from the above equation we can say that options should be priced in such a way that the difference between the call and put price at initiation is 0. Hence for a 5-year period with a dividend yield of 1.783% we calculated the forward price on the spreadsheet to be 1621.54. We compared it with the Bloomberg value of 1623.850. We will use strike at ATMF=1623.850 price to maintain consistency in creating the ladder options.

2.2.1 Rung Levels:

After selecting an appropriate level of ATMF used to price the option, we had to decide on appropriate rung levels and the distance between each of these rungs. The rungs in a ladder call option provide an opportunity to ‘lock-in’ a profits as soon as the price hits the rung levels and hence on expiry if the price of the option is below the rung levels we would have already locked-in a profit from the rungs in the ladder option thus increasing profitability and reducing price risks. The highest level and the lowest level of the rung offers the maximum payoff if the price hits the highest rung level hence the broker will be able to offer a higher return on investment. However, this return is contingent on the probability of the price hitting the particular rung level.

Hence, we used the probability of the stock price hitting the rungs in the construction of our ladder option. We simulated 10000 simulations of the stock price in MATLAB using the Monte Carlo method with Black Scholes constant volatility model. We then considered the highest price level reached by each of the simulations and counted the number of times the price was higher than the barrier level. We then divided the value by the total number of simulations to arrive at the probabilities for each of the rungs to be hit. The calculated the probabilities for two intervals i.e. 10% and 20% intervals between rungs. The results are in table 1 and table 2 and table 3.

Table 1: 2.5% Rung Interval

2.5% Rung level	**Probability of Hits**
90%	35.16%
92.50%	47.74%
95%	62.21%
97.50%	77.95%
102.5	80.95%
105%	65.34%
107.5%	52.50%
110%	41.82%
112.5%	32.17%
115%	23.94%
117.5%	17.65%
120%	12.71%
125%	6.08%

Table 2: 5% Rung Interval

5% Rung	**Probability of Hits**
80%	5.39%
85%	16.11%
90%	35.16%
95%	62.21%
105%	65.34%
110%	41.82%
115%	23.94%
120%	12.71%
125%	6.08%

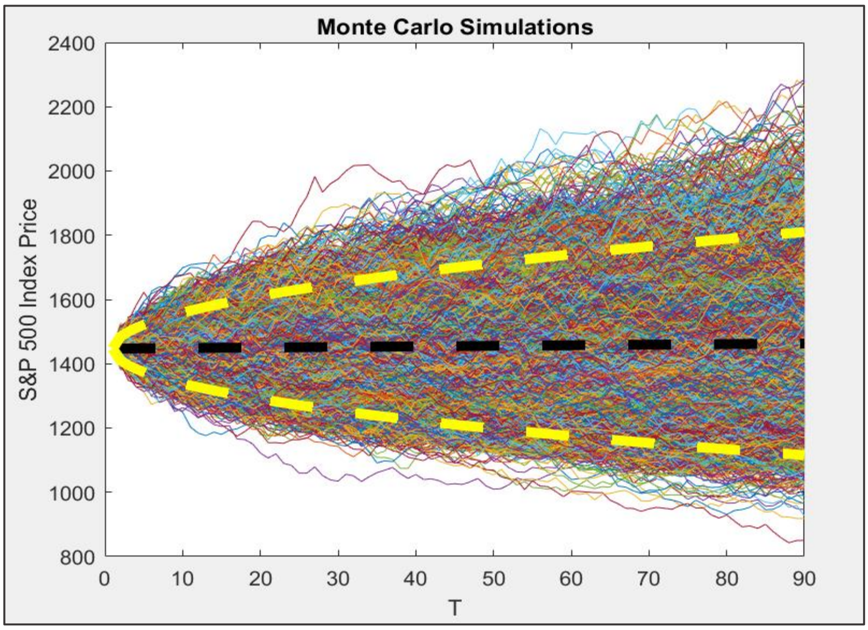
Table 3: 10% Rung Interval

10% Rung	**Probability of Hits**
60%	0.0001%
70%	0.19%
80%	5.39%
90%	35.16%
110%	41.82%
120.0%	12.71%
130.0%	29.10%
140.0%	0.50%
150.0%	0.0006%

As we can see from the above tables that the probabilities of the rungs offering the highest payoffs decreases with increasing rung levels. We calculated the probabilities of an option with 10%, 5% and 2.5% intervals between rungs. We found that the after 4 rung levels on either side of the strike, the probabilities of hitting subsequent rungs in the trials are less than 10% in table 2 and table 3. Choosing smaller intervals of 2.5% will help us capture the probabilities better but the increase in model complexity without a corresponding increase in the performance leads us to evaluate the trade-off between model complexity and performance. For instance, we see that with 4 rungs capturing up to 12.71% of the probability on the upside in table 2 will be require 8 rungs to capture the same probabilities in table 1 for a ladder call option. Hence, we choose the intervals of 5% between rungs and cap it at 4 rungs on either side of the strike.

We also plotted the Monte-Carlo simulations on MATLAB and plotted the 95 percent confidence interval to capture the upper and lower bonds as shown in figure 2.2. We calculated the maximum and minimum price points reached by the upper and lower bounds of the confidence intervals plotted. We found that 1810.30 was the maximum level of the upper bound and 1447.203 was minimum level reached by the lower bound. Hence our upper and lower most rungs at 1948.60 and 1299.10 capture all the movements in the 95% confidence interval. Confidence intervals provide the strength of statistical significance and express the possible range of true values adequately when compared to the other measures of uncertainty such as p-values (Du Prel, Hommel, Röhrig and Blettner, 2009).

Fig 2.2: Monte-Carlo Simulations with 95% Confidence Intervals



2.2.2 Ladder Call Option:

In this section, we describe the ladder call option used to benefit from the upside movement in prices. We use a 4 rung ladder call to construct our portfolio which consists of the vanilla calls, vanilla puts and knock out options. The profit locked in by a particular rung level is given by Max (R-K, 0) where R is the rung level and K is the strike. The profit is locked in if the price hits the underlying at any time before expiry and ends below the rung levels. The payoff on expiration date of our 4 rung ladder call with rungs R1, R2, R3, R4 are as below:

$$Max\ [Max\ \{S_T - K;\ R_i^* - K\},\ 0]$$

Where R_i^* denotes the maximum rung level reached until expiration.

The payoff from the ladder call option is plotted in figure 2.3. A ladder call is created using a combination of synthetic long positions, which is a combination of long and short vanilla puts which allows the holder to fully participate in the upside price movements. The remaining rung levels can be engineered using the combination of put spreads and knock out put spreads. We construct the 4 rung ladder call in the SPX index using a standard 2 rung ladder call and an additional 2 rungs. The positions are described in table 4. The offsetting positions are markets as cancelled.

Figure 2.4 describes the 4 RLC payoff profile as seen in OSA on Bloomberg. The payoff profile compares ladder call payoff with the vanilla call payoff. We notice that the market value of the 4 RLC is always higher than that of the vanilla call payoff. This is due to the additional lock-in feature which comes at an additional cost. The payoff diagram also depicts the presence of a ‘kink’ near the rung levels as the rising price has an ever increasing chance of locking in a payoff as it approaches the rung level and then subsequently has a reducing premium after the rung level is breached. The value of the ladder call asymptotically approaches the value of the of the vanilla call option for higher price levels. This is due to the intrinsic value of the option being the dominant factor in both the deep ITM calls and hence the ladder call and the vanilla calls market values are dominated by this factor.

Table 4: Composition of a Ladder Call Position

Position	Components	Strike Interval
1	$+P^{KO}(X = R_3, X^{OS} = R_4)$ $-P^{KO}(X = R_4, X^{OS} = R_4)$	Knock-out put spread Spanning $[R_3\ R_4]$
2	$+P^{KO}(X = R_2, X^{OS} = R_3)$ $-P^{KO}(X = R_3, X^{OS} = R_3)$	Knock-out put spread Spanning $[R_2\ R_3]$
3	$+P^{KO}(X = R_1, X^{OS} = R_2)$ $-P^{KO}(X = R_2, X^{OS} = R_2)$	Knock-out put spread Spanning $[R_1\ R_2]$
4	$+P^{KO}(X = S, X^{OS} = R_1)$ $-P^{KO}(X = R_1, X^{OS} = R_1)$	Knock-out put spread Spanning $[S_1\ R_1]$
5	$\cancel{-P\ (X = R_3)}$ $+P\ (X = R_4)$	Vanilla put spread Spanning $[R_3\ R_4]$
6	$\cancel{-P\ (X = R_2)}$ $\cancel{+P\ (X = R_3)}$	Vanilla put spread Spanning $[R_2\ R_3]$
7	$\cancel{-P\ (X = R_1)}$ $\cancel{+P\ (X = R_2)}$	Vanilla put spread Spanning $[R_1\ R_2]$
8	$-P\ (X = S)$ $\cancel{+P\ (X = R_1)}$	Vanilla put spread Spanning $[S\ R_1]$
9	$+C\ (X = S)$	Strike

Figure 2.3 - Ladder Call Payoff

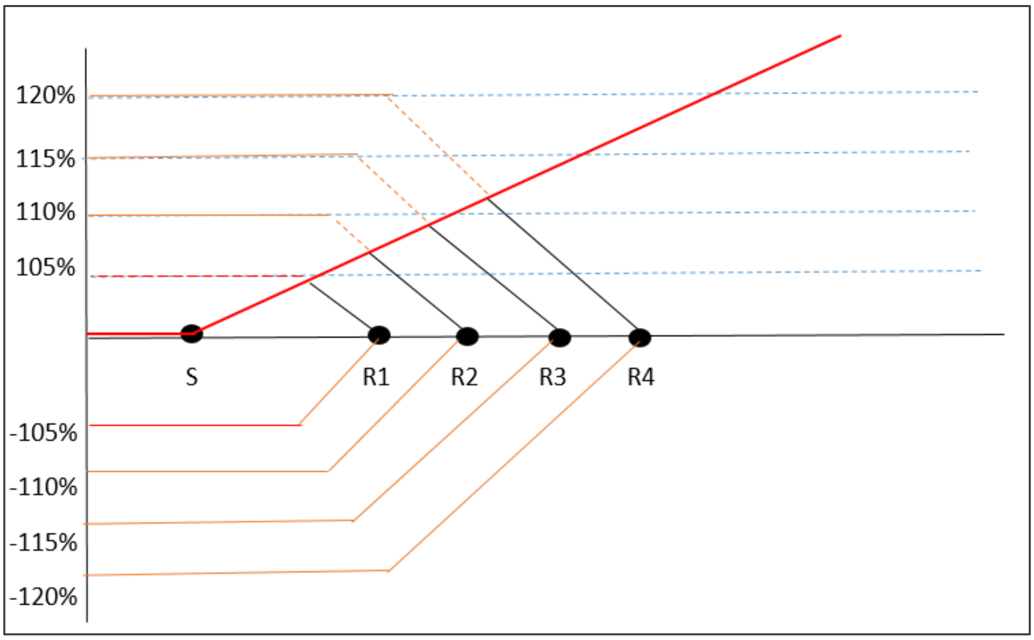
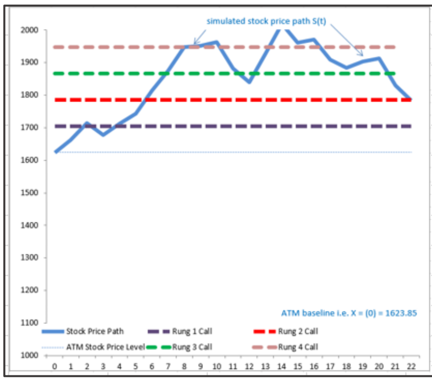


Figure 2.4 - Payoff Profile of a Replicating Portfolio for a 4 Rung Ladder Call

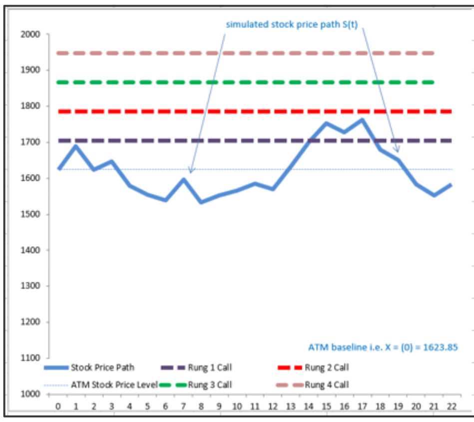
In order to gain an intuitive understanding of the ladder call option used the excel spreadsheet to simulate the 4RLC for different scenarios and simulated price paths based on Black Scholes constant volatility model. We embedded the payoffs with respect to each of the positions and calculated the total payoff of reach of the scenarios. The scenarios and the constituent payoffs are described as below:

Figure 2.5 - Scenario with all rungs breached and subsequent payoffs



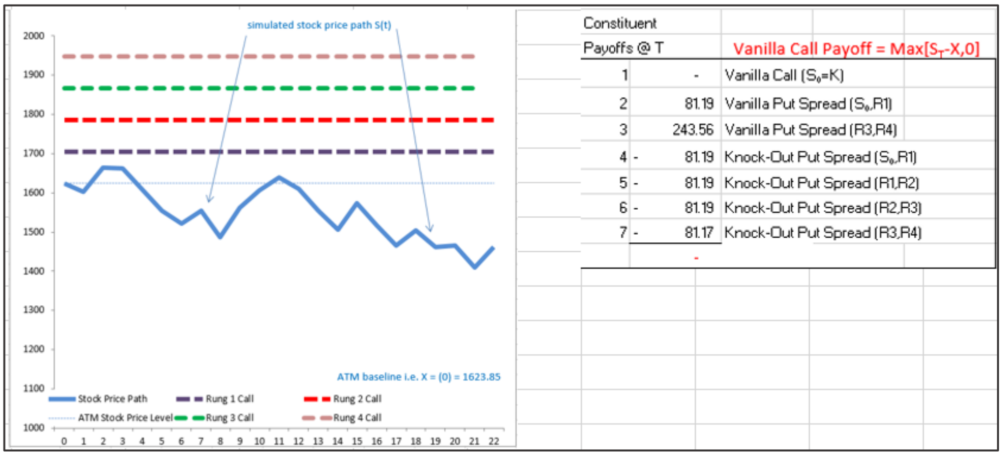
Payoffs @ T		Vanilla Call Payoff = Max[S_T-X,0]
1	160.33	Vanilla Call (S_0=K)
2	-	Vanilla Put Spread (S_0,R1)
3	164.42	Vanilla Put Spread (R3,R4)
4	-	Knock-Out Put Spread (S_0,R1)
5	-	Knock-Out Put Spread (R1,R2)
6	-	Knock-Out Put Spread (R2,R3)
7	-	Knock-Out Put Spread (R3,R4)
	324.75	

Figure 2.6 - Scenario with 1 rung breached and subsequent payoffs



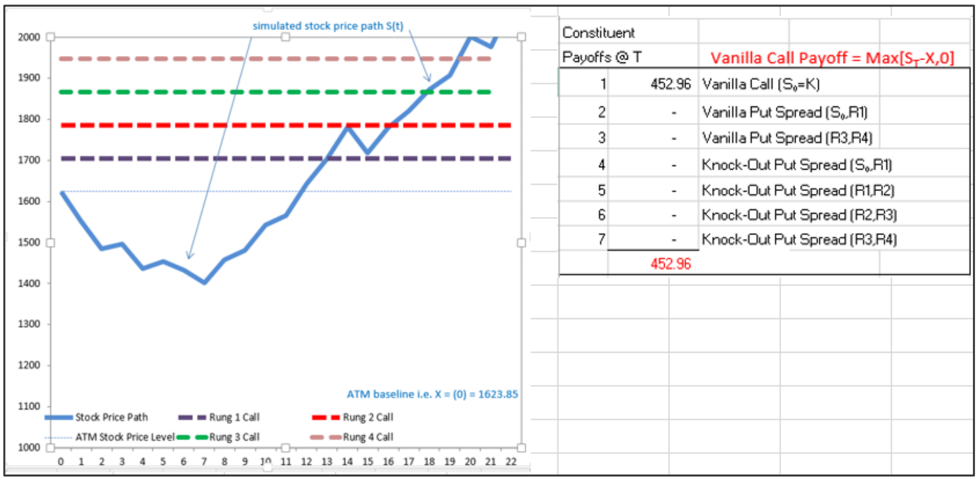
Constituent Payoffs @ T		Vanilla Call Payoff = Max[S_T-X,0]
1	-	Vanilla Call (S_0=K)
2	81.19	Vanilla Put Spread (S_0,R1)
3	243.56	Vanilla Put Spread (R3,R4)
4	-	Knock-Out Put Spread (S_0,R1)
5	- 81.19	Knock-Out Put Spread (R1,R2)
6	- 81.19	Knock-Out Put Spread (R2,R3)
7	- 81.17	Knock-Out Put Spread (R3,R4)
	81.19	

Figure 2.7: Scenario with no rungs breached and subsequent payoffs



Constituent Payoffs @ T		Vanilla Call Payoff = Max[S_T-X,0]
1	-	Vanilla Call (S_0=K)
2	81.19	Vanilla Put Spread (S_0,R1)
3	243.56	Vanilla Put Spread (R3,R4)
4	- 81.19	Knock-Out Put Spread (S_0,R1)
5	- 81.19	Knock-Out Put Spread (R1,R2)
6	- 81.19	Knock-Out Put Spread (R2,R3)
7	- 81.17	Knock-Out Put Spread (R3,R4)
	-	

Figure 2.8: Scenario with S_T > highest rung level and subsequent payoffs



Constituent Payoffs @ T		Vanilla Call Payoff = Max[S_T-X,0]
1	452.96	Vanilla Call (S_0=K)
2	-	Vanilla Put Spread (S_0,R1)
3	-	Vanilla Put Spread (R3,R4)
4	-	Knock-Out Put Spread (S_0,R1)
5	-	Knock-Out Put Spread (R1,R2)
6	-	Knock-Out Put Spread (R2,R3)
7	-	Knock-Out Put Spread (R3,R4)
	452.96	

Figure 2.5, the first scenario shows the stock price breaching all the rungs to end above the second rung level. The corresponding payoffs show that each of the rung levels have locked in a profit corresponding to rungs breached. Hence the payoff is $324.75 which is equivalent to ($81.19x4), $81.19 being the difference between each rung levels in dollar terms. Similarly, the second scenario in figure 2.6, shows one rung breached and price ending below strike on expiry. Hence the payoff is equal to $81.19. The third scenario in figure 2.7 shows none of the rung levels breached and the stock ending below the strike in which case the payoff from the 4RLC will be $0. The last scenario in figure 2.8 shows the stock price breaching all the rungs and staying above the highest rung level at expiry. Hence the payoff in this case will be equal to that of the vanilla call option i.e. $452.96.

2.2.3 Ladder Put Option:

The ladder put benefits from the downward price movements in the stock price. We use a 4 rung short ladder put which is symmetrical to the 4 rung ladder call in our replicating portfolio. The profit is locked in at each rung level is given by Max (K-R, 0). The payoff at expiry is given by the function:

- Max [Max {K - S_T; K - R_i^*}, 0]

Where R_i^* denotes the maximum rung level reached until expiration.

The payoff from the ladder put option is plotted in figure 2.9. The ladder put consists of a combination of call spreads and knock-out call spreads. Table 5 shows the components of the 4RLP.

Figure 2.9 - Ladder Put Payoff

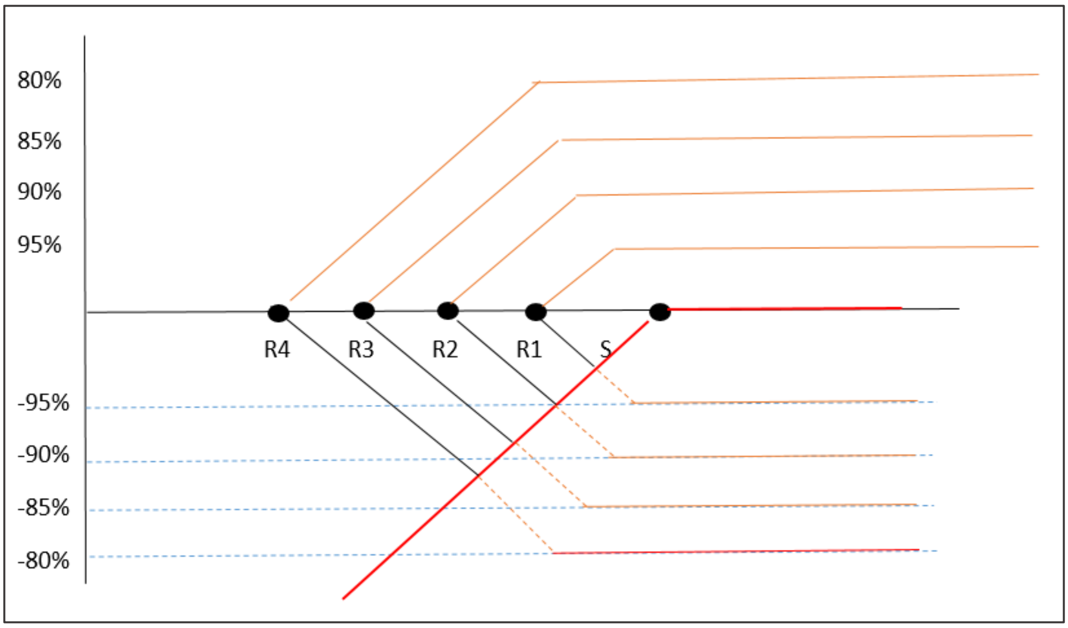
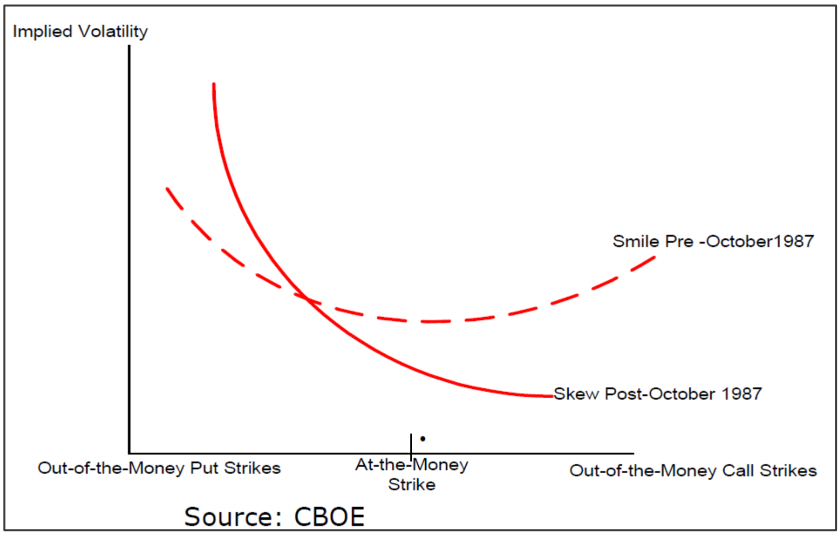


Table 5: Composition of a Ladder Put Position

Position	Components	Strike Interval
1	$+C^{KO}(X = R_3, X^{OS} = R_4)$ $-C^{KO}(X = R_4, X^{OS} = R_4)$	Knock-out put spread Spanning [R_3 R_4]
2	$+C^{KO}(X = R_2, X^{OS} = R_3)$ $-C^{KO}(X = R_3, X^{OS} = R_3)$	Knock-out put spread Spanning [R_2 R_3]
3	$+C^{KO}(X = R_1, X^{OS} = R_2)$ $-C^{KO}(X = R_2, X^{OS} = R_2)$	Knock-out put spread Spanning [R_1 R_2]
4	$+C^{KO}(X = S, X^{OS} = R_1)$ $-C^{KO}(X = R_1, X^{OS} = R_1)$	Knock-out put spread Spanning [S_1 R_1]
5	~~-C (X = R_3)~~ +C (X = R_4)	Vanilla put spread Spanning [R_3 R_4]
6	~~-C (X = R_2)~~ ~~+C (X = R_3)~~	Vanilla put spread Spanning [R_2 R_3]
7	~~-C (X = R_1)~~ ~~+C (X = R_2)~~	Vanilla put spread Spanning [R_1 R_2]
8	-C (X = S) ~~+C (X = R_1)~~	Vanilla put spread Spanning [S R_1]
9	+P (X = S)	Strike

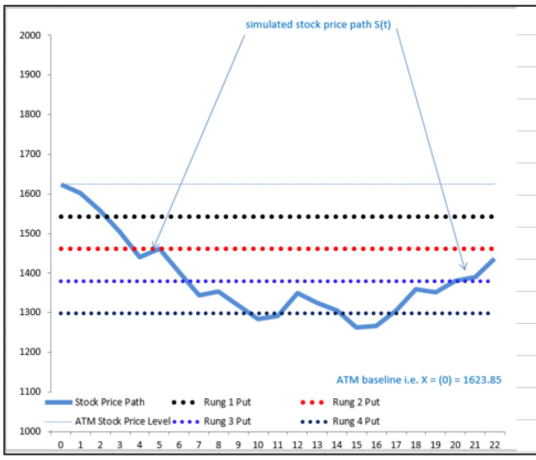
On observing the payoff profile created in OSA we notice that the price for ladder puts are higher than vanilla puts similar to the ladder call option due to the additional lock-in feature. However, we can notice that the premium for the lock-in puts is higher than the premium for the same lock-in calls. This is due to the smirk or smile pattern in the volatility curve. The at-the-money options have low volatilities and as the options goes out of money or deep in the money, the value of the local volatility increases which results in the smile shape (Zheng and Cai, 2013). This phenomenon was observed in the year and the following years of the crash of 1987 (figure 2.10). Hence the 'crash-o-phobia' which leads investors to hedge against downside price movements makes the lock in puts more expensive than the lock-in calls. Another interesting fact that is observable in the graph is that the ladder call P&L is below 0 even above the strike level. This is due to the 'hook effect' in the local volatility curves.

Figure 2.10 - S&P 500 Implied Volatility Curve Pre and Post 1987



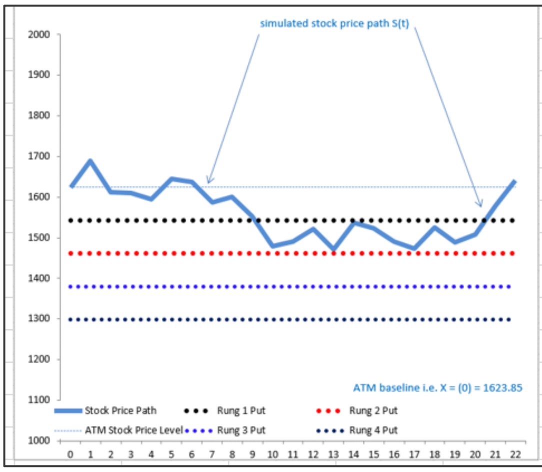
The analysis of the payoff scenarios are outlined as below for the 4RLP option. The scenarios and the constituent payoffs are described as below figures 2.11 to figure 2.14. We can see the payoffs similar to the 4RLC created previously.

Figure 2.11 - Scenario with all rungs breached and subsequent payoffs



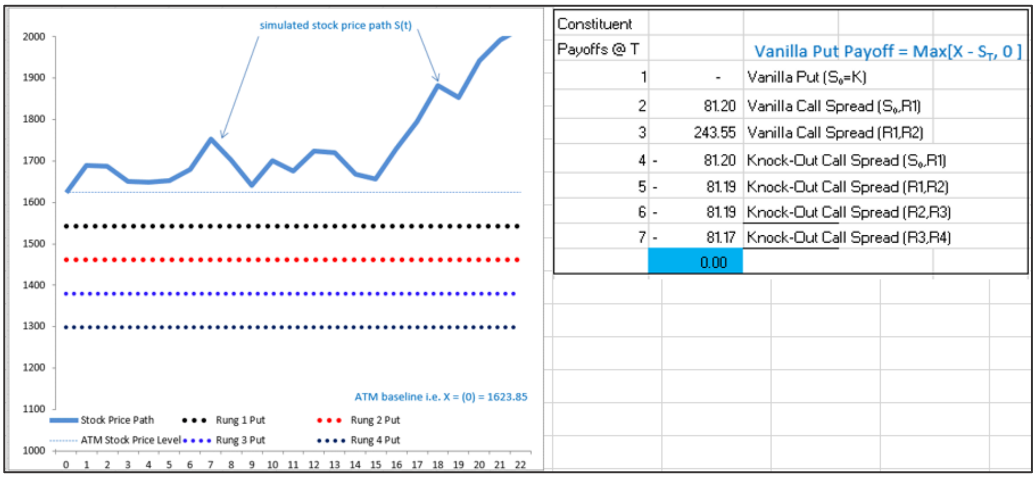
Constituent Payoffs @ T		Vanilla Put Payoff = Max[X - S_T, 0]
1	187.66	Vanilla Put (S_0=K)
2	-	Vanilla Call Spread (S_0,R1)
3	137.09	Vanilla Call Spread (R1,R2)
4	-	Knock-Out Call Spread (S_0,R1)
5	-	Knock-Out Call Spread (R1,R2)
6	-	Knock-Out Call Spread (R2,R3)
7	-	Knock-Out Call Spread (R3,R4)
	324.75	

Figure 2.12 - Scenario with 1 rung breached and subsequent payoffs



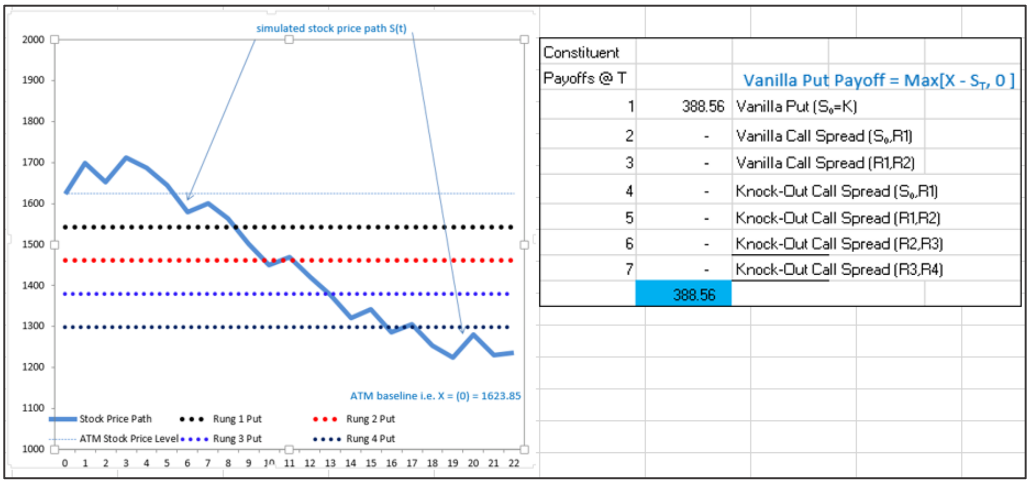
Constituent Payoffs @ T		Vanilla Put Payoff = Max[X - S_T, 0]
1	-	Vanilla Put (S_0=K)
2	81.20	Vanilla Call Spread (S_0,R1)
3	243.55	Vanilla Call Spread (R1,R2)
4	-	Knock-Out Call Spread (S_0,R1)
5 -	81.19	Knock-Out Call Spread (R1,R2)
6 -	81.19	Knock-Out Call Spread (R2,R3)
7 -	81.17	Knock-Out Call Spread (R3,R4)
	81.20	

Figure 2.13 - Scenario with no rungs breached and subsequent payoffs



Constituent Payoffs @ T			Vanilla Put Payoff = Max[X - S_T, 0]
1		-	Vanilla Put (S_0=K)
2		81.20	Vanilla Call Spread (S_0,R1)
3		243.55	Vanilla Call Spread (R1,R2)
4	-	81.20	Knock-Out Call Spread (S_0,R1)
5	-	81.19	Knock-Out Call Spread (R1,R2)
6	-	81.19	Knock-Out Call Spread (R2,R3)
7	-	81.17	Knock-Out Call Spread (R3,R4)
		0.00	

Figure 2.14 Scenario with S_T > highest rung level and subsequent payoffs



Constituent Payoffs @ T		Vanilla Put Payoff = Max[X - S_T, 0]
1	388.56	Vanilla Put (S_0=K)
2	-	Vanilla Call Spread (S_0,R1)
3	-	Vanilla Call Spread (R1,R2)
4	-	Knock-Out Call Spread (S_0,R1)
5	-	Knock-Out Call Spread (R1,R2)
6	-	Knock-Out Call Spread (R2,R3)
7	-	Knock-Out Call Spread (R3,R4)
	388.56	

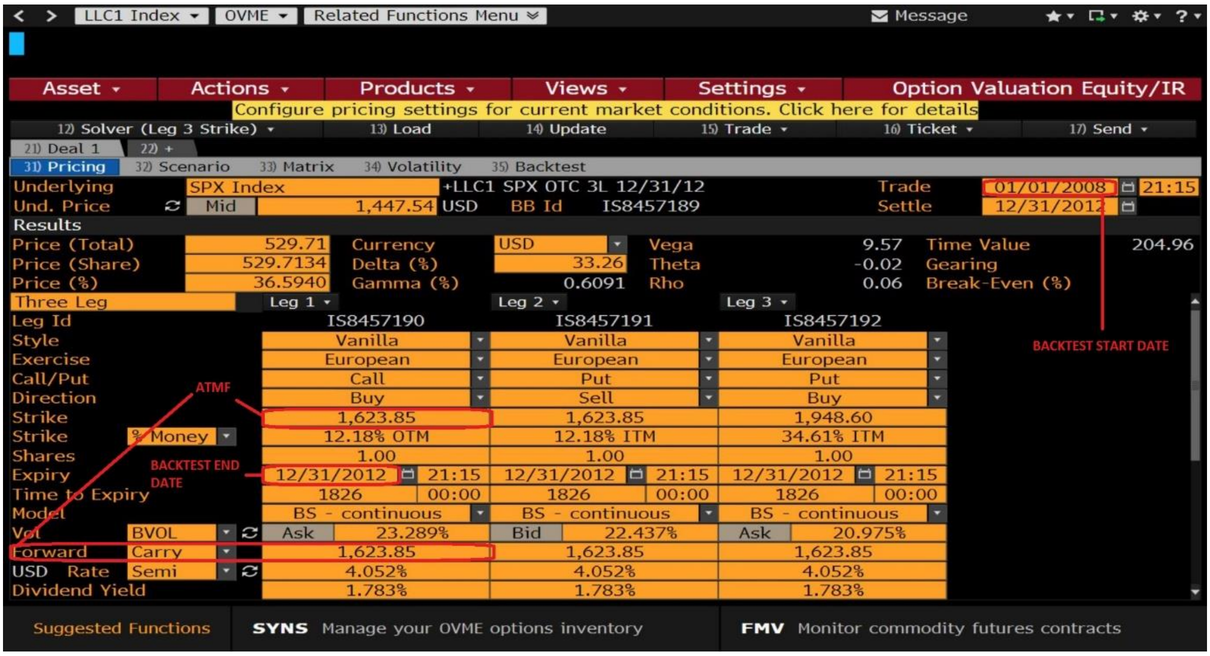
2.3 Ladder Forward Option Backtest

As calculated above using Bloomberg, the ATMF price of the ladder forward option was found to be at 1623.85. The ladder forward options are then backtested over the selected historical period i.e. 1st January 2008 to 31st December 2012. In the SPX Index graphical price chart illustrated in figure x1, it is observed that the SPX Index hits a max price of 1468.25 during the lifespan of the option, due to which none of the barriers of the knockout long ladder call component were breached, leaving only few of the barriers of the short ladder put position being breached. In figure 2.15 indicates the first rung level of the short ladder put component not being breached, in green. It also indicates the lowest rung level (R4 =1299.10) of the Short ladder put component and final price at maturity in red and yellow respectively.

Figure 2.15 – SPX Index Ladder Option Backtest (1st Jan 2008 to 31st Dec 2012)

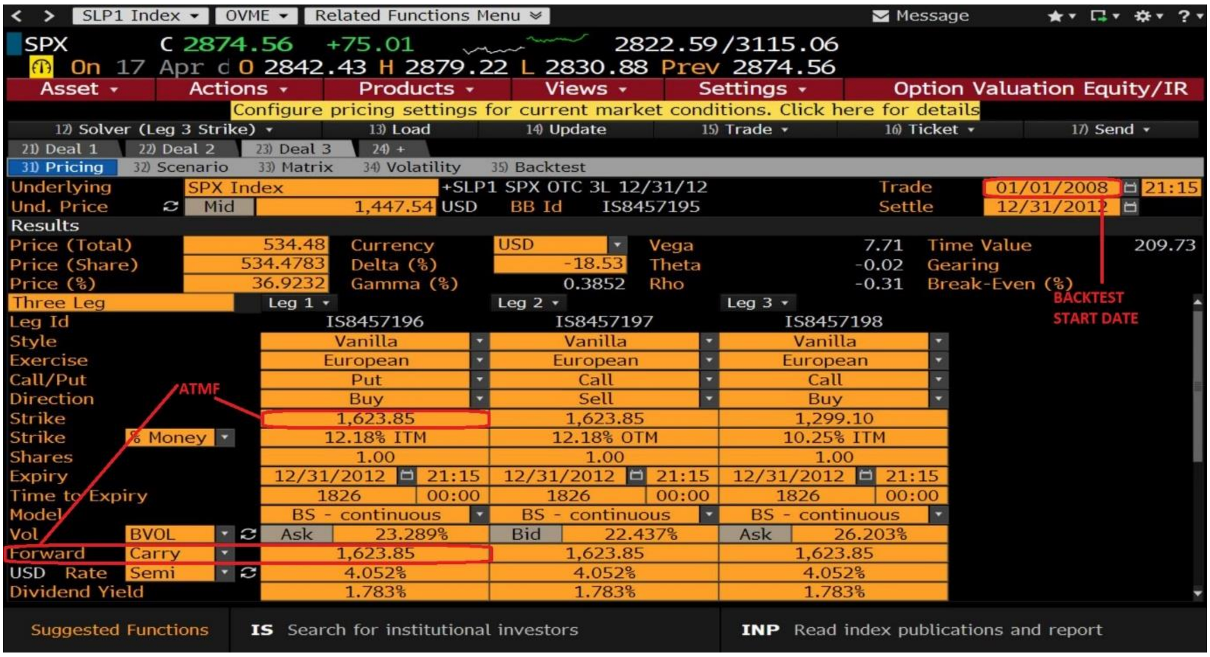
Using the OVME monitor on Bloomberg, a long 4-Rung ladder call (+4RLC) was created, with the trade date as 01/01/2008 and the expiry as 31/12/2012. The strike was set at ATMF price calculated at 1623.85 and the rung levels were set from 105% to 120% (R1 = 1705.0425, R2 = 1786.235, R3 = 1867.4275 and R4 = 1948.60).

Figure 2.16 – Backtest on +4RLC Component of Ladder Option using OVME



Similarly, a 4-rung short ladder put (-4RLP) was also created, with the trade date as 01/01/2008 and expiry at 31/12/2012. Once again, the strike is set as ATMF price calculated at 1623.85, with rung levels set between 95% and 80% (R1 = 1542.65, R2 = 1461.45, R3 = 1380.27 and R4 = 1299.10).

Figure 2.17 – Backtest on -4RLP component of Ladder Option using OVME



In figure 2.18, the five-year ladder option which is a combined product of both long 4-rung ladder call and short 4-rung ladder put, was compiled using OSA monitor on Bloomberg.

Figure 2.18 – OSA: Ladder Forward Option Payoff Profile

2.4 Validating the 4RLC and 4RLP Payoff and Replicating Portfolio:

As we discussed the payoffs of the 4RLC is given by the following expression:

$$Max\ [Max\ \{S_T - K;\ R_i^* - K\},\ 0]$$

The recorded price S_T at the expiry date was 1402.43. We selected the rung levels to be at 1705.042, 1786.235, 1867.427 and 1948.60 for the rung levels R1, R2, R3 and R4 respectively for the 4RLC. Hence, we can represent the payoff as below:

$$Max\ [Max\ \{1402.43,\ 1705.042,\ 1786.235,\ 1948.60 - 1623.85\},\ 0]$$

During the backtest we notice that the maximum index level was 1468.20. In this case no upside barriers were breached until expiry and hence the option has 0 value.

$$Max\ [\{1468.2 - 1623.85\},\ 0] = 0$$

Figure 2.19 shows the Bloomberg market value of the 4RLC which shows that the knock out put spreads offset the put spreads. These values are consistent with our calculations above.

Figure 2.19 - 4RLC Payoff Market Value

In terms of the 4RLP option the equation for the payoff is represented by the following:

$$-Max\ [Max\ \{K - S_T;\ K - R_i^*\},\ 0]$$

Hence, we represent the payoffs for all the rungs as follows:

$$Max\ [K - Min\ \{S_T,\ R_i^*\},\ 0]$$

$$Max\ [1623.85 - Min\ \{1468.20,\ 1542.65,\ 1461.465,\ 1380.27,\ 1299.1\},\ 0]$$

$$Max\ [1623.85 - 1299.10\},\ 0] = 324.75$$

The same values can be verified in figure 2.19 which confirms are calculations above for the 4RLP. On combining the 4RLC and 4RLP the combined market value of the resultant 8RL portfolio equals to $324.75.

2.5 Validating Ladder Option Replicating Portfolio with Rebate and Lookback Options:

We use an alternative replicating portfolio design to validate the price consistency of the ladder option replicating portfolio. We first use the rebate option which locks in a pre-specified rebate when the barrier is breached on the upside. The capital gain that is guaranteed in the 4RLC with 9 legs is the same as when the barrier is hit in this 5-leg strategy. Hence the capital gain is increasingly locked-in in a ratchet style fashion. Typically, a ladder option will have 2n+3 legs but using the KO barriers with rebates we reduce the number of rungs to n+1. In our 8RL option comprising of long 4RLC and short 4RLP option, we will would have 11 legs as specified by table 4. The payoffs of the 4RLC and 4RLP are shown in the table 6 and table 7 below:

Table 6: KO Call with Rebates + OTM Vanilla Call

Position	Components	Strike Interval
1	$+C^{KO}(X = R_{3,} X^{OS} = R_4)$ + 5% rebate	Knock-out with rebates Spanning $[R_3\ R_4]$
2	$+C^{KO}(X = R_{2,} X^{OS} = R_3)$ + 5% rebate	Knock-out with rebates Spanning $[R_2\ R_3]$
3	$+C^{KO}(X = R_{1,} X^{OS} = R_2)$ + 5% rebate	Knock-out with rebates Spanning $[R_1\ R_2]$
4	$+C^{KO}(X = S_, X^{OS} = R_1)$ + 5% rebate	Knock-out with rebates Spanning $[S\ R_1]$
5	$+C\,(X = R_4)$	OTM Vanilla Call with strike R_4

Table 7: KO Call with Rebates + OTM Vanilla Call

Position	Components	Strike Interval
1	$+P^{KO}(X = R_3, X^{OS} = R_4)$ + 5% rebate	Knock-out with rebates Spanning [R_3 R_4]
2	$+P^{KO}(X = R_2, X^{OS} = R_3)$ + 5% rebate	Knock-out with rebates Spanning [R_2 R_3]
3	$+P^{KO}(X = R_1, X^{OS} = R_2)$ + 5% rebate	Knock-out with rebates Spanning [R_1 R_2]
4	$+P^{KO}(X = S, X^{OS} = R_1)$ + 5% rebate	Knock-out with rebates Spanning [S R_1]
5	$+P\ (X = R_4)$	OTM Vanilla Put with strike R_4

We verify these payoffs in figure 2.20 using the OSA tool in Bloomberg. We find the Market Value of the 8RL option with rebates have the same market value as our 8RL without rebates. Hence, we verified the price consistency in our portfolio while reducing the number of legs.

Figure 2.20 - Payoff profile for the KO Call with Rebates + OTM Vanilla Call

The second alternative we have to our replicating portfolio is the use of lookback options. The payoffs from the lookback options typically depend on the maximum and minimum price levels reached until expiry i.e. a floating strike (Hull, 2008). We modify the exotic option to have a fixed strike F and lock-in payoff equal to the chosen rung level. In figure 2.21 we see the market value of the of the payoffs equal to that of our 8RL replicating portfolio.

Figure 2.21: Lookback Call Component with Rung Levels

Figure 2.22 - Lookback Put Component with Rung Levels

Figure 2.23 Lookback Portfolio Payoff

Hence overall, the ladder option structured product instrument is suited for risk averse investors. The product offers a high potential for locking in profits on the upside and downside price movements and benefiting from volatile markets. The product can be modified by setting rungs closer to the strike thus improving the chances of locking in more profits depending on the risk appetite of the investors. Alternatively using the skewness of option prices in the SPX index we can alter the short and long positions accordingly to create a structured product with 0 initial costs, excluding broker commission and transaction costs.

2.6 Ladder Forward Option Against Various Market Scenarios

The Ladder forward option is a viable product for investors looking to invest in risk adverse options with low cost protection. The performance of the Ladder Option can be tested by treating it against various market scenarios, be it a bull, bear, or flat market condition. Like the tests used in the Autocallable section, a Monte-Carlo methodology is utilised. The three historical periods of SPX index that follow the three selected market scenarios bear (approx. -10% returns), bull (approx. +10% returns) and flat (approx. 0% return) have been used as references for values such as annualized volatility and expected level of returns. By using the price on 01/01/08, which is 1447.54 as the initial price and by running 1000 simulations, we simulate price paths that the SPX index would take in the occasion of all three market scenarios to calculate the probability of particular payoffs from the Ladder Forward option. Figure 2.24, 2.25 and 2.26 illustrate the potential price paths taken for a bear market scenario, flat market scenario and bull market scenario, respectively.

With the simulated price path for the different market scenarios obtained, the potential payoff distribution of the ladder forward option for the three scenarios can be viewed by calculating the probability of the asset hitting the rung levels set in both the ladder call and put components. Figure 2.27 depicts the potential payoff distribution of the created ladder forward option for the selected three market scenarios. It can be observed that during a bull market, the product would make a profit of 20% or above, roughly 90.8% of the time, indicating a highly profitable investment product for bullish investors. Conversely, in a bear market, the investor would generate profits approximately 5.4% of the time. There is a stark difference in the payoff distributions between market conditions, a risk a potential investor must bear as they commit to this investment product.

Figure 2.24 – Histogram: Bear Market Price Scenario Simulation

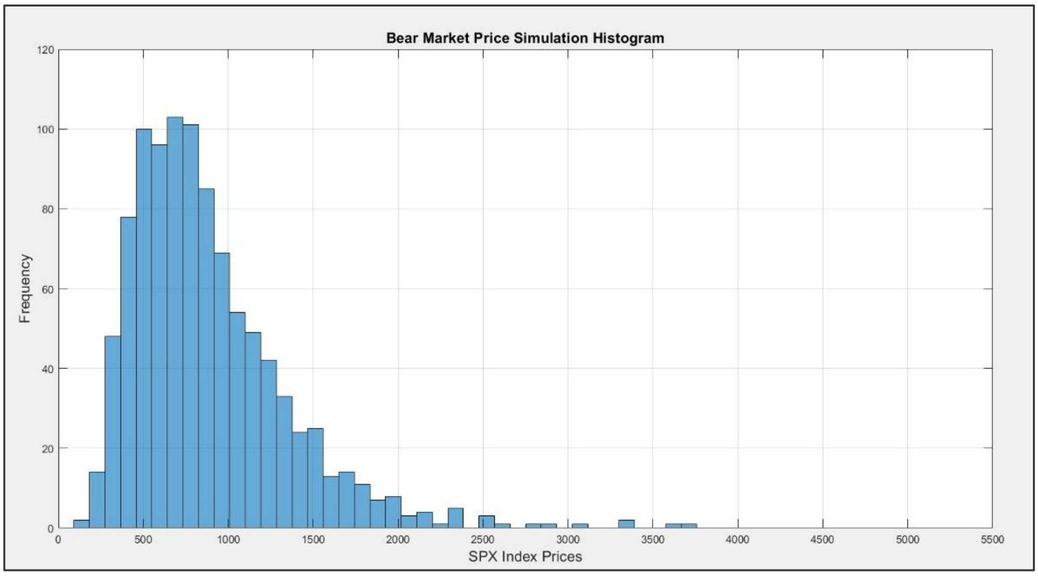


Figure 2.25 – Histogram: Flat Market Scenario Price Simulation

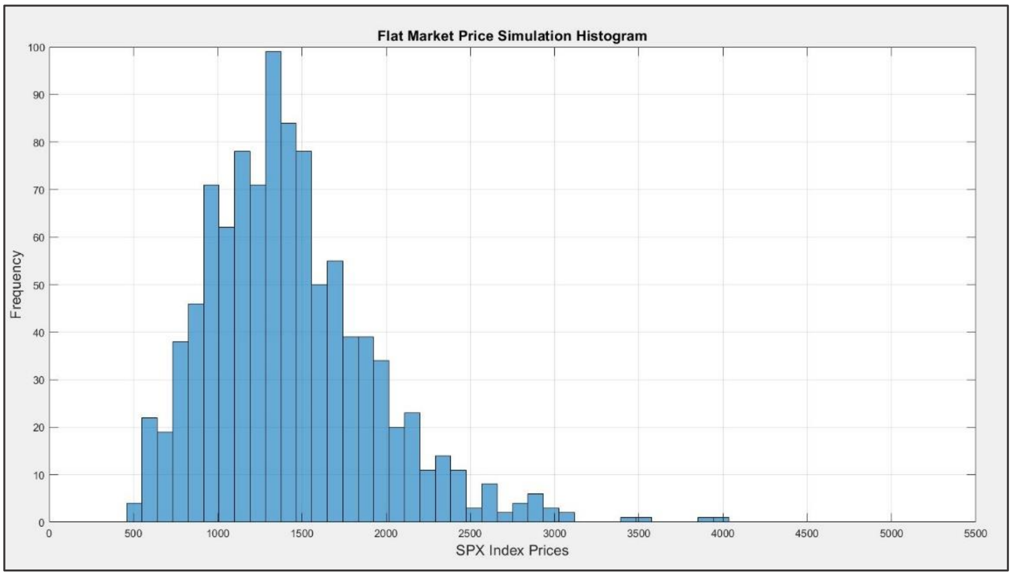


Figure 2.26 – Histogram: Bull Market Price Scenario Simulation

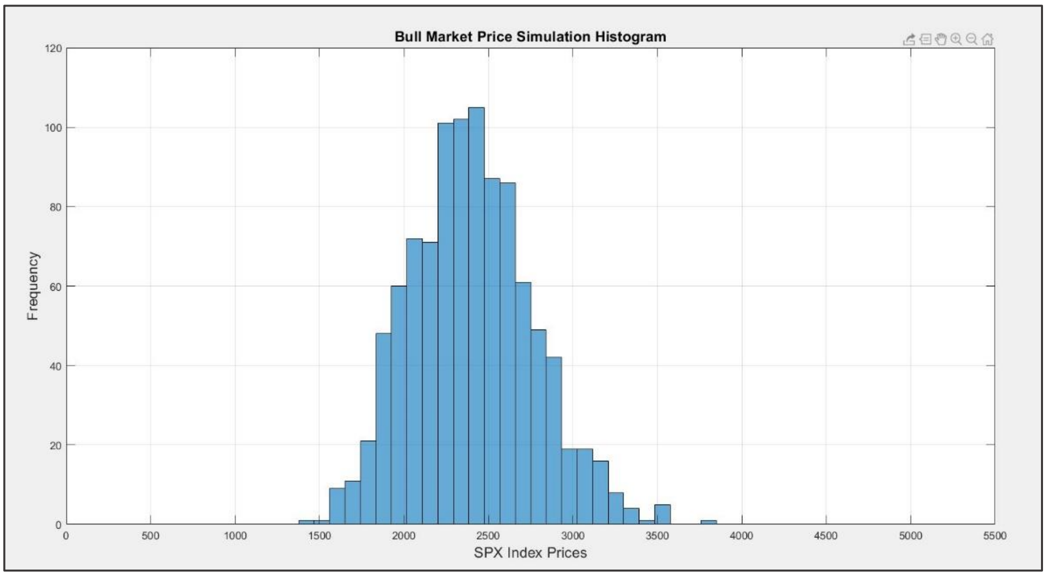
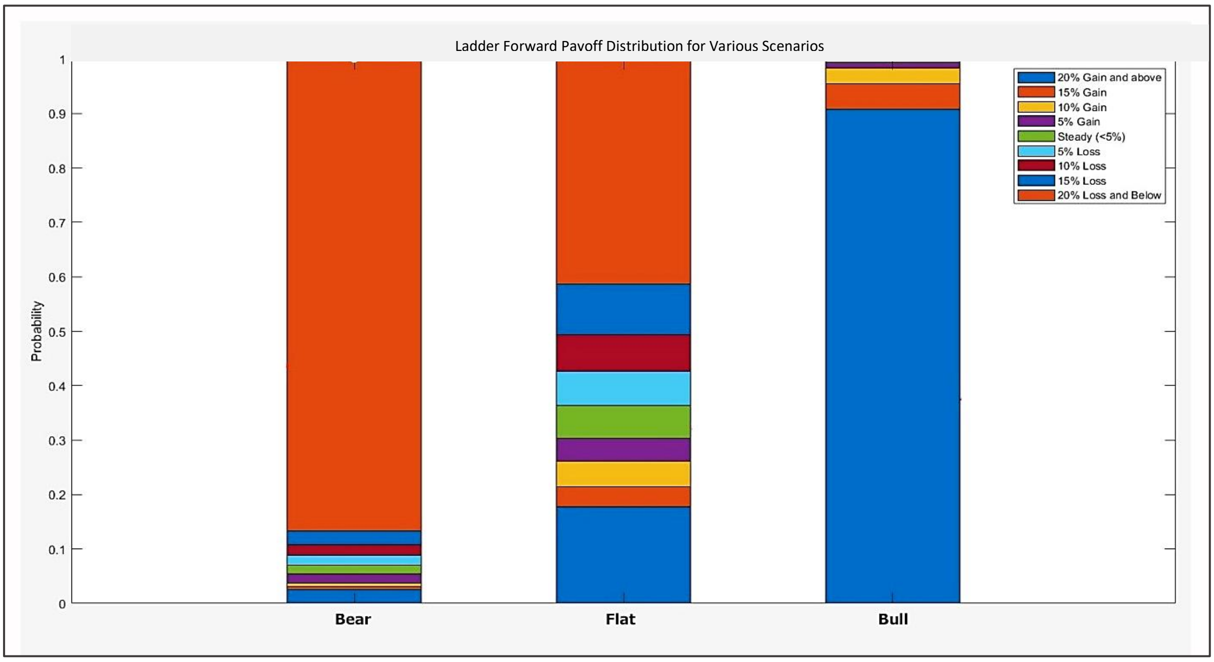


Figure 2.27 – Payoff Distribution in Various Market Conditions



Bibliography:

Du Prel, J., Hommel, G., Röhrig, B. and Blettner, M. (2009) 'Confidence Interval or P-Value?', *Dtsch Arztebl Int*, 106(19), 335-339, available: https://doi.org/10.3238/arztebl.2009.0335

Hull, J. (2008) *Options, Futures, And Other Derivatives*, 9th ed, New Jersey: Pearson Education.

Zheng, J. and Cai, Y. (2013) 'Modelling the Implied Volatility Surface: A Study for S&P 500 Index Option', *Research Journal of Applied Sciences, Engineering and Technology*, 11(9), 1973-1977, available: https://doi.org/10.19026/rjaset.5.4737

Citigroup (2007) 'Citigroup's Guide to Structured Product Terminology', *Corporate* Statement, New York: Citigroup

Deng, G., Guo, H., and McCann, C (2009) 'Structured Products in the Aftermath of Lehman Brothers', Securities Litigation & Consulting Group – Working *Paper Series,* available: https://www.slcg.com/pdf/workingpapers/Structured%20Products%20in%20the%20Aftermath%20of%20Lehman%20Brothers.pdf

Fabozzi, F. (2005) 'The Structured Finance Market: An Investor's Perspective', *Financial Analysts Journal*, 61(3), 27–40, available: https://doi.org/10.2469/faj.v61.n3.2725

Kim, K and Lim, D. (2018) 'A Recursive Method for Static Replication of Autocallable Structured Products', *Quantitative Finance*, 1-15, available: https://doi.org/10.1080/14697688.2018.1523546

Mariathasan, J. (2019) 'Factor Investing: The Paradox of Low Volatility', *IPE,* available: https://www.ipe.com/factor-investing-the-paradox-of-low-volatility/10030358.article

McCann, C. (2006) 'Are Structured Products Suitable for Retail Investors?', *Securities Litigation & Consulting Group*, available: https://www.slcg.com/pdf/workingpapers/StructuredProducts.pdf

Merrill Lynch (2018) 'Market Linked Investments – Autocallable Notes' *Investor Information,* available:https://mlaem.fs.ml.com/content/dam/ML/ipo/equity-linked/Oct19/MLUSU.pdf

MATLAB CODE: Section 1

```
% MATLAB Code | Section 1 %

% Bear Market Scenario %

n = 1000;
dt = 252;
e = randn(dt*5, n);
S0 = 100;
sigmabear = 0.2221;
mubear = -0.1;
sbear = zeros(dt*5+1,n);
sbear(1,:)=S0;
for i=2:dt*5+1;
    for j=1:n;
        sbear(i,j)=sbear(i-1,j)*exp((mubear-0.5*sigmabear^2)*(1/dt)+sigmabear*e(i-1,j)*sqrt(1/dt));
    end
end

ACBarrier=110*(ones(dt*5,1));
DownBarrier=55*(ones(dt*5,1));
Strike=100*(ones(dt*5,1));

figure
hold on
plot(sbear);
plot(DownBarrier, 'LineWidth', 4);
plot(Strike, 'LineWidth', 4);
title('S&P 500 Monte Carlo Simulations | Bear Market','FontWeight','bold')
hold off

% Autocall Year 1
Y1BEAR=find(sbear(dt+1,:)>=110);
sbear(:,Y1BEAR)=[];

%Autocall Year 2
Y2BEAR=find(sbear((dt+1)+dt,:)>=110);
sbear(:,Y2BEAR)=[];

%Autocall Year 3
Y3BEAR=find(sbear((dt+1)+dt+dt,:)>=110);
sbear(:,Y3BEAR)=[];

%Autocall Year 4
Y4BEAR=find(sbear((dt+1)+dt+dt+dt,:)>=110);
sbear(:,Y4BEAR)=[];

%Autocall Year 5
Y5BEAR=find(sbear((dt+1)+dt+dt+dt+dt,:)>=110);
sbear(:,Y5BEAR)=[];

Min_Value_Bear=min(sbear,[],1);
Barrier_Breaches=find(Min_Value_Bear<=55);
data=zeros(3,7);

Plot_Data_Bear=[size(Y1BEAR,2); size(Y2BEAR,2); size(Y3BEAR,2); ...
    size(Y4BEAR,2); size(Y5BEAR,2); size(sbear,2)-size(Barrier_Breaches,2); ...
    size(Barrier_Breaches,2)]
data(1,:)=Plot_Data_Bear;
bar(data,0.5,'stacked')
title('Payoff Probabilities | Bear Market','FontWeight','bold')
legend('Auto-Called at Year 1 with 10% Coupon',...
    'Auto-Called at Year 2 with 20% Coupon',...
    'Auto-Called at Year 3 with 30% Coupon',...
    'Auto-Called at Year 4 with 40% Coupon',...
    'Auto-Called at Year 5 with 50% Coupon',...
    'Not Auto-Called | Capital Protected',...
    'Not Auto-Called | Capital Loss',...
    'Location','southeast')
ylabel('Probability (%)','FontWeight','bold');
xlabel('Bear Market Scenario','FontWeight','bold');

bar(Plot_Data_Bear, Plot_Data_Flat, Plot_Data_Bull, 'stacked')
```

```matlab
% Flat Market Scenario %

n = 1000;
dt = 252;
e = randn(dt*5, n);
S0 = 100;
sigmaflat = 0.152;
muflat = 0;
sflat = zeros(dt*5+1,n);
sflat(1,:)=S0;
for i=2:dt*5+1;
    for j=1:n;
        sflat(i,j)=sflat(i-1,j)*exp((muflat-0.5*sigmaflat^2)*(1/dt)+sigmaflat*e(i-1,j)*sqrt(1/dt));
    end
end

ACBarrier=110*(ones(dt*5,1));
DownBarrier=55*(ones(dt*5,1));
Strike=100*(ones(dt*5,1));

figure
hold on
plot(sbear);
plot(DownBarrier, 'LineWidth', 4);
plot(Strike, 'LineWidth', 4);
title('S&P 500 Monte Carlo Simulations | Flat Market','FontWeight','bold')
hold off

% Autocall Year 1
Y1FLAT=find(sflat(dt+1,:)>=110);
sflat(:,Y1FLAT)=[];

%Autocall Year 2
Y2FLAT=find(sflat((dt+1)+dt,:)>=110);
sflat(:,Y2FLAT)=[];

%Autocall Year 3
Y3FLAT=find(sflat((dt+1)+dt+dt,:)>=110);
sflat(:,Y3FLAT)=[];

%Autocall Year 4
Y4FLAT=find(sflat((dt+1)+dt+dt+dt,:)>=110);
sflat(:,Y4FLAT)=[];

%Autocall Year 5
Y5FLAT=find(sflat((dt+1)+dt+dt+dt+dt,:)>=110);
sflat(:,Y5FLAT)=[];

Min_Value_Flat=min(sflat,[],1);
Barrier_Breaches=find(Min_Value_Flat<=55);
data=zeros(3,7);

Plot_Data_Flat=[size(Y1FLAT,2); size(Y2FLAT,2); size(Y3FLAT,2); ...
    size(Y4FLAT,2); size(Y5FLAT,2); size(sflat,2)-size(Barrier_Breaches,2); ...
    size(Barrier_Breaches,2)]
data(1,:)=Plot_Data_Flat;
bar(data,0.5,'stacked')
title('Payoff Probabilities | Flat Market','FontWeight','bold')
legend('Auto-Called at Year 1 with 10% Coupon',...
    'Auto-Called at Year 2 with 20% Coupon',...
    'Auto-Called at Year 3 with 30% Coupon',...
    'Auto-Called at Year 4 with 40% Coupon',...
    'Auto-Called at Year 5 with 50% Coupon',...
    'Not Auto-Called | Capital Protected',...
    'Not Auto-Called | Capital Loss',...
    'Location','southeast')
ylabel('Probability (%)','FontWeight','bold');
xlabel('Flat Market Scenario','FontWeight','bold');
```

```matlab
% Bull Market Scenario %

n = 1000;
dt = 252;
e = randn(dt*5, n);
s0 = 100;
sigmabull = 0.0679;
mubull = 0.1;
sbull = zeros(dt*5+1,n);
sbull(1,:)=s0;
for i=2:dt*5+1;
    for j=1:n;
        sbull(i,j)=sbull(i-1,j)*exp((mubull-0.5*sigmabull^2)*(1/dt)+sigmabull*e(i-1,j)*sqrt(1/dt));
    end
end

ACBarrier=110*(ones(dt*5,1));
DownBarrier=55*(ones(dt*5,1));
Strike=100*(ones(dt*5,1));

figure
hold on
plot(sbear);
plot(DownBarrier, 'LineWidth', 4);
plot(Strike, 'LineWidth', 4);
title('S&P 500 Monte Carlo Simulations | Bull Market','FontWeight','bold')
hold off

% Autocall Year 1
Y1BULL=find(sbull(dt+1,:)>=110);
sbull(:,Y1BULL)=[];

%Autocall Year 2
Y2BULL=find(sbull((dt+1)+dt,:)>=110);
sbull(:,Y2BULL)=[];

%Autocall Year 3
Y3BULL=find(sbull((dt+1)+dt+dt,:)>=110);
sbull(:,Y3BULL)=[];

%Autocall Year 4
Y4BULL=find(sbull((dt+1)+dt+dt+dt,:)>=110);
sbull(:,Y4BULL)=[];

%Autocall Year 5
Y5BULL=find(sbull((dt+1)+dt+dt+dt+dt,:)>=110);
sbull(:,Y5BULL)=[];

Min_Value_Bull=min(sbull,[],1);
Barrier_Breaches=find(Min_Value_Bull<=55);
data=zeros(3,7);

Plot_Data_Bull=[size(Y1BULL,2); size(Y2BULL,2); size(Y3BULL,2); ...
    size(Y4BULL,2); size(Y5BULL,2); size(sbull,2)-size(Barrier_Breaches,2); ...
    size(Barrier_Breaches,2)]
data(1,:)=Plot_Data_Bull;
bar(data,0.5,'stacked')
title('Payoff Probabilities | Bull Market','FontWeight','bold')
legend('Auto-Called at Year 1 with 10% Coupon',...
    'Auto-Called at Year 2 with 20% Coupon',...
    'Auto-Called at Year 3 with 30% Coupon',...
    'Auto-Called at Year 4 with 40% Coupon',...
    'Auto-Called at Year 5 with 50% Coupon',...
    'Not Auto-Called | Capital Protected',...
    'Not Auto-Called | Capital Loss',...
    'Location','southeast')
ylabel('Probability (%)','FontWeight','bold');
xlabel('Bull Market Scenario','FontWeight','bold');
```

```
% Bear Flat Bull Market Stacked Bar Graph %

bear = [16.4, 5.1, 3.6, 1.7, 1.34, 13.4, 58.6];
flat = [23.3, 13.3, 6.7, 4.1, 4.1, 37.6, 10.9];
bull = [51.9, 33.8, 10.8, 3, 0.3, 0.2, 0];

matrix = [bear; flat; bull];
figure(1)
bar(matrix, 0.5, 'stacked')
title('Distribution of Payoff Probabilities | 5-Year Autocallable Note','FontWeight','bold');
ylabel('Probability (%)', 'FontWeight', 'bold');
xlabel('Market Scenario', 'FontWeight', 'bold');
set(gca, 'XTickLabel', {'Bear Market', 'Flat Market', 'Bull Market'});
legend('Auto-Called at Year 1 with 10% Coupon',...
    'Auto-Called at Year 2 with 20% Coupon',...
    'Auto-Called at Year 3 with 30% Coupon',...
    'Auto-Called at Year 4 with 40% Coupon',...
    'Auto-Called at Year 5 with 50% Coupon',...
    'Not Auto-Called | Capital Protected',...
    'Not Auto-Called | Capital Loss');
```

MATLAB CODE: Section 2

```
% MATLAB Code | Section 2 %

% Calculating value of ladder call using B-S formula (Monte Carlo Method).
N= 90;  %time-steps
n= 10000; %simulations
So = 1623.85; %Starting Stock Price
dt=1/365; %Time step, 1/act
dW=randn(N,n); %Random Component for GBM equation
mu=0.0445; %US003M ICE LIBOR Index, average rate of last three months. Any local Money Market policy rate will work here.
strike = 1623.85; %Strike Price of KI option
R1 = So*(1.05);
R2 = So*(1.10);
R3 = So*(1.15);
R4 = So*(1.20);
KO_Barrier= [So*(1.05); So*(1.10); So*(1.15); So*(1.20)]; % 105% Ladder barrier price
r=0.0425; %Risk-free rate. Policy rates should be used here (central bank).
sigma=0.2480; %Constant Implied Volatility for Black-Scholes model price

% Monte Carlo Simulation using Black-Scholes Constant Volatility Model %
Stt=zeros(N,1);

  for i=1:n
        Stt(1,i)=So;
  end

      for i=1:n %simulations
    for j=2:N % %Time-Steps
        Stt(j,i)= Stt(j-1,i)*exp((mu-0.5*sigma^2)*dt+sqrt(dt)*sigma*dW(j,i));
    end
      end
 %Log returns from each simulation
BSReturns=zeros(1,n);
for i= 1:n
   BSReturns(1,i)= log(Stt(N,i)/Stt(1,i));
end

BSMEAN=mean(BSReturns); %Mean return
BSSTD=std(BSReturns); %Standard Deviation
BSStError= BSSTD/sqrt(length(BSReturns)); %Standard Error


 BS_Call_Payoff = zeros(5,n);
  for i= 1:n
      BS_Call_Payoff(1,i) = max(0,Stt(N,i)-strike);
```

```
      BS_Call_Payoff(2,i) = max(0,max(Stt(N,i),R1)-strike)
      BS_Call_Payoff(3,i) = max(0,max(Stt(N,i),R2)-strike)
      BS_Call_Payoff(4,i) = max(0,max(Stt(N,i),R3)-strike)
      BS_Call_Payoff(5,i) = max(0,max(Stt(N,i),R4)-strike)
  end
 maxStt=max(Stt); %max values in each column
 minStt=min(Stt);

 count = 0;
 for(i=1:n)
 if minStt(1,i)<(So*0.925)
     count=count+1;
 else
     count = count
 end
 end
 prob = count/n;
disp(prob)

 % Calculating a price for each barrier price set by KO_Barrier above %

 BS_Call_PayoffT=zeros(length(KO_Barrier),n);
 for k = 1:length(KO_Barrier)
     for i = 1:n %Number of Simulations
         if maxStt(:,i) >= KO_Barrier(k,1) && Stt(N,i) >= KO_Barrier(k,1)
             BS_Call_PayoffT(k,i)= exp(-r*dt*N)*BS_Call_Payoff(k,i);
         elseif maxStt(:,i) >= KO_Barrier(k,1) && Stt(N,i) < KO_Barrier(k,1)
             BS_Call_PayoffT(k,i)= exp(-r*dt*N)*(KO_Barrier(k,1)-strike);
         elseif maxStt(:,i) < KO_Barrier(k,1) && Stt(N,i) > strike
             BS_Call_PayoffT(k,i)= exp(-r*dt*N)*BS_Call_Payoff(k,i);
         else
             BS_Call_PayoffT(k,i)=0;
         end
     end
 end

 BS_Call_Price=zeros(length(KO_Barrier),1);
 for i= 1:length(KO_Barrier)
     BS_Call_Price(i,1)=mean(BS_Call_PayoffT(i,:));
 end
 BS_Vanilla_Call_Payoff=exp(-r*dt*N)*mean(max(0,Stt(end,:)-strike));

% Graphs %

 % Black-Scholes Constant Volatility Model %
 figure(1)
 plot(Stt);
        title('Monte Carlo Simulation using BS Constant Volatility Model');
        xlabel('Time in Days');
        ylabel('S&P 500 Index Price');
        hold all

% 95 Percent Confidence Interval %
simulations = ones(N,n);
simulations(1,:) = simulations(1,:)*So;
E = randn(90,n)
ESt = [So zeros(1,90-1)];
LSt = [So zeros(1,90-1)];
USt = [So zeros(1,90-1)];

for i=1:1:89
    ESt(i+1)=simulations(1,1)*exp(mu*i*dt);
    USt(i+1)=ESt(i+1)+1.96*sigma*simulations(1,1)*sqrt(i*dt);
    LSt(i+1)=ESt(i+1)-1.96*sigma*simulations(1,1)*sqrt(i*dt);
    simulations(i+1,:) = simulations(i,:).*exp((mu-.5*sigma.^2)*dt+sigma.*sqrt(dt).*dW(i,:));
end

figure
plot(simulations);
hold all
plot(ESt,'--','Color','black','Linewidth',5.0)
plot(USt,'--','Color','yellow','Linewidth',5.0)
```

```
plot(LSt,'--','Color','yellow','Linewidth',5.0)
title('Monte Carlo Simulations');
xlabel('T');
ylabel('S&P 500 Index Price');


X = ['Average PV Payoff of ', num2str(n), ' BS simulations is ', num2str(mean(BS_Vanilla_Call_Payoff)),...
    ' (Vanilla Call Price). Price of ', num2str((KO_Barrier(k,1)/So)*100), '% Ladder Call = ',
num2str(BS_Call_Price(k,1)),...
    '.'];
disp(X)
toc


% Bear Market Scenario %

n = 1000;
dt = 252;
e = randn(dt*5, n);
S0 = 1447.54;
sigmabear = 0.2221;
mubear = -0.1;
sbear = zeros(dt*5+1,n);
sbear(1,:)=S0;
for i=2:dt*5+1;
    for j=1:n;
        sbear(i,j)=sbear(i-1,j)*exp((mubear-0.5*sigmabear^2)*(1/dt)+sigmabear*e(i-1,j)*sqrt(1/dt));
    end
end

BearFinal = sbear(1260,1:end);

% Plot the Histogram %
edges = linspace(0,5500,61); % Create 20 bins.
histogram(BearFinal, 'BinEdges',edges);

% Format Graph %
grid on;
xlim([0, 5500]);
xlabel('SPX Index Prices', 'FontSize', 14);
ylabel('Frequency', 'FontSize', 14);
title('Bear Market Price Simulation Histogram', 'FontSize', 14);


% Flat Market Scenario %

n = 1000;
dt = 252;
e = randn(dt*5, n);
S0 = 1447.54;
sigmaflat = 0.1520;
muflat = 0;
sflat = zeros(dt*5+1,n);
sflat(1,:)=S0;
for i=2:dt*5+1;
    for j=1:n;
        sflat(i,j)=sflat(i-1,j)*exp((muflat-0.5*sigmaflat^2)*(1/dt)+sigmaflat*e(i-1,j)*sqrt(1/dt));
    end
end

FlatFinal = sflat(1260,1:end);

% Plot the Histogram %
edges = linspace(0,5500,61); % Create 20 bins.
histogram(FlatFinal, 'BinEdges',edges);

% Format Graph %
grid on;
xlim([0, 5500]);
xlabel('SPX Index Prices', 'FontSize', 14);
ylabel('Frequency', 'FontSize', 14);
title('Flat Market Price Simulation Histogram', 'FontSize', 14);
```

```
% Bull Market Scenario %

n = 1000;
dt = 252;
e = randn(dt*5, n);
S0 = 1447.54;
sigmabull = 0.0679;
mubull = 0.1;
sbull = zeros(dt*5+1,n);
sbull(1,:)=S0;
for i=2:dt*5+1;
    for j=1:n;
        sbull(i,j)=sbull(i-1,j)*exp((mubull-0.5*sigmabull^2)*(1/dt)+sigmabull*e(i-1,j)*sqrt(1/dt));
    end
end

BullFinal = sbull(1260,1:end);

% Plot the Histogram %
edges = linspace(0,5500,61); % Create 20 bins.
histogram(BullFinal, 'BinEdges',edges);

% Format Graph %
grid on;
xlim([0, 5500]);
xlabel('SPX Index Prices', 'FontSize', 14);
ylabel('Frequency', 'FontSize', 14);
title('Bull Market Price Simulation Histogram', 'FontSize', 14);


% Payoff Distribution %
BearMkt = [0.025,0.006,0.007,0.016,0.016,0.018,0.020,0.025,0.867];
FlatMkt = [0.178,0.036,0.047,0.042,0.061,0.062,0.068,0.092,0.414];
BullMkt = [0.908,0.047,0.028,0.011,0.003,0.001,0.001,0.001,0];

scenarioMatrix = [BearMkt;FlatMkt;BullMkt];

bar(scenarioMatrix,0.5,'stacked');
title('Ladder Forward Payoff Distribution for various scenarios');
xlabel('Market Scenarios');
ylabel('Probability');
legend('20% Gain and above','15% Gain','10% Gain','5% Gain','Steady (<5%)','5% Loss','10% Loss','15% Loss','20% Loss and
Below');
```