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MODENA E REGGIO EMILIA

Simulation

Simulation overview

Goal of simulation

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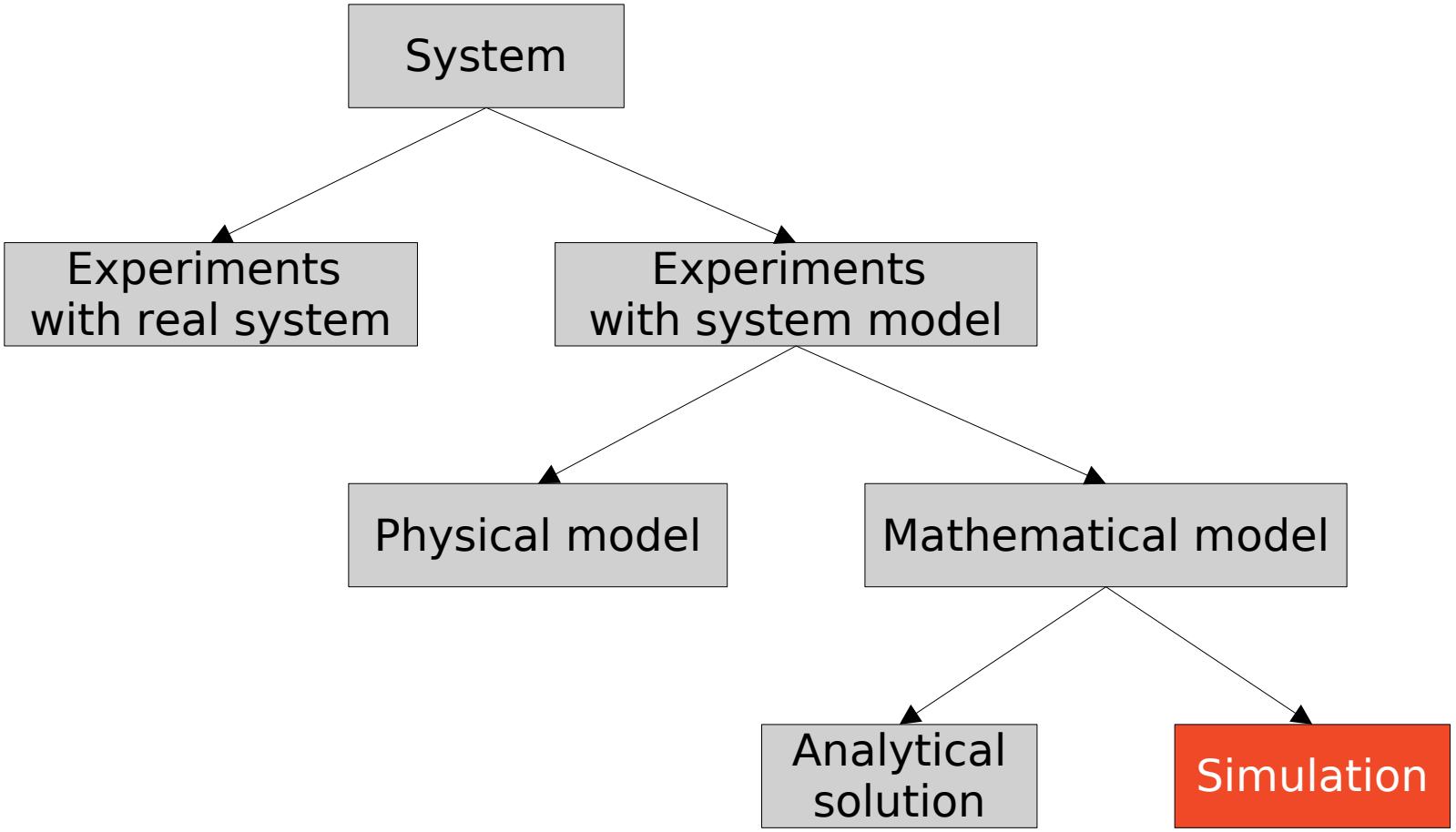
- Build model of a system that captures its behavior
  - Can test under different conditions
  - Can evaluate modification to the system
- System: set of parts cooperating for a common goal
- Model: abstract representation of system
  - Typically simplified version
  - Level of simplification depends on goal of the analysis
- Model typically implemented as computer program

Possible approaches



Motivations for simulation

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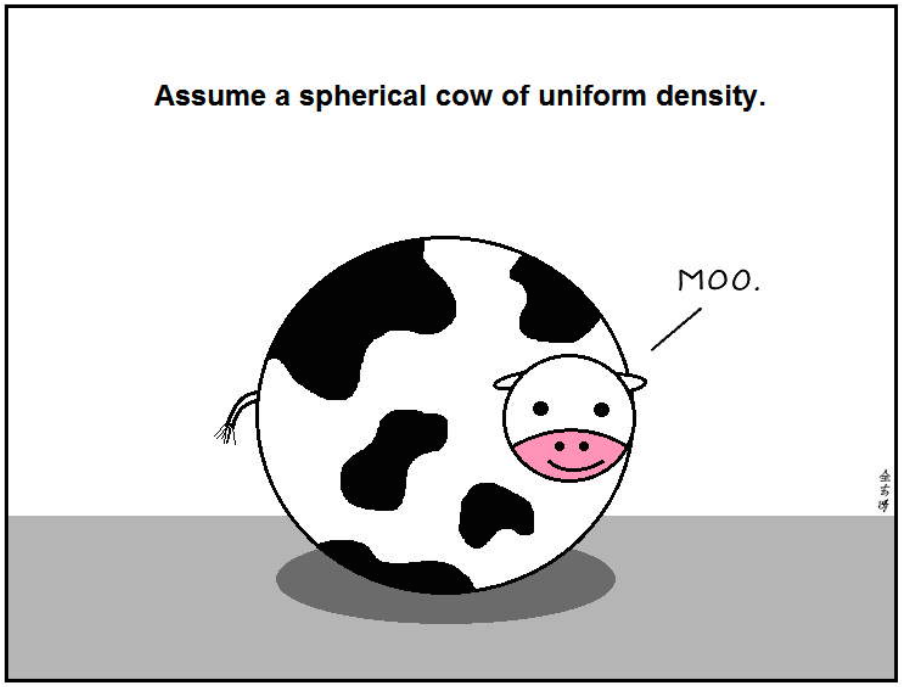
- Actual system *vs*. System model
  - Easier to do/Less dangerous (if system exists)
  - Less expensive than building a real system
- Physical *vs*. Mathematical model
  - Easier and less expensive to implement
  - Easier to modify
  - Mathematical model must be accurate enough
  - Remember the Spherical Cow
- Analytical solution *vs.* Simulation
  - Analytical is more precise
  - Simulation is more flexible



Types of simulations

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- Static vs. Dynamic
  - Time is considered or not?
- Deterministic vs. Stochastic model
  - Typically random variables are used in the simulation
- Discrete vs. Continuous model
  - Discrete → event-based simulation
  - Continuous → time cannot be split into events
  - Depends on the system and what we are looking for
  - Example: car traffic
    - Model each car → model is discrete (movement of car)
    - Model traffic → flow-based model (differential equations)
  - See also fluid models in large systems

Discrete simulator

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- Several components:
- System state
  - Includes system clock (“now” variable)
- Statistical counters (data collectors)
- Routines:
  - Initialization (setup for t=0)
  - Timing (select next event)
  - Event management
- Report generation
- Main()
  - Startup and shutdown

Steps for good simulation

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- Problem and study formulation
  - Define the problem of interest
  - Define:
    - Goal of the study
    - Questions to be answered
    - KPI to measure
    - Scope of model
    - Configurations to consider (scenarios)
- Model definition
  - Collect information on system
    - Several people to involve
    - Check for inaccurate information
    - Check for not-formalized aspects of problem

Steps for good simulation

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- Model definition (continued)
  - Collect data for model parameters (if possible)
  - Document the model assumptions
  - Collect data on performance of existing system (for validation)
  - Define level of detail (remember the Spherical Cows)
- Model validation
  - Discuss with other people (experts, management, ...)
- Software development
- Simulator validation
  - Compare with data (if available) or theoretical model
  - Run sensitivity analyses on parameters

Steps for good simulation

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- Experiments design
  - Run lengths
  - Warm-up period (avoid transients)
  - Number of runs
- Experiments
- Data analysis
- (Good) reporting
  - Simulation presents data, scenarios, ...
  - Management takes decisions
  - Point out assumptions and limits of validity
  - Get paid

Advantages of simulation

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- Can deal with complex systems
  - Hard to manage random variables with analytical models
  - Hard to find analytical models to describe them
  - Analytical models are hard to solve in closed form
- Can analyze several scenarios
  - Existing system with different operating parameters (what-if)
  - Variations of existing system (capacity planning)
- Easier to control than prototype systems
  - External interferences like network status
- Explore large time windows

Drawbacks of simulation

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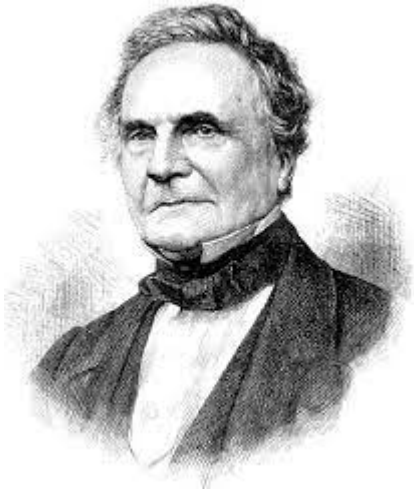
- Use of stochastic processes
  - Simulation provides just an estimation
- (Good) simulators are complex to develop
- Simulators ar as good as the model they implement
  - GIGO systems → Garbage In Garbage Out
  - A word of wisdom from the past:



“ I have been asked: ‘Mr. Babbage, if you put into the machine wrong figures, will the right answers come out?' I am not able rightly to apprehend the kind of confusion of ideas that could provoke such a question.”

Some common pitfalls

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- Not having well-defined objectives for the simulation
  - "everything" is not a well-defined objective
- Wrong level of detail
  - Too much detail → simulation not scalable (common newbie mistake, often related to previous point)
  - Too little detail → inaccurate/wrong results
- Wrong model
  - Input variables models?
  - KPI?
- Considering simulation an exercise in computer programming
  - Not considering simulation as a complex set of tasks
  - Error in statistical analysis (e.g., just one run)

Recall of statistics

Random variables

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- Experiment: a sample of a random variable X
- Sample space S: the set of possible outcomes (Co-dominion of random variable)
  - Each sample belongs to S
  - Example:
    - Coin toss → S={H, T}
    - 1d6 roll → S={1, 2, …, 6}
  - S can be a set of finite values, an interval or R
- Probability density:
  - P(x) = P(X=x)
  - The sum or probabilities is 1
    - $\Sigma_{x \in S} p(x)=1$
    - $\int p(x)\, dx = 1$

Relevant metrics

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- Average value $\mu=E(X)$
  - $E(X) = \Sigma x\ p(x)$
  - Relevant properties:
    - $E(cX) = cE(X)$
    - $E(\Sigma_i\ c_iX_i) = \Sigma_i\ c_i\ E(X_i)$
- Median value
  - Value x: $P(X \leq x) = 0.5$
- Quantiles are defined in a similar way
- Variance $\sigma^2=E[(X-\mu)^2]=E(X^2) -\mu^2$
  - $E[(X-E(X))^2]=E[X^2 -2X\ E(X) +E(X)^2]=$
    $=E(X^2) -2E(X)E(X) + E(X)^2=E(X^2) -2E(X)^2 + E(X)^2$
- Standard Deviation $\sigma= sqrt(\sigma^2)$

Variable independence

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- Variables can be:
  - Independent
  - Correlated
- Independent variables can be studied separately
- For correlated variables the probability distribution are intertwined
  - P(X=x)=f(P(Y=y))
- In simulation variables are often correlated
- To obtain statistically valid results we need to achieve variable independence

Repeated experiments

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- In simulation we repeat experiments:
  - Each experiments starts with a different seed for RNG
  - Experiments are statistically independent
- From central limit theorem
  - $F(X)=P(X<x)$
  - $F(X) \rightarrow G(X)$ for $N \rightarrow \infty$
  - $G(X)$ = Gaussian function
- Law of large numbers
  - N= number of observations
  - $N \rightarrow \infty$ means that sample average converges to $E(X)$

Gaussian functions

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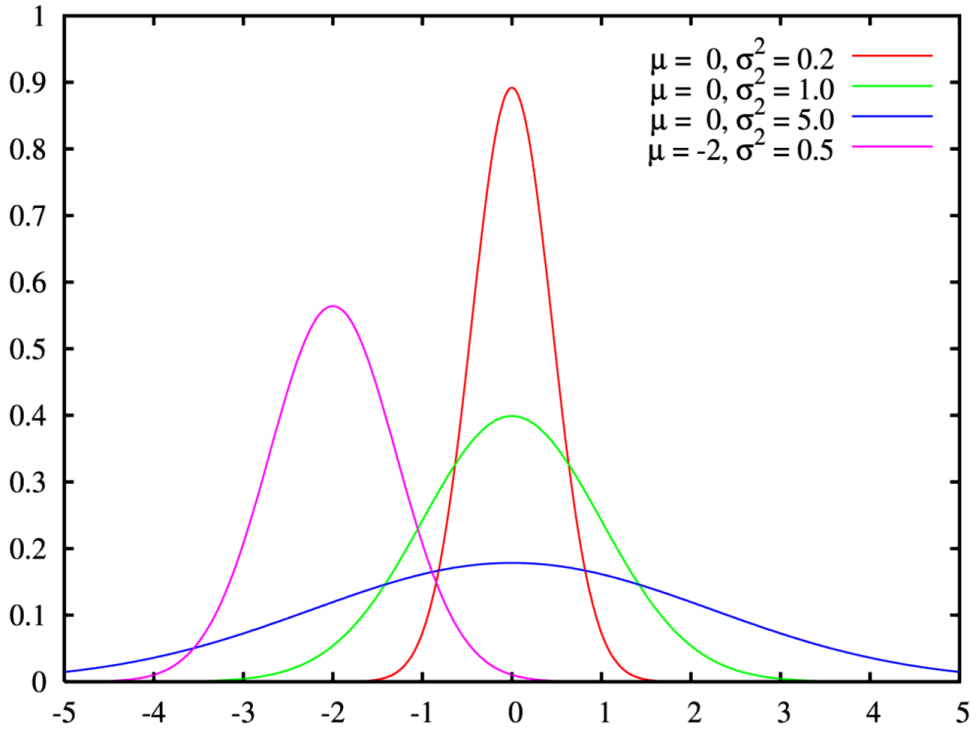
- Meaning of variance:
- Low variance:
  → High and narrow peak
- High variance:
  → Low and wide peak



Confidence intervals

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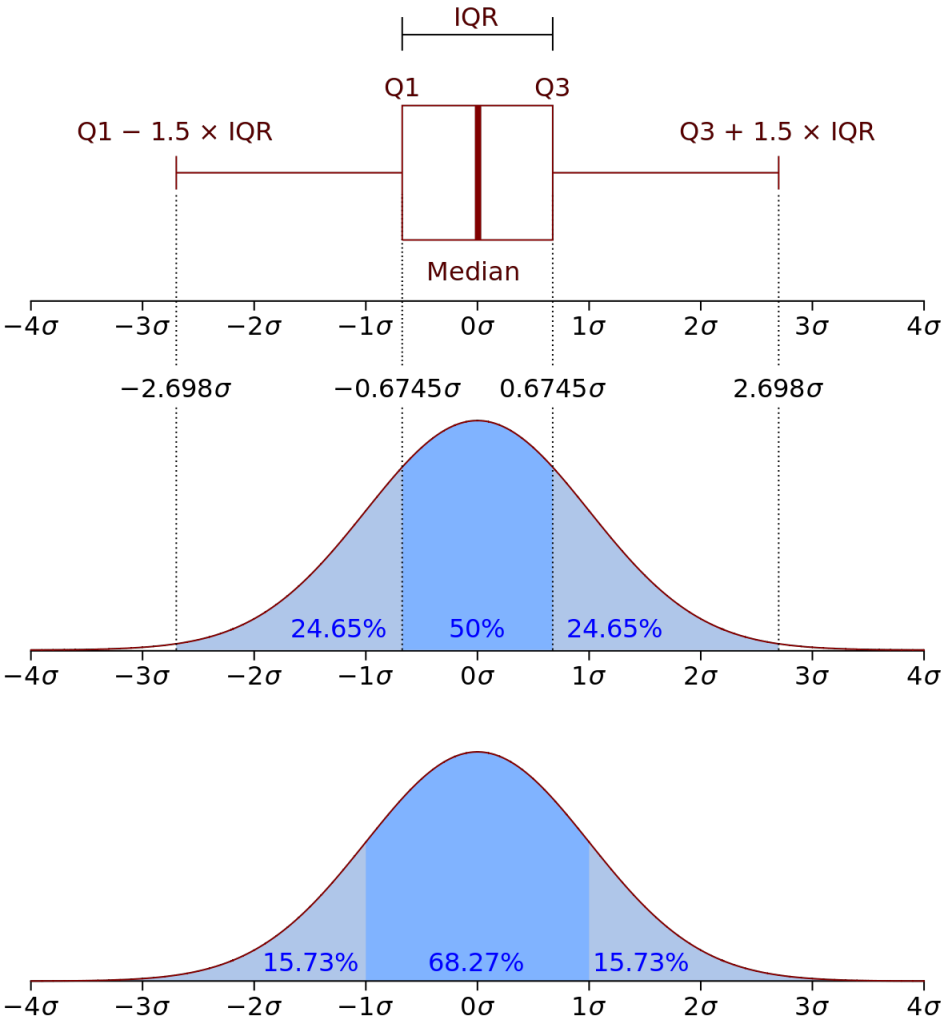
- Analysis of a Gaussian distribution
- Median=Average
- Confidence:
  - μ±σ→ 68% confidence
  - μ± 3σ→ 99% confidence
- Need to estimate μ and σ from repeated experiments



Building models

Sound process

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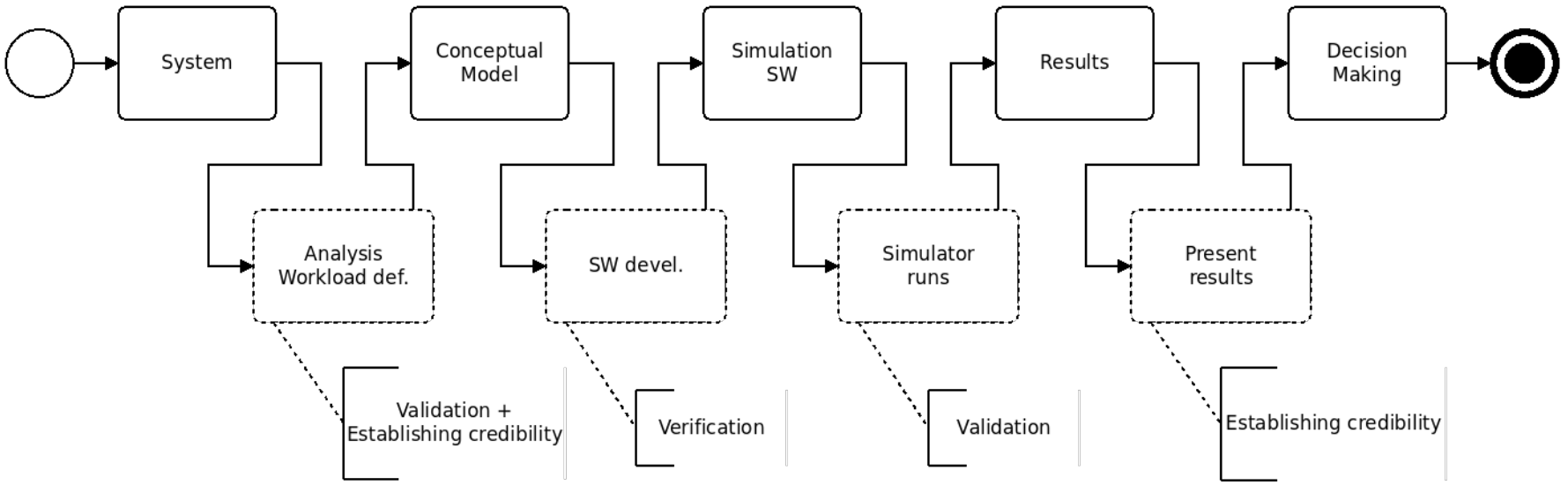
- Validation
  - Model is an accurate representation of the system,
- Verification
  - Concepts maps into SW implementation
- Establishing credibility
  - Information accepted as credible from experts/management



Accreditation

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- Example:
  - Accreditation with DoD in USA
- Key elements to be considered
  - Verification and validation correctly carried out
  - Quality of SW development (including historical data)
  - Quality of available data
  - Quality of documentation
  - Known problems and limitation in simulator or model
- Sources of errors → potentially inaccurate results
  - Approximations in system/model
  - Approximations in simulator runs (e.g., # of runs, length)

Some remarks

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- Common newbie mistake: add every detail
  - Un-scalable simulation
  - Huge parameter space to explore
- Detail depends on the data available
  - No need to model input variables parameters if no data are available
  - Rely on simple, standard assumptions

Verification techniques

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- Use modular approach
  - Unit testing of components
- Use external code review
  - Walk-through sessions with other developers
- Explore large parameter space
  - See if the results make sense
  - Compare with theoretical models when possible
  - Compare with historical results if possible
- Use traces
  - Easier to reproduce experiments and to analyze problems

Validation / Credibility-improving techniques

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- Seek wisdom from experts
  - No need to make a fool of yourself
  - Especially if you are not an expert on the subject
- Observe the system
  - Collect data
  - Evaluate data Quality (Missing points, variance issues, ...)
- Rely on existing theory/studies
  - Two weeks in lab can save you a 30-min visit to the library…
  - Consider previous simulation projects

Validation / Credibility-improving techniques

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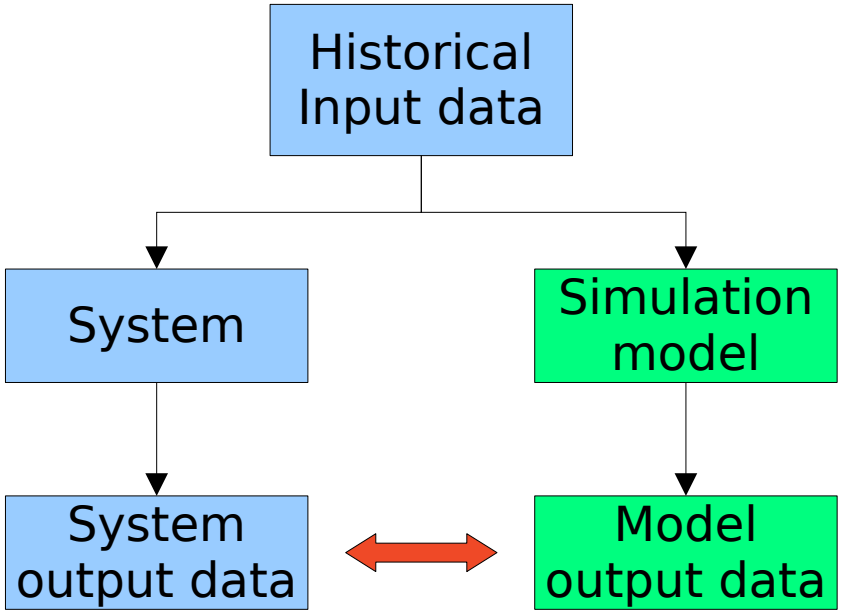
- Correlation-inspection approach
- Strong validation for existing/known scenario



Describing input

Input variables

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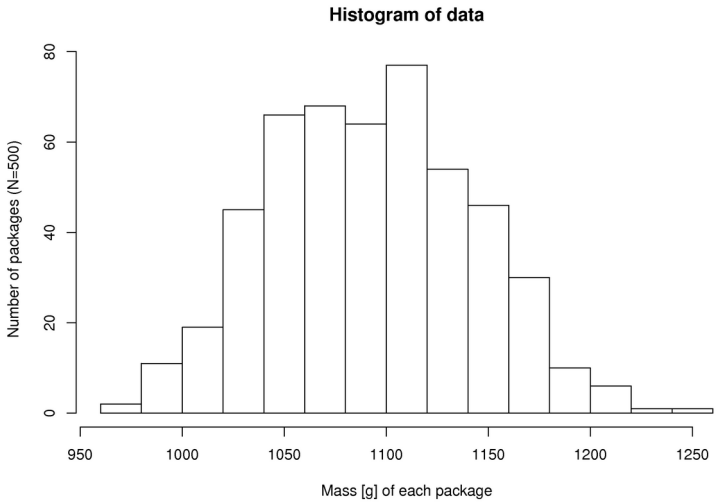
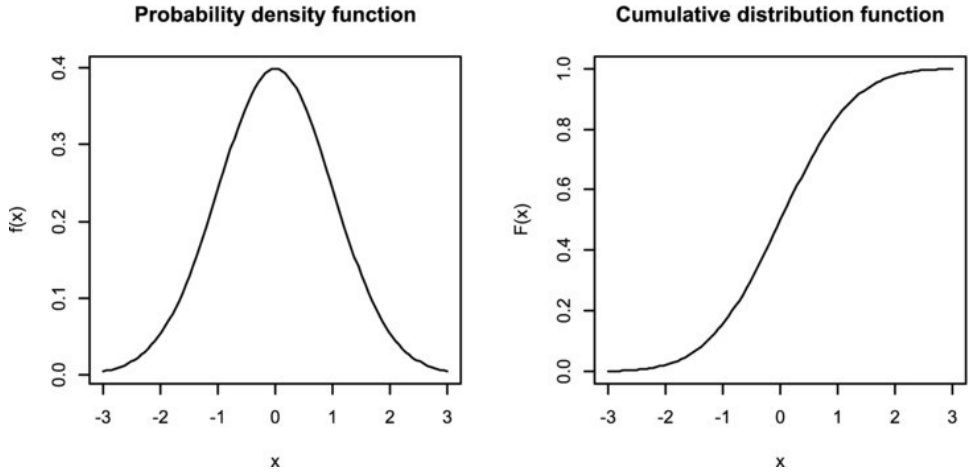
- Input variables are random variables
- Typically described statistically
- Use of a probability distributions
  - Derived from real observation (histogram)
  - Mathematical model





Some useful distributions

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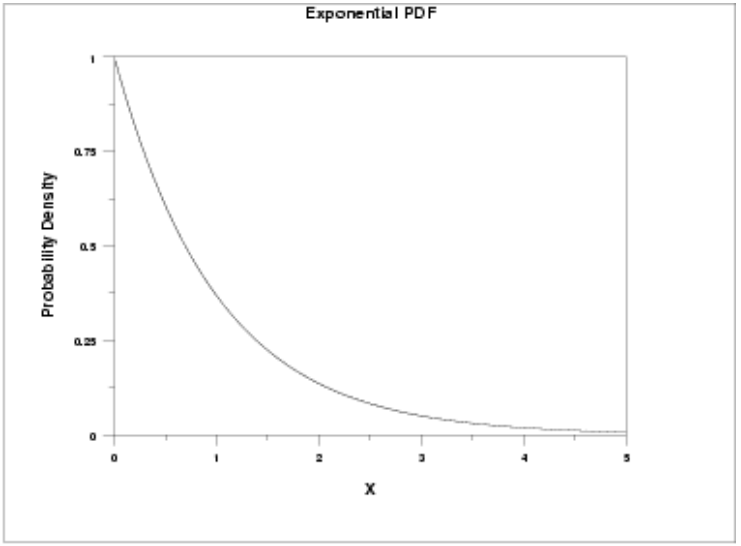
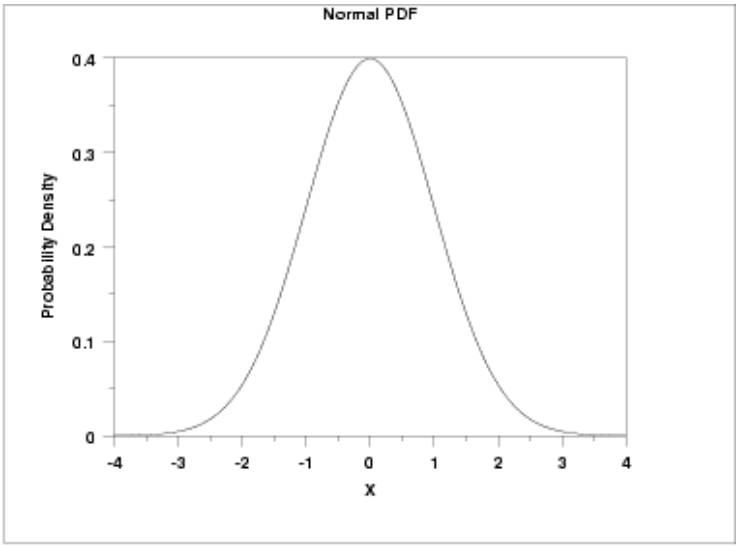
- Exponential
  - Typical inter-arrival time
  - Poisson process
- Gaussian (Normal)
  - Independent samples of same process





Some useful distributions

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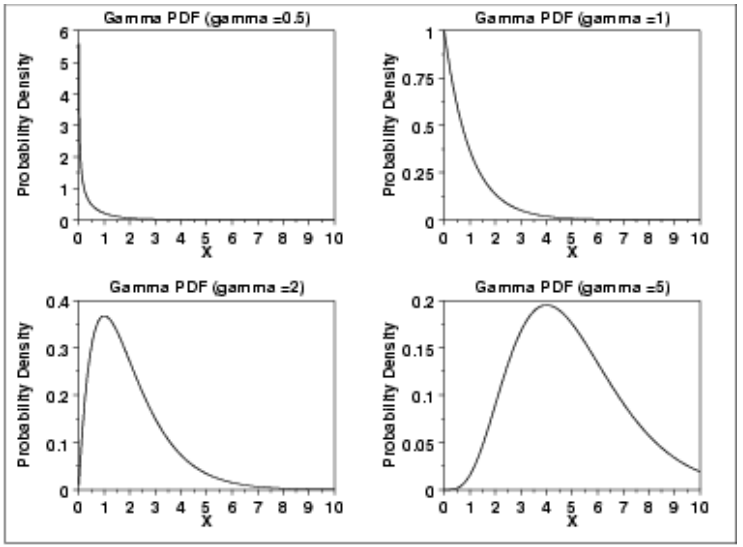

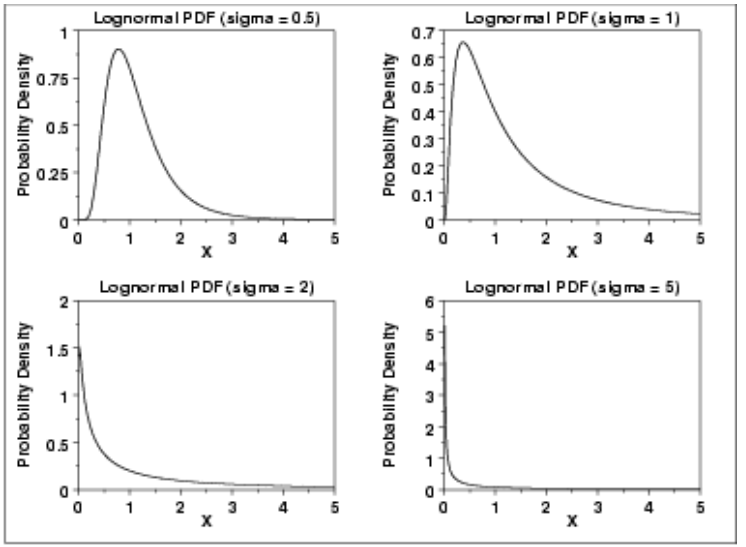

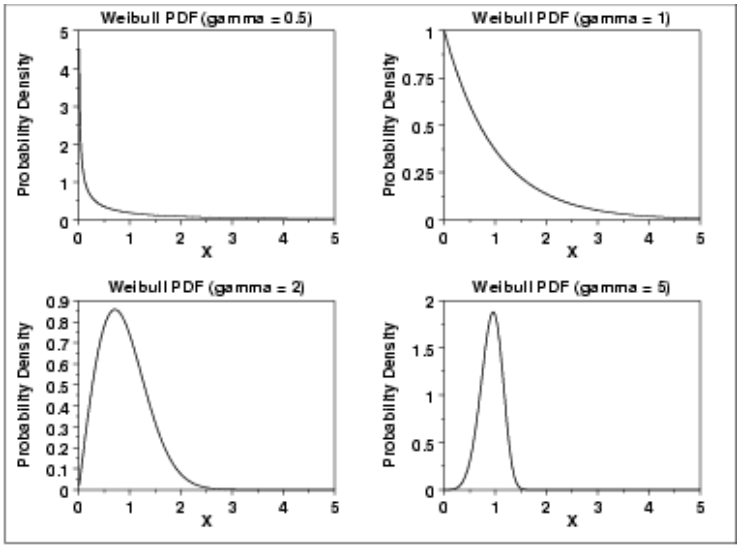

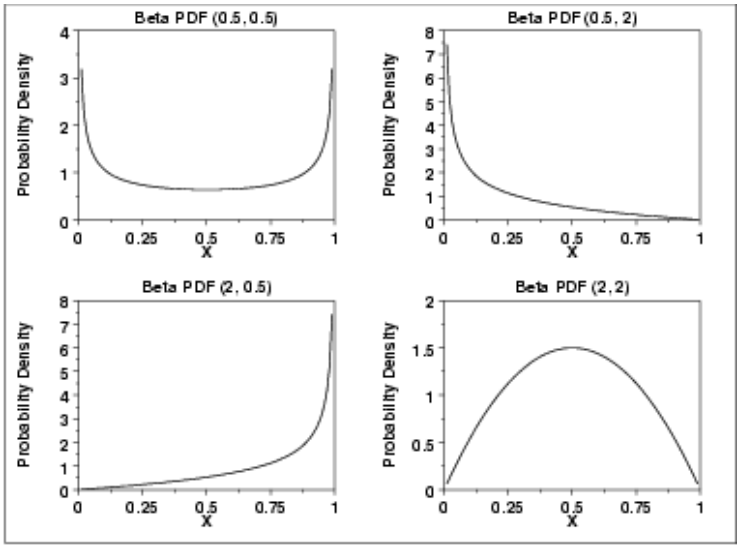

A remark on RNG

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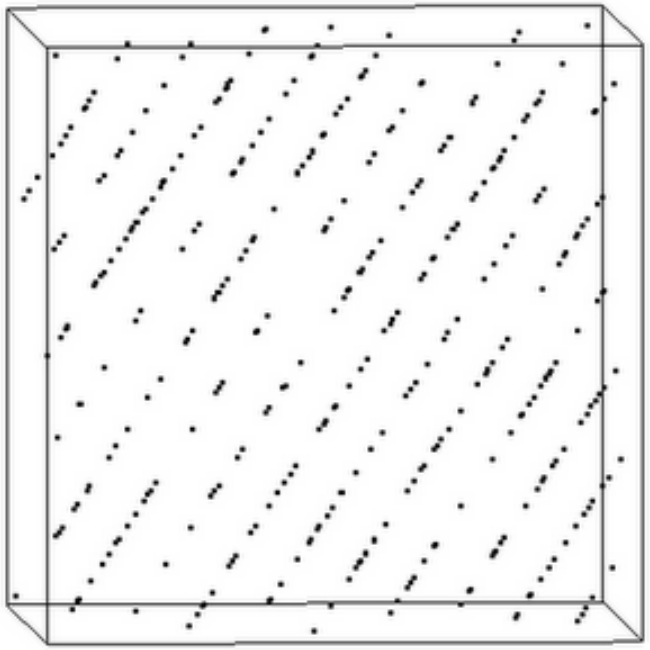
- RNG: Random Number Generators
  - Used to produce input variables
  - Starts from a seed value like `srand()`
- Not really random!
- The good:
  - Reproducible experiments
  - Multiple seeds → independent experiments
- The bad:
  - Can be really un-random-like
  - Need to select the RNG
  - Need to select the seed



Assessing sample independence

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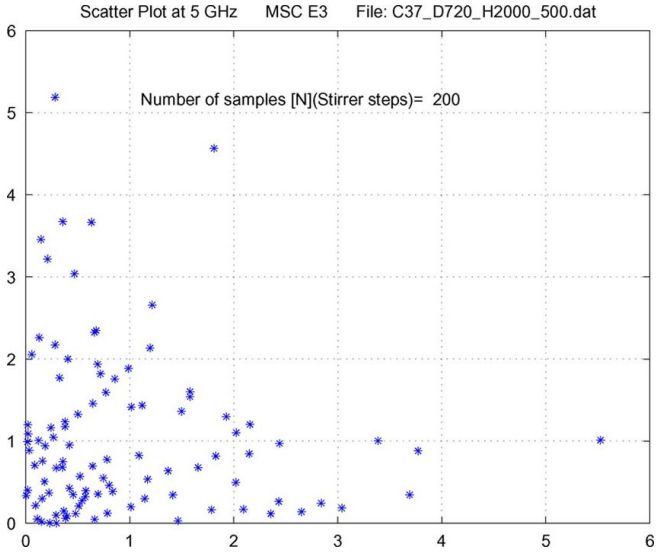
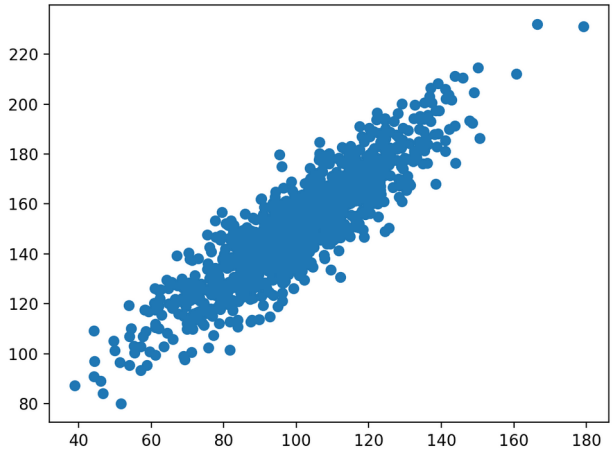
- Variable independence
  - Among variables
  - Correlation analysis
- Techniques:
  - Correlation value
  - Visual inspection
- Correlation (scatter) plot
  - Two examples
  - Uncorrelated data
  - Correlated data





Assessing sample independence

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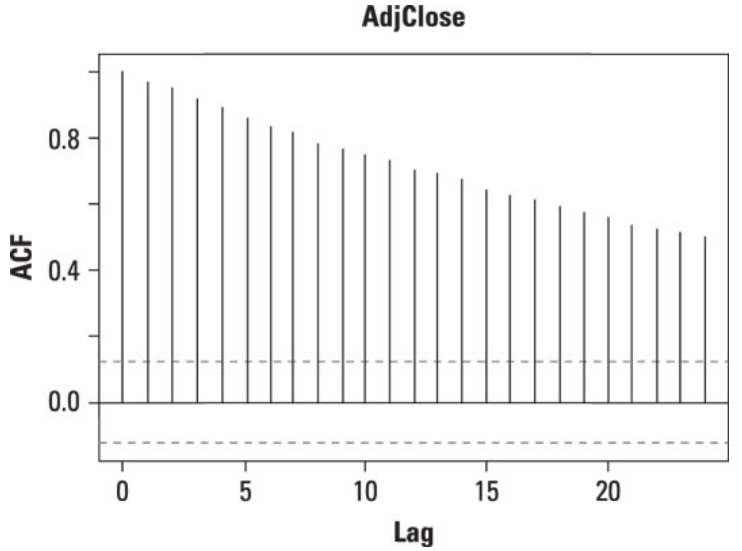
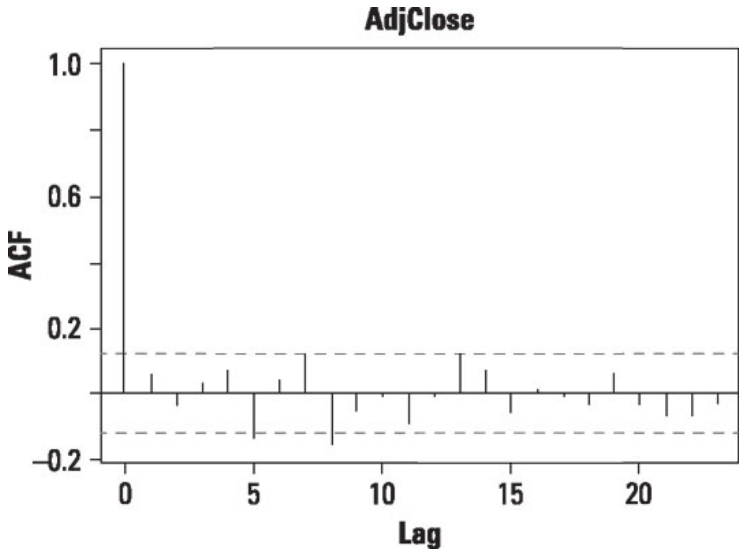
- Auto-correlation
- Correlation between
  - Current series
  - Time-shifted series
- Examples of:
  - Auto-correlated values
  - Non auto-correlated values





Short representation of random variable

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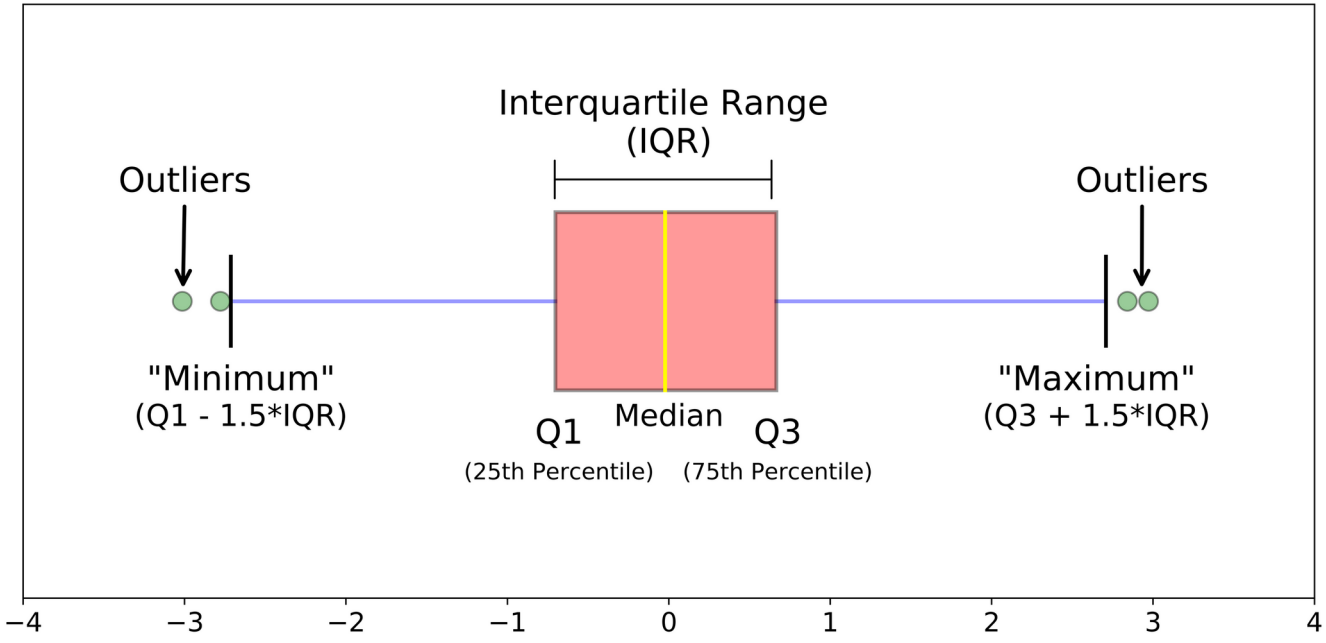
- Boxplot
- Compact representation of a random variable
- Main key points of
  - PDF function
  - Samples

- Median
- Quartiles (25-perc, 75-perc)
- Estimation of min/max
  - Useful for Gaussian var.
- Outliers



Short representation of random variable

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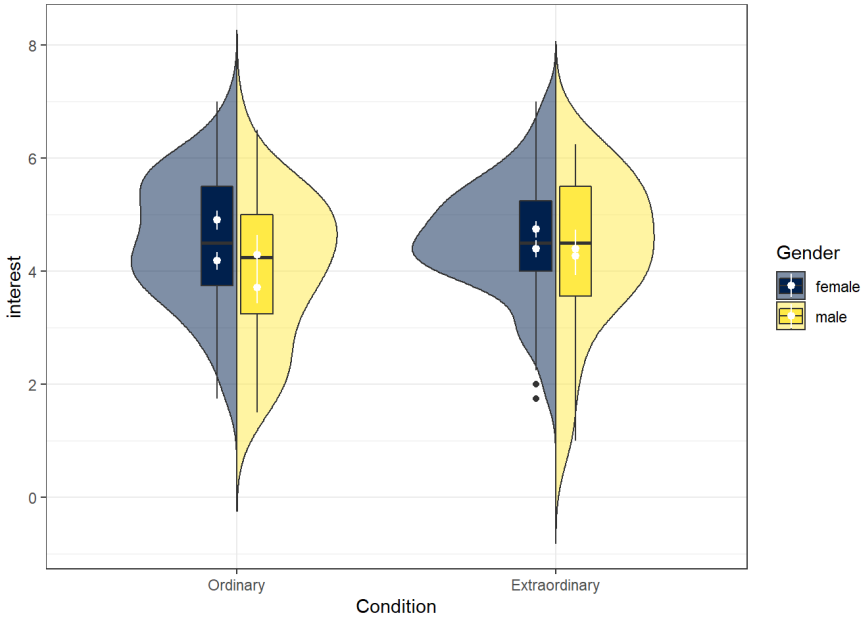
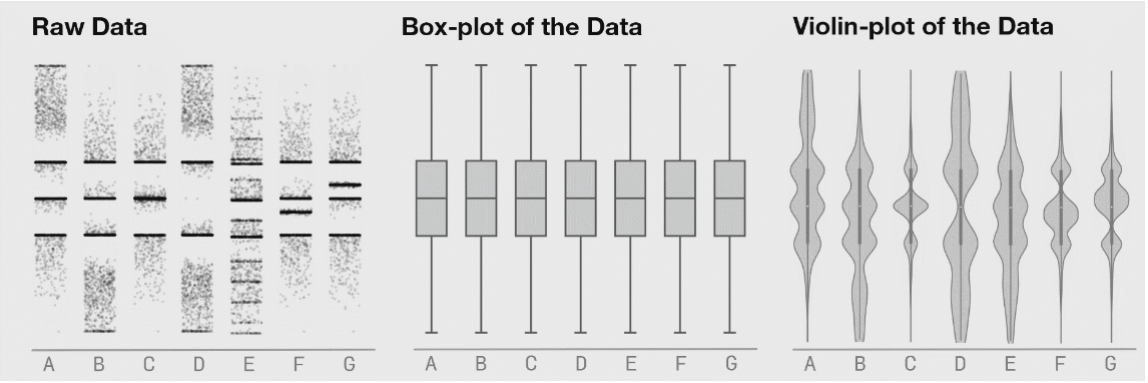
- Violin plot
- Composite plot
  - Boxplot +
  - Probability density
- Useful to:
  - Provide detailed statistical description
  - Compare different variables
- Can be valuable when distribution is irregular





Variable comparison

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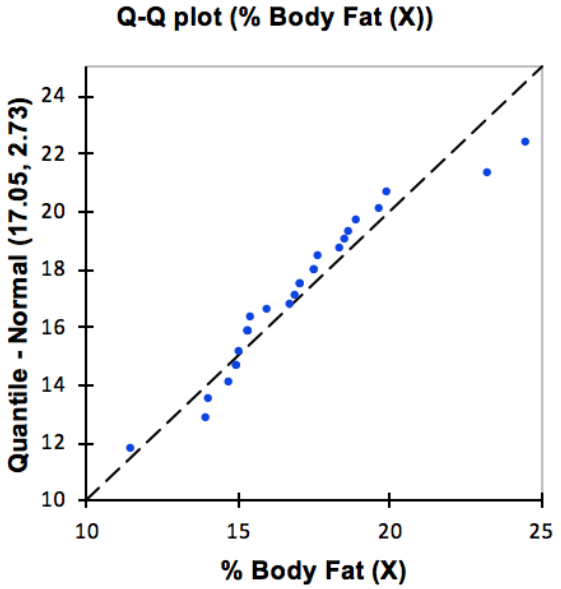
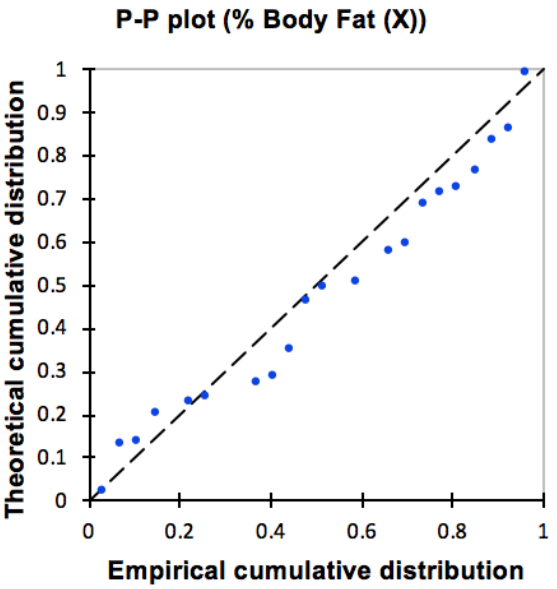
- Given two variables with PDF
  - $F(x) = P(F \leq x)$
  - $G(x) = P(G \leq x)$
- P-P plot
  - Plot (F(x), G(x)) for all x
- Percentile view
  - Q(i, F(x)) → i-th percentile
- Q-Q plot
  - Plot (Q(i, F(x)), Q(i, G(x))) for i in [1..100]
- More emphasis on head and tail





Data analysis

Transient and steady-state

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- Simulation goals:
  - Analysis of steady state
  - Analysis of transient behavior
- Examples of steady state analyses:
  - Throughput of a computing system under a known workload
  - Steady state is not necessarily constant
    - Workload changes over the time of day
    - Workload evolution is much slower than client requests
  - Transient effect are present but not wanted
- Examples of transient analysis:
  - Throughput evolution during traffic surge and cloud scale-up

Types of simulation

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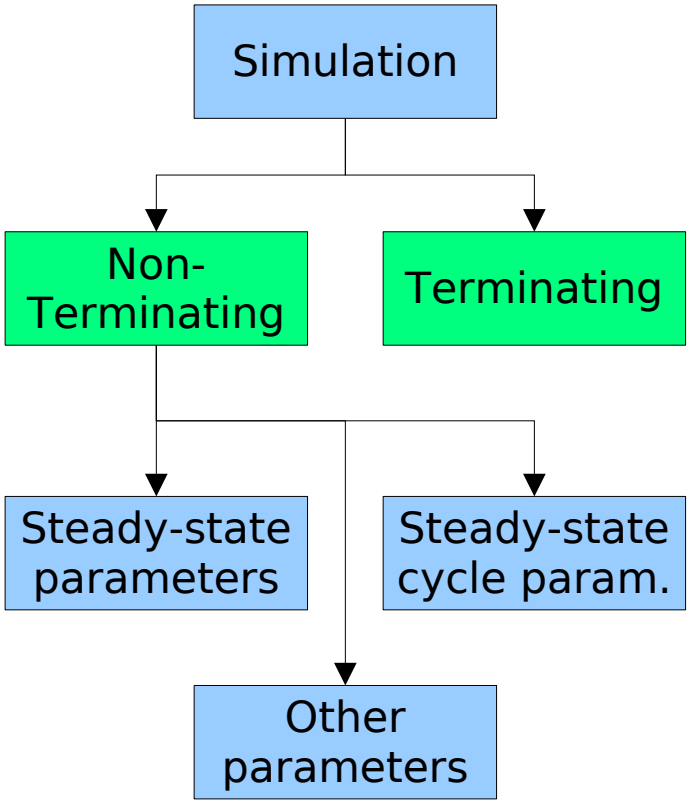
- Types of simulation based on output analyses
- Terminating analyses
  - Natural termination event
  - Focus on transient behavior
  - Examples:
    - A day in a retail establishment
    - Manufacturing a batch of items
- Non-terminating simulations
  - No termination event
  - Normal (long run) operation setup



Types of simulation

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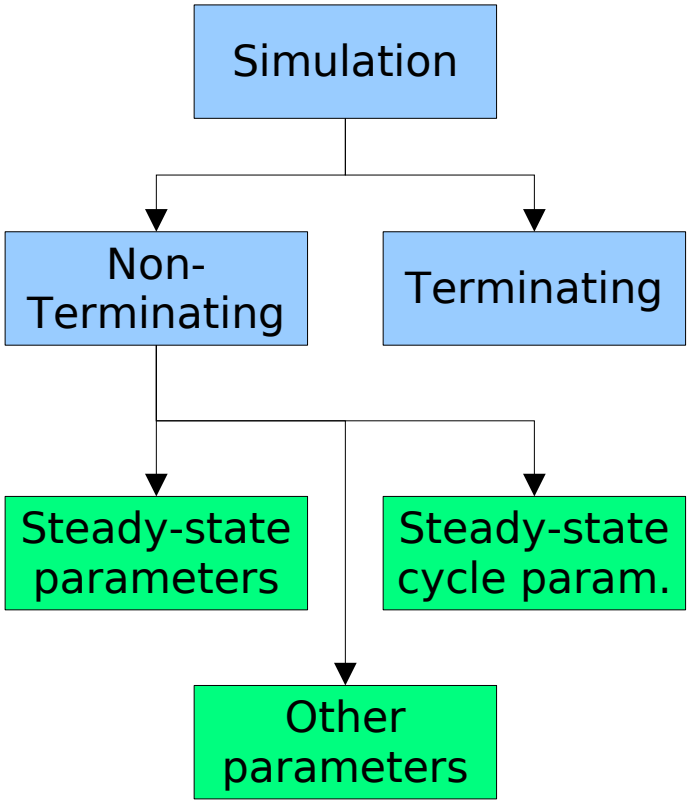
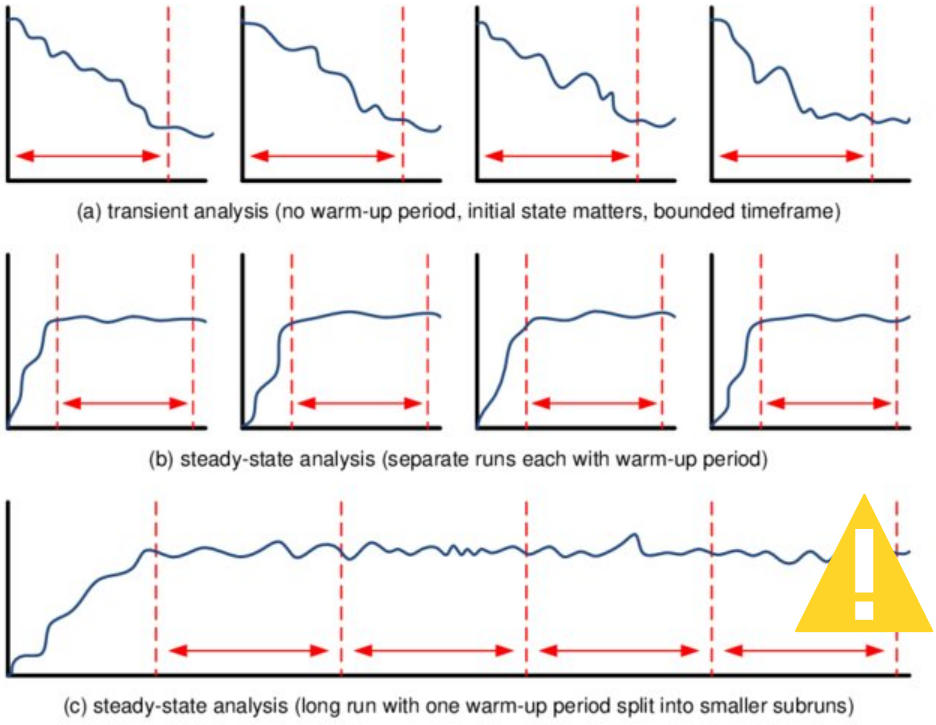
- Steady state parameters analyses
  - Avoids transient effects
  - We assume system status constant over time
- Steady state cycle parameters analyses
  - Focus on returning patterns
  - E.g., scaling behavior of cloud over daily patterns
    - Not interested in single requests, fluid models
- Other parameters
  - Everything else (es, transient)



Steady state analysis

- Some critical elements to remember
- Presence of initial transient periods (cold system)
  - Reducing impact of transient using long simulations
  - Avoid collecting data during the initial period (warm-up)
- Simulation length:
  - Must collect enough system data
  - Remember law of large numbers



Steady state cycle analysis

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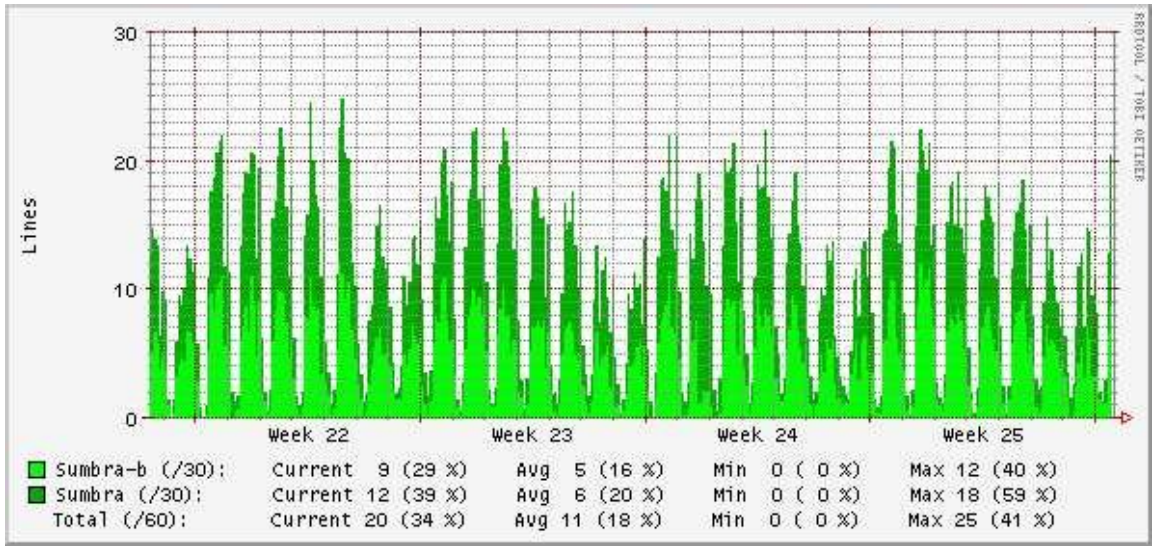
- Dual focus:
  - Variance analysis (focus on oscillations)
  - Trend analysis (focus on long term patterns)
- Use of filters/ICA techniques to separate patterns
- Smoothing (EWMA) for finding trends



Variance reduction

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- Increasing accuracy of simulations
- Some approaches:
- CRN: Common Random Numbers
  - When comparing alternative setup use the same seeds
  - Environment as similar as possible between configurations
  - Even RNG settings!
  - Better if we use separate sequences for different parameter

Variance reduction

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- More complex approaches
- AV: Antithetic Variates
  - Use RNG such that time series are negatively correlated
  - Average is more stable
- Control variates
  - Usable if we know two variables to be correlated
  - We can use this correction to reduce variance
  - Use correlation to estimate the shift from the real average
  - Needs knowledge of statistical properties of output