Aufgabe 1:   a)   normal sind $\tilde{A}, \tilde{C}$, subnormal ist $\tilde{B}$

b) $\tilde{B}_{norm} = \{(1, 0.25), (2, 0.5), (3, 0.75), (4, 1), (5, 1), (6, 0.5)\}$

c) $\tilde{B} \subseteq \tilde{A}$ gilt, aber nicht $\tilde{B}_{norm} \subseteq \tilde{A}$

d) $\operatorname{supp}(\tilde{C}) = \{x \mid 0 < x < 2\pi, \; x \neq \pi\}$

e) $C^{\geq 0.5} = \{x \mid \frac{\pi}{6} \leq x \leq \frac{5\pi}{6}$ bzw. $\frac{7\pi}{6} \leq x \leq \frac{11\pi}{6}\}$

f)

	1	2	3	4	5	6
$\tilde{A}^c$	0.6	0.3	0.1	0	0.4	0.8
$\tilde{A} \cap \tilde{B}$	0.1	0.2	0.3	0.4	0.4	0.2
$\tilde{A} \cup \tilde{B}$	0.4	0.7	0.9	1.0	0.6	0.2
$\tilde{A} \sqcap_a \tilde{B}$	0.04	0.14	0.27	0.4	0.24	0.04
$\tilde{A} \sqcup_a \tilde{B}$	0.46	0.76	0.93	1.0	0.76	0.36
$\tilde{A} \sqcap_b \tilde{B}$	0	0	0.2	0.4	0	0
$\tilde{A} \sqcup_b \tilde{B}$	0.5	0.9	1.0	1.0	1.0	0.4
$\tilde{A} \sqcap_d \tilde{B}$	0	0	0	0.4	0	0
$\tilde{A} \sqcup_d \tilde{B}$	1.0	1.0	1.0	1.0	1.0	1.0

g)

	1	2	3	4	5	6
$\frac{\tilde{A}+\tilde{B}}{2}$	0.25	0.45	0.6	0.7	0.5	0.2
$\sqrt{\tilde{A} \cdot \tilde{B}}$	0.2	0.37	0.52	0.63	0.49	0.2
$\tilde{A} \cdot_{\gamma=} \tilde{B}$	0.08	0.23	0.39	0.53	0.34	0.08

h)   nein

Aufgabe 2:   Antwort ist subjektiv bedingt, jeder darf sein eigenes Modell haben

Aufgabe 3:   a)

$$\tilde{A} \cap \tilde{B}: \quad \mu(x) = \begin{cases} \dfrac{x}{5} & 0 \leq x < \dfrac{25}{6} \\[2mm] \dfrac{10-x}{7} & \dfrac{25}{6} \leq x \leq 10 \end{cases}$$

$$\tilde{A} \cup \tilde{B}: \quad \mu(x) = \begin{cases} \dfrac{x}{3} & 0 \leq x < 3 \\[2mm] \dfrac{10-x}{7} & 3 \leq x \leq \dfrac{25}{6} \\[2mm] \dfrac{x}{5} & \dfrac{25}{6} \leq x < 5 \\[2mm] \dfrac{10-x}{5} & 5 \leq x \leq 10 \end{cases}$$

$$\tilde{A} \sqcap_a \tilde{B}: \quad \mu(x) = \begin{cases} \dfrac{x^2}{15} & 0 \leq x < 3 \\[2mm] \dfrac{(10-x)x}{35} & 3 \leq x < 5 \\[2mm] \dfrac{(10-x)^2}{35} & 5 \leq x \leq 10 \end{cases}$$

$$\tilde{A} \sqcup_a \tilde{B}: \quad \mu(x) = \begin{cases} \dfrac{8x - x^2}{15} & 0 \le x < 3 \\[2mm] \dfrac{x^2 - 8x + 50}{35} & 3 \le x < 5 \\[2mm] \dfrac{(10-x)(x+2)}{35} & 5 \le x \le 10 \end{cases}$$

$$\tilde{A} \sqcap_d \tilde{B}: \quad \mu(3) = \frac{3}{5}, \quad \mu(5) = \frac{5}{7}, \quad \mu(x) = 0 \text{ sonst}$$

$$\tilde{A} \sqcup_d \tilde{B}: \quad \mu(0) = 0, \quad \mu(10) = 0, \quad \mu(x) = 1 \text{ sonst}$$

b) $\operatorname{supp}(\tilde{A} \sqcap_d \tilde{B}) = \{3, 5\}$, $\operatorname{supp}(\tilde{A} \sqcup_d \tilde{B}) = \{x \mid 0 < x < 10\}$

Aufgabe 4: $\tilde{A} \subseteq \tilde{B} \quad \Leftrightarrow \quad \forall x \in X : \mu_A(x) \le \mu_B(x)$

Sei $x \in \operatorname{supp}(\tilde{A}) \quad \Rightarrow \quad \mu_A(x) > 0$

Wegen $\mu_A(x) \le \mu_B(x)$ ist dann auch $\mu_B(x) > 0$, also $x \in \operatorname{supp}(\tilde{B})$

Aufgabe 5: a) siehe 1 f)

b) γ für arithmetisches Mittel: 0.6777 , γ für geometrisches Mittel: 0.6035 ,

Aufgabe 6: a) $\tilde{A} \otimes \tilde{B} = \begin{Bmatrix} ((3,2),0.3),((3,3),0.1),((3,4),0.3),((3,5),0.3),((4,2),0.4),((4,3),0.1), \\ ((4,4),0.7),((4,5),0.7),((5,2),0.4),((5,3),0.1),((5,4),1),((5,5),0.7), \\ ((6,2),0.4),((6,3),0.1),((6,4),0.5),((6,5),0.5) \end{Bmatrix}$

b) $\tilde{C} = \begin{Bmatrix} (6,0.3),(8,0.4),(9,0.1),(10,0.4),(12,0.4),(15,0.3), \\ (16,0.7),(18,0.1),(20,1.0),(24,0.5),(25,0.7),(30,0.5) \end{Bmatrix}$

c) Nur $\tilde{A}$ ist diskrete Fuzzy-Zahl

d) $\tilde{D} = \{(3,0.3),(4,0.7),(5,1.0),(6,0.5)\}$

Aufgabe 7: a) $\operatorname{supp}(\tilde{A}) = \{x \mid 3 < x < 9\} \quad \Rightarrow \quad \operatorname{supp}(\tilde{B}) = \{y \mid 4 < y < 12\}$

$$\mu_B(y) = \mu_A\left(f^{-1}(y)\right) = \mu_A\left(\frac{36}{y}\right) = \begin{cases} 9\left(\dfrac{1}{2} - \dfrac{2}{y}\right) & 4 \le y < \dfrac{36}{7} \\[2mm] 1 & \dfrac{36}{7} \le y < \dfrac{36}{5} \\[2mm] 3\left(\dfrac{6}{y} - \dfrac{1}{2}\right) & \dfrac{36}{5} \le y \le 12 \end{cases}$$

b) $f^{-1}(0) = \{3,5,7,9\} \quad \Rightarrow \quad \mu_B(0) = 1$

$f^{-1}(1) = \{6\} \quad \Rightarrow \quad \mu_B(1) = 1$

$f^{-1}(-1) = \{4,8\} \quad \Rightarrow \quad \mu_B(-1) = 0.5$

$f^{-1}(-0.5) = \left\{\dfrac{10}{3}, \dfrac{14}{3}, \dfrac{22}{3}, \dfrac{26}{3}\right\} \quad \Rightarrow \quad \mu_B(-0.5) = \dfrac{5}{6}$

Aufgabe 8: $\mathrm{supp}\big(\tilde{A}\big) = \{x \mid 2 < x < 6\} \;\Rightarrow\; \mathrm{supp}\big(\tilde{B}\big) = \{y \mid 0 \le y < 6.25\}$

Fall 1: $2.25 < y < 6.25$

$$f^{-1}(y) = \left\{4.5 - \sqrt{y}\right\} \;\Rightarrow\; \mu_B(y) = \mu_A\left(4.5 - \sqrt{y}\right) = \frac{2.5 - \sqrt{y}}{3}$$

Fall 2: $0 \le y < 2.25$

$$f^{-1}(y) = \left\{4.5 - \sqrt{y}, 4.5 + \sqrt{y}\right\} \;\Rightarrow\; \mu_B(y) = \max\left\{\mu_A\left(4.5 - \sqrt{y}\right), \mu_A\left(4.5 + \sqrt{y}\right)\right\}$$

$$\mu_B(y) = \begin{cases} \frac{1}{3}\left(2.5 + \sqrt{y}\right) & 0 \le y < 0.25 \\ 1.5 - \sqrt{y} & 0.25 \le y < 1 \\ \frac{1}{3}\left(2.5 - \sqrt{y}\right) & 1 \le y \le 2.25 \end{cases}$$

insgesant

$$\mu_B(y) = \begin{cases} \frac{1}{3}\left(2.5 + \sqrt{y}\right) & 0 \le y < 0.25 \\ 1.5 - \sqrt{y} & 0.25 \le y < 1 \\ \frac{1}{3}\left(2.5 - \sqrt{y}\right) & 1 \le y < 6.25 \end{cases}$$

Aufgabe 9: $\tilde{B} = (85, 7, 10)_{tri}$, $\tilde{C} = (57, 11, 6)_{tri}$

Aufgabe10: $\tilde{A}, \tilde{B} \subseteq \tilde{C} \;\Rightarrow\; \forall x \in X: \mu_A(x) \le \mu_C(x)$ und $\mu_B(x) \le \mu_C(x)$

$\Rightarrow\; \forall x \in X: \mu_C(x) \ge \max\left\{\mu_A(x), \mu_B(x)\right\} = \mu_{A \cup B}(x) \;\Rightarrow\; \tilde{A} \cup \tilde{B} \subseteq \tilde{C}$

Aufgabe11: $t(x,y) = \dfrac{xy}{1 + (1-x)(1-y)}$

Aufgabe12: $\tilde{B} = (88, 13, 17)_{tri}$, $\tilde{C} = (20, 19, 11)_{tri}$

Aufgabe13: $\mu_k(x) = \begin{cases} 1 & 0 \le x \le 120 \\ 5 - \frac{1}{30}x & 120 \le x \le 150 \\ 0 & 150 \le x \le 180 \end{cases}$

$$\mu_{nsl}(x) = \begin{cases} 1 & 0 \le x \le 120 \\ 1 - \left(\frac{1}{30}x - 4\right)^2 & 120 \le x \le 150 \\ 0 & 150 \le x \le 180 \end{cases}$$

$$\mu_{lonsk}(x) = \begin{cases} 0 & 0 \le x \le 120 \\ 1 - \left(5 - \frac{1}{30}x\right)^2 & 120 \le x \le 150 \\ 1 & 150 \le x \le 180 \end{cases}$$

Aufgabe 14: $\mu_{nsl}(x) = \begin{cases} 0 & 0 \le x \le 30 \\ 1 - \frac{1}{400}(50-x)^2 & 30 \le x \le 50 \\ 1 & 50 \le x \le 100 \end{cases}$

$\mu_{zs}(x) = \begin{cases} 0 & 0 \le x \le 30 \\ \frac{1}{10}\sqrt{5x-150} & 30 \le x \le 50 \\ 1 & 50 \le x \le 100 \end{cases}$

Aufgabe 15: $\tilde{R} \circ_{MP} \tilde{S}$:

	1	2
α	0.7	0.63
β	0.6	0.63
γ	0.8	0.9

Aufgabe 16:

a) $\tilde{R} \circ_{MM} \tilde{S}$

	α	β	γ	δ
a	0.7	0.2	0.1	0.1
b	0.9	0.5	0.5	0.1
c	0.9	0.9	0.5	0.3

b) $\tilde{R} \circ_{MP} \tilde{S}$

	α	β	γ	δ
a	0.63	0.14	0.05	0.01
b	0.9	0.5	0.25	0.05
c	0.9	0.9	0.5	0.15

c) $\{(1, 1.0), (2, 0.5), (3, 0.4)\}$

Aufgabe 17: $\mu_{B''}(x) = \begin{cases} \dfrac{x-22}{7} & 22 \le x < \dfrac{77}{3} \\ \dfrac{11}{21} & \dfrac{77}{3} \le x \le \dfrac{659}{21} \\ \dfrac{34-x}{5} & \dfrac{659}{21} < x \le 12 \end{cases}$

Aufgabe 18:

a)

	10	20	30
1	1	0.3	0
2	0	0.3	0
3	0	0.6	1

b) $\{(10, 0.2), (20, 0.3), (30, 0.1)\}$

c) $\{(10, 0.4), (20, 0.5), (30, 0.5)\}$

Aufgabe 19: $\tilde{R}$ ist Lösung

Keine Garantie für Fehlerfreiheit !