Computationele logica

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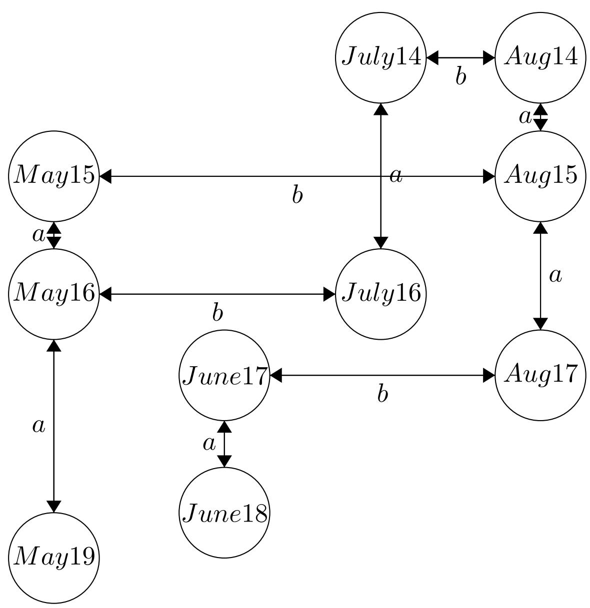
Exercise 1

ϕ = the date of Cheryl's birthday, a = Albert, b = Bernard, c = Cheryl

With arrows we are representing the children's knowledge relations, so we'll get an epistemic model: all relations R1, R2, R3 are equivalence relations. So in particular they are reflexive, but for simplicity of drawing we skipped the loops.

(a) Model M of the situation immediately after Cheryl gives the boys their pieces of information:



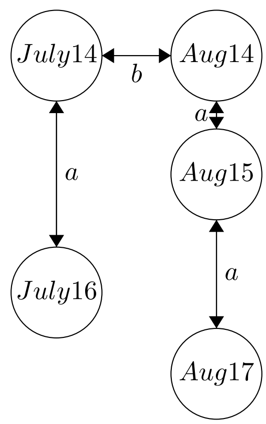
(b) Epistemic sentence encoding Albert's first announcement:

$!(\neg K_a \phi \wedge K_a \neg K_b \phi)$

Albert announces that he doesn't know the date of Cheryl's birthday, and that he knows that Bernard doesn't know the date of Cheryl's birthday either.

(c) Updated model M' after Albert's first announcement:

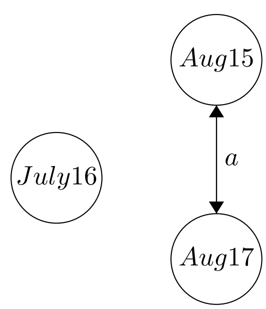
If Bernard either knew that the date was the 19th of May or the 18th of June, he would've instantly known the date of Cheryl's birthday because no other dates exists with these day numbers. Therefore, May and June are no longer an option after the public announcement.



(d) Epistemic sentence and updated model M" after Bernard's announcement:

$!(K_b\phi)$

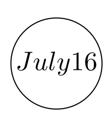
Bernard announces that he knows the date of Cheryl's birthday now.



(e) Epistemic sentence and updated model M"' after Albert's second announcement:

$!(K_a\phi)$

Albert announces that he too knows.



Cheryl's birthday is July 16.

Exercise 2

Prove formally that, for every sentence φ, the sentence

$$\neg K_a \varphi \Rightarrow K_a \neg K_a \varphi$$

(expressing "Negative Introspection of Knowledge") is *valid* on (the family of all) **epistemic** models.

Let $M = \{W, R_a, R_b, \ldots, \nu\}$ be any epistemic model and let $w \in W$ be any world in it.

To prove the claim, suppose that $\neg K_a \varphi$ is true at w, i.e.

$$w \models_M \neg K_a \varphi. \tag{1}$$

We need to prove that

$$w \models_M K_a \neg K_a \varphi. \tag{?}$$

Let W' be all worlds reachable from w by actor a, i.e. $W' = \{w' \in W | w R_a w'\}$. Then from (1) it follows that $\exists v \in W' | v \models \varphi$ and $\exists s \in W' | s \not\models \varphi$. Because of symmetry, $v R_a w$ and because of transitivity, $v R_a s$. Because actor a cannot distinguish between v and s, it holds that $\neg K_a \varphi$ is true in v and s. Because W' only exists of worlds like v and s, $\neg K_a \varphi$ is true in all worlds of W'. From the semantics of K_a, it follows that for each $w' \in W'$, $w' \models_M K_a \neg K_a \varphi$. Because $w \in W'$ ($w R_a w$ (reflexivity)), $w \models_M K_a \neg K_a \varphi$ and (?) is proven.

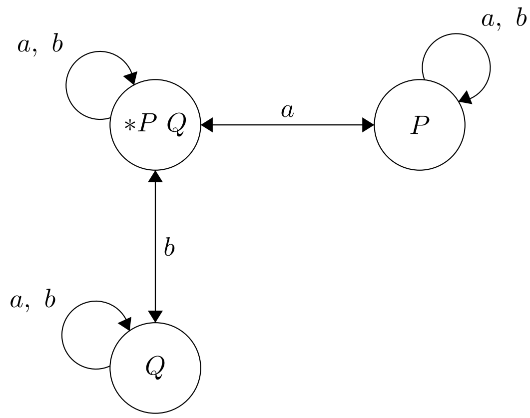
Exercise 3

Using the semantics of knowledge K_a and common knowledge Ck, show that the following is NOT valid on *epistemic models with (only) 2 agents a and b:*

$$(K_a K_b \phi \wedge K_b K_a \psi) \Rightarrow Ck(\phi \wedge \psi)$$

$*$ = The representation of the world
P = ϕ
Q = ψ
a = Albert, b = Bernard, c = Cheryl

The epistemic model holds the beliefs that both a and b know P and Q, but they are not sure whether they know the fact that both a and b know P and Q.


a, b
a, b
*P Q
a
P
b
a, b
Q