Computationele logica

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Exercise 1

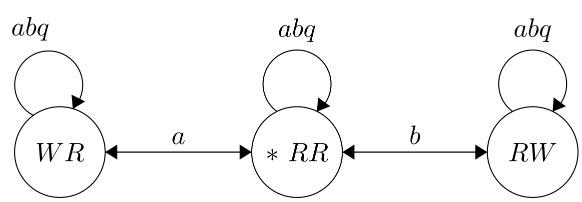
1. The sentence θ encoding all information:

   Alice and Bob have red hats, Alice knows that Bob has a red hat, Bob knows Alice has a red hat, the queen knows Alice and Bob have red hats, the queen knows that Alice knows that Bob has a red hat, the queen knows that Bob knows that Alice has a red hat. It is common knowledge that the queen knows the color of Alice's and Bob's hats, it is common knowledge that Alice knows the color of Bob's hat, it is common knowledge that Bob knows the color of Alice's hat, it is common knowledge that Alice and Bob don't know the color of their own hats, it is common knowledge that both Alice and Bob know that they have either a red or white hat and it is common knowledge that there are two red hats and one white hat.

   $\theta = r_a \wedge r_b \wedge K_a(r_b) \wedge K_b(r_a) \wedge K_q(r_a) \wedge K_q(r_b) \wedge K_q(K_a(r_b)) \wedge K_q(K_b(r_a)) \wedge CK(K_q(r_a \vee w_a)) \wedge CK(K_q(r_b \vee w_b)) \wedge CK(K_a(r_b \vee w_b)) \wedge CK(K_b(r_a \vee w_a)) \wedge CK(\neg K_a(r_a)) \wedge CK(\neg K_a(\neg r_a)) \wedge CK(\neg K_a(w_a)) \wedge CK(\neg K_a(\neg w_a)) \wedge CK(\neg K_b(r_b)) \wedge CK(\neg K_b(\neg r_b)) \wedge CK(\neg K_b(w_b)) \wedge CK(\neg K_b(\neg w_b)) \wedge CK((r_a \wedge r_b) \vee (r_a \wedge w_b) \vee (w_a \wedge r_b))$

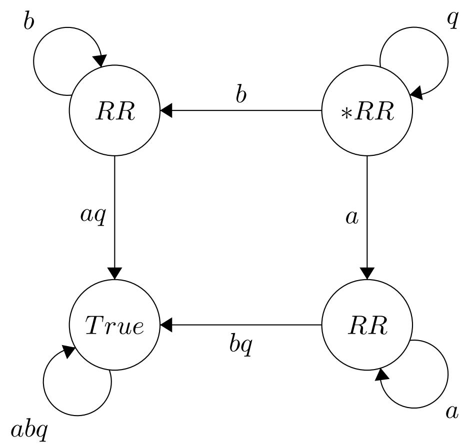
2. A representation of the situation model $\mathbf{M}$:

   $\mathcal{A}$ = {a, b, q} the agents Alice, Bob, and the Queen
   $\Phi = \{r_a, w_a, r_b, w_b\}$ written as WR for: a is white and b is red



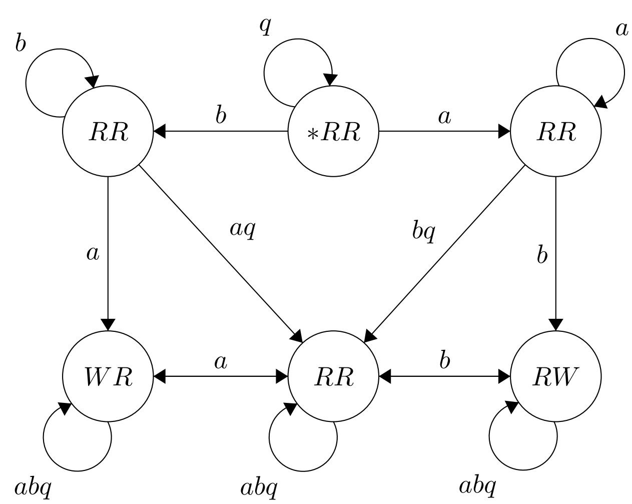
This is an epistemic model: YES

3. Seperately a and b look in their mirrors and see their red hats, the queen sees everything, represented in the event model Σ with four actions:



This is an epistemic model: NO
This is a doxasic model: YES

4. The update product of the two models $\mathbf{M} \bigotimes \Sigma$:

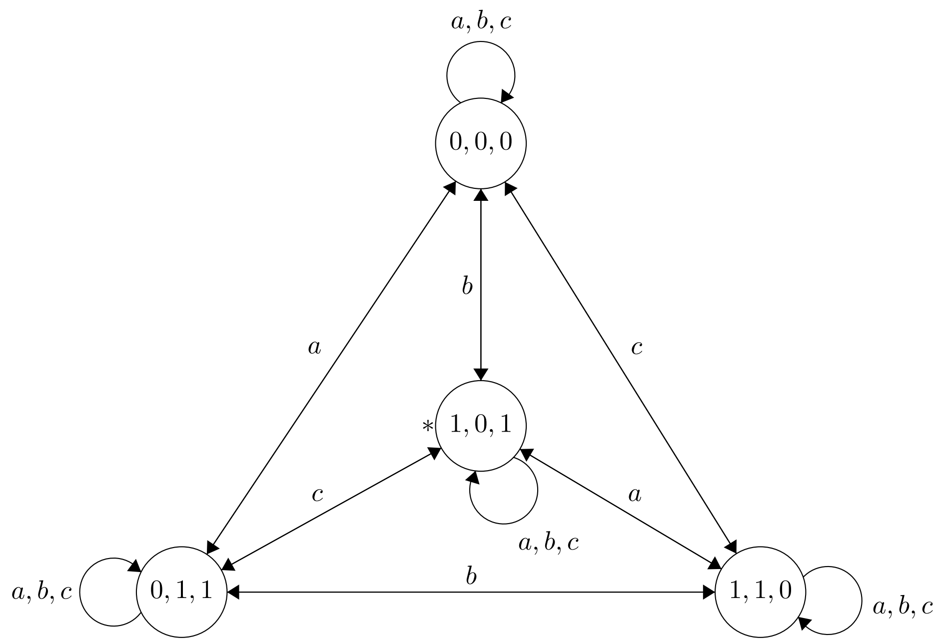


This is an epistemic model: NO
This is a doxasic model: YES

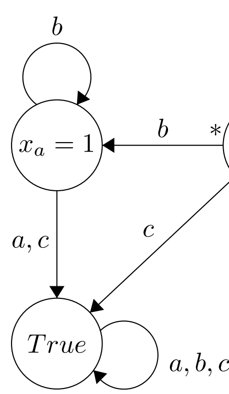
Exercise 2

We mark real worlds/events with an $*$.

1. Representation of the bits world as an epistemic model $\mathbf{M}$:

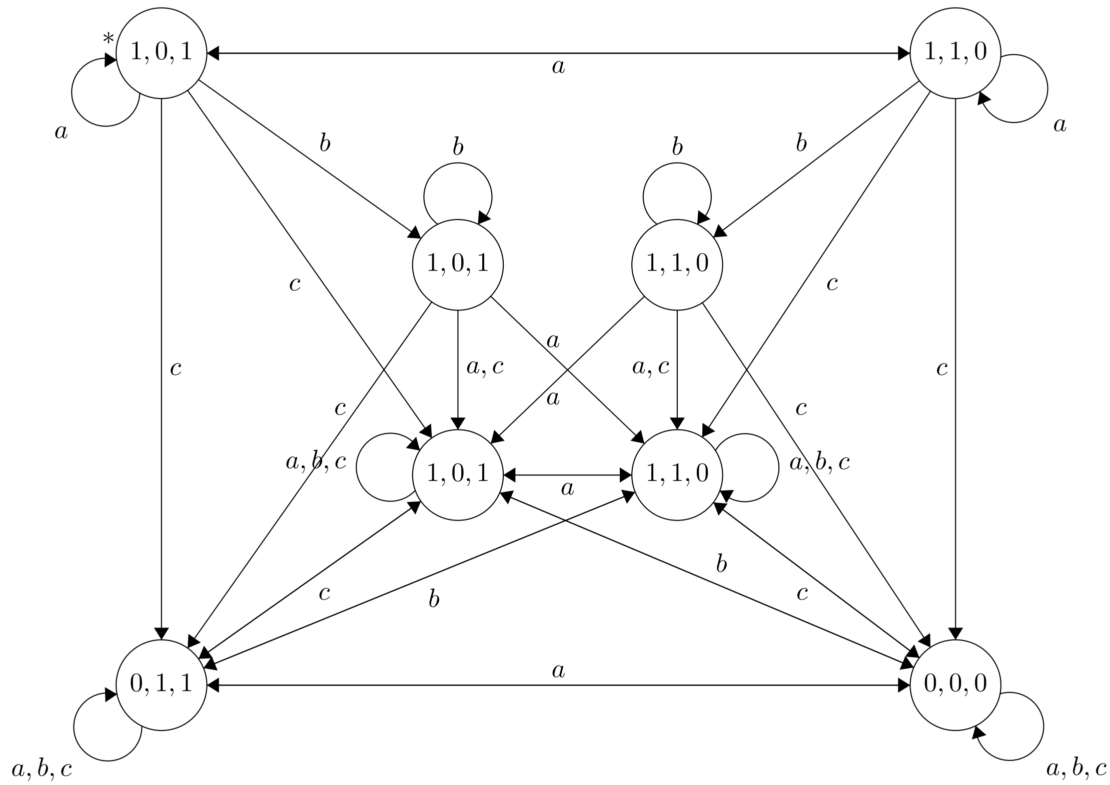


2. Representation of event model Σ:

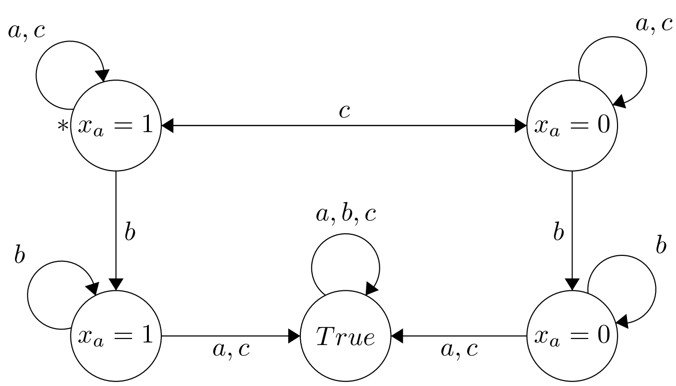


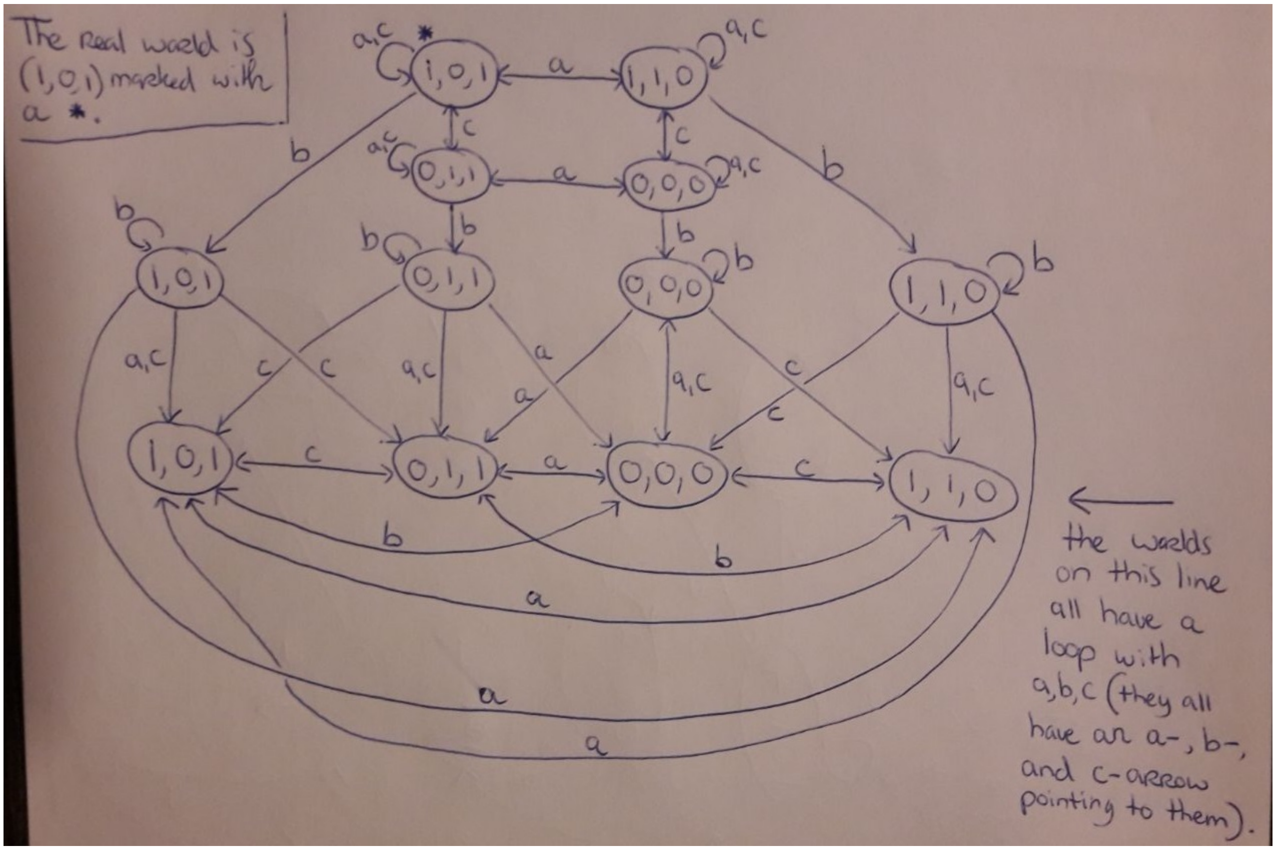
This is a doxastic model.

3. Representation of model **M'**:



4. Representation of event model Σ':





5. The update product of the two models $\mathbf{M"} = \mathbf{M} \bigotimes \Sigma'$:

Exercise 3

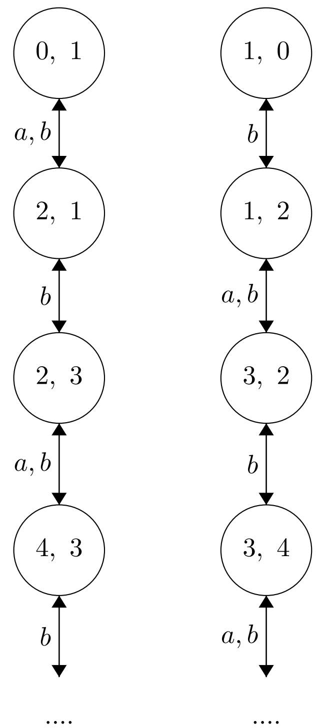
Each of two children, Alice and Bob, has a natural number written in the *back of his/her head*. Let $n_a \in \{0, 1, 2, \dots\}$ be Alice's number and $n_b \in \{0, 1, 2, \dots\}$ be Bob's number. It is common knowledge that: (i) *no child can see his/her own number*, (ii) Alice stands in the back of Bob, so *she can see Bob's number n_b, but Bob cannot see any of the numbers*, and (iii) one of the numbers is the immediate successor of the other (in any order): i.e. either $n_a = n_b + 1$ or $n_b = n_a + 1$. In the following, each of the children will be asked a number of questions, which they are required to answer publicly and truthfully.

1. *How many possible worlds* are there (that are consistent with the story above)?

   There are infinite possible worlds.

2. *Represent* **(draw) the above situation as an *epistemic model* $\mathbf{M}_1$**, with two agents (a for Alice, b for Bob), using pairs of numbers (n_a,n_b) as "names" for the possible worlds. **Draw the epistemic accessibility relations** for each agent, but do *not* worry about the valuation (yet), since no atomic sentences are given yet.

   For simplicity we skipped the loops (every world has an a- and b-arrow to itself) and we skipped a lot of b-arrows (every world has a b-arrow to every other world).



3. For your model, consider now the following *four atomic sentences* $0_a, 0_b, 1_a, 1_b$. Here, 0_a means "Alice's number is equal to 0" (i.e. $n_a = 0$), and 0_b means "Bob's number is equal to 0" (i.e. $n_b = 0$); 1_a means "Alice's number is equal to 1" (i.e. $n_a = 1$), and 1_b means "Bob's number is equal to 1" (i.e. $n_b = 1$). **Specify the valuation for these atomic sentences** in the above model.

   We only learned how to valuate worlds, and the valuation of a world is the set of basic facts that are true in that world. So when we only consider the four atomic sentences above, we have as valuations:

   $\nu((0,1)) = \{0_a, 1_b\}$

   $\nu((1,0)) = \{1_a, 0_b\}$

   $\nu((2,1)) = \{1_b\}$

   $\nu((1,2)) = \{1_a\}$

   in all other worlds v: $\nu(v) = \emptyset$

   If we assume "valuation of an atomic sentence" means "specify the set of worlds in which the atomic sentence is true", then the valuations are:

   $\nu(0_a) = \{(0,1)\}$

   $\nu(0_b) = \{(1,0)\}$

$\nu(1_a) = \{(1,0),(1,2)\}$

$\nu(1_b) = \{(0,1),(2,1)\}$

4. Alice is asked the following question: *"Do you know whether your own number is equal to* 0 *or not, and if so then which of the two?"* So her answers can be: (a) *I don't know*, (b) *I know that my number is equal to* 0, or (c) *I know that my number is NOT equal to* 0.

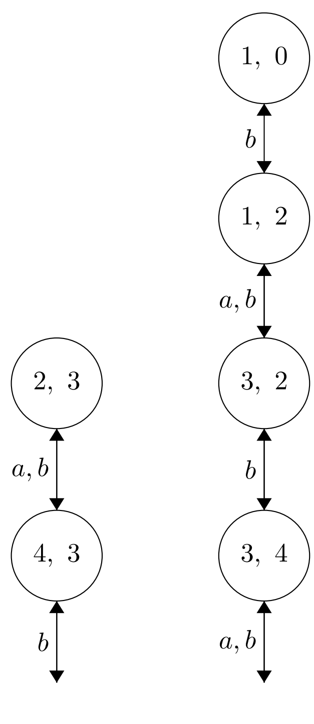
   Let us suppose that in fact *Alice answers (c) "I know that my number is NOT equal to* 0*"*.

   Write down a sentence φ in epistemic logic that expresses her *answer*.

   $\varphi = \mathrm{K}_a(\neg 0_a)$

5. Interpreting the above answer as a truthful public announcement $!\varphi$ of the sentence written in the previous part, **represent (draw) the updated model $\mathbf{M}_2 = \mathbf{M}_1^{!\varphi}$ after this public announcement**.

   For simplicity we skipped the loops (every world has an a- and b-arrow to itself) and we skipped a lot of b-arrows (every world has a b-arrow to every other world).



6. Suppose now that, *after* Alice answered as above, Bob is asked the "same"

question: *"Do you (Bob) know whether your own number is equal to* 0 *or not, and if so then which of the two?"*

What will Bob answer? Also, **what will be updated model M_3 representing the situation after he answers**? Justify your answers!

Bob will answer (a) *I don't know*, because in all worlds of M_2 Bob thinks every other world is possible (there are b-arrows going from each world to each other world). $M_3 = M_2$ (we cannot remove worlds), because there is no world in which Bob's answer ("I don't know") is false.

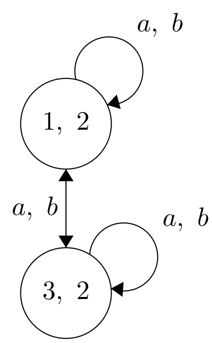
7. After the previous round of questions, Alice is now asked the following question: *"Do you know whether your own number is equal to* 1 *or not, and if so then which of the two?"* The asnwers can be: (a) *I don't know*, (b) *I know that my number is equal to* 1, or (c) *I know that my number is NOT equal to* 1.

   Let us suppose that in fact *Alice answers (a) "I don't know"*.

   Write down a sentence ψ in epistemic logic that expresses her *answer*.

   $\psi = \neg K_a(1_a) \wedge \neg K_a(\neg 1_a)$

8. Interpreting the above answer as a truthful public announcement $!\psi$ of the sentence written in the previous part, **represent (draw) the updated model $M_4 = M_3^{!\psi}$ after this public announcement**.



9. Suppose now that, *after* Alice answered as above, Bob is asked the "same" question: *"Do you (Bob) know whether your own number is equal to* 1 *or not, and if so then which of the two?"*

   What will Bob answer? Also, **what is his number**? Justify your answers!

   Bob will answer (c) *I know that my number is NOT equal to* 1, because in M_4 there is no world in which Bob knows his number is 1 (there is no b-arrow to a world where $n_b = 1$). $n_b = 2$, because in all possible worlds in M_4 Bob's number is 2.