Computationele logica

Kamans, Jim	Roosingh, Sander	Stefan, Schenk
-000000001	11983957	11881798

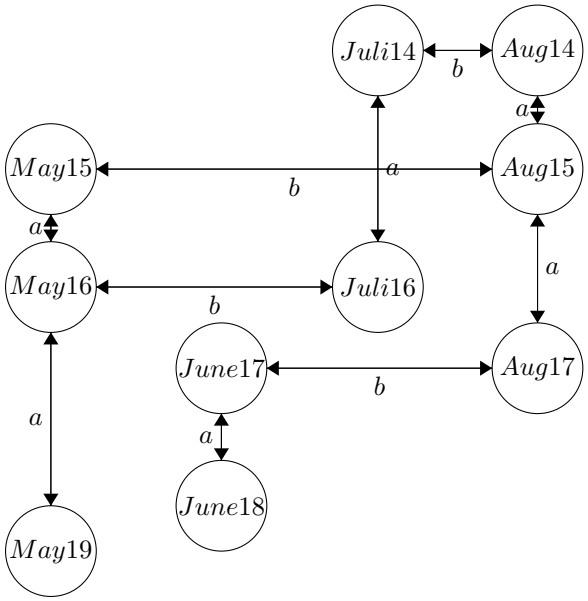
November 2017

1 Exercise 1: Singapore problem

ϕ = date of Cheryl's birthday
a = Albert, b = Bernard, c = Cheryl

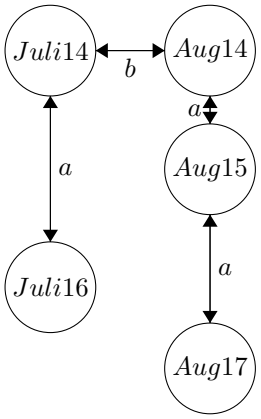
With arrows we are representing the children's knowledge relations, so we'll get an epistemic model: all relations R1, R2, R3 are equivalence relations. So in particular they are reflexive, but for simplicity of drawing we skipped the loops.

(a) Model M of the situation immediately after Cheryl gives the boys their pieces of information:

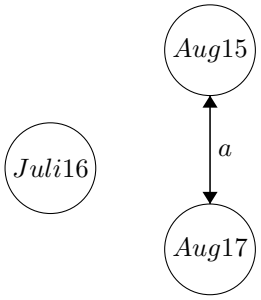


(b) Epistemic sentence encoding Albert's first announcement:
$!_a(\neg K_a\phi \wedge K_a\neg K_b\phi)$

(c) Updated model M' after Albert's first announcement:



(d) Epistemic sentence and updated model M" after Bernard's announcement:
$!_b(K_b\phi)$



(e) Epistemic sentence and updated model M"' after Albert's second announcement:
$!_a(K_a\phi)$

2 Exercise 2

Prove formally that, for every sentence φ, the sentence

$$\neg K_a\varphi \Rightarrow K_a\neg K_a\varphi$$

(expressing "Negative Introspection of Knowledge") is *valid* on (the family of all) **epistemic** models.

3 Exercise 3

Using the semantics of knowledge K_a and common knowledge Ck, show that the following is NOT valid on *epistemic models with (only) 2 agents a and b:*

$$(K_a K_b \phi \wedge K_b K_a \psi) \Rightarrow Ck(\phi \wedge \psi)$$

$*$ = The representation of the world
P = ϕ
Q = ψ

The epistemic model holds the beliefs that both a and b know P and Q, but they are not sure whether they know the fact that both a and b know P and Q.