

Computationele logica

Kamans, Jim
10302905

Roosinigh, Sander
11983957

Schenk, Stefan
11881798

November 2017

1 Exercise 1: Singapore problem

ϕ = date of Cheryl's birthday

- (a) Model M of the situation immediately after Cheryl gives the boys their pieces of information:
- (b) Epistemic sentence encoding Albert's first announcement:
 $!_a(\neg K_a \phi \wedge K_a \neg K_b \phi)$
- (c) Updated model M' after Albert's first announcement:
- (d) Epistemic sentence and updated model M'' after Bernard's announcement:
 $!_b(K_b \phi)$
- (e) Epistemic sentence and updated model M''' after Albert's second announcement:
 $!_a(K_a \phi)$

2 Exercise 2

Prove formally that, for every sentence φ , the sentence

$$\neg K_a \varphi \Rightarrow K_a \neg K_a \varphi$$

(expressing “Negative Introspection of Knowledge”) is *valid* on (the family of all) **epistemic** models.

Let $M = \{W, R_a, R_b, \dots, \nu\}$ be any epistemic model and let $w \in W$ be any world in it.

To prove the claim, suppose that $\neg K_a \varphi$ is true at w , i.e.

$$(1) \quad w \models_M \neg K_a \varphi.$$

We need to prove that

$$(?) \quad w \models_M K_a \neg K_a \varphi.$$

Let v be an arbitrary world such that $w R_a v$. By the semantics of K_a , (1) implies

3 Exercise 3

Using the semantics of knowledge K_a and common knowledge Ck , show that the following is NOT valid on *epistemic models with (only) 2 agents a and b*:

$$(K_a K_b \phi \wedge K_b K_a \psi) \Rightarrow Ck(\phi \wedge \psi)$$

* = The representation of the world

P = ϕ

Q = ψ

The epistemic model holds the beliefs that both a and b know P and Q, but they are not sure whether they know the fact that both a and b know P and Q.

