

# Computationele logica

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## Exercise 1

(a) Show the following equivalence:

$$K_a\varphi \Leftrightarrow B_a^{\neg\varphi} \text{ false}$$

[ANSWER HERE]

(b) Prove semantically the equivalence claimed on Slide 24 of Lecture Notes 4.2:

$$B_a\phi \Leftrightarrow \Diamond_a \Box_a \phi$$

where  $\Diamond_a\phi : \neg\Box_a\neg\phi$  is the dual modality to safe belief  $\Box_a$ .

[ANSWER HERE]

(c) Prove (via a counterexample) that safe belief does NOT imply strong belief; i.e. that

$$\Box_a\varphi \not\Rightarrow Sb_a\varphi$$

[ANSWER HERE]

**HINT:** The positive statements (a) and (b) need general proofs: you need to show that, for every plausibility model and every sentence  $\phi$ , the desired formula is true at all the worlds of that model. But the negative statement (c) must be shown by giving a counterexample: construct some plausibility model and find some sentence  $\varphi$  for which the implication fails to be true at some world of that model (which we can think of as the “real world”).

## Exercise 2

1. Write down a logical formula in the language of beliefs, knowledge and conditional beliefs to encode all the above assumptions.

[ANSWER HERE]

2. *Represent the agent's beliefs (and conditional beliefs), using a plausibility model with four possible worlds. Specify the valuation (which atomic sentences of the two atomic sentences  $d$  and  $t$  are true at which worlds). Represent the agent's plausibility relation on these worlds, by drawing arrows going from the less plausible worlds to the more plausible ones.*

[ANSWER HERE]

3. *Suppose somebody who never lies tells our agent "You are close to the target if and only if you believe that you are in a dangerous zone." Write down formally this sentence as a formula  $\varphi$  in doxastic logic (using the atomic sentences).*

[ANSWER HERE]

4. *Interpreting the above truthful announcement  $!\varphi$  as an update with the sentence  $\varphi$  in the previous part, represent the updated model.*

[ANSWER HERE]

5. *After the previous announcement, another truthful announcement is made: "You are in a dangerous zone if and only if you don't believe that you are in a dangerous zone." Write down formally this sentence as a formula  $\psi$  in doxastic logic (using the atomic sentences).*

[ANSWER HERE]

6. *What is the real world? (In other words, answer the question: is the agent in a dangerous zone or not, and is he close to the target or not?) Justify your answer, by interpreting the announcement in the previous part as a new update  $\neg\psi$  and representing the updated model.*

[ANSWER HERE]