Computationele logica

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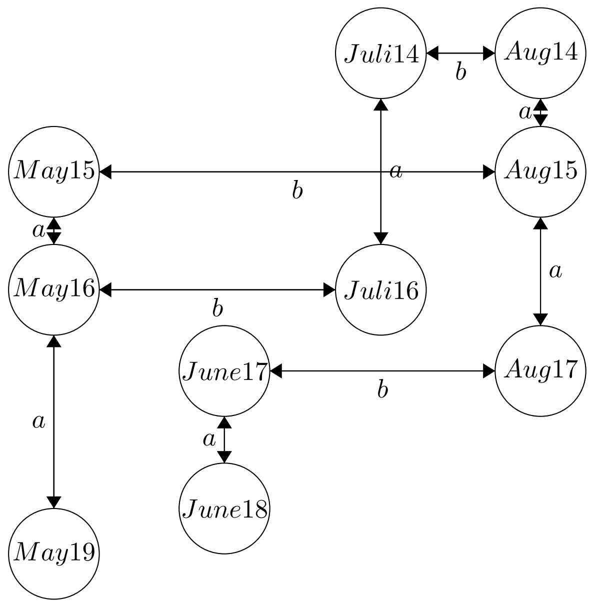
1 Exercise 1: Singapore problem

ϕ = date of Cheryl's birthday
a = Albert, b = Bernard, c = Cheryl

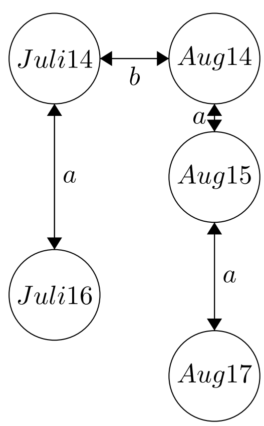
With arrows we are representing the children's knowledge relations, so we'll get an epistemic model: all relations R1, R2, R3 are equivalence relations. So in particular they are reflexive, but for simplicity of drawing we skipped the loops.

(a) Model M of the situation immediately after Cheryl gives the boys their pieces of information:

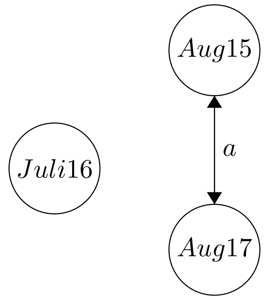


(b) Epistemic sentence encoding Albert's first announcement:
$!_a(\neg K_a\phi \land K_a\neg K_b\phi)$

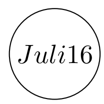
(c) Updated model M' after Albert's first announcement:



(d) Epistemic sentence and updated model M" after Bernard's announcement:
$!_b(K_b\phi)$



(e) Epistemic sentence and updated model M"' after Albert's second announcement:
$!_a(K_a\phi)$



2 Exercise 2

Prove formally that, for every sentence φ, the sentence

$$\neg K_a\varphi \Rightarrow K_a\neg K_a\varphi$$

(expressing "Negative Introspection of Knowledge") is *valid* on (the family of all) **epistemic** models.

Let $M = \{W, R_a, R_b, \ldots, \nu\}$ be any epistemic model and let $w \in W$ be any world in it.

To prove the claim, suppose that $\neg K_a\varphi$ is true at w, i.e.

(1) $$w \models_M \neg K_a\varphi.$$

We need to prove that

(?) $$w \models_M K_a\neg K_a\varphi.$$

Let v be an arbitrary world such that wR_av. By the semantics of K_a, (1) implies

3 Exercise 3

Using the semantics of knowledge K_a and common knowledge Ck, show that the following is NOT valid on *epistemic models with (only) 2 agents a and b:*

$$(K_aK_b\phi \wedge K_bK_a\psi) \Rightarrow Ck(\phi \wedge \psi)$$

$*$ = The representation of the world
P = ϕ
Q = ψ

The epistemic model holds the beliefs that both a and b know P and Q, but they are not sure whether they know the fact that both a and b know P and Q.