Assignment 3

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Note: In all plausibility model the reflexive and transitive arrows are omitted (just like in the slides).

1 Proofs

1.1

To proof: $K_a\varphi \Leftrightarrow B_a^{\neg\varphi} false$

Proof

$\Rightarrow$ Let $S = (S, \leq a, \sim_a, ||\cdot||)$ be an arbitrary plausibility model, and let $s \in S$ be arbitrary. Assume that $s \models K_a\varphi$. By semantic definition of K, this means that $s(a) \subseteq ||\varphi||$ (where $s(a) = \{t \subseteq S : s \sim_a t\}$ is world s's information cell in a's partition). By definition of $best_a$, $best_a(\neg\varphi \cap s(a)) \subseteq (\neg\varphi \cap s(a)) \subseteq s(a) \subseteq ||\varphi||$. But because φ is true for all worlds in $s(a)$, $best_a(\neg\varphi \cap s(a))$ is an empty set ($\varnothing$). If there are no worlds there can only contradictory propositions and by the semantic definition of conditional belief, this means that $s \models B_a^{\neg\varphi} false$.

$\Leftarrow$ Let $S = (S, \leq a, \sim_a, ||\cdot||)$ be an arbitrary plausibility model, and $s \in S$ be arbitrary. Assume that $s \models B_a^{\neg\varphi} false$. This means the agent believes in a contradiction when $\neg\varphi$ is announced. So $\neg\varphi$ must contradict with a proposition that is in the model before the announcement. $\neg(\neg\varphi)$ is φ. Because there are no worlds left in the model when $\neg\varphi$ is announced, before the announcement in all the worlds accessible by agent a from s, φ must be true: $t \models \varphi$ for all t such that $s \sim_a t$. By the semantic definition of knowledge in a plausibility model, this means that $K_a\varphi$. $\square$

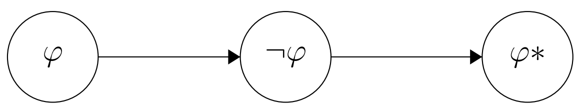
1.2

To proof: $B_a\phi \Leftrightarrow \Diamond_a\Box_a\phi$ *Proof*

$\Diamond_a\Box_a\phi := \neg\Box_a\neg\Box_a\phi$. $\neg\Box_a\phi$ means that believe ϕ can be lost, it is not safe to believe ϕ. Therefore $\neg\Box_a\neg\Box_a\phi$ means that the believe that ϕ can be lost can be lost as well. Because the agent believes that ϕ.

1.3

In the world denoted by the *, $\Box_a\varphi$ is true but $Sb_a\varphi$ is false because there exists a world where $\neg\varphi$ that is "better" than another world where φ.



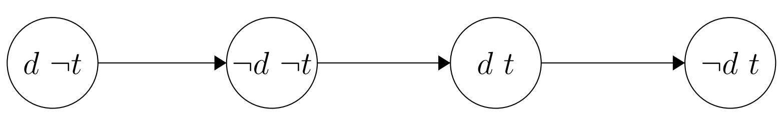
2 Virtual agent

2.1

Write down a logical formula, ϕ, in the language of beliefs, knowledge and conditional beliefs to encode all the above assumptions.

$$
\begin{aligned}
\phi = \\
&\neg Kd \land \neg K\neg d \land \neg Kt \land \neg K\neg t \land \\
&B(\neg d \land t) \land \\
&B^{\neg(\neg d \land t)}(d \land t) \land \\
&B^{(\neg t)}(\neg t \land \neg d)
\end{aligned}
$$

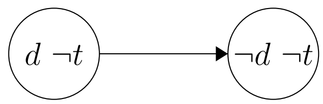
2.2



2.3

$\varphi = t \Leftrightarrow B(d)$

2.4



2.5

$\psi = d \Leftrightarrow \neg B(d)$

2.6

In the right world of the model in 2.4, ψ doesn't hold because $\neg B(d)$ is true but d is not. In the left world ψ holds because d is true and $\neg B(d)$ is true.