

Computationele logica

Kamans, Jim
10302905

Roosingh, Sander
11983957

Schenk, Stefan
11881798

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Exercise 1

1. The sentence θ encoding all information:

The Queen knows the following:

Alice knows Bob has a red hat. Alice knows Bob doesn't know it, and she knows the Queen knows this. Alice doesn't know her own hat.

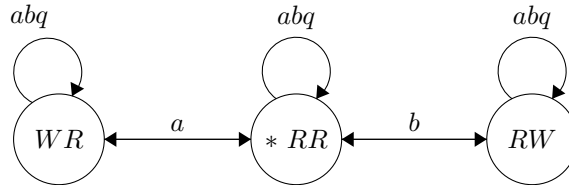
Bob knows Alice has a red hat. Bob knows Alice doesn't know it, and he knows the Queen knows this. Bob doesn't know his own hat.

$$\theta = K_q(K_a(r_b \wedge \neg K_b(r_b \vee w_b)) \wedge K_q((r_a \vee r_w) \wedge (r_b \vee r_w))) \wedge \neg K_a(r_a \vee w_a) \wedge K_b(r_a \wedge \neg K_a(r_a \vee w_a) \wedge K_q((r_a \vee r_w) \wedge (r_b \vee r_w))) \wedge \neg K_b(r_b \vee w_b))$$

2. A representation of the situation model \mathbf{M} :

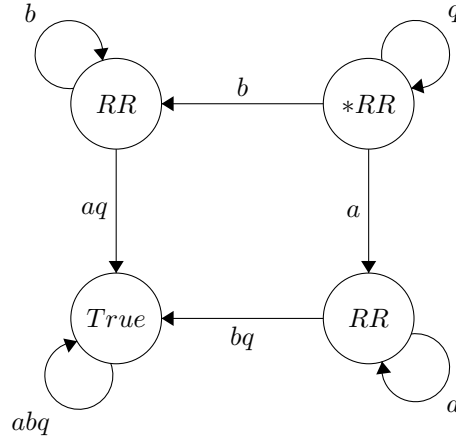
$\mathcal{A} = \{a, b, q\}$ the agents Alice, Bob, and the Queen

$\Phi = \{r_a, w_a, r_b, w_b\}$ written as WR for: a is white and b is red



This is an epistemic model: YES

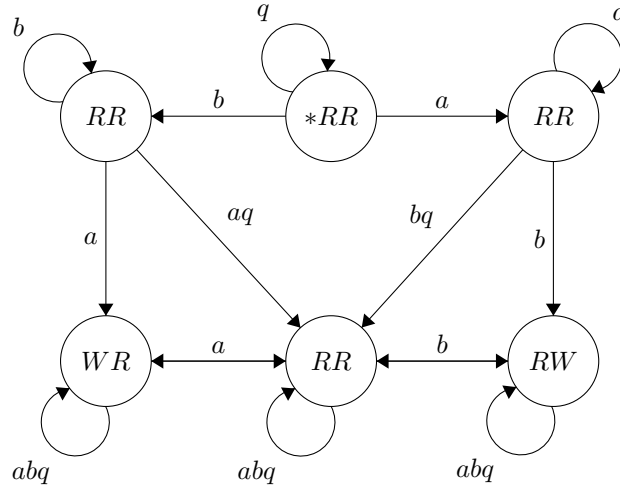
3. Separately a and b look in their mirrors and see their red hats, the queen sees everything, represented in the event model Σ with four actions:



This is an epistemic model: NO

This is a doxastic model: YES

4. The update product of the two models $\mathbf{M} \otimes \Sigma$:

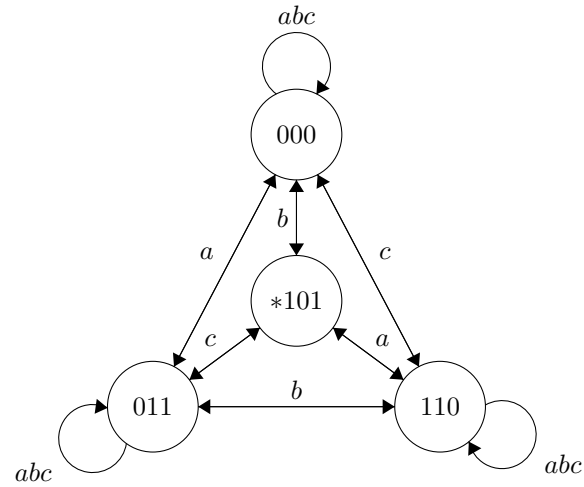


This is an epistemic model: NO

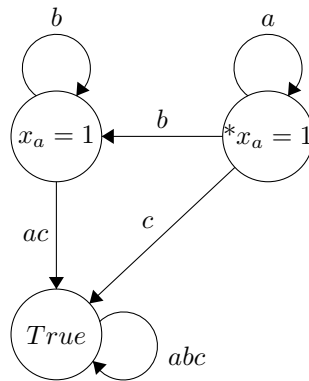
This is a doxastic model: YES

Exercise 2

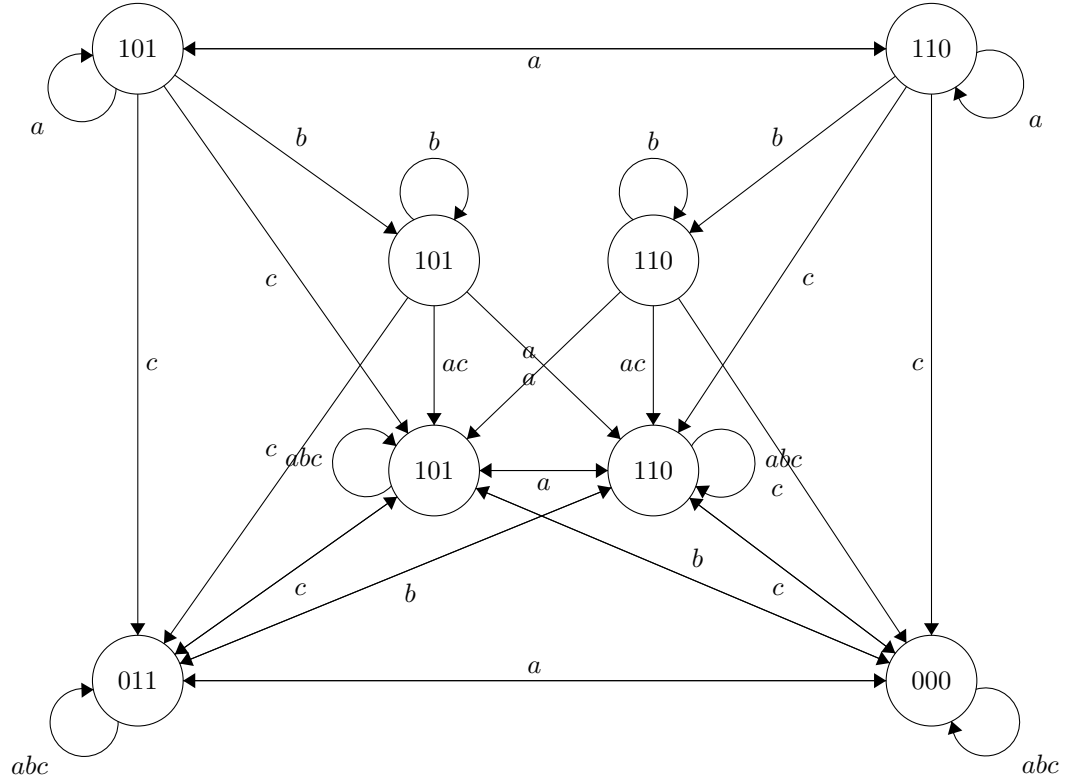
1. Representation of the bits world as an epistemic model \mathbf{M} :



2. Representation of event model Σ :



3. Representation of model \mathbf{M}' :



4. Representation of event model Σ' :

5. The update product of the two models $\mathbf{M}'' = \mathbf{M} \otimes \Sigma'$:

Exercise 3

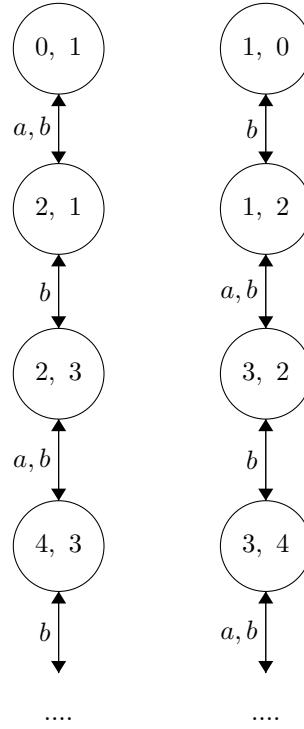
Each of two children, Alice and Bob, has a natural number written in the *back of his/her head*. Let $n_a \in \{0, 1, 2, \dots\}$ be Alice's number and $n_b \in \{0, 1, 2, \dots\}$ be Bob's number. It is common knowledge that: (i) *no child can see his/her own number*, (ii) Alice stands in the back of Bob, so *she can see Bob's number n_b , but Bob cannot see any of the numbers*, and (iii) one of the numbers is the immediate successor of the other (in any order): i.e. either $n_a = n_b + 1$ or $n_b = n_a + 1$. In the following, each of the children will be asked a number of questions, which they are required to answer publicly and truthfully.

1. *How many possible worlds* are there (that are consistent with the story above)?

There are infinite possible worlds.

2. **Represent (draw) the above situation as an epistemic model M_1** , with two agents (a for Alice, b for Bob), using pairs of numbers (n_a, n_b) as “names” for the possible worlds. **Draw the epistemic accessibility relations** for each agent, but do *not* worry about the valuation (yet), since no atomic sentences are given yet.

For simplicity we skipped the loops (every world has an a - and b -arrow to itself) and we skipped a lot of b -arrows (every world has a b -arrow to every other world).



3. For your model, consider now the following *four atomic sentences* $0_a, 0_b, 1_a, 1_b$. Here, 0_a means “Alice’s number is equal to 0” (i.e. $n_a = 0$), and 0_b means “Bob’s number is equal to 0” (i.e. $n_b = 0$); 1_a means “Alice’s number is equal to 1” (i.e. $n_a = 1$), and 1_b means “Bob’s number is equal to 1” (i.e. $n_b = 1$). **Specify the valuation for these atomic sentences** in the above model.

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4. Alice is asked the following question: “*Do you know whether your own number is equal to 0 or not, and if so then which of the two?*” So her answers can be: (a) *I don’t know*, (b) *I know that my number is equal to 0*, or (c) *I know that my number is NOT equal to 0*.

Let us suppose that in fact *Alice answers (c) “I know that my number is NOT equal to 0”*.

Write down a sentence φ in epistemic logic that expresses her answer.

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5. Interpreting the above answer as a truthful public announcement $!\varphi$ of the sentence written in the previous part, **represent (draw) the updated model $M_2 = M_1^{!\varphi}$ after this public announcement.**

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6. Suppose now that, *after* Alice answered as above, Bob is asked the “same” question: “*Do you (Bob) know whether your own number is equal to 0 or not, and if so then which of the two?*”

What will Bob answer? Also, **what will be updated model M_3 representing the situation after he answers?** Justify your answers!

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7. After the previous round of questions, Alice is now asked the following question: “*Do you know whether your own number is equal to 1 or not, and if so then which of the two?*” The answers can be: (a) *I don’t know*, (b) *I know that my number is equal to 1*, or (c) *I know that my number is NOT equal to 1*.

Let us suppose that in fact *Alice answers (a) “I don’t know”*.

Write down a sentence ψ in epistemic logic that expresses her *answer*.

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8. Interpreting the above answer as a truthful public announcement $!\psi$ of the sentence written in the previous part, **represent (draw) the updated model $M_4 = M_3^{! \psi}$ after this public announcement.**

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9. Suppose now that, *after* Alice answered as above, Bob is asked the “same” question: “*Do you (Bob) know whether your own number is equal to 1 or not, and if so then which of the two?*”

What will Bob answer? Also, **what is his number?** Justify your answers!

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