Computationele logica

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Exercise 1

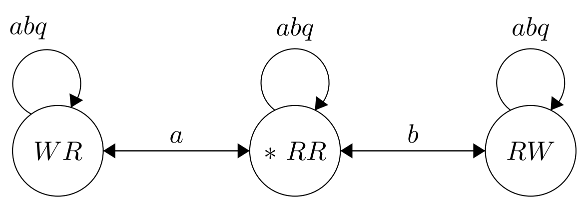
1. The sentence θ encoding all information:

   The Queen knows the following:
   Alice knows Bob has a red hat. Alice knows Bob doesn't know it, and she knows the Queen knows this. Alice doesn't know her own hat.
   Bob knows Alice has a red hat. Bob knows Alice doesn't know it, and he knows the Queen knows this. Bob doesn't know his own hat.

   $\theta = K_q(K_a(r_b \land \neg K_b(r_b \lor w_b) \land K_q((r_a \lor r_w) \land (r_b \lor r_w))) \land \neg K_a(r_a \lor w_a) \land K_b(r_a \land \neg K_a(r_a \lor w_a) \land K_q((r_a \lor r_w) \land (r_b \lor r_w))) \land \neg K_b(r_b \lor w_b))$

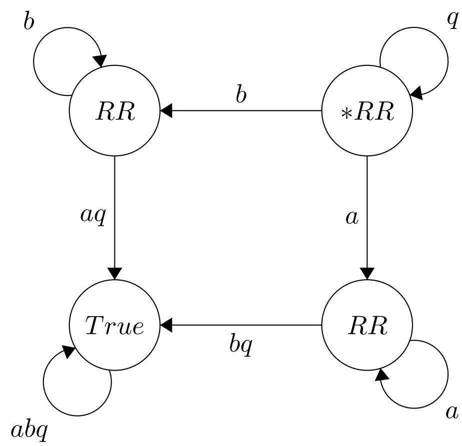
2. A representation of the situation model $\mathbf{M}$:

   $\mathcal{A}$ = {a, b, q} the agents Alice, Bob, and the Queen
   $\Phi = \{r_a, w_a, r_b, w_b\}$ written as WR for: a is white and b is red



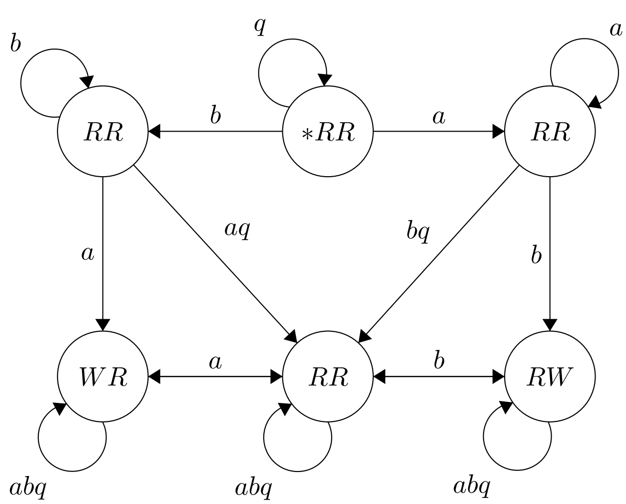
   This is an epistemic model: YES

3. Seperately a and b look in their mirrors and see their red hats, the queen sees everything, represented in the event model Σ with four actions:



This is an epistemic model: NO
This is a doxasic model: YES

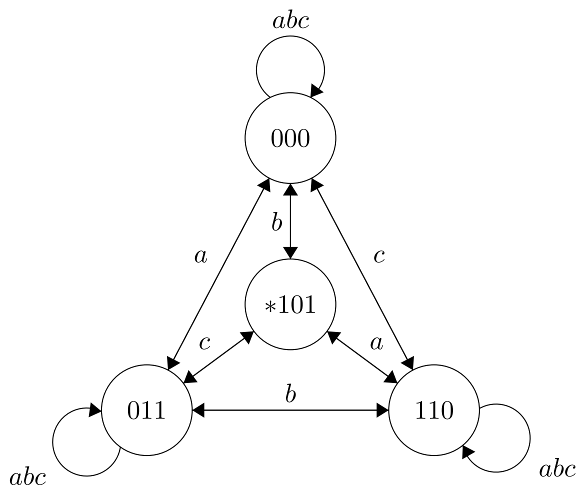
4. The update product of the two models $\mathbf{M} \bigotimes \Sigma$:



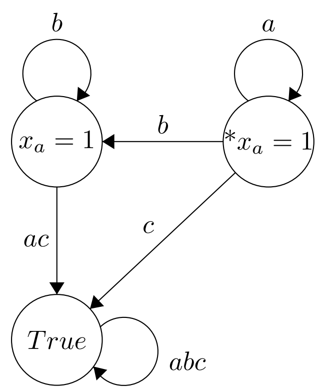
This is an epistemic model: NO
This is a doxasic model: YES

Exercise 2

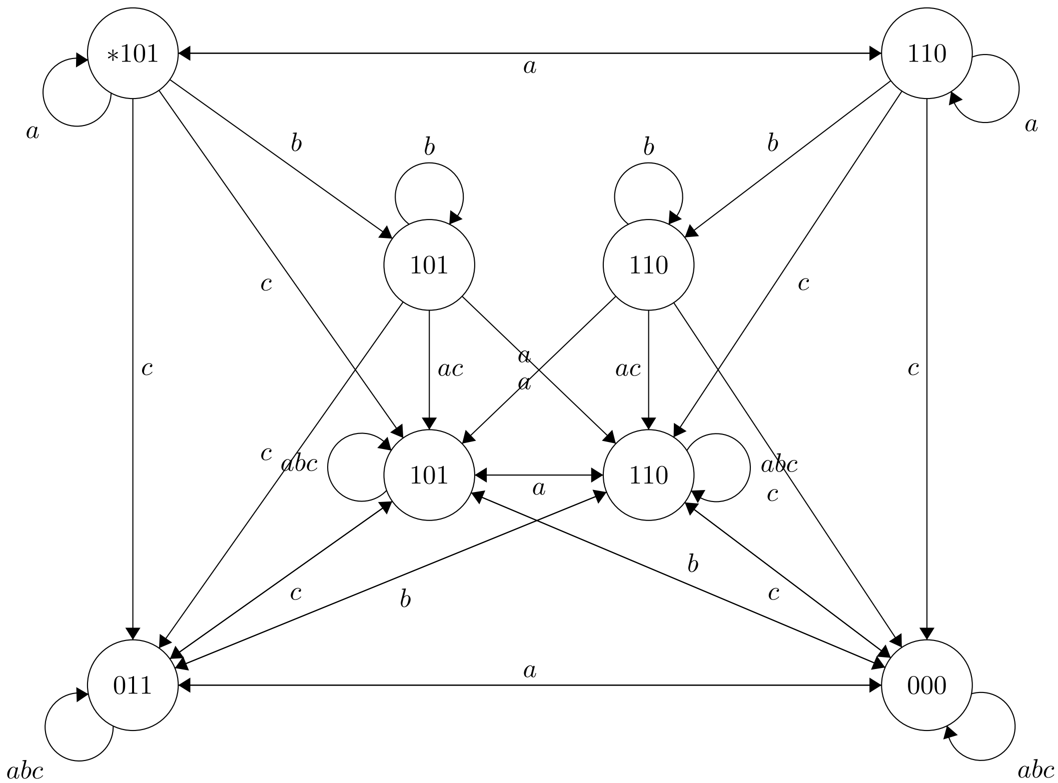
1. Representation of the bits world as an epistemic model **M**:



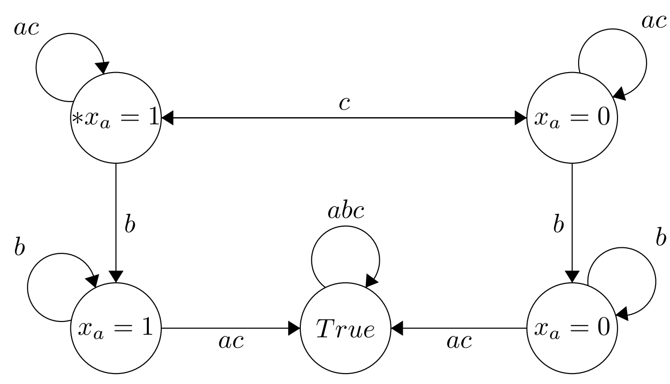
2. Representation of event model Σ:



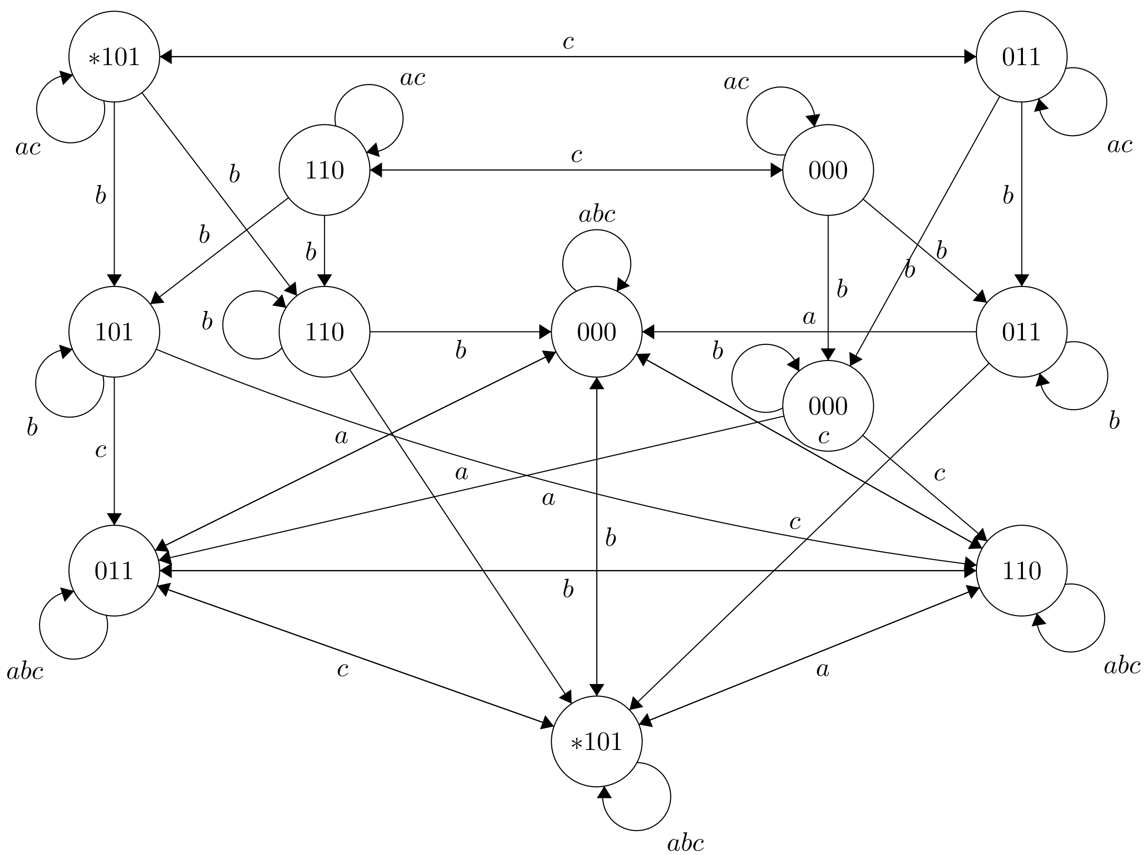
3. Representation of model **M'**:



4. Representation of event model **Σ'**:



5. The update product of the two models $\mathbf{M''} = \mathbf{M} \bigotimes \Sigma'$:



Exercise 3

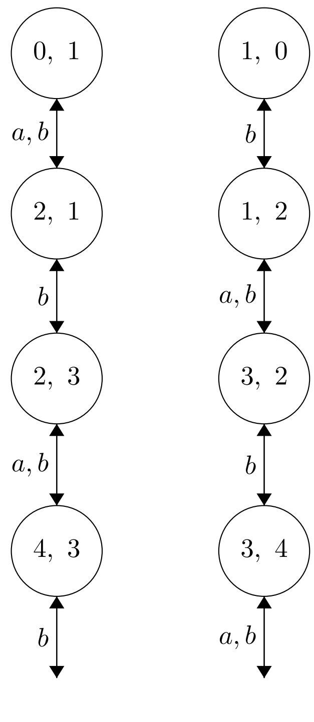
Each of two children, Alice and Bob, has a natural number written in the *back of his/her head*. Let $n_a \in \{0, 1, 2, \dots\}$ be Alice's number and $n_b \in \{0, 1, 2, \dots\}$ be Bob's number. It is common knowledge that: (i) *no child can see his/her own number*, (ii) Alice stands in the back of Bob, so *she can see Bob's number n_b, but Bob cannot see any of the numbers*, and (iii) one of the numbers is the immediate successor of the other (in any order): i.e. either $n_a = n_b + 1$ or $n_b = n_a + 1$. In the following, each of the children will be asked a number of questions, which they are required to answer publicly and truthfully.

1. *How many possible worlds* are there (that are consistent with the story above)?

   There are infinite possible worlds.

2. *Represent* **(draw) the above situation as an *epistemic model* $\mathbf{M}_1$**, with two agents (a for Alice, b for Bob), using pairs of numbers (n_a,n_b) as "names" for the possible worlds. **Draw the epistemic accessibility relations** for each agent, but do *not* worry about the valuation (yet), since no atomic sentences are given yet.

   For simplicity we skipped the loops (every world has an a- and b-arrow to itself) and we skipped a lot of b-arrows (every world has a b-arrow to every other world).



3. For your model, consider now the following *four atomic sentences* $0_a, 0_b, 1_a, 1_b$. Here, 0_a means "Alice's number is equal to 0" (i.e. $n_a = 0$), and 0_b means "Bob's number is equal to 0" (i.e. $n_b = 0$); 1_a means "Alice's number is equal to 1" (i.e. $n_a = 1$), and 1_b means "Bob's number is equal to 1" (i.e. $n_b = 1$). **Specify the valuation for these atomic sentences** in the above model.

   BlaBlaBla

4. Alice is asked the following question: *"Do you know whether your own number is equal to* 0 *or not, and if so then which of the two?"* So her answers can be: (a) *I don't know*, (b) *I know that my number is equal to* 0, or (c) *I know that my number is NOT equal to* 0.

   Let us suppose that in fact *Alice answers (c) "I know that my number is NOT equal to* 0*"*.

   Write down a sentence φ in epistemic logic that expresses her *answer*.

   BlaBlaBla

5. Interpreting the above answer as a truthful public announcement $!\varphi$ of the sentence written in the previous part, **represent (draw) the updated model $\mathbf{M}_2 = \mathbf{M}_1^{!\varphi}$ after this public announcement**.

BlaBlaBla

6. Suppose now that, *after* Alice answered as above, Bob is asked the "same" question: *"Do you (Bob) know whether your own number is equal to* 0 *or not, and if so then which of the two*?"

   What will Bob answer? Also, **what will be updated model $\mathbf{M}_3$ representing the situation after he answers**? Justify your answers!

   BlaBlaBla

7. After the previous round of questions, Alice is now asked the following question: *"Do you know whether your own number is equal to* 1 *or not, and if so then which of the two*?" The asnwers can be: (a) *I don't know*, (b) *I know that my number is equal to* 1, or (c) *I know that my number is NOT equal to* 1.

   Let us suppose that in fact *Alice answers (a) "I don't know"*.

   Write down a sentence ψ in epistemic logic that expresses her *answer*.

   BlaBlaBla

8. Interpreting the above answer as a truthful public announcement $!\psi$ of the sentence written in the previous part, **represent (draw) the updated model $\mathbf{M}_4 = \mathbf{M}_3^{!\psi}$ after this public announcement**.

   BlaBlaBla

9. Suppose now that, *after* Alice answered as above, Bob is asked the "same" question: *"Do you (Bob) know whether your own number is equal to* 1 *or not, and if so then which of the two*?"

   What will Bob answer? Also, **what is his number**? Justify your answers!

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