Lyapunov Orbits

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0.1 Lyapunov Orbits

0.1.1 Dynamical model

The dynamical model employed in this study is the planar circular restricted three-body problem (PCR3BP). In this framework, the equation of motion are written in a non-dimensional form as follows:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = 2 \begin{bmatrix} \dot{y} \\ -\dot{x} \end{bmatrix} + \begin{bmatrix} U_x \\ U_y \end{bmatrix} \tag{1}$$

where x and y are the non-dimensional position coordinates of the spacecraft. U is the gravitational potential of the system, which is given by:

$$U(x, y) = \frac{x^2 + y^2}{2} + \frac{1 - \mu}{r_1} + \frac{\mu}{r_2} \tag{2}$$

where μ is the mass ratio of the two primary bodies, and r_1 and r_2 are the distances from the spacecraft to the primary bodies.

The model is rewritten in the form $\dot{\boldsymbol{x}} = f(\boldsymbol{x})$ with:

$$\boldsymbol{x} = \begin{bmatrix} x \\ y \\ u \\ v \end{bmatrix} \qquad \text{and} \qquad \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{u} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} u \\ v \\ 2v + U_x \\ -2u + U_y \end{bmatrix} \tag{3}$$

0.1.2 Lagrangian points

The Lagrangian points are equilibrium points in the phase space of the PCR3BP. The five Lagrangian points are denoted by L_1, L_2, L_3, L_4, and L_5. The first three points are collinear with the two primary bodies, while the last two are the third vertex of the equilateral triangle formed by the two primary bodies.

For Lyapunov orbits, we are interested in the L_1 and L_2 points. The phase space around these points is of the type saddle-center in the planar case, with two real eigenvalues (one positive unstable and one negative stable) and two complex conjugate eigenvalues.

The choosen binary system is the Earth-Moon system, but the results can be generalized as long as the mass parameter μ is such that collinear points are unstable.
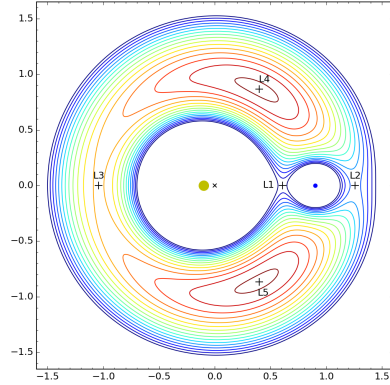

Figure 1: Lagrange points in the PCR3BP

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0.1.3   Linearized motion around collinear equilibrium points

With $\boldsymbol{x_L}$ the state of the generic collinear equilibrium point, the generic state can be linearized around $\boldsymbol{x_L}$ as $\boldsymbol{x} = \boldsymbol{x_L} + \delta\boldsymbol{x}$

Chapter 1

Methodology

Chapter 2

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