Lyapunov Orbits

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0.1 Lyapunov Orbits

0.1.1 Dynamical model

The dynamical model employed in this study is the planar circular restricted three-body problem (PCR3BP). In this framework, the equation of motion are written in a non-dimensional form as follows:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = 2 \begin{bmatrix} \dot{y} \\ -\dot{x} \end{bmatrix} + \begin{bmatrix} U_x \\ U_y \end{bmatrix} \tag{1}$$

where x and y are the non-dimensional position coordinates of the spacecraft. U is the gravitational potential of the system, which is given by:

$$U(x,y) = \frac{x^2+y^2}{2} + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} \tag{2}$$

where μ is the mass ratio of the two primary bodies, and r_1 and r_2 are the distances from the spacecraft to the primary bodies.
The model is rewritten in the form $\dot{\boldsymbol{x}} = f(\boldsymbol{x})$ with:

$$\boldsymbol{x} = \begin{bmatrix} x \\ y \\ u \\ v \end{bmatrix} \qquad \text{and} \qquad \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{u} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} u \\ v \\ 2v + U_x \\ -2u + U_y \end{bmatrix} \tag{3}$$

0.1.2 Lagrangian points

The Lagrangian points are equilibrium points in the phase space of the PCR3BP. The five Lagrangian points are denoted by L_1, L_2, L_3, L_4, and L_5. The first three points are collinear with the two primary bodies, while the last two are the third vertex of the equilateral triangle formed by the two primary bodies.

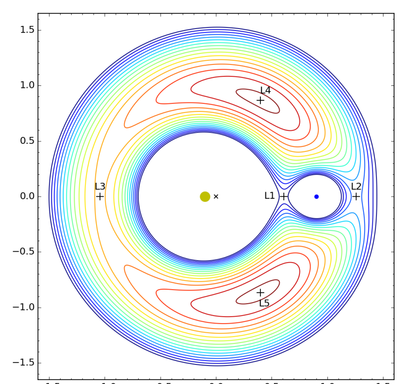

Figure 1: Lagrange points in the PCR3BP

For Lyapunov orbits, we are interested in the L_1 and L_2 points. The phase space around these points is of the type saddle-center in the planar case, with two real eigenvalues (one positive unstable and one negative stable) and two complex conjugate eigenvalues.
The choosen binary system is the Earth-Moon system, but the results can be generalized as long as the mass parameter μ is such that collinear points are unstable.

0.1.3 Linearized motion around collinear equilibrium points

With $\boldsymbol{x_L}$ the state of the generic collinear equilibrium point, the generic state can be linearized around $\boldsymbol{x_L}$ as $\boldsymbol{x} = \boldsymbol{x_L} + \delta\boldsymbol{x}$

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