EEDG/CE 6303: Testing and Testable Design

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Session 08

Memory Testing

Key Issues

- Motivation for testing memories
- Modeling memory chips
- Reduced functional fault models
- Traditional tests
- March tests
- Pseudorandom memory tests

Classical Tests

Functional RAM Chip Testing

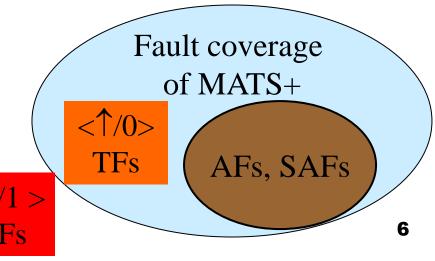
- Purpose
 - 1. Cover traditional tests
 - Zero-One (MSCAN)
 - Checkerboard
 - GALPAT and Walking 1/0
 - 2. Cover tests for stuck-at, transition and coupling faults
 - MATS and MATS+
 - March C-
 - March A and March B
 - 3. Comparison of march tests

Fault Coverage of Tests

- When a test detects faults of a particular type, it detects:
 - all subtypes of that type; e.g., if it detects TFs it has to detect all $<\uparrow/0>$ and $<\downarrow/1>$ TFs
 - all positions of each subtype (addr. a-cell < or > v-cell)
- A complete test detects all faults it is designed for
 It may, additionally, and unintentionally, detect also other faults
 But not all subtypes and not all positions of each of these faults

Example: MATS+ : $\{M0: (w0); M1: (r0,w1); M2: (v1,w0)\}$

- Detects all AFs
- Detects all SAFs
- Detects all <↑/0> TFs
- Does not detect all <↓/1> TFs
- ⇒ MATS+ does **not** detect TFs



Traditional Tests

- Traditional tests are older tests
 - Usually developed without explicitly using fault models
 - Usually they also have a relatively long test time
 - —Some have special properties in terms of:
 - detecting dynamic faults
 - locating (rather than only detecting) faults
- Many traditional tests exist:
 - 1. Zero-One (Usually referred to as *Scan Test* or *MSCAN*)
 - 2. Checkerboard
 - 3. GALPAT and Walking 1/0
 - 4. Sliding Diagonal
 - 5. Butterfly
 - 6. Surround Disturb
 - 7. Many, many others

Zero-One Test (Scan Test, (M)SCAN)

Minimal test, consisting of writing & reading 0s and 1s

```
Step 1: write 0 in all cells
Step 2: read all cells
Step 3: write 1 in all cells
Step 4: read all cells
```

- March notation for Scan test: {\(\psi(w0);\\psi(r0);\\psi(w1);\\psi(r1)\)}
- Test length: 4n operations; which is O(n)

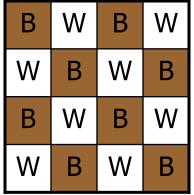
Zero-One Test (cont.)

- MSCAN: {M0: \(\psi(w0); M1: \(\psi(r0); M2: \(\psi(w1); M3: \(\psi(r1) \)}
- Fault detection capability: AFs not detected
 - *Condition AF* not satisfied: 1. $\uparrow (rx,...,wx^*)$ 2. $\downarrow (rx^*,...,wx)$. So, not all AFs are detected.
 - If address decoder maps all addresses to a single cell, then it can only be guaranteed that one cell is fault free
 - Not all TFs are detected. E.g. Not all $< \downarrow /1 >$ TFs are detected because not all \downarrow transitions are generated.
 - Not all CFs are detected because not all \downarrow transitions are generated.
 - <↑;↑> CFids are not detected because in M3, the expected value is the same as the value induced by CFs.
 - $<\uparrow;\downarrow>$ and $<\uparrow;\downarrow>$ CFs are detected only if a-cell has lower address than v-cell (otherwise, w1 in M2 will mask the fault)
 - Special property: Stresses read/write & precharge circuits when Fast X addressing is used and sequence of write/read 0101.... data in a column!

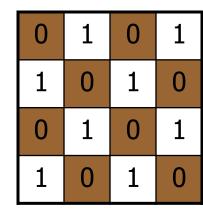
Row	000000	Checker 010101
stripe	111111	board 101010
•	000000	010101
	111111	101010
		stripe 111111 000000

Checkerboard

- It is a SCAN test, using *checkerboard* data background pattern:
 - Step 1: w1 in all cells-W w0 in all cells-B
 - *Step 2*: read all cells
 - Step 3: w0 in all cells-Ww1 in all cells-B
 - Step 4: read all cells



	VV	D	VV	
(Ched	ckerl	boar	\overline{d}

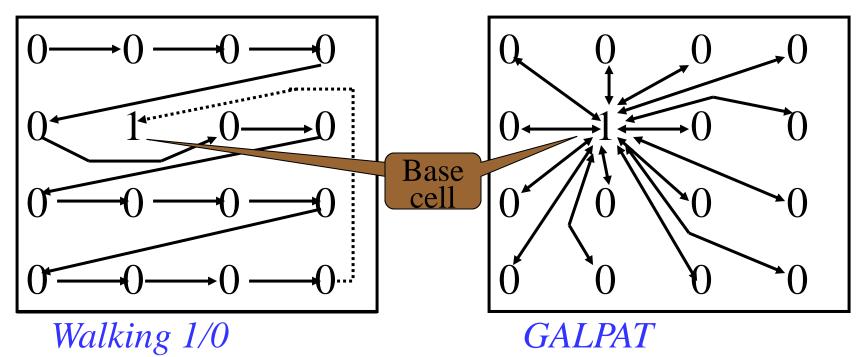


Step1 pattern

- Test length: 4n operations; which is O(n)^{data background}
- Fault detection capability:
 - Condition AF not satisfied: 1. $\uparrow (rx,...,wx^*)$; 2. $\downarrow (rx^*,...,wx)$. So, not all AFs are detected.
 - If address decoder maps all cells-W to one cell, and all cells-B to another cell, then only 2 cells guaranteed fault free
 - SAFs are detected if it can be guaranteed (through other tests) that the address decoder functions correctly. Otherwise, only two cells can be guaranteed to be free of SAFs.
 - Similar to Zero-One test, not all TFs and CFs are detected.
 - Special property: Maximizes leakage between physically adjacent cells. Used for DRAM retention test!!

GALPAT and Walking 1/0

- GALPAT (GALloping PATterns) and Walking 1/0 are similar algorithms
 - They walk a base-cell through the memory
 - The memory cell is filled with 0s (or 1s) except the based cell which contains a 1 (or 0).
 - After each step of the base-cell, the contents of all other cells is verified, followed by verification of the base-cell
 - Difference between GALPAT and Walking 1/0 is when, and how often, the base-cell is read

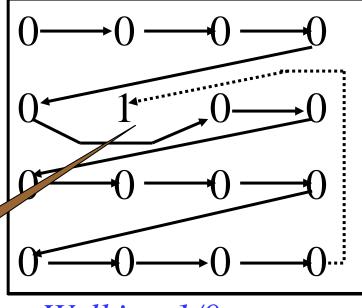


GALPAT Algorithm

```
Step 1: for d := 0 to 1 do
        begin
             for i := 0 to n - 1 do
                  A[i] := d;
             for base-cell := 0 to n-1 do
             begin
                  A[base-cell] := \overline{d};
Step 2:
                  perform READ ACTION;
                  A[base-cell] := d;
Step 3:
             end;
         end;
        READ ACTION for GALPAT:
        begin
             for cell := 0 to n - 1 (base-cell excluded) do
             begin
                 if (A[cell] \neq d) then output ('Error at cell', cell);
Step 4:
                 if (A[base-cell] \neq \overline{d}) then output ('Error at cell', cell);
Step 5:
             end;
         end;
                                                         Base
```

Walking 1/0 Algorithm

```
Step 1: for d := 0 to 1 do
        begin
             for i := 0 to n - 1 do
                 A[i] := d;
             for base-cell := 0 to n-1 do
             begin
                 A[base-cell] := \overline{d};
Step 2:
                 perform READ ACTION;
                 A[base-cell] := d;
Step 3:
             end;
        end;
        READ ACTION for Walking 1/0:
        begin
             for cell := 0 to n - 1 (base-cell excluded) do
             begin
                 if (A[cell] \neq d) then output ('Error at cell', cell);
Step 6:
             end:
             if (A[base-cell] \neq \overline{d}) then output ('Error at cell', cell);
Step 7:
        end;
                                                             Base
```

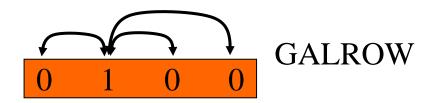


GALPAT and Walking 1/0: Properties

- All AFs are detected and *located*.
 - Step 5 (or 7) locates the problem if it is in the base-cell
 - Step 4 (or 6) locates the problem if it is in the other cells.
- All SAFs will be located because the base-cell is written (Step 2) and read (Step 5 and 7) with values 0 and 1.
- All TFs are located because the base-cell will make a ↑ and a ↓ transitions (Step 2) after which it is read (Step 5 and 7).
- CFids are located. In Step 2, <↑;↑> and <↓;↓> CFs may be sensitized (depending on the value of d in Step 1 to be 1 or 0, respectively) and located in Step 4 or Step 6. In Step 3, <↓;↑> and <↑;↓> CFids may be sensitized and located in Step 4 or Step 6.

GALPAT and Walking 1/0: Properties (cont.)

- GALPAT detects write recovery faults (Cause: slow addr. decoders)
- Test length: $O(n^2)$: Not acceptable for practical purposes
- Most coupling faults in a memory are due to sharing
 - a WL and the column decoder: cells in the *same row*
 - BLs and row decoder: cells in the same column
- Subsets of GALPAT and Walking I/O used (BC= Base-Cell)
 - GALROW and WalkROW: Read Action on cells in row of BC

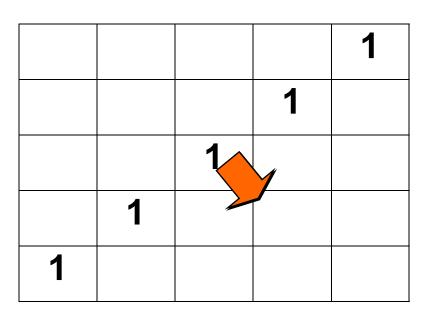


— GALCOL and WalkCOL: Read Action on cells in column of BC Test length (assuming $n^{1/2}$ rows and $n^{1/2}$ columns): O($n^{3/2}$) *Note:* Test time for 4Mb 150 ns memory

— For $O(n^2)$ test = O(20 days), and for $O(n^{3/2})$ test = O(14 sec.)

Sliding Diagonal Algorithm

- Sliding (Galloping) Row/Column/Diagonal
 - —Based on GALPAT, but instead of shifting a 1 through the memory, a complete diagonal of 1s is shifted:
 - The whole memory is read after each shift
 - —Detects all faults as GALPAT, except for some CFs
 - —Complexity is 4n^{1.5}.



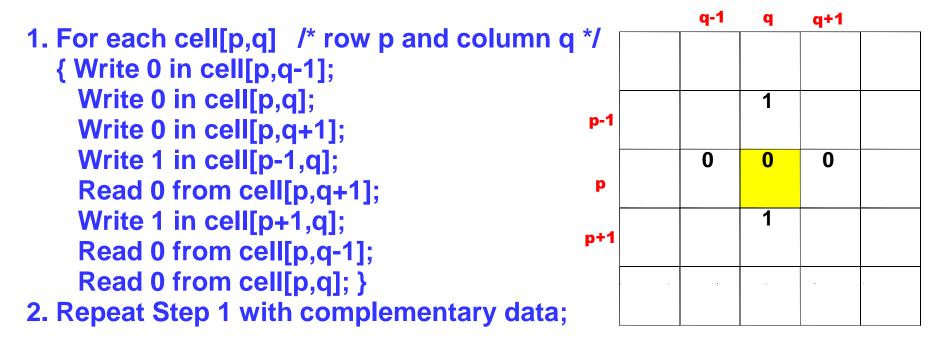
Butterfly Algorithm

- Butterfly Algorithm
 - Complexity is 5nlogn
 - All SAFs and some AFs are detected

```
1. Write background 0;
2. For BC = 0 to n-1
  { Complement BC; dist = 1;
    While dist <= mdist /* mdist < 0.5 col/row length */
      { Read cell @ dist north from BC;
        Read cell @ dist east from BC;
                                                    1
        Read cell @ dist south from BC;
                                                   5,10
                                                        2
                                                            7
        Read cell @ dist west from BC;
                                                    3
        Read BC; dist *= 2; }
                                                   8
    Complement BC; }
3. Write background 1; repeat Step 2;
```

Surround Disturb Algorithm

- Surround Disturb (SD) Algorithm
 - Examine how the cells in a row are affected when complementary data are written into adjacent cells of neighboring rows.
 - Designed on the premise that DRAM cells are most susceptible to interference from their nearest neighbors (eliminates global sensitivity checks).



Moving Inversion Algorithm

- Moving Inversion (MOVI) Algorithm
 - For functional and AC parametric test
 - Functional (13n): for AF, SAF, TF, and most CF $\{ \psi(w0); \uparrow (r0, w1, r1); \uparrow (r1, w0, r0); \psi(r0, w1, r1); \psi(r1, w0, r0) \}$
 - Parametric (12nlogn): for Read access time
 - + 2 successive Reads @ 2 different addresses with different data for all 2-address sequences differing in 1 bit
 - + Repeat M2~M5 for each address bit
 - + GALPAT---all 2-address sequences

March Tests

March Tests

- The simplest, and most efficient tests for detecting AFs,
 SAFs, TFs and CFs are march tests
- The following march tests are covered:
 - MATS+
 - Detects AFs and SAFs
 - March C-
 - Detects AFs, SAFs, TFs, and unlinked CFins, CFsts, CFids
 - March A
 - Detects AFs, SAFs, TFs, CFins, CFsts, CFids, linked CFids (but not linked with TFs)
 - March B
 - Detects AFs, SAFs, TFs, CFins, CFsts, CFids, linked CFids

History of MATS Tests

- ATS: Algorithmic Test Sequence
 - By Knaizuk and Hartmann (1977)
 - It requires 4x2ⁿ memory accesses.
 - Notations:
 - Addresses:

Let A_{μ} be the memory address μ ,

$$0 \le \mu < 2^n$$
.

Let

$$\pi_0 = \{A_{\mu} | \mu \equiv 0 \text{(modulo 3)}\},\$$

$$\pi_1 = \{A_{\mu} | \mu \equiv 1 \text{ (modulo 3)} \},$$

$$\pi_2 = \{A_{\mu} | \mu \equiv 2 \text{(modulo 3)} \}.$$

Tabulated algorithm

+ Wr: Write

+ R: Read

Step Partition	1	2	3	4	5	6	7	. 8
$\pi_{\scriptscriptstyle 0}$		Wr W ₁				RW_1	$Wr W_0, R W_0$	
$\pi_{_1}$	Wr Wo		$R W_o$	$Wr W_1$		RW_1		
$\pi_{_2}$	Wr W _o				R W _o	-		$Wr W_1, R W_1$

Algorithm

Step 1: Write the all 0 word, W_0 , at all locations

$$A_j \in \pi_1$$
 and $A_k \in \pi_2$.

Step 2: Write the all 1 word, W_1 , at all locations

$$A_i \in \pi_0$$
.

Step 3: Read all locations $A_j \in \pi_1$:

if output
$$\begin{cases} = W_0; & \text{no fault indicated;} \\ \neq W_0; & \text{RAM fault indicated.} \end{cases}$$

Step 4: Write the all 1 word W_1 at all locations

$$A_j \in \pi_1$$
.

Step 5: Read all locations $A_k \in \pi_2$:

if output
$$\begin{cases} = W_0; & \text{no fault indicated;} \\ \neq W_0; & \text{RAM fault indicated.} \end{cases}$$

Step 6: Read all locations $A_i \in \pi_0$ and $A_j \in \pi_1$:

if output
$$\begin{cases} = W_1; & \text{no fault indicated;} \\ \neq W_1; & \text{RAM fault indicated.} \end{cases}$$

Step 7: Write and then read the all 0 word W_0 at all locations

$$A_i \in \pi_0$$

If output
$$\begin{cases} = W_0; & \text{no fault indicated;} \\ \neq W_0; & \text{RAM fault indicated.} \end{cases}$$

Step 8: Write and then read the all 1 word W_1 at all locations

$$A_k \subset \pi_2$$
.

If output
$$\begin{cases} = W_1; & \text{no fault indicated;} \\ \neq W_1; & \text{RAM fault indicated.} \end{cases}$$

END.

History of MATS Tests (cont.)

- MATS: Modified Algorithmic Test Sequence
 - —By Nair (1979)
 - —MATS $\{ \hat{\Pi}(w0); \hat{\Pi}(r0,w1); \hat{\Pi}(r1) \}$
 - Complexity is 4n.

- Tabulated algorithm
 - + Wr: Write
 - + R: Read

```
program testmemory;

const = N; "the number of words in the memory"

var i: integer;

begin

for i := 0 \text{ to } N - 1 \text{ do writeOinlocation}(i);

for i := 0 \text{ to } N - 1 \text{ do}

begin

readOfromlocation(i);

writeIinlocation(i);

end;

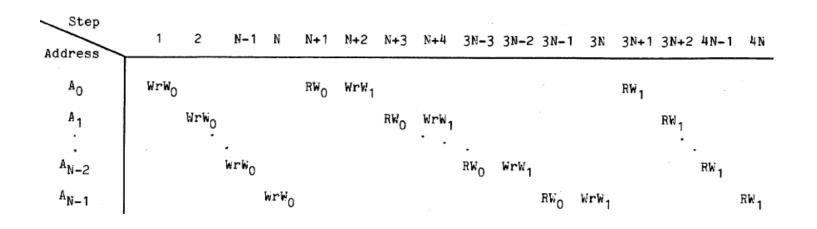
for i := 0 \text{ to } N - 1 \text{ do readIfromlocation}(i);

end;

for i := 0 \text{ to } N - 1 \text{ do readIfromlocation}(i);

end.
```

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MATS+

- MATS+ algorithm: $\{M0: \updownarrow(w0); M1: \updownarrow(r0,w1); M2: \lor(r1,w0)\}$
- Fault coverage
 - AFs detected because MATS+ satisfies Cond. AF (When reads, accessing multiple cells, return a random value) Cond. AF: 1. ↑(rx,...,wx*) and 2. ↓(rx*,...,wx) (1) satisfied by: M1:↑(r0,w1) and (2) by: M2:↓(r1,w0)
 - SAFs are detected: from each cell the value 0 and 1 is read
 - SAFs on Read/Write logic will be detected as both 0 and 1 are written and read.
- Test length: 5*n*

Note: If fault model is *symmetric* with respect to 0/1, \uparrow/\downarrow , \uparrow/\downarrow and with respect to address a-cell < v-cell and address a-cell > v-cell, then **each march tests has 3 equivalent tests**

```
\begin{array}{ll} \textbf{Os} \Leftrightarrow \textbf{1s} & \{ \updownarrow (w0); \pitchfork (r0,w1); \Downarrow (r1,w0) \} \Rightarrow \{ \updownarrow (w1); \pitchfork (r1,w0); \Downarrow (r0,w1) \} \\ & \Uparrow s \Leftrightarrow \Downarrow s : & \{ \updownarrow (w0); \pitchfork (r0,w1); \Downarrow (r1,w0) \} \Rightarrow \{ \updownarrow (w0); \Downarrow (r0,w1); \pitchfork (r1,w0) \} \\ & \textbf{Os} \Leftrightarrow \textbf{1s}, \pitchfork s \Leftrightarrow \Downarrow s : & \{ \updownarrow (w0); \pitchfork (r0,w1); \Downarrow (r1,w0) \} \Rightarrow \{ \updownarrow (w1); \Downarrow (r1,w0); \pitchfork (r0,w1) \} \\ & \textbf{Os} \Leftrightarrow \textbf{1s}, \pitchfork s \Leftrightarrow \Downarrow s : & \{ \updownarrow (w0); \pitchfork (r0,w1); \Downarrow (r1,w0) \} \Rightarrow \{ \updownarrow (w1); \Downarrow (r1,w0); \pitchfork (r0,w1) \} \\ & \textbf{Os} \Leftrightarrow \textbf{1s}, \pitchfork s \Leftrightarrow \Downarrow s : & \{ \updownarrow (w0); \pitchfork (r0,w1); \Downarrow (r1,w0) \} \Rightarrow \{ \updownarrow (w1); \Downarrow (r1,w0); \pitchfork (r0,w1) \} \\ & \textbf{Os} \Leftrightarrow \textbf{1s}, \pitchfork s \Leftrightarrow \Downarrow s : & \{ \updownarrow (w0); \pitchfork (r0,w1); \Downarrow (r1,w0) \} \Rightarrow \{ \updownarrow (w1); \Downarrow (r1,w0); \pitchfork (r0,w1) \} \\ & \textbf{Os} \Leftrightarrow \textbf{1s}, \pitchfork s \Leftrightarrow \Downarrow s : & \{ \updownarrow (w0); \pitchfork (r0,w1); \Downarrow (r1,w0) \} \Rightarrow \{ \updownarrow (w1); \Downarrow (r1,w0); \pitchfork (r0,w1) \} \\ & \textbf{Os} \Leftrightarrow \textbf{1s}, \pitchfork s \Leftrightarrow \Downarrow s : & \{ \updownarrow (w0); \pitchfork (r0,w1); \Downarrow (r1,w0) \} \Rightarrow \{ \updownarrow (w1); \Downarrow (r1,w0); \pitchfork (r0,w1) \} \\ & \textbf{Os} \Leftrightarrow \textbf{1s}, \pitchfork s \Leftrightarrow \Downarrow s : & \{ \updownarrow (w0); \pitchfork (r0,w1); \Downarrow (r1,w0) \} \Rightarrow \{ \updownarrow (w1); \Downarrow (r1,w0); \pitchfork (r0,w1) \} \\ & \textbf{Os} \Leftrightarrow \textbf{1s}, \pitchfork s \Leftrightarrow \Downarrow s : & \{ \updownarrow (w0); \pitchfork (r0,w1); \Downarrow (r1,w0) \} \Rightarrow \{ \updownarrow (w1); \Downarrow (r1,w0); \pitchfork (r0,w1) \} \\ & \textbf{Os} \Leftrightarrow \textbf{1s}, \pitchfork s \Leftrightarrow \Downarrow s : & \{ \updownarrow (w0); \pitchfork (r0,w1); \Downarrow (r1,w0) \} \Rightarrow \{ \updownarrow (w1); \Downarrow (r1,w0); \pitchfork (r0,w1) \} \\ & \textbf{Os} \Leftrightarrow \textbf{1s}, \pitchfork s \Leftrightarrow \Downarrow s : & \{ \updownarrow (w0); \pitchfork (r0,w1); \Downarrow (r1,w0) \} \Rightarrow \{ \updownarrow (w1); \Downarrow (r1,w0); \pitchfork (r0,w1) \} \\ & \textbf{Os} \Leftrightarrow \textbf{1s}, \pitchfork s \Leftrightarrow \textbf{1s}, \dotsc s \Leftrightarrow \textbf
```

March C-

- March C (Marinescu,1982): an 11n algorithm
 {\$\psi(w0);\$\psi(r0,w1);\$\psi(r1,w0);\$\psi(r0);\$\psi(r0,w1);\$\psi(r1,w0);\$\psi(r0)\$
 It can be shown that middle `\$\psi(r0)' march element is redundant
- March C- (van de Goor,1991): a 10*n* algorithm
 {\$\psi(w0);\$\psi(r0,w1);\$\psi(r1,w0);\$\psi(r0,w1);\$\psi(r1,w0);\$\psi(r0)\$
 M0 M1 M2 M3 M4 M5
- Fault coverage of March C- (Summary)
 - AFs: Cond. AF satisfied by M1 and M4, or by M2 and M3
 - SAFs: Detected by M1 (SA1 faults) and M2 (SA0 faults)
 - TFs: $<\uparrow/0>$ TFs sensitized by M1, detected by M2 (and M3+M4) $<\downarrow/1>$ TFs sensitized by M2, detected by M3 (and M4+M5)
 - CFins $(\langle\uparrow;\downarrow\rangle,\langle\downarrow;\downarrow\rangle)$ detected
 - CFsts (<1;0>, <1;1>, <0;0>, <0;1>) detected
 - CFids $(<\uparrow;0>=<\uparrow;\downarrow>,<\uparrow;1>=<\uparrow;\uparrow>,<\downarrow;0>=<\downarrow;\downarrow>,<\downarrow;1>=<\downarrow;\uparrow>)$ detected

March C- Detects SAF, TF, AF

March C-:
$$\{ (w0); (r0,w1); (r1,w0); (r0,w1); (r1,w0); (r0,w1); (r1,w0); (r0,w1); ($$

Fault	Condition	Sensitizing	Detection	Comments
SAF <∀/0>		M1 (when operating on a cell, it tries to write 1)	M2 (when operating on a cell, it reads and expects 1)	M3+M4 also sensitizes and detects SA0.
SAF <∀/1>		M0 (when operating on a cell, it tries to write 0)	M1 (when operating on a cell, it reads and expects 0)	M2+M3 also sensitizes and detects SA1.
TF <↑/0>		M1	M2	M3+M4 together do the same.
TF <↓/1>		M2	M3	M4+M5 together do the same.
AFs				M1+M4 together satisfy Condition for detecting AFs
				M2+M3 together satisfy Condition for detecting AFs

March C- Detects CFids

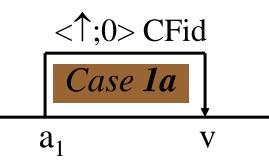
March C-:
$$\{ (w0); (r0,w1); (r1,w0); (r0,w1); (r1,w0); (r0,w1); (r1,w0); (r0,w1); ($$

Proof for detecting CFs are all similar. **Analyze all cases**:

- Relative positions of a-cell and v-cell
 - 1. address of a-cell < v-cell;
 - 2. address of a-cell > v-cell
- Fault subtype
 - **a.** CFid $<\uparrow;0>$; **b.** CFid $<\uparrow;1>$; **c.** CFid $<\downarrow;0>$; **d.** CFid $<\downarrow;1>$

- Fault sensitized by M3 and detected by M4
- Other cases need to be argued similarly one by one.
- Can be summarized like this:

Fault	Condition	Sensitizing	Detection	Comments	
<1;0>	a-cell < v-cell	M3 (when operating on a changes v to faulty value [v=0])	M4 (when operating on v expecting 1 but reading 0)		



March C- Cannot Detect Linked CFids

March C-:
$$\{ \updownarrow (w0); \pitchfork (r0,w1); \pitchfork (r1,w0); \Downarrow (r0,w1); \Downarrow (r1,w0); \updownarrow (r0) \}$$

$$0 \quad M1 \quad M2 \quad M3 \quad M4 \quad M5$$

$$1 \quad A_2 \quad A_3 \quad A_4 \quad A_5$$

$$2 \quad A_2 \quad A_1 \quad V$$

- If the CFid <↑;0>a₁ (a-cell is a₁) is linked to CFid <↑;1>a₂, and address of a₂ < a₁ then linked fault will not be detected
 Reason: M3 will sensitize both faults, such that masking occurs
- Can be summarized like this:

Fault	Condition	Sensitizing	Detection	Comments
< \uparrow ;0>a ₁ linked to < \uparrow ;1>a ₂	a ₂ -cell <a<sub>1-cell<v-cell< td=""><td>M3 (when operating on a_1 changes v to faulty value [v=0], but when operating on a_2 changes it back to fault-free value [v=1])</td><td>None</td><td>It cannot be detected</td></v-cell<></a<sub>	M3 (when operating on a_1 changes v to faulty value [v =0], but when operating on a_2 changes it back to fault-free value [v =1])	None	It cannot be detected

March C- Detects Unlinked CFin

March C-:
$$\{ \updownarrow (w0); \pitchfork (r0,w1); \pitchfork (r1,w0); \Downarrow (r0,w1); \Downarrow (r1,w0); \updownarrow (r0) \}$$

$$M0 \quad M1 \quad M2 \quad M3 \quad M4 \quad M5$$

$$<\uparrow; \updownarrow > CFin$$

$$Case 1: j < i$$

- Let C_i be coupled to any number of cells with addresses lower than i and let C_i be the highest of those cells (j<i)
 - (a) C_i is $<\uparrow$; $\updownarrow>$ coupled to C_j ; then M1 will sensitize and detect the fault, as well as M3 followed by M4.
 - (b) C_i is $< \downarrow$; $\updownarrow>$ coupled to C_j ; then M2 will sensitize and detect the fault, as well as M4 followed by M5. In summary:

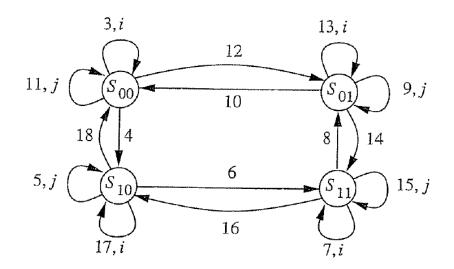
Fault	Condition	Sensitizing	Detection	Comments
<^; \$>	a-cell (C _j) <v-cell (c<sub="">i)</v-cell>	M1 (when operating on C_j changes C_i to faulty value $[C_i=1]$	M1 (when operating on C _i expecting 0 but reading 1)	Similarly M3 can sensitize the fault and M4 can detect it
<↓; \$>	a-cell (C _j) <v-cell (c<sub="">i)</v-cell>	M2 (when operating on C_j changes C_i to faulty value $[C_i=0]$	M2 (when operating on C _i expecting 1 but reading 0)	Similarly M4 can sensitize the fault and M5 can detect it

- Case 2: j>i
 - The proof is similar to Case 1.

March C- Detects State Coupling Faults

March C-: $\{ \updownarrow (w0); \pitchfork (r0,w1); \pitchfork (r1,w0); \Downarrow (r0,w1); \Downarrow (r1,w0); \updownarrow (r0) \}$ M0 M1 M2 M3 M4 M5

- All CFst are detected as the four states of any two cells i and j are reached.
 - Example here assumes i<j (C_i is victim)
 - All 4 states are generated and verified because in each state the values of cell C_i and C_j are read. For example, in state S₀₀ cell C_j is read in Step 3 and cell C_j in step 11.



Step	March element	State S _{ij} befor operatio	Operation e on	State S _{ij} afte operat	
1	M_0		w0 into i	_	
2			w0 into j	S_{00}	
3	M_1	S_{00}	r0 from i	S ₀₀	
4		S_{00}	w1 into i	S_{10}	
5		S_{10}	r0 from j	S ₁₀	
6		S_{10}	w1 into j	S_{11}	
7	M_2	S_{11}	r1 from i	S ₁₁	
8		S_{11}	w0 into i	S_{01}	
9		S_{01}	rl from j	S_{01}	
10		S_{01}	w0 into <i>j</i>	S ₀₀	
11	M_3	S ₀₀	r0 from <i>j</i>	S_{00}	
12		S_{00}	w1 into j	S ₀₁	
13		S_{01}	r0 from i	S_{01}	
14		S_{01}	w1 into i	S_{11}	
15	M_4	S_{11}	r1 from j	S_{11}	
16		S_{11}	w0 into j	S ₁₀	
17		S ₁₀	r1 from i	S ₁₀	
18		S ₁₀	w0 into i	S_{00}	

March A & March B

- March A algorithm (Suk,1981)
 {\$\psi(w0);\$\psi(r0,w1,w0,w1);\$\psi(r1,w0,w1);\$\psi(r1,w0,w1,w0);\$\psi(r0,w1,w0)\$
 M0 M1 M2 M3 M4
- March A (Test length: 15**n*) detects
 - AFs, SAFs, TFs, CFins, CFsts, CFids
 - Linked CFids, but **not linked with TFs**. For example, M1 detects $<\uparrow;\uparrow>$ linked with $<\downarrow;\downarrow>$ Odd number of transitions prevents masking.
 - March A is complete: detects all intended faults
 - March A is *irredundant*: no operation can be removed
- March B algorithm (Test length: 17*n)

$$\{ (w0); (r0,w1,r1,w0,r0,w1); (r1,w0,w1); (r1,w0,w1,w0); (r0,w1,w0) \}$$

M0 M1 M2 M3 M4

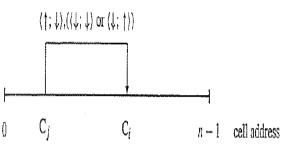
- Detects all faults of March A
- Detects **CFids linked with TFs**, because M1 detects *all* TFs (e.g. $<\uparrow/0>$ or $<\downarrow/1>$)

March A Detects CFin

- March A algorithm (Suk,1981)
 {\$\psi(w0);\$\psi(r0,w1,w0,w1);\$\psi(r1,w0,w1);\$\psi(r1,w0,w1,w0);\$\psi(r0,w1,w0)\$
 M0 M1 M2 M3 M4
- Case 1: j<i (j: aggressor's address and i for victim's address)
 - Let C_i be coupled to any odd number of cells with addresses lower than i and let C_i be the highest of those cells (j<i)
 - (a) C_i is $<\uparrow$; $\updownarrow>$ coupled to C_i ; then M2 will sensitize and detect the fault.
 - (b) C_i is $<\downarrow$; $\updownarrow>$ coupled to C_j ; then M1 or M2 will sensitize and detect the fault.
 - (c) C_i is $<\uparrow$; $\updownarrow>$ and $<\downarrow$; $\updownarrow>$ coupled to C_j ; then M1 will sensitize and detect the fault.
- Case 2: j>i
 - The proof is similar to Case 1.
 - (a) will be sensitized and detected by M3, (b) by M4 and (c) by M3.

March A Detects CFid

- March A algorithm (Suk,1981)
 {\$\psi(w0)\$; \$\psi(r0,w1,w0,w1)\$; \$\psi(r1,w0,w1)\$; \$\psi(r1,w0,w1,w0)\$; \$\psi(r0,w1,w0)\$; \$\psi(r0,w1,w0
- The proof that March A can detect linked CFids and certain CFins linked with CFids follows.
- Case 1: j<i (j: aggressor's address and i for victim's address)
 - Let C_i be coupled to any number of cells with addresses lower than i and let C_i be the highest of those cells (j<i). There are 4 cases (for 4 CFids):
 - (a) C_i is $<\uparrow;\downarrow>$ coupled to C_j (and possibly **also** $<\downarrow;\downarrow>$ or $<\downarrow;\uparrow>$ coupled to C_i); then M2 will detect the $<\uparrow;\downarrow>$ fault because:



Fault: CFid $<\uparrow;\downarrow>$ (unlinked)

Faults: Linked ($\langle \uparrow; \downarrow \rangle$, $\langle \downarrow; \downarrow \rangle$)

Faults: Linke	! (<↑;↓> ,	· <↓;↑>)
---------------	------------	-----------

	M2 oper. on C _j /Ci	C _j	C _i	Comment	M2 oper. on C _j /Ci	C _j	C _i	Comment	M2 oper. on C _j /Ci	C _j	C _i	Comment
	r1 (C _j)	1	1	Initial state	r1 (C _j)	1	1	Initial state	r1 (C _j)	1	1	Initial state
Ī	w0 (C _j)	0	1		w0 (C _j)	0	0		w0 (C _j)	0	1	
	w1 (C _j)	1	0		w1 (C _j)	1	0		w1 (C _j)	1	0	
	r1 (C _i)	1	0	<↑; ↓> detected	r1 (C _i)	1	0	Linked faults detected	r1 (C _i)	1	0	Linked faults detected

March A Detects CFid (cont.)

- March A algorithm (Suk,1981)
 {\$(w0);\$(r0,w1,w0,w1);\$(r1,w0,w1);\$(r1,w0,w1,w0);\$\$(r0,w1,w0)}
 M0
 M1
 M2
 M3
 M4
- Case 1: j<i (cont.)
 - (b) C_i is $<\uparrow;\uparrow>$ coupled to C_j (and possibly **also** $<\downarrow;\downarrow>$ or $<\downarrow;\uparrow>$ coupled to C_i); then M1 will detect the $<\uparrow;\uparrow>$ fault.
 - (c) C_i is $<\downarrow;\downarrow>$ coupled to C_j (and **not** $<\uparrow;\downarrow>$ or $<\uparrow;\uparrow>$ coupled to C_j); then M2 will detect the $<\downarrow;\downarrow>$ fault.
 - (d) C_i is $<\downarrow;\uparrow>$ coupled to C_j (and **not** $<\uparrow;\downarrow>$ or $<\uparrow;\uparrow>$ coupled to C_j); then M1 will detect the $<\downarrow;\uparrow>$ fault.
- Case 2: (j>i)
 - The proof is similar to Case 1, using M3 and M4 instead of M1 and M2.

March B

- March A algorithm (Test length: 15n)
 {\$\psi(w0)\$; \$\psi(\mathbf{r0},\w1,\w0,\w1)\$; \$\psi(\mathbf{r1},\w0,\w1)\$; \$\psi(\mathbf{r1},\w1)\$; \$\psi(\mathbf{r1},\w
- March B algorithm (Test length: 17n)

```
\{ (w0); \hat{\mathbf{1}}(\mathbf{r0,w1,r1,w0,r0,w1}); \hat{\mathbf{1}}(\mathbf{r1,w0,w1}); \forall (\mathbf{r1,w0,w1,w0}); \forall (\mathbf{r0,w1,w0}) \}
M0 M1 M2 M3 M4
```

- Detects all faults of March A
- Detecting SoPF (open fault) required (...,rx,...,rx*). M1 satisfies this condition.
- Detects **CFids linked with TFs**, because M1 detects *all* TFs.
- Two extra reads in M1 are intended to prevent TFs to be masked by CFs because no write operations to other cells, which may be potential coupling cells, take place.
 - For TF $<\uparrow/0>$: The first w1 stimulates it and r1 will detect it.
 - For TF< \downarrow /1>: w0 stimulates it and r0 will detect it.

Other March Tests

 Marching 1/0: {↑(w0); ↑(r0,w1,r1); ↓(r1,w0,r0); ↑(w1); ↑(r1,w0,r0); ↓(r0,w1,r1)}

• MATS++:

```
\{ (w0); (r0,w1); \forall (r1,w0,r0) \}
```

March X:

```
\{ \updownarrow (w0); \, \uparrow (r0,w1); \, \downarrow (r1,w0); \, \updownarrow (r0) \}
```

March Y:

```
\{ \updownarrow (w0); \uparrow (r0,w1,r1); \lor (r1,w0,r0); \uparrow (r0) \}
```

Test Requirements for Detecting SOpFs

- An SOpF is caused by an open WL which makes the cell inaccessible
- To detect SOpFs, assuming a non-transparent sense amplifier, a march test has to verify that a 0 and a 1 has to be read from every cell.
- This will be the case when the march test contains the March Element 'ME' of the form: $(..., rx, ..., rx^*, ...)$, for x = 0 and x = 1.

Example: The ME "↑(r0,w1,r1,w0,r0,w1)" satisfies the above requirement

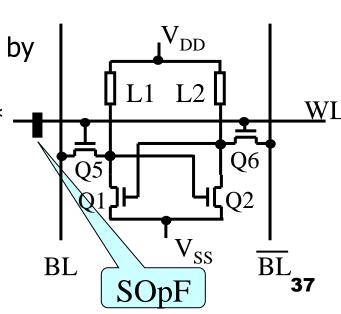
— This ME may be broken down into two MEs of the form:

(...,rx,...) & $(...,rx^*,...)$, for x = 0 and x = 1.

Example: Two MEs " \uparrow (r0,w1,r1); \downarrow (r1,w0,r0)" satisfy the above requirement

Note: Any test can be changed to detect SOpFs by making sure that the above requirement is satisfied by possibly adding a rx and/or a rx* — operation to a ME

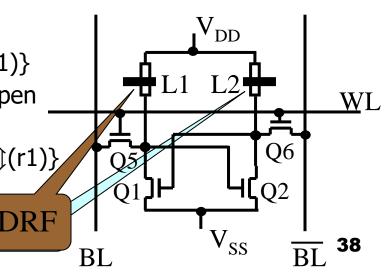
Example: MATS+ $\{ (w0); \uparrow (r0,w1); \lor (r1,w0) \}$ becomes $\{ (w0); \uparrow (r0,w1,r1); \lor (r1,w0,r0) \}$



Test Requirements for Detecting DRFs

Any march test can be extended to detect DRFs

- 1. Every cell has to be brought into one state
- 2. A time period (*Del*) has to be waited for the fault to develop
 - Note: The time for Del is typically between 100 and 500 ms
- 3. The cell contents has to be verified (should not be changed)
- Above three steps to be done for both states of every cell
 - Example: MATS+ { $$(w0);$(r0,w1);$$$(r1,w0)}$ becomes { $$(w0);$De/;$$$$(r0,w1);$De/;$$$$$$$$$(r1,w0)}$
 - <u>Example</u>: Assuming the existing test ends with all cells in state 0: {Existing March Test; *Del*; \$\partial(r0,w1); *Del*; \$\partial(r1)\$}
 - Example: If both pull-up devices may be open DRF behaves like an SOpF: {Existing March Test; Del; \(\psi(r0,w1,r1); \textit{Del}; \(\psi(r1)\)\)}



How to Analyze March Tests

- For each fault in the following list, find out:
 - 1. which element(s) sensitizes the fault
 - which element(s) detects the fault
 (For coupling faults (victim C_i and aggressor C_i), consider two cases i<j and i>j).
- List of faults include:
 - Stuck at faults [2 cases]: SA0 ($\langle \forall /0 \rangle$), SA1 ($\langle \forall /1 \rangle$)
 - Transition faults [2]: $<\uparrow/0>$, $<\downarrow/1>$
 - Inversion coupling faults [2]: $\langle \uparrow; \downarrow \rangle$, $\langle \downarrow; \downarrow \rangle$
 - Idempotent coupling faults [4]: $<\uparrow;\downarrow>$, $<\uparrow;\uparrow>$, $<\downarrow;\downarrow>$, $<\downarrow;\uparrow>$
 - State coupling faults [4]: <0;0>, <0;1>, <1;0>, <1;1>
 - Two linked faults within the same group: CFin-CFin [3], CFid-CFid [6], CFst-CFst [6]
 - Two linked faults across the groups: CFin-CFid [8], CFin-CFst [8], CFid-CFst [16]
 - Stuck-open faults (SopF)
 - Data retention fault (DTR)
 - ...

Comparison of March Tests

Name	Faults detected
Algorithm	
MATS++	SAF/AF
\updownarrow (w0); \uparrow (r	$(r_0, w_1); \downarrow (r_1, w_0, r_0)$
March X	AF/SAF/TF/CFin
\updownarrow (w0); \uparrow (r	$(r_0, w_1); \downarrow (r_1, w_0); \uparrow (r_0)$
March Y	AF/SAF/TF/CFin
\updownarrow (w0); \uparrow (r	$(r_0, w_1, r_1); \downarrow (r_1, w_0, r_0); \uparrow (r_0)$
March C-	SAF/AF/TF/CF
\updownarrow (w0); \uparrow (r	$(r_0,w_1); \uparrow (r_1,w_0); \downarrow (r_0,w_1); \downarrow (r_1,w_0); \uparrow (r_0)$

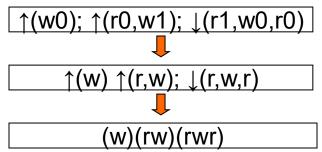
	MATS++	March X	March Y	March C-
SAF	~	V	V	V
TF	~	~	~	~
AF	~	~	~	~
SOF	~		~	
CFin		~	~	~
CFid				~
CFst				<u> </u>

Test Algorithm Generation by Simulation (TAGS)

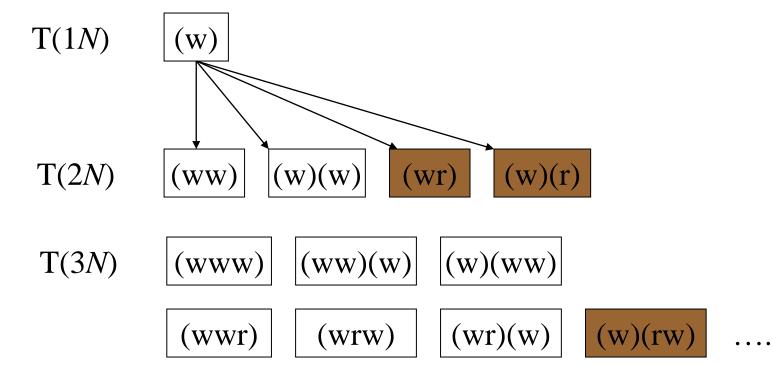
- Target fault models (SAF, TF, AF, SOF, CFin, CFid, CFst), time constraints ∞.
 - Given a set of target fault models, generate a test with 100% fault coverage
 - Given a set of target fault models and a test length constraint, generate a test with the highest fault coverage
- Priority setting for fault models
 - Test length/test time can be reduced
- Diagnostic test generation
 - Need longer test to distinguish faults
- March template abstraction

TAGS Template and Heuristics

March template abstraction:



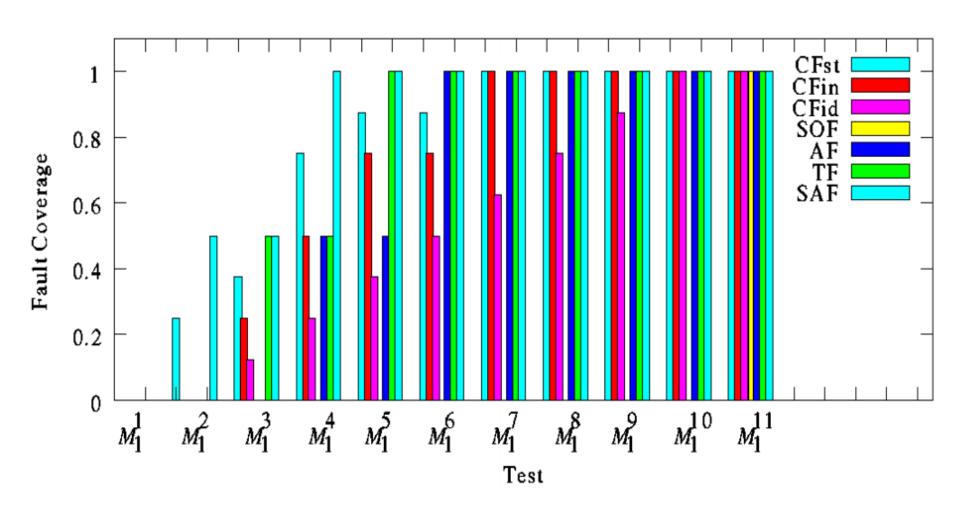
- Exhaustive generation: complexity is very high, e.g., 6.7 million templates when N = 9
- Heuristics should be developed to select useful templates



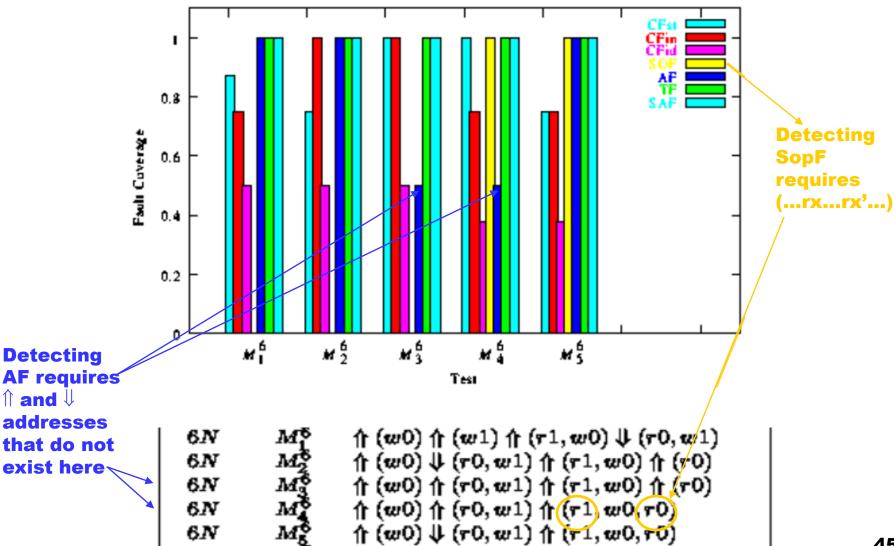
TAGS Results

T(N)	Name	March algorithm
1 N	M_1^1	ሰ (w0)
2N	M_1^2	ሰ (w0) ሰ (r0)
3.N	M_1^3	$\uparrow (w0) \uparrow (w1) \uparrow (r1)$
3.N	M_2^3	$\uparrow (w0) \uparrow (r0, w1)$
3.N	M_1^3	
3.N	M_2^3	$\uparrow (w0) \downarrow (r0, w1)$
4.N	M_1^2	$\uparrow (w0) \downarrow (r0, w1) \uparrow (r1)$
4.N	M_2^4	$\Uparrow (w0) \Downarrow (r0, w1, r1)$
5.N	M_1^3	$\uparrow (w0) \uparrow (w1) \uparrow (r1, w0) \uparrow (r0)$
5.N	M_2^5	$\uparrow (w0) \Downarrow (r0, w1) \uparrow (r1, w0)$
5N	M_3^3	$\uparrow (w0) \uparrow (w1) \uparrow (r1, w0, r0)$
6N	$M_{\mathbf{L}}^{\mathbf{S}}$	$\uparrow (w0) \uparrow (w1) \uparrow (r1, w0) \downarrow (r0, w1)$
6N	M_2^6	$\uparrow (w0) \downarrow (r0, w1) \uparrow (r1, w0) \uparrow (r0)$
6N	M_3°	$\uparrow (w0) \uparrow (r0, w1) \uparrow (r1, w0) \uparrow (r0)$
6N	M_{4}^{6}	$\uparrow (w0) \uparrow (r0, w1) \uparrow (r1, w0, r0)$
6N	M_{5}^{6}	$\uparrow (w0) \Downarrow (r0,w1) \uparrow (r1,w0,r0)$
7N	M_1^7	$\uparrow (w0) \uparrow (r0, w1) \uparrow (r1, w0) \downarrow (r0, w1)$
7N	M_1^{γ}	$\uparrow (w0) \uparrow (r0, w1) \uparrow (r1, w0) \Downarrow (r0, w1)$
7N	M_2^{27}	$\uparrow (w0) \uparrow (w1) \downarrow (r1, w0) \uparrow (r0, w1, r1)$
7N	M_3^{γ}	$\uparrow (w0) \downarrow (r0, w1) \uparrow (r1, w0, r0) \uparrow (r0)$
7N	M_{\bullet}^{3}	$\uparrow (w0) \uparrow (r0, w1) \uparrow (r1, w0, r0) \uparrow (r0)$
8.N	M_1^8	$\uparrow (w0) \uparrow (r0, w1) \uparrow (r1, w0) \downarrow (r0, w1)$
	-	介 (r1)
8.N	M_2^8	$\mathring{\uparrow}$ (w0) $\mathring{\uparrow}$ (r0, w1) $\mathring{\uparrow}$ (r1, w0)
	-	ψ $(r0, w1, r1)$
9.N	M_1^9	$\uparrow (w0) \uparrow (r0, w1) \uparrow (r1, w0) \downarrow (r0, w1)$
	•	$\psi(r1,w0)$
9N	M_2^9	$\uparrow (w0) \uparrow (r0, w1) \uparrow (r1, w0)$
	-	$\Downarrow (r0, w1, r1) \uparrow (r1)$
10N	M_1^{10}	$\uparrow (w0) \uparrow (r0, w1) \uparrow (r1, w0) \downarrow (r0, w1)$
	•	Ψ (r1, w0) ↑ (r0)
10N	M_2^{10}	$\uparrow (w0) \uparrow (r0, w1) \uparrow (r1, w0) \downarrow (r0, w1)$
	-	$\psi(r_1, w_0, r_0)$
11 <i>N</i>	M_1^{11}	$\uparrow (w0) \uparrow (r0, w1) \uparrow (r1, w0) \downarrow (r0, w1)$
	•	ψ (r1, w0, r0) \uparrow (r0)

Simulation Results for TAGS (1N to 11N)



Simulation Results for TAGS (6N)



Word-Oriented Memory Test

- A word-oriented memory has Read/Write operations that access the memory cell array by a word instead of a bit.
- Word-oriented memories can be tested by applying a bit-oriented test algorithm repeatedly with a set of different data backgrounds:
 - —The repeating procedure multiplies the testing time

Word-Oriented Memory Test (cont.)

- Background bit is replaced by background word
 - Bit level MATS++: $\{ \updownarrow (w0); \uparrow (r0,w1); \lor (r1,w0,r0) \}$
 - Word level MATS++: {\$(wa); \$(ra,wb); \$(rb,wa,ra)}

 (a is complement of b)
- Conventional method is to use logm+1 different backgrounds for m-bit words
 - Called standard backgrounds
 - m=8 bits: 00000000, 01010101, 00110011, and 00001111
 - Apply the test algorithm logm+1=4 times, so complexity is 4*6N/8=3N

Cocktail-March Algorithm

- Motivation:
 - Repeating the same algorithm for all logm+1 backgrounds is redundant as far as intra-word coupling faults are concerned
 - Different algorithms target different faults.
- Approaches:
 - 1. Use multiple backgrounds in a single algorithm run
 - Merge and forge different algorithms and backgrounds into a single algorithm
- Good for word-oriented memories

Cocktail-March Algorithm (cont.)

- Algorithm (by Wu et al. TCAD 04/2002):
 - March C- (complexity of 10N) for solid background (0000)
 - Then a 5N March for each of other standard backgrounds (0101, 0011):

 (wa,wb,rb,wa,ra)

Results:

- Complexity is (10+5logm)N, where m is word length and N is word count
- Test time is reduced by 39% if m=4, as compared with extended March C-
- Improvement increases as m increases

Pseudorandom Memory Tests

Pseudo-Random 'PR' Memory Tests

Purpose

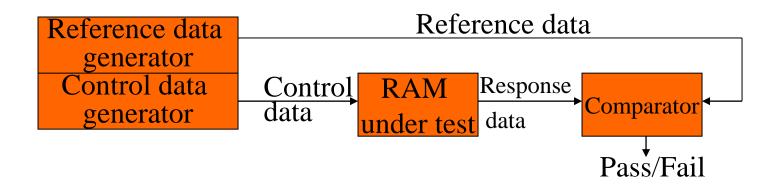
- Explain concept of pseudo-random (PR) testing
- —Compute test length of PR tests for SAFs and *k*-CFs
- —Evaluation of PR tests
- PR pattern generators and test response evaluators

Sources of material

- Mazumder, P. and Patel, J.H. (1992). An Efficient Design of Embedded Memories and their Testability Analysis using Markov Chains. JETTA, Vol. 3, No. 3; pp. 235-250
- Krasniewski, A. and Krzysztof, G. (1993). Is There Any Future for Deterministic Self-Test of Embedded RAMs? In Proc. ETC'93; pp. 159-168
- van de Goor, A.J. (1998). Testing Semiconductor Memories, Theory and Practice. ComTex Publishing, Gouda, The Netherlands
- van de Goor, A.J. and de Neef, J. (1999). Industrial Evaluation of DRAM Tests. In Proc. Design and Test in Europe (DATe'99), March 8-13, Munich; pp. 623-630
- van de Goor, A.J. and Lin, Mike (1997). The Implementation of Pseudo-Random Tests on Commercial Memory Testers. In Proc. IEEE Int. Test Conf., Washington DC, 1997, pp. 226-235

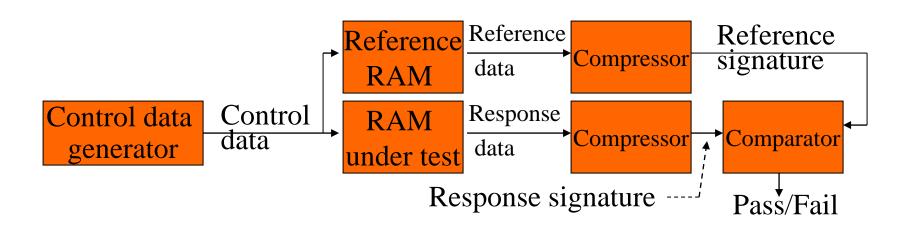
Concepts of PR Memory Testing

- Deterministic tests
 - —Control & Reference data for the RAM under test have predetermined values
 - —The Response data of the RAM under test is compared with the expected data, in order to make Pass/Fail decision



Concepts of PR Memory Testing (cont.)

- Pseudo-random tests
 - Control data on some or all inputs established pseudorandomly
 - —Reference data can be obtained from a Reference RAM or, as shown, from a compressor



Concepts of PR Memory Testing (cont.)

- Memory tests use
 - control values for:
 - Address lines (N)
 - R/W line (1)
 - write data values (B)

Address N RAM under R/W line test 1 B

Deterministic test method

Uses deterministic control and write data values

- In a test the following can be Deterministic (D) or PR (R)
 - The Address (A): DA or RA
 - The Write (W) operation: DW or RW
 - The Data (D) to be written: DD or RD
 - ⇒ MATS+ is a DADWDD (Det. Addr, Det. Write, Det. Data) test
- In a PR test at least ONE component has to be PR
 - This can be: A (Address) &/or W (Write operation) &/or D (Data)
 - PR tests are preferred over random tests: PR tests are repeatable

Pseudo-Random Tests for SAFs

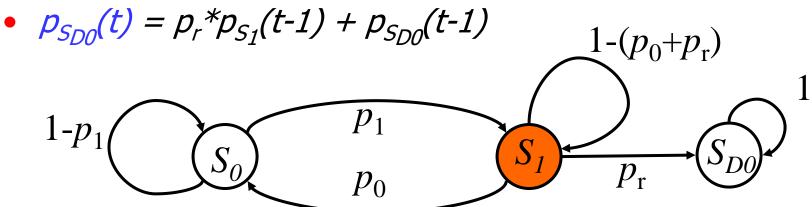
Some probabilities for computing the *test length* (*TL*)

- The TL is a function of the escape probability `e'
- p: probability that a line has the value 1
- p_a: probability that an address line has the value 1
- p_d: probability that a data line has the value 1
- p_w: probability that the write line has the value 1
- p_A : probability of selecting address A (with z 0s and N-z 1s) $p_A = (1 p_a)^z * p_a^{(N-z)}$
- p_1 : probability of writing 1 to address A; $p_1 = p_d * p_w * p_A$
- p_0 : prob. of writing 0 to address A; $p_0 = (1 p_d) * p_w * p_A$
- p_r : probability of *reading* address A_r : $p_r = (1 p_w) * p_A$
- Given that a particular address A has been selected, the operation will either be a w1, w0 or r. Therefore, $p_A = p_1 + p_0 + p_r$

Test Length of PR Test for SAFs

Markov chain for detecting a SAO fault (SA1 fault is similar)

- S_0 : state in which a 0 is stored in the cell
- S₁: state in which a 1 should be in the cell
- S_{D0} : state in which SAO fault is detected (absorbing state)
- $p_{S_0}(t)$: probability of being in state S_0 at time t
- Initial conditions: $p_{S_0}(0) = 1 p_{II}$, $p_{S_1}(0) = p_{II}$, $p_{S_{D_0}}(0) = 0$
- $p_{S_0}(t) = (1 p_1) * p_{S_0}(t-1) + p_0 * p_{S_1}(t-1)$
- $p_{S_1}(t) = p_1 * p_{S_0}(t-1) + (1-p_0-p_r) * p_{S_1}(t-1)$



Test Length of PR Test for SAFs (cont.)

- With deterministic testing fault detected with certainty
- With PR testing fault detected with an escape probability `e'
 - —SA0 fault is detected when: $p_{SDO}(t) \ge 1-e$
 - $-T_0(e)$ is TL (test length) for SA0 faults
- T(e): The test length for SAFs is: $T(e) = max(T_0(e), T_1(e))$

$$T_{0}(e) = \left\lceil \frac{ln\left(\frac{2 \cdot \alpha \cdot e}{1 + \alpha - 2 \cdot (1 - p_{w}) \cdot p_{I1}}\right)}{ln\left(1 - \frac{(1 - \alpha) \cdot p_{A}}{2}\right)} \right\rceil \qquad T_{1}(e) = \left\lceil \frac{ln\left(\frac{2 \cdot \beta \cdot e}{1 + \beta - 2 \cdot (1 - p_{w}) \cdot (1 - p_{I1})}\right)}{ln\left(1 - \frac{(1 - \beta) \cdot p_{A}}{2}\right)} \right\rceil$$

$$\beta = [1-4 (1-p_d) p_w (1-p_w)]^{1/2}$$

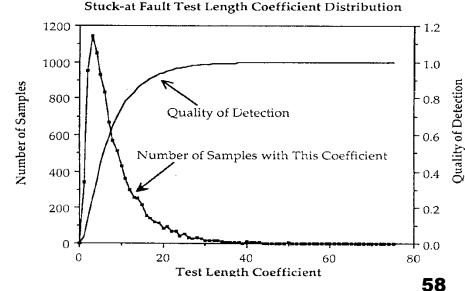
$$\alpha = [1-4 p_d p_w (1-p_w)]^{1/2}$$

Test Length of PR Test for SAFs (cont.)

- Test length is a function of the escape probability and memory size.
- Test length coefficient =T(e)/n (using $p_a=p_d=p_w=1/2$)
 - T(e): total number of operations
 - T(e)/n: test length in terms of number of operations per cell. It is independent of memory size n & proportional to ln(e)
- e.g. MATS+ test requires 5 operations per cell to detect all SAFs, while PR test requires 48+1(for initialization)=49 operations per cell to detect SAFs with an escape probability of e=0.001.

Test length coefficient

e	Memory size			
	n=32	<i>n</i> =1k	<i>n</i> =32k	<i>n</i> =1024k
0.1	17	17	17	17
0.01	33	33	33	33
0.001	48	48	48	48
0.0001	64	64	64	64
0.00001	80	80	80	80



Test Lengths: Deterministic -- PR Tests

Fault	Test length coefficient			
	Deterministic	Pseudo-random		
		e=0.01	e=0.001	<i>e</i> =0.000001
SAF	5* <i>n</i> (MATS+)	33*n	46*n	93*n
CFid	10* <i>n</i> (March C-)	145*n	219*n	445*n
ANPSF <i>k</i> =3	28*n	294*n	447*n	905*n
APSF $k=3$	$n+32*n*log_2n$	294*n	447*n	905*n
ANPSF <i>k</i> =5	195*n	1200*n	1805*n	3625*n
APSF $k=5$?	1200*n	1805*n	3625*n

Observations

- Note: ANPSFs have *k*-2 cells in only *one* position
- For simple fault models deterministic tests more efficient
 - Detect all faults of **some** fault models with e = 0
- For complex fault models PR tests do exist
 - PR tests detect all faults of all fault models, however with e > 0

Strengths/Weaknesses of PR Tests

- Deterministic tests based on a-priori fault models
 - Models usually restricted to the memory cell array
 - 5% of real defects not explained (Krasniewski, ETC'93)
 - Tests detect 100% of targeted faults only
- Pseudo-random tests
 - Not targeted towards a particular fault model
 - PR tests detect faults of *all fault models*; however, with some e > 0
 - Long test time: Test length (TL) proportional to ln(e) and 2^{k-2}
 - For CFids: 445*n ($e = 10^{-5}$) versus 10*n (for March C-)
 - Less of a problem for SRAMs (e.g.,1 Mword, 1ns, 1000n test takes 1s)
 - Random pattern resistant faults
 - with a large data state (e.g., bit line imbalance)
 - requiring a large address/operation state (e.g., Hammer tests)
 - Cannot *locate faults* easily (For laser/dynamic repair)
 - Well suited for BIST
 - Very useful for verification purposes
 - Used for production SRAM testing (together with deterministic tests)
 - Unknown fault models, short time to volume, high speed SRAM