EEDG/CE 6303: Testing and Testable Design

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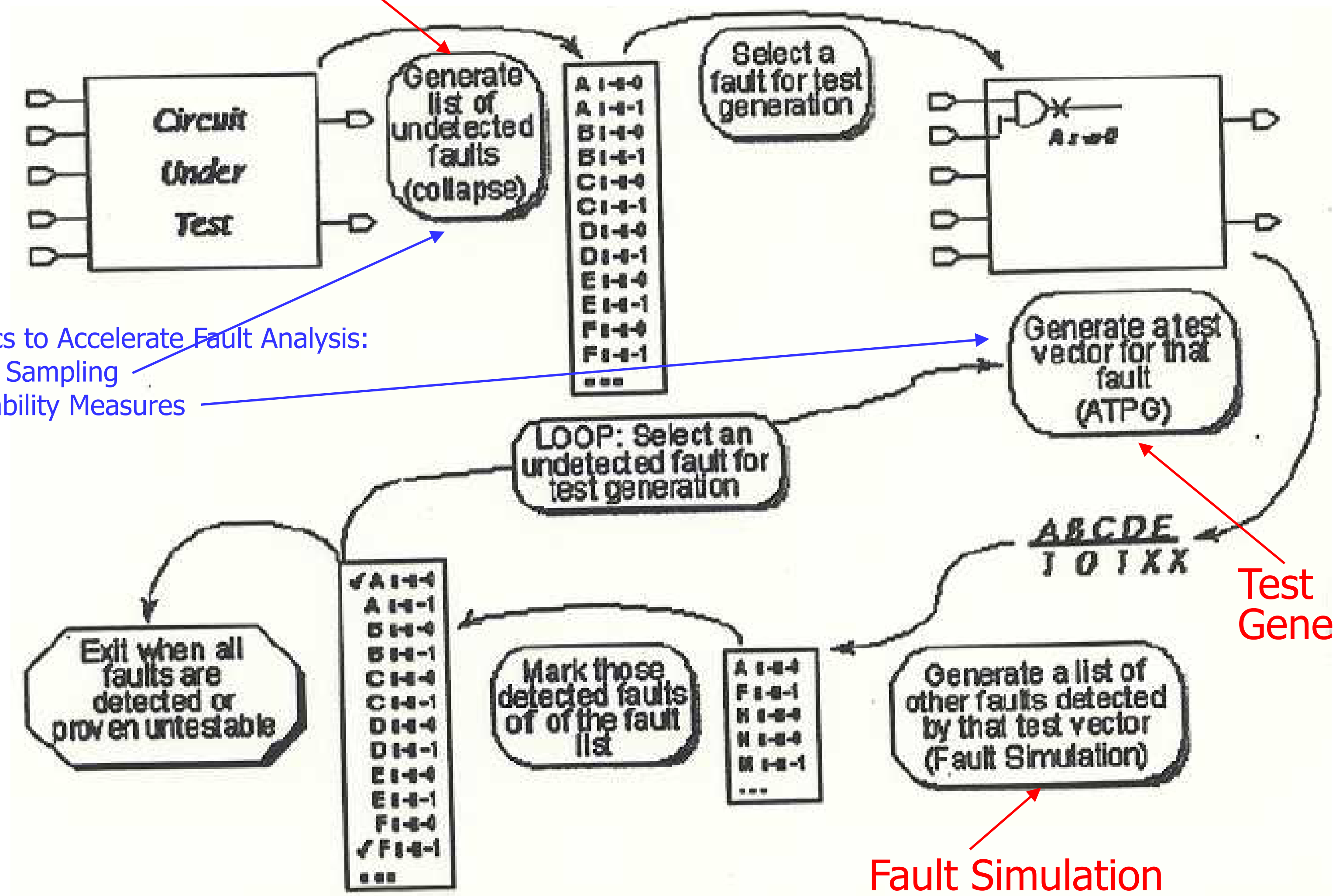
Session 05

Acceleration Heuristics for Test Generation

Fault Analysis System (Review)

Fault Collapsing



Heuristics to Accelerate Fault Analysis:

- Fault Sampling
- Testability Measures

Test Generation

Fault Simulation

Fault Sampling
(A Statistical Method for Fault Simulation)

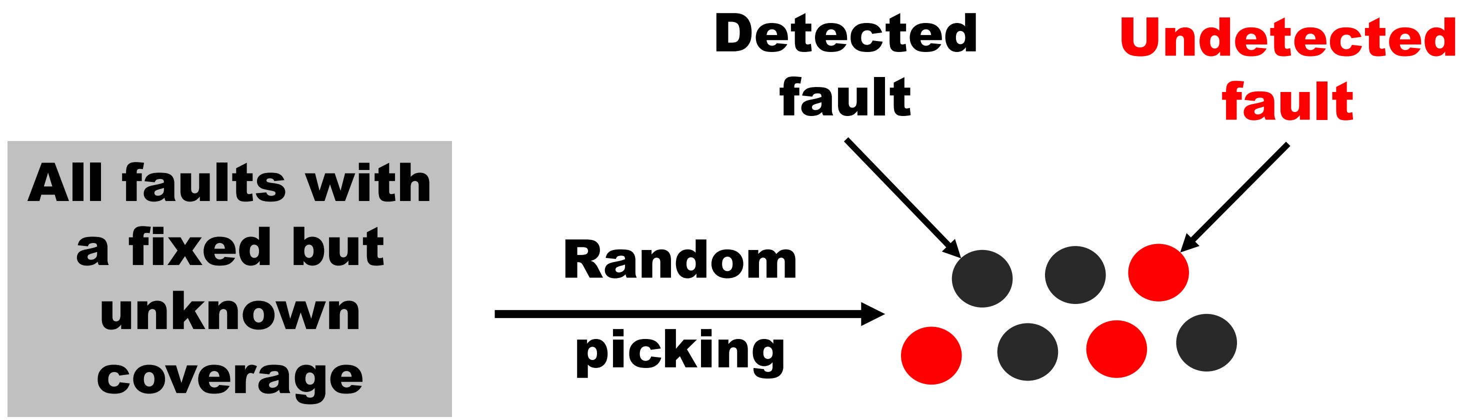
Basic Idea

- A randomly selected subset (sample) of faults is simulated.
- Measured coverage in the sample is used to estimate fault coverage in the entire circuit.
- **Advantage**: Saving in computing resources (CPU time and memory.)
- **Disadvantage**: Limited data on undetected faults.

Motivation for Sampling

- Complexity of fault simulation depends on:
  - Number of gates
  - **Number of faults**
  - Number of vectors
- Complexity of fault simulation with fault sampling depends on:
  - Number of gates
  - Number of vectors

Random Sampling Model



N_p = total number of faults (population size)

C = fault coverage (unknown ; $0 \leq C \leq 1$)

N_s = sample size

$N_s \ll N_p$

c = sample coverage (a random variable – $0 \leq c \leq 1$)

- The challenge is to sample enough such that
  - You save time compared to simulating all faults and
  - c (estimated fault coverage) is close to C (real fault coverage).

Key Parameters

- **N_p**: Total number of faults in the circuit for which coverage is to be determined
- **C**: Unknown but true fault coverage of given vectors, $0 \le C \le 1$. This is the quantity being estimated.
- **CN_p**: Actual (but unknown) number of faults detectable by the given vectors.
- **N_s**: Number of randomly sampled faults from the set of N_p faults. N_s is known and normally $N_s << N_p$.
- **c**: Sample coverage, a random variable with range, $0 \le c \le 1$.
- **x**: Value of c determined from sample fault simulation, $0 \le x \le 1$.
- **xN_s**: Number of sampled faults detected by given vectors. This is a known quantity that is determined by the fault simulator.

Probabilistic Analysis

- Using the key parameters:

$$\text{Ways of obtaining sample of size } N_s = \binom{N_p}{N_s} = \frac{N_p!}{N_s!(N_p - N_s)!}$$

Number of ways to choose detectable faults

$$\text{Ways of obtaining sample coverage } x = \binom{CN_p}{xN_s} \cdot \binom{(1-C)N_p}{(1-x)N_s}$$

Number of ways to choose undetectable faults

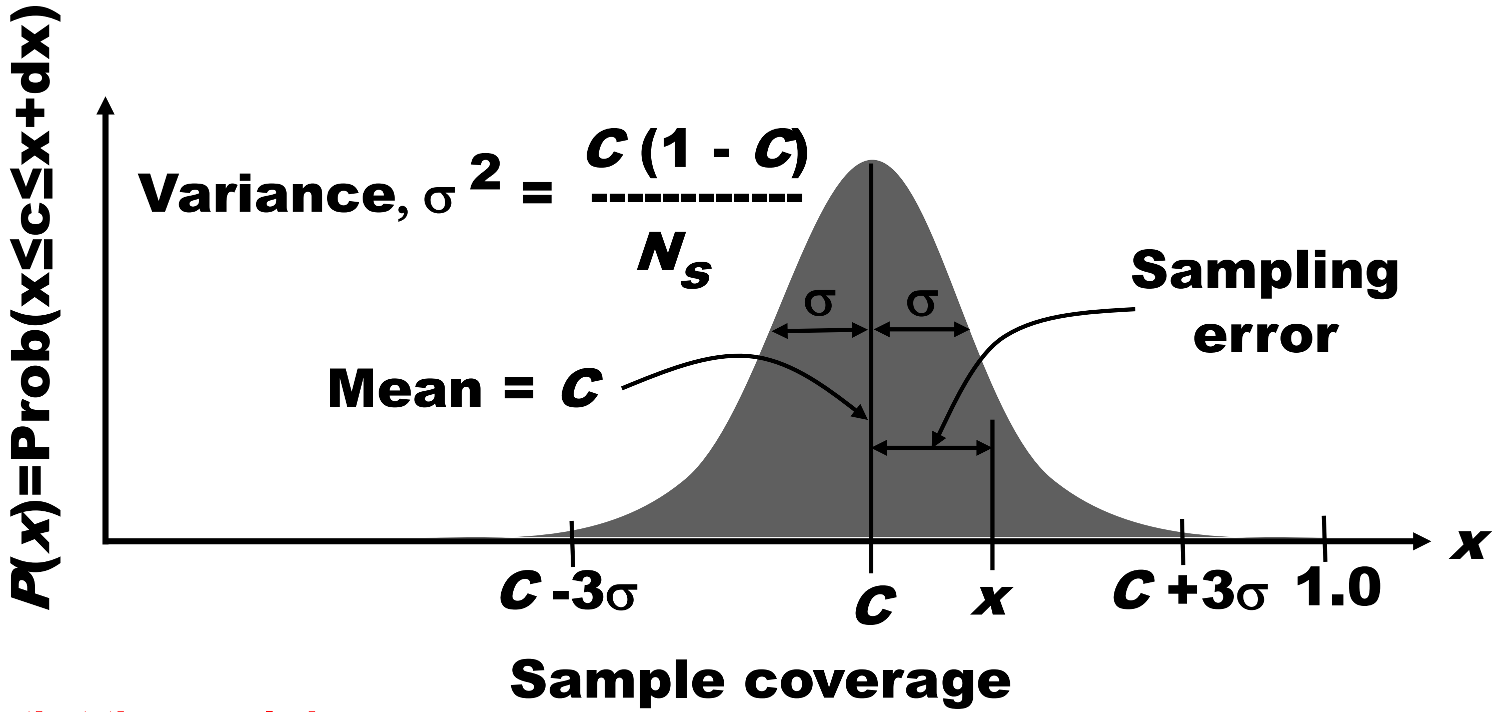
$$p(x) = \Pr ob(\text{sample coverage}, c = x) = \frac{\binom{CN_p}{xN_s} \cdot \binom{(1-C)N_p}{(1-x)N_s}}{\binom{N_p}{N_s}}$$

- This is known as the **hypergeometric** probability density function of a discrete-valued random variable. The random variable c can take discrete values $0, 1/N_s, 2/N_s, \ldots, 1$. When N_s is large, c can be treated as a continuous variable and the above p(x) can be approximated by a **Gaussian (normal)** probability density function with mean E(c)=C and variance σ^2:

$$p(x) = \Pr ob(x \le c \le x + dx) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-C)^2}{2\sigma^2}}$$

Probability Density of Sample Coverage

$$p(x) = \Pr ob(x \le c \le x + dx) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-C)^2}{2\sigma^2}}$$



Probability that the sampled
fault coverage stays within a limit

Sampling Error

- The variance of c can be determined as:

$$\sigma^2 = \frac{C(1-C)}{N_s}(1-\frac{N_s}{N_p}) \approx \frac{C(1-C)}{N_s}$$

- The sampling error is defined as |x-C| and its high confidence (0.997 probability) can be determined by limiting it to 3σ.

$$|x-C| = 3\sigma$$

$$= 3\sqrt{\frac{C(1-C)}{N_s}}$$

- For sampling error λσ in general (instead of 3σ) solve this quadratic equation for C:

$$(x-C)^2 = \lambda^2 \frac{C(1-C)}{N_s}$$

- When λ=3, using approximation of $N_s \geq 1000$

$$3\sigma \text{ coverage estimate} = C_{3\sigma} = x \pm \frac{4.5}{N_s}\sqrt{1+0.44N_s x(1-x)}$$

Example I

- A circuit with 39,096 faults has an actual fault coverage of C=87.1%. This is found by an accurate fault simulation in 94sec CPU time.
- The measured coverage in a random sample of N_s=1,000 faults is x=88.7%.
- CPU time for sample simulation was 11sec, i.e. about 10% of that for all faults.
- The $C_{3\sigma}$ formula gives an estimate of
  - 88.7% $\pm$ 3%.

Example II

- For the same circuit with 39,096 faults, suppose we want the 3σ sampling error not to exceed $\pm\Delta$

$$\Delta^2 = \frac{4.5^2}{{N_s}^2}(1 + 0.44 N_s x(1-x)) \approx \frac{4.5^2}{N_s} 0.44 x(1-x)$$

- The sample size is assumed to be large. Since maximum value of x(1-x) is 0.25 which occurs at x=0.5, we get:

$$N_s = \frac{4.5^2 \times 0.44 \times 0.25}{\Delta^2} = \frac{2.2275}{\Delta^2}$$

- For Δ=0.02, we obtain N_s=5,569, that is the sample size for the worst case. If x=0.90, then $C_\Delta = C_{0.02} = 0.90 \pm 0.02$.
- If x=0.90, the 3σ range will be $C_{3\sigma} = 0.90 \pm 0.012$.