EEDG/CE 6303: Testing and Testable Design

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Session 01

Introduction

Failure Rate
(Fault Occurrence Frequency)

Fault Occurrence Frequency

- Can be explained using reliability theory
- The point in time t which a failure occurs can be considered a random variable u
- The probability of a failure before time t, F(t), is the unreliability of the system: $F(t) = Prob(u \leq t)$
  - $F(0) = 0$: Initially the system is operable
  - $F(\infty) = 1$: Ultimately the system will fail
- The reliability of a system, R(t), is the probability of a correct functioning system at time t: $R(t) = 1 - F(t)$
  - $R(0)=1$ and $R(\infty)=0$
- $R(t)+F(t)=1$: system is either operable or failing

Fault Occurrence Frequency (cont'd)

- Reliability can also be defined as:

$$R(t) = \frac{\text{Number of components surviving at time t}}{\text{Number of components at time 0}}$$

- The derivative of *F(t), f(t),* is called the failure probability density function:

$$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt}$$

- Hence $F(t) = \int_0^t f(t)dt$ and $R(t) = \int_t^\infty f(t)dt$

Fault Occurrence Frequency (cont'd)

- The failure rate , $z(t)$, is defined as the conditional probability that the system fails during the period (t, $t+\Delta t$); given that the system was operational at time t

$$z(t) = \frac{\text{Number of failing components per unit time at time t}}{\text{Number of surviving components at time t}}$$

- Alternatively, z(t) can be expressed as follow:

$$z(t) = \lim_{\Delta t \to 0} \frac{F(t+\Delta t) - F(t)}{\Delta t} * \frac{1}{R(t)} = \frac{dF(t)}{dt} * \frac{1}{R(t)} = \frac{f(t)}{R(t)}$$

Fault Occurrence Frequency (cont'd)

- *R(t)* can be expressed in terms of *z(t)* as follows

$$\int_0^t z(t)dt = \int_0^t \frac{f(t)}{R(t)}dt = -\int_{R(0)}^{R(t)} \frac{dR(t)}{R(t)} = -\ln\frac{R(t)}{R(0)}$$

or $\quad R(t) = R(0)e^{-\int_0^t z(t)dt}$

- The average lifetime of a system, Θ, can be expressed as the mathematical expectation of *t* to be:

$$\Theta(t) = \int_0^\infty t * f(t)dt$$

Fault Occurrence Frequency (cont'd)

- For a non-maintained system, *Θ*, is called the Mean Time To Failure, MTTF. Using partial integration, and assuming $\lim_{T \to \infty} T * R(T) = 0$

$$\mathrm{MTTF} = \Theta = \int_0^{\infty} t * f(t)dt = -\int_0^{\infty} t * \frac{dR(t)}{dt} dt = -\int_{R(0)}^{R(\infty)} t * dR(t)$$

$$= \lim_{T \to \infty} \left(-t * R(t) \Big|_0^T + \int_0^T R(t)dt \right) = \int_0^{\infty} R(t)dt$$

- Given a system with the following reliability:

$$R(t) = e^{-\lambda t}$$

The failure rate, *z(t),* of that system is computed below, and has a constant value λ :

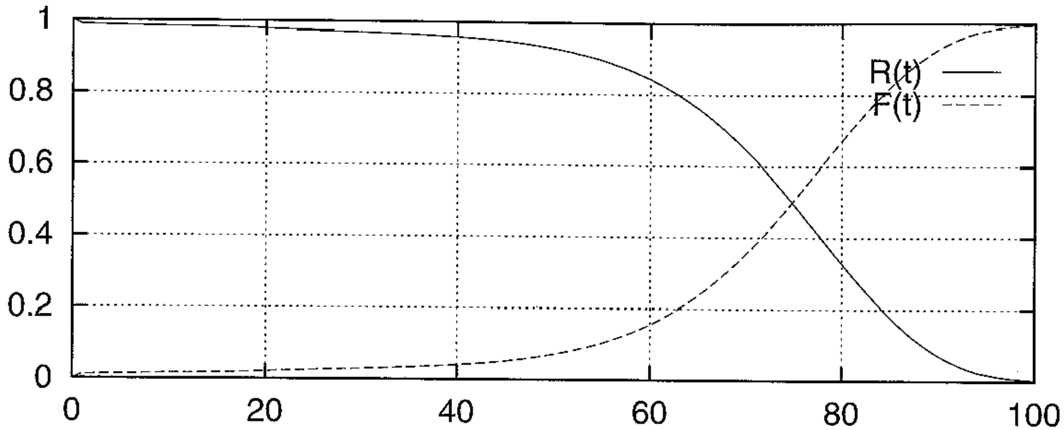
$$z(t) = \frac{f(t)}{R(t)} = \frac{dF(t)}{dt} / R(t) = \frac{d(1 - e^{-\lambda t})}{dt} / e^{-\lambda t} = \lambda e^{-\lambda t} / e^{-\lambda t} = \lambda$$

Fault Occurrence Frequency (cont'd)

- Assuming failures occur randomly with a constant rate λ, the MTTF can be expressed as:

$$\text{MTTF} = \Theta = \int_0^\infty R(t)dt = \int_0^\infty e^{-\lambda t}dt = \frac{1}{\lambda}$$

- Example: *R(t)* & *F(t)* for life (& death) expectancy of Dutch male population (over years: 1976– 1980)



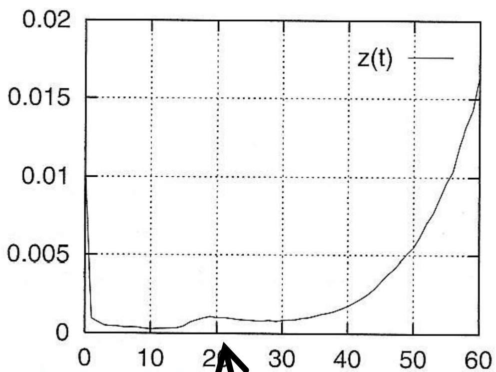
(a) $R(t)$, $F(t)$ vs years

Note: Number of people > 100 yrs old too small

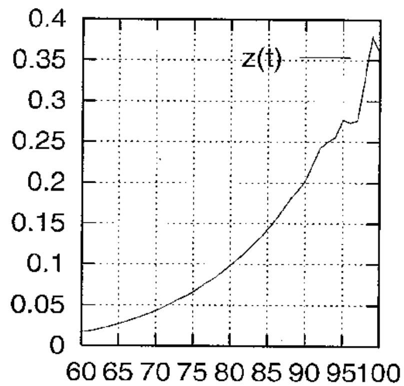
Fault Occurrence Frequency (cont'd)

- z(t) and f(t) for same example:

z(t)

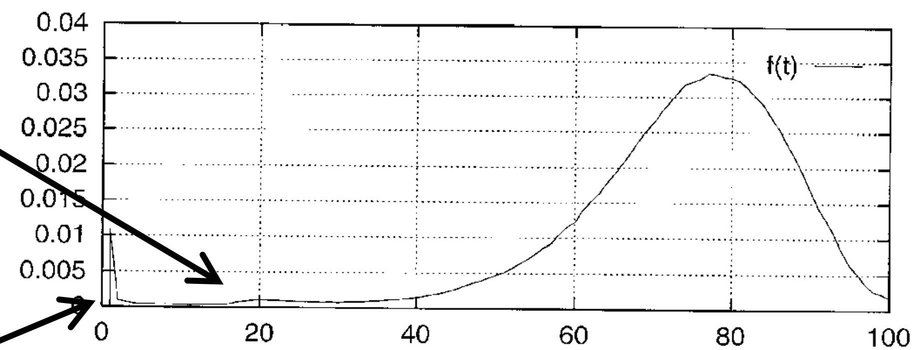


(b) $z(t)$ vs younger years



(c) $z(t)$ vs older years

f(t)



(d) $f(t)$ vs years

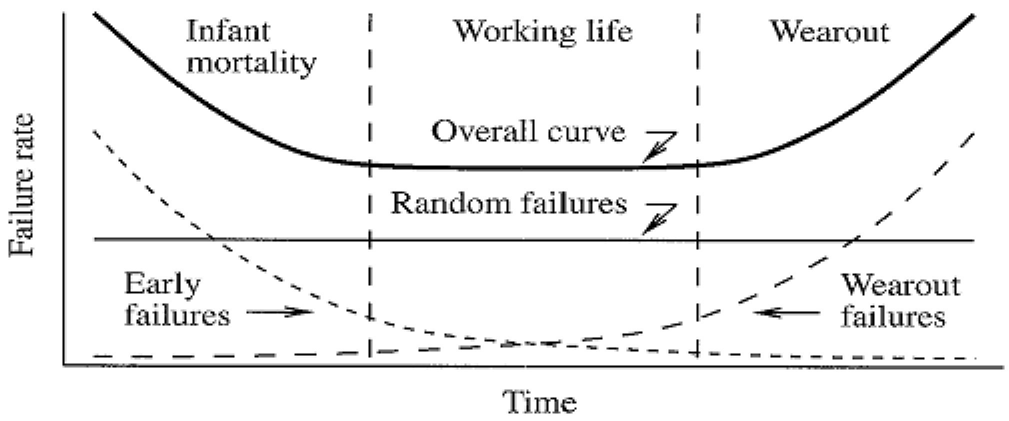
Note: Increase of *z(t)* & *f(t)* between ages 18—20 due to driving accidents

Note: Infant mortality rate

Failure Rate Over Product Lifetime

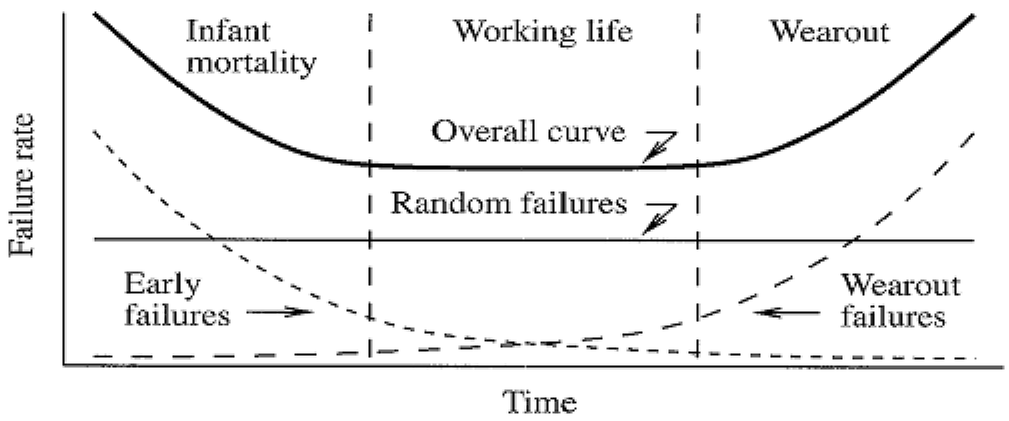
- A well-know graphical representation of the failure rate, *z(t),* is the bathtub curve. It consists of three regions:

  1. Infant Mortality
  2. Working Life
  3. Wearout



Failure Rate Over Product Lifetime (cont'd)

- Infant mortality: failures in this region are attributed to poor quality due to variations in the production process.
- Working life: Constant failure rate λ. Failures are considered to occur randomly in time.
- Wearout: Increasing failure rate. This represents the end-of-life period of a system.



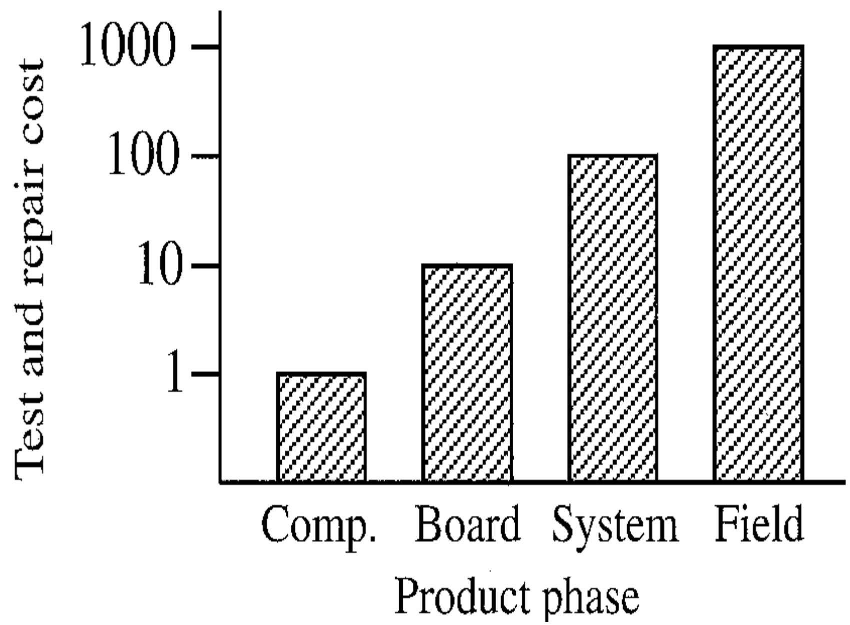
Failure Rate Over Product Lifetime (cont'd)

- It should be clear that a system should be shipped after it has passed the infant mortality period, in order to reduce the number of field returns.
- Shipping a system after the infant mortality period can be done by:
  1. Aging the system for that period (this can be several months)
  2. Aging the system under stress (this accelerates the aging process – e.g. Burn-in test)

Test Economics

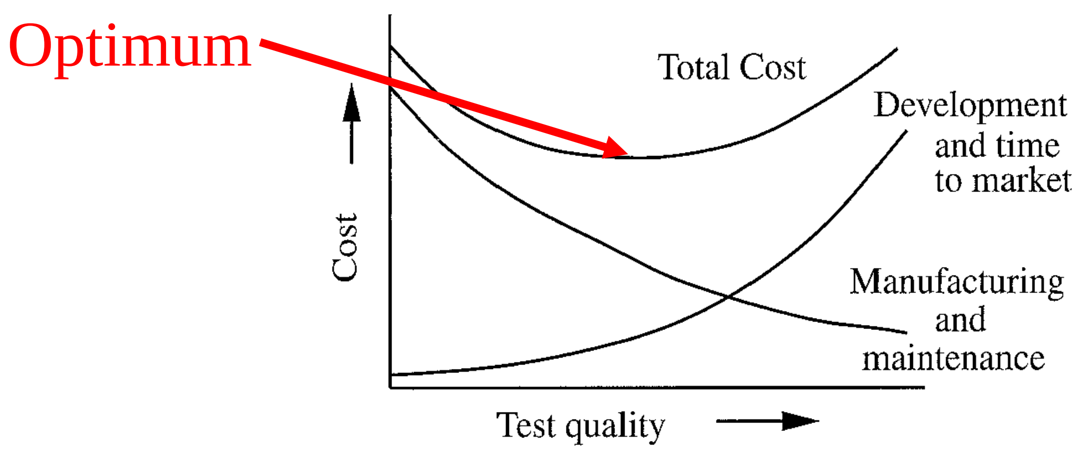
Repair Cost During the Product Phases

- A move from one product phase to the next causes the volume of parts and the test & repair cost to increase by a factor of *10*. *This is the rule-of-ten.*



Economics and Liability of Testing

- *Good tests*
  - reduce test & repair cost (see rule-of-ten)
  - can reduce development time & time-to market
  - can reduce field maintenance costs
  - reduce personal injury and lawsuits
- There is an optimum in test development cost and its contribution to profit. Too many tests require a long test development time and test cost

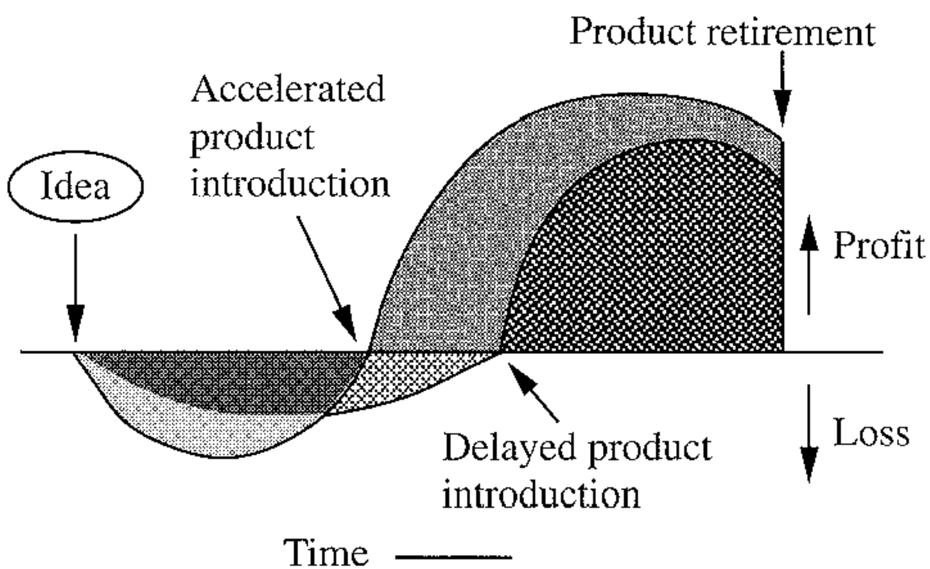


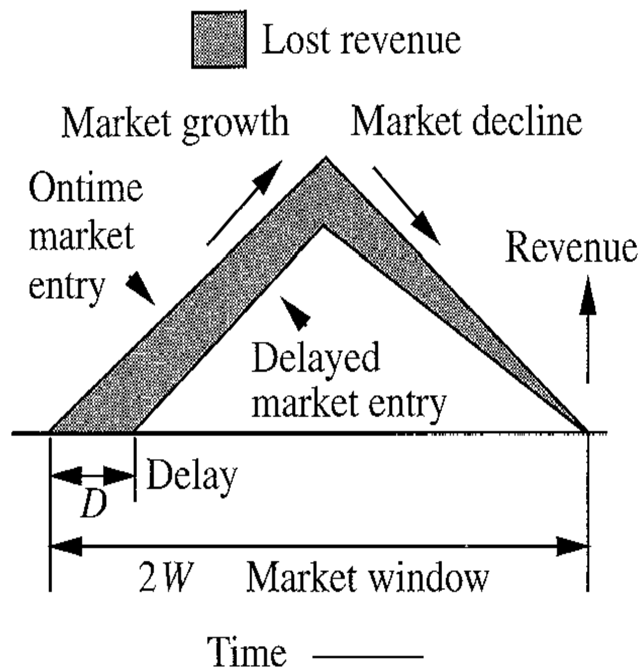
Total Profit

- The life time of a product has several economic phases
  - The development phase
    - Product design takes place
    - No income; only expenses
    - Area under zero-line is development cost
  - The market growth phase
    - Market acceptance increases with time
  - The market decline phase
    - Product becomes less attractive
    - Market share decreases
    - Price may have to be reduced

Total Profit (cont.)

- The total profit over the life time of a product is the area above the zero-line (revenue) – area below the zero-line (development cost)
- In case of a delay '*D*' in product development, the development cost is higher, while the revenue is reduced, because the obsolescence point will not change





Product Development Delay Cost

- Assuming *M* is the maximum market growth, which is reached after time *W*, the revenue lost due to a delay *D* (hatched area in next slide) can be computed as follows:
  - The Expected Revenue '*ER*' is: $ER = \frac{1}{2} * 2W * M = W * M$
  - The Revenue of the Delayed Product '*RDP*' is:

$$RDP = \frac{1}{2} * (2W - D) * (\frac{W - D}{W} * M)$$

  - The Lost Revenue '*LR*' is:

$$LR = ER - RDP$$

$$LR = W * M - \frac{2W^2 - 3D * W + D^2}{2W} * M = ER * \frac{D * (3W - D)}{2W^2}$$