
EEDG/CE 6303: Testing and Testable Design

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Session 10

Built-In Self-Test (BIST)

Basic Concept

Key Issues

- Motivation and economics
- Definitions for components and procedures
- *Built-in self-testing* (BIST) process
- BIST *pattern generation* (PG)
- BIST *response compaction* (RC)
- *Aliasing probability*

Motivation

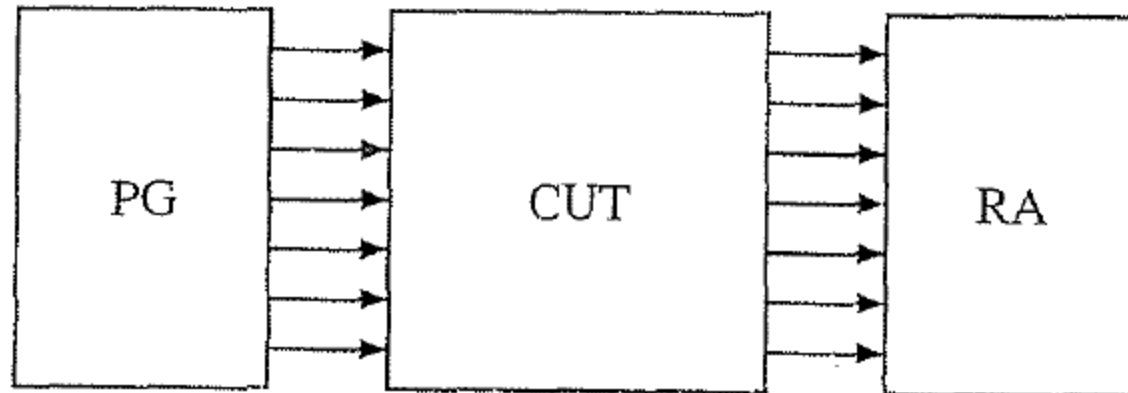
- The problems in today's semiconductor testing
 - Traditional test techniques become quite expensive
 - No longer provide sufficiently high fault coverage
- Why do we need built-in self-test (BIST)?
 - For mission-critical applications
 - Detect un-modeled faults
 - Provide remote diagnosis
- Useful for field test and diagnosis (less expensive than a ATE)
- Software tests for field test and diagnosis:
 - Low hardware fault coverage
 - Low diagnostic resolution
 - Slow to operate
- Hardware BIST benefits:
 - Lower system test effort
 - Improved system maintenance and repair
 - Improved component repair
 - Better diagnosis

Motivation (cont.)

- Costly test problems can be alleviated by BIST
 - Increasing chip logic-to-pin ratio – harder observability
 - Increasingly dense devices and faster clocks
 - Increasing test generation and application times
 - Increasing size of test vectors stored in ATE
 - Expensive ATE needed for above 1 GHz clocking chips
 - Hard testability insertion – designers unfamiliar with gate-level logic, since they design at behavioral level
 - In-circuit testing* no longer technically feasible
 - Shortage of test engineers
 - Circuit testing cannot be easily partitioned

BIST Concept

- Built-in self-test refers to techniques and circuit configurations that enable a chip to test itself
- In this methodology, test patterns are generated and test responses are analyzed on-chip.
- The simplest of BIST designs has a pattern generator (PG) and a response analyzer (RA)



BIST Advantages

- BIST offers several advantages over testing using automatic test equipment (ATE)
 1. In BIST the test circuitry is incorporated on-chip and no external tester is required (especially attractive for high-speed circuits).
 2. Self-test can be performed at the circuit's normal clock rate.
 3. Self-testable chip has the ability to perform self-test even after it is incorporated into a system (either for periodic testing or to diagnose system failures).

Main BIST Techniques

- Logic BIST Techniques

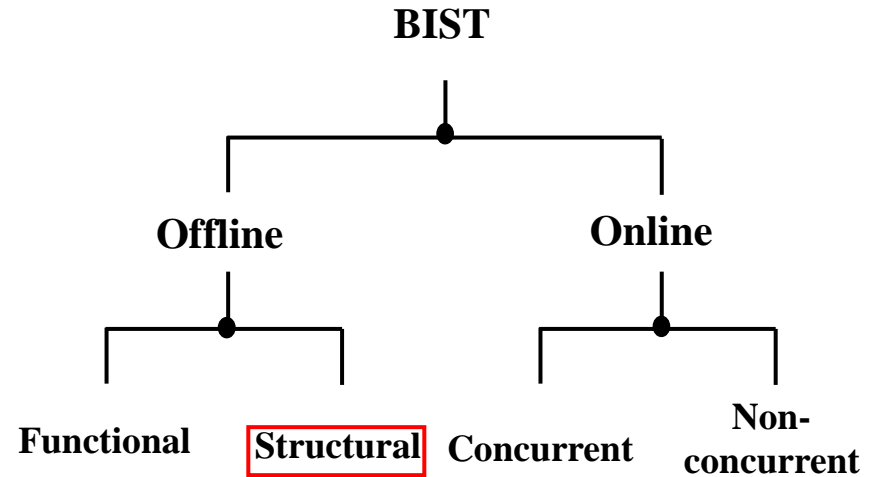
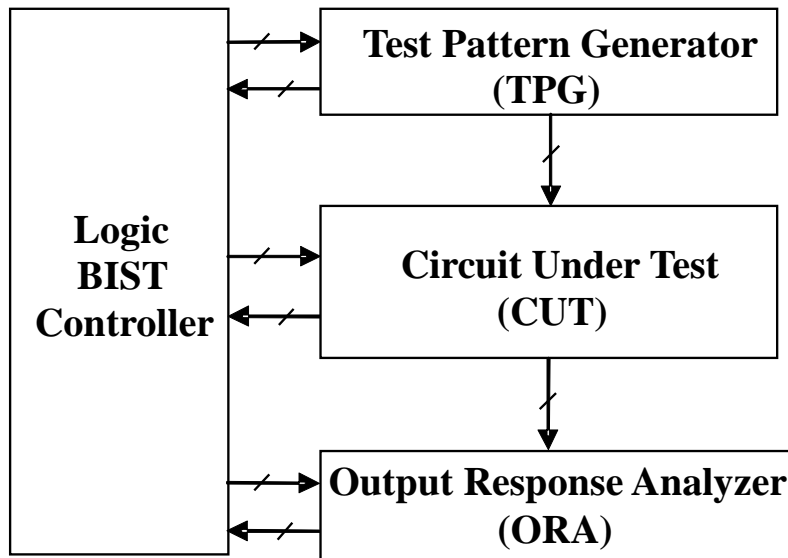
1. Online BIST

- Concurrent online BIST
- Non Concurrent online BIST

2. Offline BIST

- Functional offline BIST
- Structural offline BIST

- A typical logic BIST system



[Abramovici 1994]

Economics – BIST Costs

- Chip area overhead for:
 - Test controller
 - Hardware pattern generator
 - Hardware response compacter
 - Testing of BIST hardware
- Pin overhead -- At least 1 pin needed to activate BIST operation
- Performance overhead – extra path delays due to BIST
- *Yield loss* – due to increased chip area or more chips in system because of BIST
- Reliability reduction – due to increased area
- Increased BIST hardware complexity – happens when BIST hardware is made testable

Benefits

- Faults tested:
 - Single combinational / sequential stuck-at faults
 - Delay faults
 - Single stuck-at faults in BIST hardware
- BIST benefits
 - Reduced testing and maintenance cost
 - Lower test generation cost
 - Reduced storage / maintenance of test patterns
 - Simpler and less expensive ATE
 - Can test many units in parallel
 - Shorter test application times
 - Can test at functional system speed

Definitions

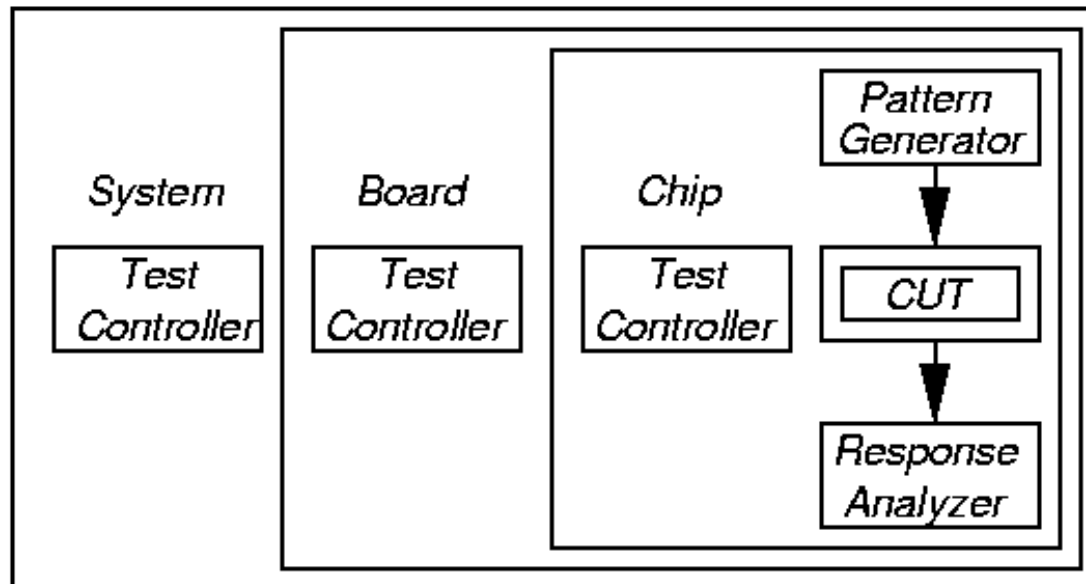
- **BILBO** – *Built-in logic block observer*, extra hardware added to flip-flops so they can be reconfigured as an LFSR pattern generator or response compacter, a scan chain, or as flip-flops
- **Concurrent testing** – Testing process that detects faults during normal system operation
- **CUT** – *Circuit-under-test*
- **Exhaustive testing** – Apply all possible 2^n patterns to a circuit with n inputs
- **Irreducible polynomial** – Boolean polynomial that cannot be factored
- **LFSR** – *Linear feedback shift register*, hardware that generates pseudo-random pattern sequence

Definitions (cont.)

- ***Primitive polynomial*** – Boolean polynomial $p(x)$ that can be used to compute increasing powers n of $x^n \text{ modulo } p(x)$ to obtain all possible non-zero polynomials of degree less than $p(x)$
- ***Pseudo-exhaustive testing*** – Break circuit into small, overlapping blocks and test each exhaustively
- ***Pseudo-random testing*** – Algorithmic pattern generator that produces a subset of all possible tests with most of the properties of randomly-generated patterns
- ***Signature*** – Any statistical circuit property distinguishing between bad and good circuits
- **TPG** – Hardware *test pattern generator*

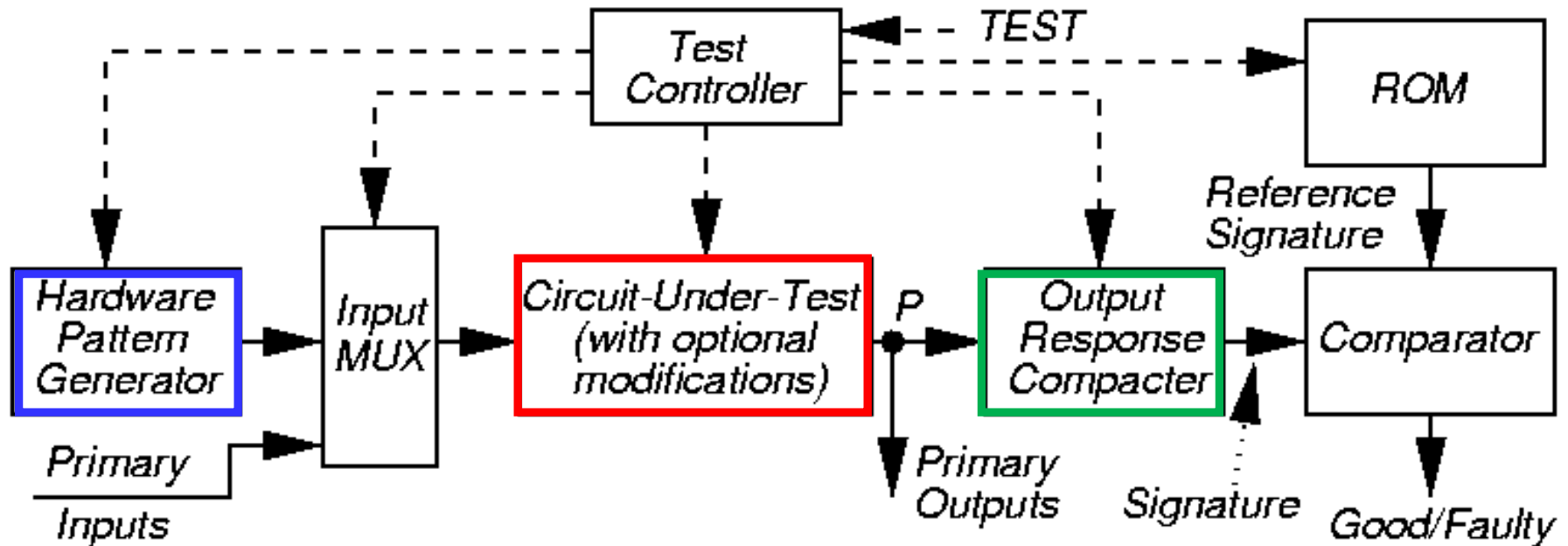
BIST Structure & Process

- *Test controller* – Hardware that activates self-test simultaneously on all PCBs
- Each board controller activates parallel chip BIST Diagnosis effective only if very high fault coverage



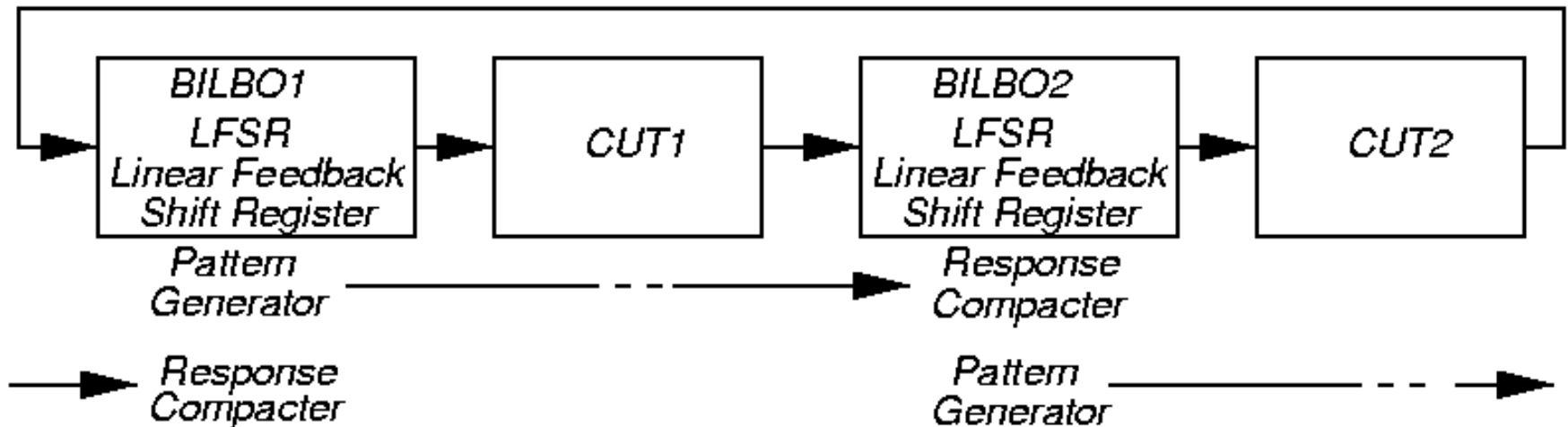
BIST Architecture

- BIST cannot test wires and transistors
 - From PI pins to Input MUX
 - From POs to output pins



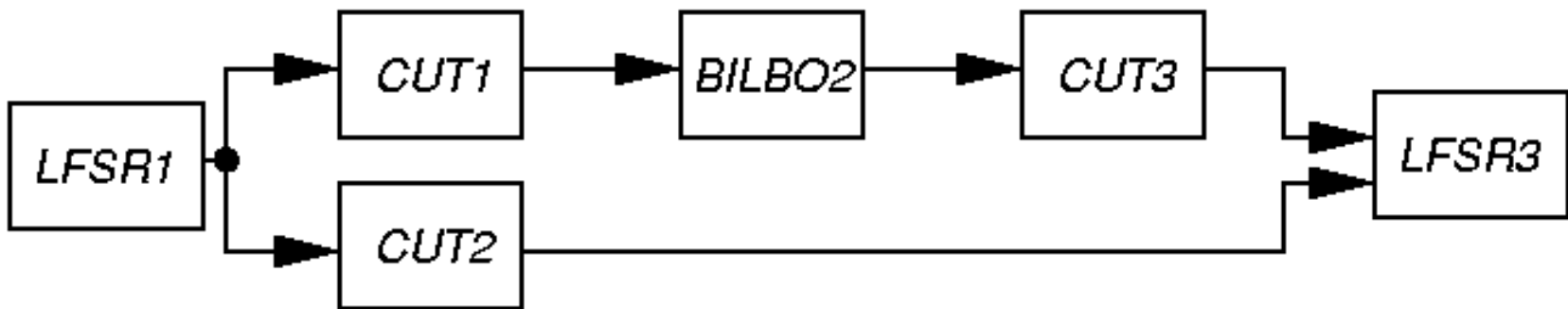
Built In Logic Block Observer (BILBO)

- Can work as a PG or RC. It has 4 modes:
 1. Flip-flop
 2. LFSR pattern generator
 3. LFSR response compacter
 4. Scan chain for flip-flops



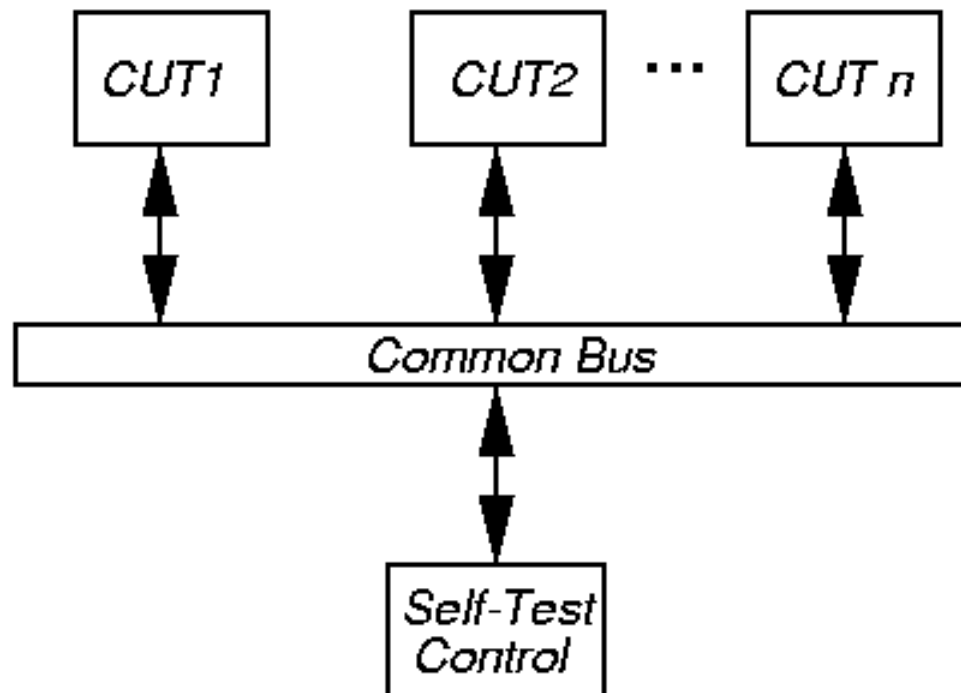
Complex BIST Architecture

- Testing session I
 - LFSR1 generates tests for CUT1 and CUT2
 - BILBO2 (LFSR3) compacts CUT1 (CUT2)
- Testing session II
 - BILBO2 generates test patterns for CUT3
 - LFSR3 compacts CUT3 response



Bus-Based BIST Architecture

- *Self-test control* broadcasts patterns to each CUT over bus – parallel pattern generation
- Awaits bus transactions showing CUT's responses to the patterns: serialized compaction



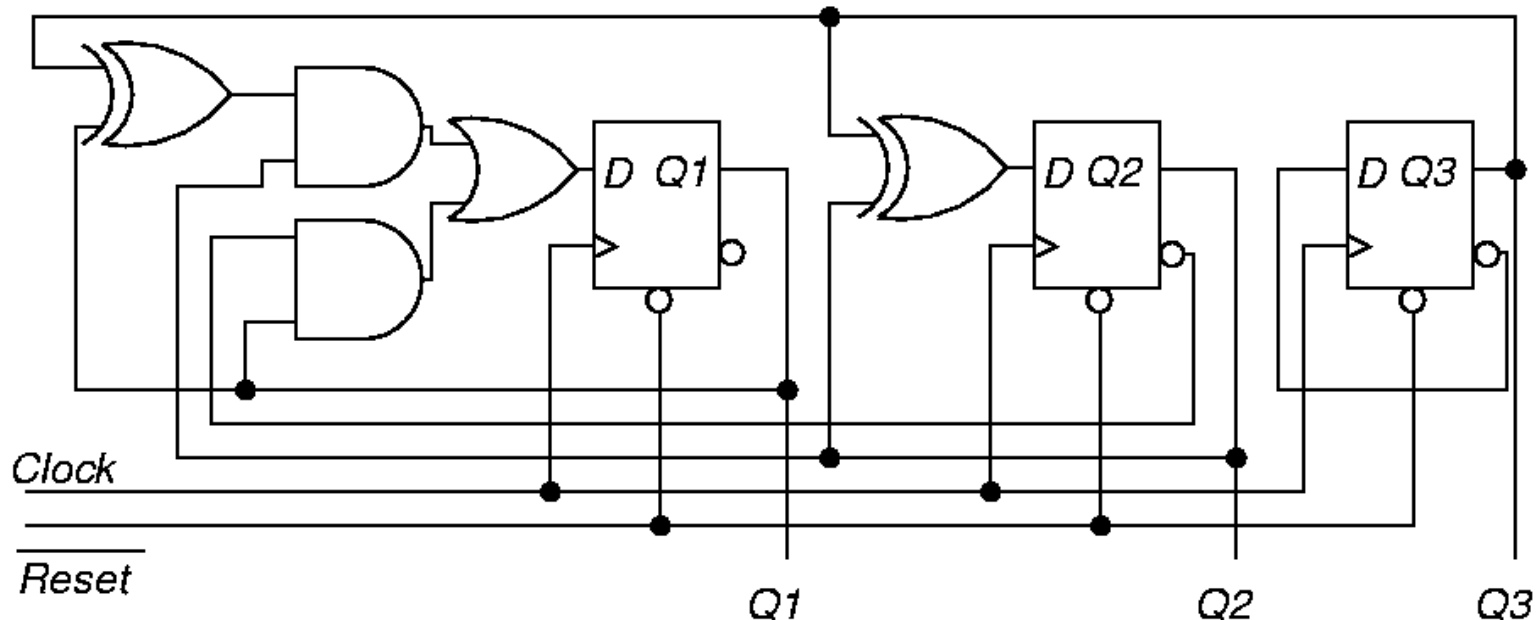
Pattern Generation in BIST

Mechanisms for Pattern Generation

- Store in ROM – too expensive
- *Exhaustive*
- *Pseudo-exhaustive*
- *Pseudo-random* (LFSR) – Preferred method
- Binary counters – use more hardware than LFSR
- Modified counters
- Test pattern *augmentation*
 - LFSR combined with a few patterns in ROM
 - *Hardware diffracter* – generates pattern cluster in neighborhood of pattern stored in ROM

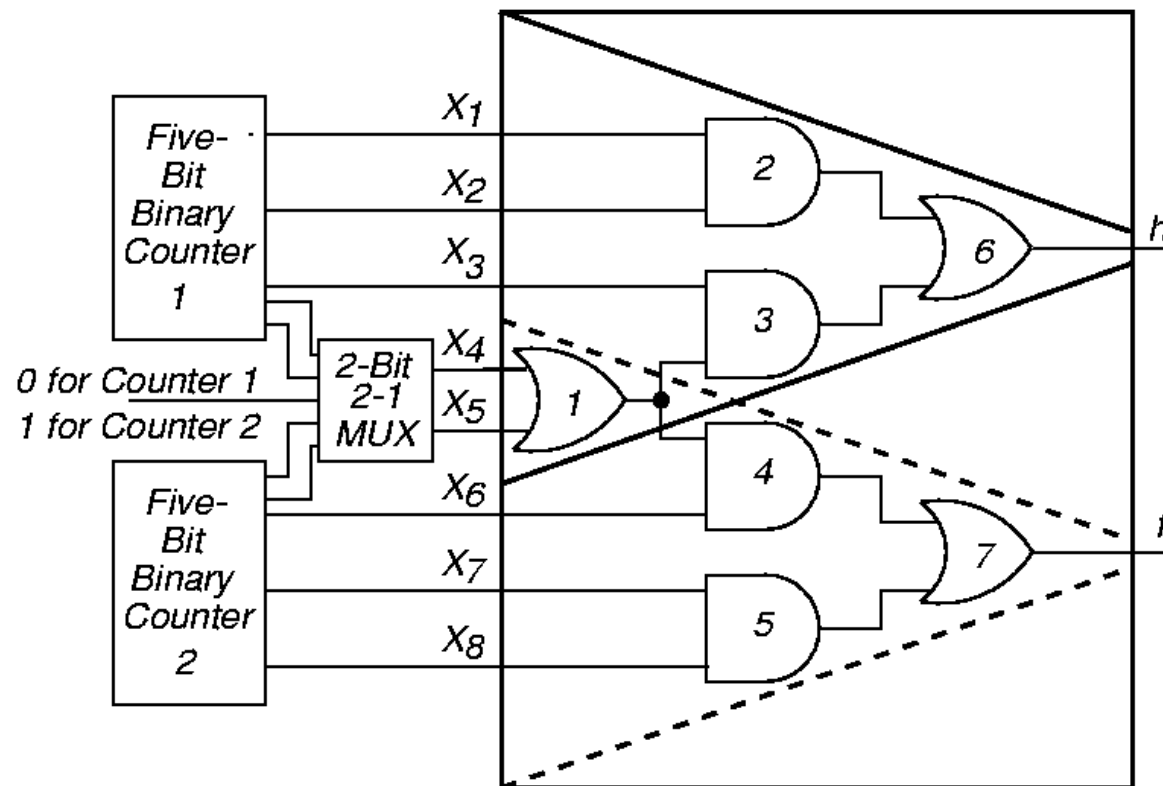
Exhaustive Pattern Generation

- Shows that every state and transition works
- For n -input circuits, requires all 2^n vectors
- An n -bit counter can do the job
- Impractical (in terms of running time) for $n > 20$



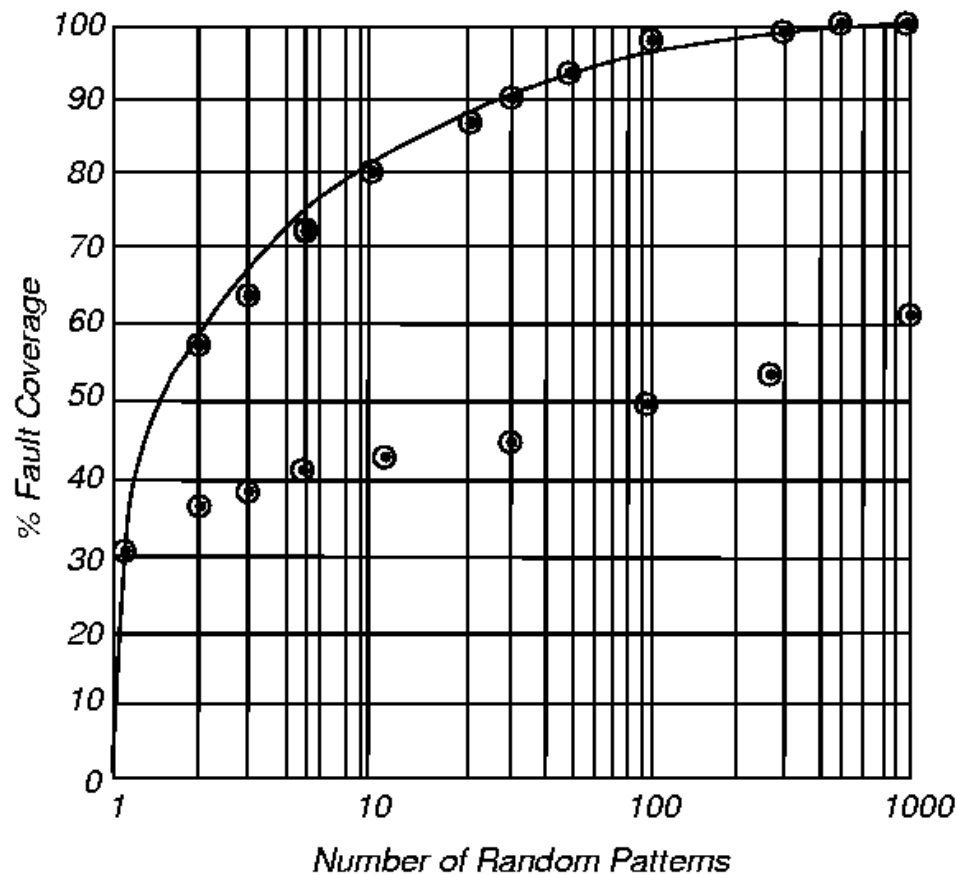
Pseudo-Exhaustive Method

- Partition large circuit into *fanin cones*
 - Backtrace from each PO to PIs influencing it
 - Test fanin cones in parallel
- Reduced # of tests from $2^8 = 256$ to $2^5 \times 2 = 64$
 - Incomplete fault coverage



Quality of Random Pattern Testing

- Some circuits may have random-pattern resistant faults. Thus, the quality of test may be low.



(a) Top curve -- random pattern testing with acceptable fault coverage.

(b) Bottom curve -- unacceptable random pattern testing.

Linear Feedback Shift Register (LFSR)

- Used to generate pseudo-random patterns
- Produces patterns algorithmically – repeatable
 - Has most of desirable random # properties
 - Near-exhaustive (maximal length) LFSR
 - Cycles through $2^n - 1$ states (excluding all-0)
- Need not cover all 2^n input combinations
- Long sequences needed for good fault coverage
- Two types of LFSR
 1. Internal XOR (modular LFSR)
 2. External XOR (standard LFSR)
- Every LFSR is described by its feedback (characteristic) polynomial represented as: $\varphi(x) = \varphi_n x^n + \dots + \varphi_1 x + \varphi_0$

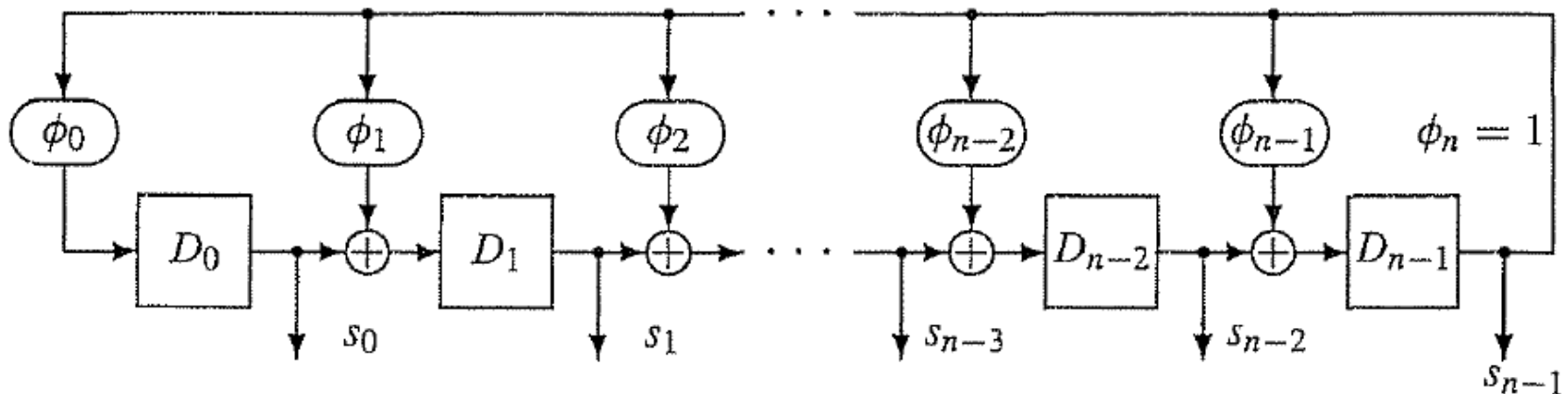
LFSR with Internal XOR

- The polynomial $\phi(x) = \phi_n x^n + \dots + \phi_1 x + \phi_0$
- The state transition relationship: $s(t+1) = A * s(t)$, where $*$ is matrix multiplication using modulo-2 arithmetic and A is an $n \times n$ matrix:

$$\begin{pmatrix} 0 & 0 & 0 & \dots & 0 & \phi_0 \\ 1 & 0 & 0 & \dots & 0 & \phi_1 \\ 0 & 1 & 0 & \dots & 0 & \phi_2 \\ 0 & 0 & 1 & \dots & 0 & \phi_3 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & \phi_{n-1} \end{pmatrix}$$

(n-1) X (n-1) Identity Matrix

Note the order of ϕ_i 's



LFSR with External XOR

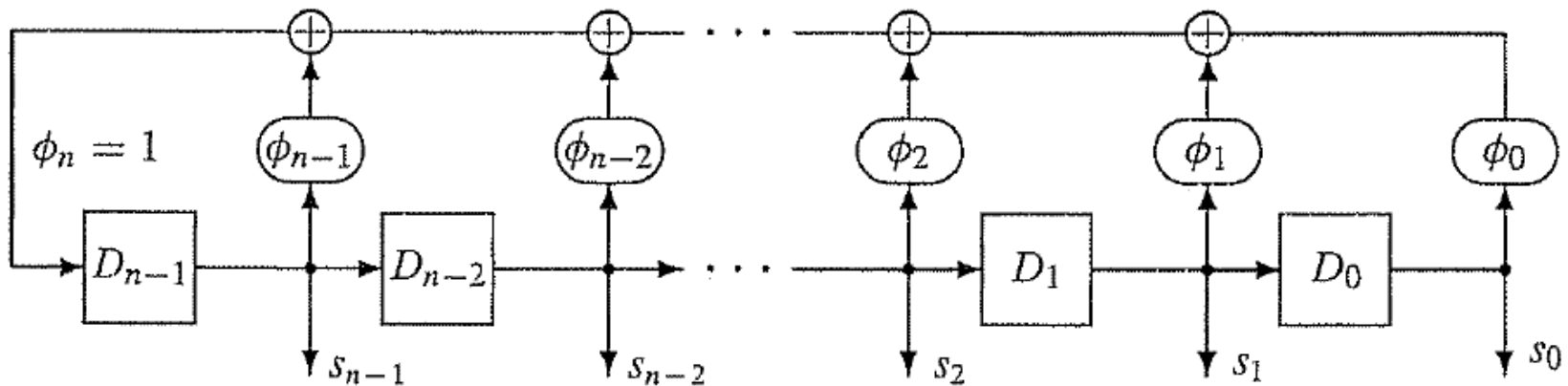
- The polynomial $\phi(x) = \phi_n x^n + \dots + \phi_1 x + \phi_0$
- The state transition relationship: $s(t+1) = A * s(t)$, where $*$ is matrix multiplication using modulo-2 arithmetic and A is an $n \times n$ matrix:

$$A = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix}$$

(n-1) X (n-1) Identity Matrix

$\phi_0 \quad \phi_1 \quad \phi_2 \quad \dots \quad \phi_{n-2} \quad \phi_{n-1}$

Note the order of ϕ_i 's



LFSR Example – Internal XOR

- The polynomial: $\phi(x) = x^4 + x + 1$

- The transition matrix/relation:

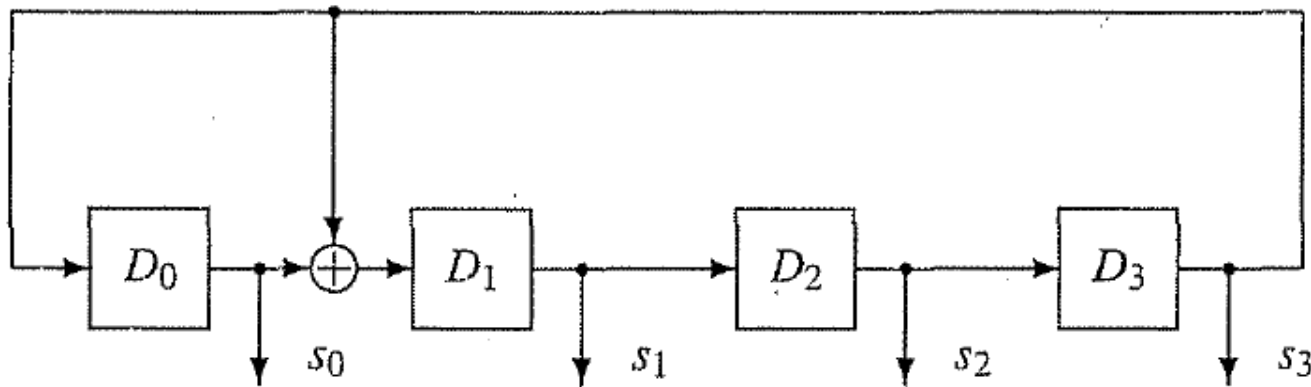
$$\begin{bmatrix} s_0(t+1) \\ s_1(t+1) \\ s_2(t+1) \\ s_3(t+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} s_0(t) \\ s_1(t) \\ s_2(t) \\ s_3(t) \end{bmatrix}$$

- The sequence:

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |

Initial Value

Cyclic Repetition



LFSR Example – External XOR

- The polynomial: $\phi(x) = x^4 + x + 1$

- The transition matrix:

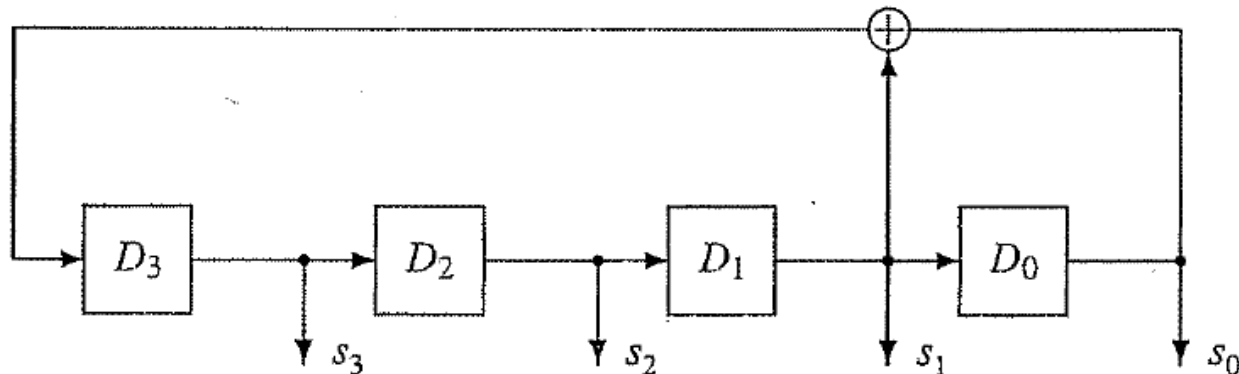
$$\begin{bmatrix} s_0(t+1) \\ s_1(t+1) \\ s_2(t+1) \\ s_3(t+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} s_0(t) \\ s_1(t) \\ s_2(t) \\ s_3(t) \end{bmatrix}$$

- The sequence:

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |

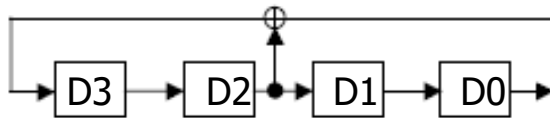
Initial Value

Cyclic Repetition

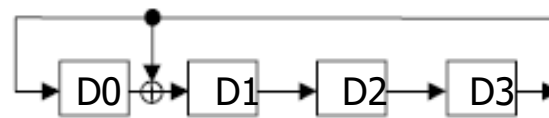


Primitive vs. Non-Primitive Polynomial

- (a) uses a **non-primitive** polynomial $\phi(x) = x^4 + x^2 + 1$
- (b) uses a **primitive** polynomial $\phi(x) = x^4 + x + 1$



(a) A 4-stage standard LFSR



(b) A 4-stage modular LFSR

0 0 0 1

1 0 0 0

0 1 0 0

1 0 1 0

0 1 0 1

0 0 1 0

0 0 0 1

1 0 0 0

0 1 0 0

1 0 1 0

0 1 0 1

0 0 1 0

0 0 0 1

1 0 0 0

0 1 0 0

1 0 1 0

0 0 0 1

1 1 0 0

0 1 1 0

0 0 1 1

1 1 0 1

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0 1 1 1

1 1 1 1

1 0 1 1

1 0 0 1

1 0 0 0

0 1 0 0

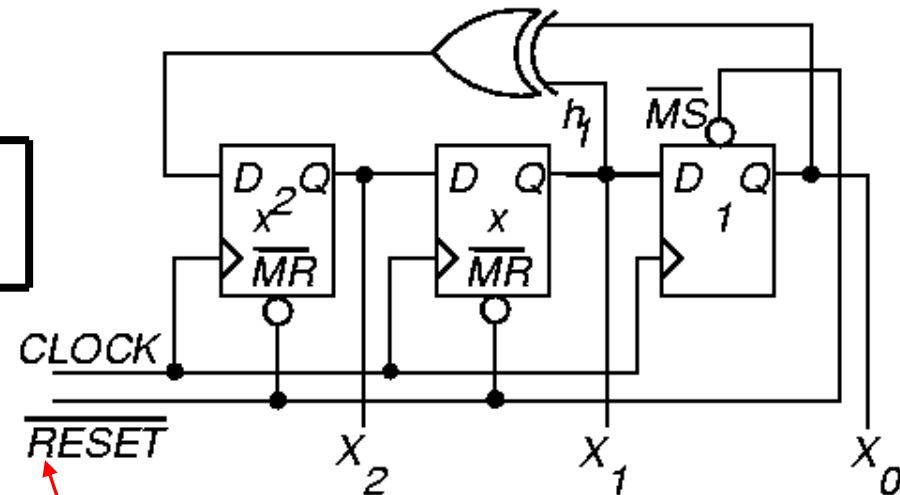
0 0 1 0

0 0 0 1

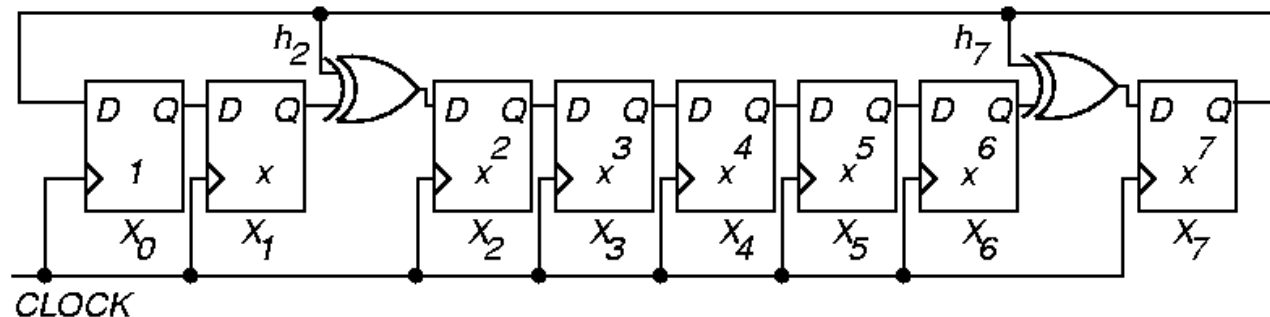
Other Examples

- External XOR with polynomial $\phi(x) = x^3 + x + 1$
(read taps from right to left)

$$\begin{bmatrix} s_0(t+1) \\ s_1(t+1) \\ s_2(t+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} s_0(t) \\ s_1(t) \\ s_2(t) \end{bmatrix}$$



- $\phi(x) = x^8 + x^7 + x^2 + 1$
(read taps from left to right)



LFSR Theory

LFSR and Field Theory

- The foundation of LFSR is a mature field theory used in many fields such as coding, encryption and testing.
- *Galois field* (mathematical system):
 - Multiplication by x same as right shift of LFSR
 - Addition operator is XOR
- Due to its linearity
 - LFSR with all-zero state will remain there forever
 - starting from a non-zero state cycles through at most $2^n - 1$ states

Primitive (Irreducible) Polynomials

- The maximum length sequence (2^n-1) can be generated only if the LFSR uses a **primitive polynomial**.
- For a given length n , the primitive polynomial is not always unique.

| Degree | Primitive polynomial |
|--------|-----------------------------|
| 2 | $x^2 + x + 1$ |
| 3 | $x^3 + x + 1$ |
| 3 | $x^3 + x^2 + 1$ |
| 4 | $x^4 + x + 1$ |
| 4 | $x^4 + x^3 + 1$ |
| 5 | $x^5 + x^2 + 1$ |
| 6 | $x^6 + x + 1$ |
| 7 | $x^7 + x + 1$ |
| 8 | $x^8 + x^4 + x^3 + x^2 + 1$ |
| 9 | $x^9 + x^4 + 1$ |
| 10 | $x^{10} + x^3 + 1$ |

Primitive Polynomials of Degree n up to 100

| <i>n</i> | Exponents | <i>n</i> | Exponents | <i>n</i> | Exponents | <i>n</i> | Exponents |
|----------|-----------|----------|-----------|----------|-----------|----------|-----------|
| 1 | 0 | 26 | 8 7 1 0 | 51 | 16 15 1 0 | 76 | 36 35 1 0 |
| 2 | 1 0 | 27 | 8 7 1 0 | 52 | 3 0 | 77 | 31 30 1 0 |
| 3 | 1 0 | 28 | 3 0 | 53 | 16 15 1 0 | 78 | 20 19 1 0 |
| 4 | 1 0 | 29 | 2 0 | 54 | 37 36 1 0 | 79 | 9 0 |
| 5 | 2 0 | 30 | 16 15 1 0 | 55 | 24 0 | 80 | 38 37 1 0 |
| 6 | 1 0 | 31 | 3 0 | 56 | 22 21 1 0 | 81 | 4 0 |
| 7 | 1 0 | 32 | 28 27 1 0 | 57 | 7 0 | 82 | 38 35 3 0 |
| 8 | 6 5 1 0 | 33 | 13 0 | 58 | 19 0 | 83 | 46 45 1 0 |
| 9 | 4 0 | 34 | 15 14 1 0 | 59 | 22 21 1 0 | 84 | 13 0 |
| 10 | 3 0 | 35 | 2 0 | 60 | 1 0 | 85 | 28 27 1 0 |
| 11 | 2 0 | 36 | 11 0 | 61 | 16 15 1 0 | 86 | 13 12 1 0 |
| 12 | 7 4 3 0 | 37 | 12 10 2 0 | 62 | 57 56 1 0 | 87 | 13 0 |
| 13 | 4 3 1 0 | 38 | 6 5 1 0 | 63 | 1 0 | 88 | 72 71 1 0 |
| 14 | 12 11 1 0 | 39 | 4 0 | 64 | 4 3 1 0 | 89 | 38 0 |
| 15 | 1 0 | 40 | 21 19 2 0 | 65 | 18 0 | 90 | 19 18 1 0 |
| 16 | 5 3 2 0 | 41 | 3 0 | 66 | 10 9 1 0 | 91 | 84 83 1 0 |
| 17 | 3 0 | 42 | 23 22 1 0 | 67 | 10 9 1 0 | 92 | 13 12 1 0 |
| 18 | 7 0 | 43 | 6 5 1 0 | 68 | 9 0 | 93 | 2 0 |
| 19 | 6 5 1 0 | 44 | 27 26 1 0 | 69 | 29 27 2 0 | 94 | 21 0 |
| 20 | 3 0 | 45 | 4 3 1 0 | 70 | 16 15 1 0 | 95 | 11 0 |
| 21 | 2 0 | 46 | 21 20 1 0 | 71 | 6 0 | 96 | 49 47 2 0 |
| 22 | 1 0 | 47 | 5 0 | 72 | 53 47 6 0 | 97 | 6 0 |
| 23 | 5 0 | 48 | 28 27 1 0 | 73 | 25 0 | 98 | 11 0 |
| 24 | 4 3 1 0 | 49 | 9 0 | 74 | 16 15 1 0 | 99 | 47 45 2 0 |
| 25 | 3 0 | 50 | 27 26 1 0 | 75 | 11 10 1 0 | 100 | 37 0 |

Note: “24 4 3 1 0” means $p(x) = x^{24} + x^4 + x^3 + x^1 + x^0$

Galois Fields

- finite fields play a key role in cryptography
- can show number of elements in a finite field **must** be a power of a prime, i.e. p^n where n is a positive integer
- known as Galois fields and denoted by $GF(p^n)$
- in particular often use these fields:
 - $n=1 \rightarrow GF(p)$
 - $p=2 \rightarrow GF(2^n)$

Polynomial Arithmetic with Modulo Coefficients

- when computing value of each coefficient do calculation modulo some value
- could be modulo any prime
- but we are most interested in $\text{mod } 2$
 - i.e. all coefficients are 0 or 1
 - Addition $\text{mod } 2$ is the same as **XOR**
 - Multiplication $\text{mod } 2$ is the same as **AND**
 - e.g. let $f(x) = x^3 + x^2$ and $g(x) = x^2 + x + 1$
 - $f(x) + g(x) = x^3 + x + 1$ (e.g. $2x^2$ does not exist because $x^2 + x^2 \rightarrow 1 + 1 \pmod{2} = 1 \oplus 1 = 0$)
 - $f(x) \times g(x) = x^5 + x^2$

Polynomial Arithmetic Over GF(2)

$$\begin{array}{r}
 x^7 \quad + x^5 + x^4 + x^3 \quad + x + 1 \\
 \quad \quad \quad + (x^3 \quad + x + 1) \\
 \hline
 x^7 \quad + x^5 + x^4
 \end{array}$$

(a) Addition

$$\begin{array}{r}
 x^7 \quad + x^5 + x^4 + x^3 \quad + x + 1 \\
 \quad \quad \quad - (x^3 \quad + x + 1) \\
 \hline
 x^7 \quad + x^5 + x^4
 \end{array}$$

(b) Subtraction

$$\begin{array}{r}
 \begin{array}{r}
 x^7 \quad + x^5 + x^4 + x^3 \quad + x + 1 \\
 \quad \quad \quad \times (x^3 \quad + x + 1) \\
 \hline
 x^7 \quad + x^5 + x^4 + x^3 \quad + x + 1
 \end{array} \\
 x^8 \quad + x^6 + x^5 + x^4 \quad + x^2 + x \\
 x^{10} \quad + x^8 + x^7 + x^6 \quad + x^4 + x^3 \\
 \hline
 x^{10} \quad \quad \quad + x^4 \quad + x^2 \quad + 1
 \end{array}$$

(c) Multiplication

$$\begin{array}{r}
 \begin{array}{r}
 x^4 + 1 \\
 \hline
 x^3 + x + 1 \overline{) x^7 \quad + x^5 + x^4 + x^3 \quad + x + 1} \\
 \underline{x^7 \quad + x^5 + x^4} \\
 x^3 \quad + x + 1 \\
 \underline{x^3 \quad + x + 1} \\
 0
 \end{array}
 \end{array}$$

(d) Division

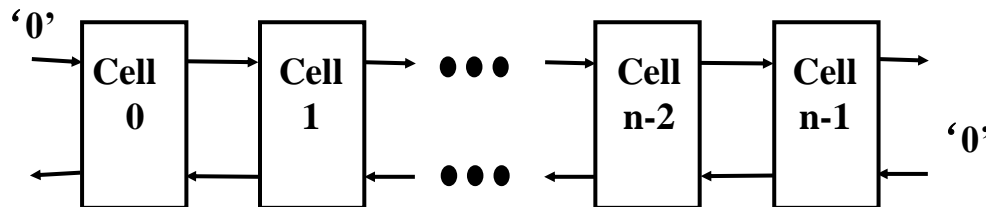
Modular Polynomial Arithmetic

- can write any polynomial in the form:
 - $f(x) = q(x)g(x) + r(x)$
 - can interpret $r(x)$ as being a remainder
 - $r(x) = f(x) \bmod g(x)$
- if have no remainder say $g(x)$ divides $f(x)$
written as $g(x)|f(x)$
- if $g(x)$ has no divisors other than itself & 1 say it is **irreducible** (called also **prime** or **primitive**) polynomial
- arithmetic modulo an irreducible polynomial forms a field

Using Cellular Automata

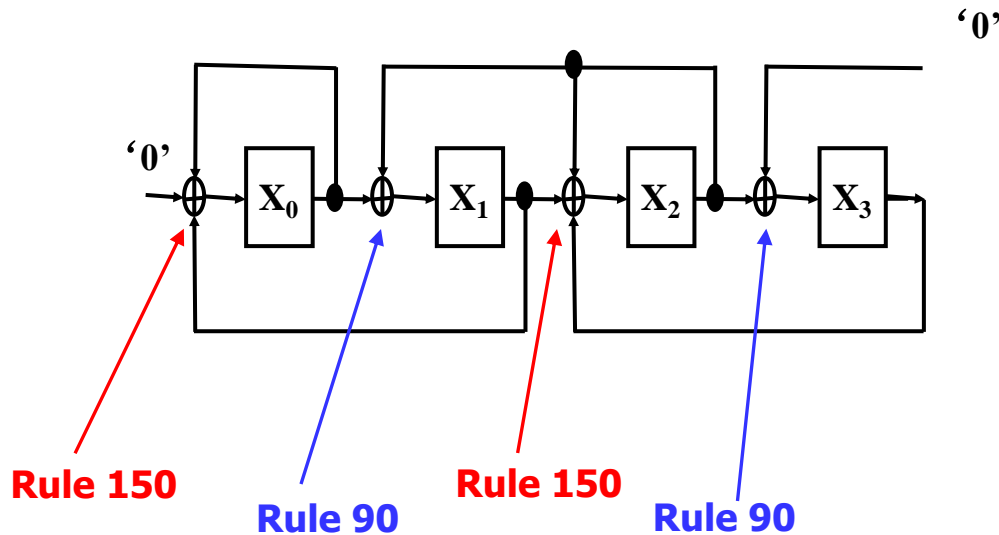
Cellular Automata (CA)

- Provide more random test patterns
- Provide high fault coverage in a random-pattern resistant (RP-resistant) circuit
- Implementation advantage
- A general structure of an n-stage CA is based on some rules
 - Rule 90: $x_i(t+1) = x_{i-1}(t) + x_{i+1}(t)$
 - Rule 150: $x_i(t+1) = x_{i-1}(t) + x_i(t) + x_{i+1}(t)$
 - Each rule determines the next state of a cell based on the state of the cell and its neighbors



Example of Cellular Automaton

A 4-stage CA



CA Rule Notation: 05 = **0101**

Test sequence

0001
0010
0111
1111
0011
0101
1000
1100
0110
1101
0100
1010
1011
1001
1110

Construction Rules for CAs of Length n up to 53

| <i>n</i> | <i>Rule *</i> | <i>n</i> | <i>Rule *</i> | <i>n</i> | <i>Rule *</i> | <i>n</i> | <i>Rule *</i> |
|----------|---------------|----------|---------------|----------|--------------------|----------|-------------------------|
| 4 | 05 | 17 | 175,763 | 30 | 7,211,545,075 | 43 | 035,342,704,132,622 |
| 5 | 31 | 18 | 252,525 | 31 | 04,625,575,630 | 44 | 074,756,556,045,302 |
| 6 | 25 | 19 | 0,646,611 | 32 | 10,602,335,725 | 45 | 151,315,510,461,515 |
| 7 | 152 | 20 | 3,635,577 | 33 | 03,047,162,605 | 46 | 0,112,312,150,547,326 |
| 8 | 325 | 21 | 3,630,173 | 34 | 036,055,030,672 | 47 | 0,713,747,124,427,015 |
| 9 | 625 | 22 | 05,252,525 | 35 | 127,573,165,123 | 48 | 0,606,762,247,217,017 |
| 10 | 0,525 | 23 | 32,716,432 | 36 | 514,443,726,043 | 49 | 02,675,443,137,056,631 |
| 11 | 3,252 | 24 | 77,226,526 | 37 | 0,226,365,530,263 | 50 | 23,233,006,150,544,226 |
| 12 | 2,525 | 25 | 136,524,744 | 38 | 0,345,366,317,023 | 51 | 04,135,241,323,505,027 |
| 13 | 14,524 | 26 | 132,642,730 | 39 | 6,427,667,463,554 | 52 | 031,067,567,742,172,706 |
| 14 | 17,576 | 27 | 037,014,415 | 40 | 00,731,257,441,345 | 53 | 207,121,011,145,676,625 |
| 15 | 44,241 | 28 | 0,525,252,525 | 41 | 15,376,413,143,607 | | |
| 16 | 152,525 | 29 | 2,512,712,103 | 42 | 11,766,345,114,746 | | |

[Hortensius 1989]

**For n=7, Rule=152=001,101,010=1,101,010, where "0" denotes a rules 90 and "1" denotes a rule 150 cell, or vice versa*