Compiling from F_i^+ to JavaScript

Yaozhu Sun

October 8, 2022

Syntax of F_i^+

Types $\qquad A, B, C ::= \top \mid \bot \mid \mathbb{B} \mid X \mid A \to B \mid \forall X * A.\ B \mid \{\ell : A\} \mid A \& B$

Type indices $\qquad T ::= \mathbb{B} \mid \overrightarrow{T} \mid T^\forall \mid \{\ell : T\} \mid T_1 \& T_2$

Expressions $\qquad e ::= \{\} \mid b \mid x \mid \mathbf{fix}\ x \colon A.\ e \mid \lambda x \colon A.\ e \colon B \mid e_1\ e_2 \mid \Lambda X * A.\ e \colon B \mid e\ A \mid \{\ell = e\} \mid e.\ell$
$\qquad\qquad\quad \mid e_1 ,,\ e_2 \mid e : A$

Values $\qquad v ::= \{\} \mid b \mid \lambda x \colon A.\ e \colon B \mid \Lambda X * A.\ e \colon B \mid \{\ell = v\} \mid v_1 ,,\ v_2$

$\boxed{|A| = T}$ $\hfill$ *(Type translation)*

$$|\mathbb{B}| = \mathbb{B} \qquad |X| = X \qquad |\forall X * A.\ B| = |B|^\forall \qquad |A \to B| = \overrightarrow{|B|} \qquad |\{\ell : A\}| = \{\ell : |A|\} \qquad |A \& B| = |A| \& |B|$$

$\boxed{\rceil A \lceil}$ $\hfill$ *(Top-like types)*

$$
\frac{}{\rceil \top \lceil}\ \text{TL-Top}
\qquad
\frac{\rceil A \lceil \quad \rceil B \lceil}{\rceil A \& B \lceil}\ \text{TL-And}
\qquad
\frac{\rceil B \lceil}{\rceil A \to B \lceil}\ \text{TL-Arrow}
\qquad
\frac{\rceil B \lceil}{\rceil \forall X * A.\ B \lceil}\ \text{TL-All}
\qquad
\frac{\rceil A \lceil}{\rceil \{\ell : A\} \lceil}\ \text{TL-Rcd}
$$

$\boxed{A^\circ}$ $\hfill$ *(Ordinary types)*

$$
\frac{}{\top^\circ}\ \text{O-Top}
\quad
\frac{}{\bot^\circ}\ \text{O-Bot}
\quad
\frac{}{\mathbb{B}^\circ}\ \text{O-Base}
\quad
\frac{}{X^\circ}\ \text{O-Var}
\quad
\frac{B^\circ}{(A \to B)^\circ}\ \text{O-Arrow}
\quad
\frac{B^\circ}{(\forall X * A.\ B)^\circ}\ \text{O-All}
\quad
\frac{A^\circ}{\{\ell : A\}^\circ}\ \text{O-Rcd}
$$

$$\boxed{\Gamma \vdash e \Leftrightarrow A \quad \leadsto J \mid z^{\pm}} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \textit{(Type-directed compilation)}$$

J-GEN
$$\frac{\Gamma \vdash e \Leftrightarrow A \quad \leadsto J \mid z^{-}}{\Gamma \vdash e \Leftrightarrow A \quad \leadsto S_0 \mid z^{+}}$$

J-TOP
$$\frac{}{\Gamma \vdash \{\} \Rightarrow \top \quad \leadsto \varnothing \mid z^{-}}$$

J-TOPABS
$$\frac{\rceil B \lceil}{\Gamma \vdash \lambda x{:}A.\ e{:}B \Rightarrow A \to B \quad \leadsto \varnothing \mid z^{-}}$$

J-TOPTABS
$$\frac{\rceil B \lceil}{\Gamma \vdash \Lambda X{*}A.\ e{:}B \Rightarrow \forall X{*}A.\ B \quad \leadsto \varnothing \mid z^{-}}$$

J-TOPRCD
$$\frac{\Gamma \vdash e \Rightarrow A \qquad \rceil A \lceil}{\Gamma \vdash \{\ell = e\} \Rightarrow \{\ell : A\} \quad \leadsto \varnothing \mid z^{-}}$$

J-BASE
$$\frac{T = |\mathbb{B}|}{\Gamma \vdash b \Rightarrow \mathbb{B} \quad \leadsto S_1 \mid z^{-}}$$

J-VAR
$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \quad \leadsto S_2 \mid z^{-}}$$

J-VARGEN
$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \quad \leadsto S_3 \mid z^{+}}$$

J-FIX
$$\frac{\Gamma, x : A \vdash e \Leftarrow A \quad \leadsto J \mid z^{-}}{\Gamma \vdash \mathbf{fix}\ x{:}A.\ e \Rightarrow A \quad \leadsto S_4 \mid z^{-}}$$

J-ABS
$$\frac{T = \overrightarrow{|B|} \qquad \Gamma, x : A \vdash e \Leftarrow B \quad \leadsto J \mid y^{+}}{\Gamma \vdash \lambda x{:}A.\ e{:}B \Rightarrow A \to B \quad \leadsto S_5 \mid z^{-}}$$

J-APP
$$\frac{\begin{array}{c}\Gamma \vdash e_1 \Rightarrow A \quad \leadsto J_1 \mid x^{+} \\ A \rhd B \to C \\ \Gamma \vdash e_2 \Leftarrow B \quad \leadsto J_2 \mid y^{+} \\ x : A \bullet y_0 \quad \leadsto J_3 \mid z\end{array}}{\Gamma \vdash e_1\ e_2 \Rightarrow C \quad \leadsto S_6 \mid z^{-}}$$

J-TABS
$$\frac{T = |B|^{\forall} \qquad \Gamma, X * A \vdash e \Leftarrow B \quad \leadsto J \mid y^{+}}{\Gamma \vdash \Lambda X{*}A.\ e{:}B \Rightarrow \forall X{*}A.\ B \quad \leadsto S_7 \mid z^{-}}$$

J-TAPP
$$\frac{\begin{array}{c}\Gamma \vdash e \Rightarrow B \quad \leadsto J_1 \mid y^{+} \\ B \rhd \forall X * C_1.\ C_2 \\ \Gamma \vdash A * C_1 \\ y : B \bullet A \quad \leadsto J_2 \mid z\end{array}}{\Gamma \vdash e\ A \Rightarrow C_2[X \mapsto A] \quad \leadsto J_1; J_2 \mid z^{-}}$$

J-RCD
$$\frac{T = \{\ell : |A|\} \qquad \Gamma \vdash e \Rightarrow A \quad \leadsto J \mid y^{+}}{\Gamma \vdash \{\ell = e\} \Rightarrow \{\ell : A\} \quad \leadsto S_8 \mid z^{-}}$$

J-PROJ
$$\frac{\begin{array}{c}\Gamma \vdash e \Rightarrow A \quad \leadsto J_1 \mid y^{+} \\ A \rhd \{\ell : B\} \\ y : A \bullet \ell \quad \leadsto J_2 \mid z\end{array}}{\Gamma \vdash e.\ell \Rightarrow B \quad \leadsto J_1; J_2 \mid z^{-}}$$

J-MERGE
$$\frac{\begin{array}{c}\Gamma \vdash e_1 \Rightarrow A \quad \leadsto J_1 \mid z^{-} \\ \Gamma \vdash e_2 \Rightarrow B \quad \leadsto J_2 \mid z^{-} \\ \Gamma \vdash A * B\end{array}}{\Gamma \vdash e_1 ,,\ e_2 \Rightarrow A \& B \quad \leadsto J_1; J_2 \mid z^{-}}$$

J-ANNO
$$\frac{\Gamma \vdash e \Leftarrow A \quad \leadsto J \mid z^{-}}{\Gamma \vdash e : A \Rightarrow A \quad \leadsto J \mid z^{-}}$$

J-ANNOGEN
$$\frac{\Gamma \vdash e \Leftarrow A \quad \leadsto J \mid z^{+}}{\Gamma \vdash e : A \Rightarrow A \quad \leadsto J \mid z^{+}}$$

J-SUB
$$\frac{\begin{array}{c}\Gamma \vdash e \Rightarrow A \quad \leadsto J_1 \mid x^{+} \\ x : A <: y : B \quad \leadsto J_2\end{array}}{\Gamma \vdash e \Leftarrow B \quad \leadsto J_1; J_2 \mid y^{-}}$$

J-SUBEQ
$$\frac{\Gamma \vdash e \Rightarrow A \quad \leadsto J \mid z^{-}}{\Gamma \vdash e \Leftarrow A \quad \leadsto J \mid z^{-}}$$

J-SUBEQGEN
$$\frac{\Gamma \vdash e \Rightarrow A \quad \leadsto J \mid z^{+}}{\Gamma \vdash e \Leftarrow A \quad \leadsto J \mid z^{+}}$$

```
/* S0 */                  /* S4 */                  };
var z = {};   J;          var x = () => z;   J;     J3;


/* S1 */                  /* S5 */                  /* S7 */
z[T] = b;                 z[T] = x => {             z[T] = X => {
                            J;   return y;              J;   return y;
/* S2 */                  };                        };
Object.assign(z, x());
                          /* S6 */                  /* S8 */
/* S3 */                  J1;                       z[T] = () => {
var z = x();              var y0 = () => {              J;   return y;
                            J2; return y;           };
```

$$\boxed{x : A \bullet \mathit{arg} \;\boxed{\leadsto\; J \mid z}} \hspace{4cm} \textit{(Distributive application)}$$

$$\frac{\text{A-Top}}{\quad \lceil A \rceil \quad}{x : A \bullet \mathit{arg} \;\boxed{\leadsto\; \varnothing \mid z}} \qquad \frac{\text{A-Arrow}}{T = \overrightarrow{|B|}}{x : A \to B \bullet y \;\boxed{\leadsto\; S_9 \mid z}} \qquad \frac{\text{A-All}}{T_1 = |B|^{\forall} \qquad T_2 = |C|}{x : \forall X * A.\, B \bullet C \;\boxed{\leadsto\; S_9' \mid z}}$$

$$\frac{\text{A-Rcd}}{T = \{\ell : |A|\}}{x : \{\ell : A\} \bullet \ell \;\boxed{\leadsto\; S_9'' \mid z}} \qquad \frac{\text{A-And} \quad x : A \bullet \mathit{arg} \;\boxed{\leadsto\; J_1 \mid z} \quad x : B \bullet \mathit{arg} \;\boxed{\leadsto\; J_2 \mid z}}{x : A \,\&\, B \bullet \mathit{arg} \;\boxed{\leadsto\; J_1; J_2 \mid z}}$$

```
/* S9 */                       /* S9' */                          /* S9'' */
Object.assign(z, x[T](y));     Object.assign(z, x[T1](T2));       Object.assign(z, x[T]());
```

$$\boxed{x : A <: y : B \;\boxed{\leadsto J}}$$ 　　　　　　　　　　　　　　　　　　　　　　　　 *(Coercive subtyping)*

S-Arrow
$$\frac{\begin{array}{cc} T_1 = \overrightarrow{|A_2|} & T_2 = \overrightarrow{|B_2|} \\ B_2{}^\circ & B_1 <: A_1 \\ x_2 : A_2 <: y_2 : B_2 \;\boxed{\leadsto J_2} \end{array}}{x : A_1 \to A_2 <: y : B_1 \to B_2 \;\boxed{\leadsto S_{11}}}$$

S-Top
$$\frac{B^\circ \qquad \lceil B \rceil}{x : A <: y : B \;\boxed{\leadsto \varnothing}}$$

S-Base
$$\frac{T = |\mathbb{B}|}{x : \mathbb{B} <: y : \mathbb{B} \;\boxed{\leadsto S_{10}}}$$

S-All
$$\frac{\begin{array}{cc} T_1 = |A_2|^\forall & T_2 = |B_2|^\forall \\ B_2{}^\circ & B_1 <: A_1 \\ x_2 : A_2 <: y_2 : B_2 \;\boxed{\leadsto J_2} \end{array}}{x : \forall X * A_1.\, A_2 <: y : \forall X * B_1.\, B_2 \;\boxed{\leadsto S_{12}}}$$

S-Rcd
$$\frac{\begin{array}{c} T_1 = \{\ell : |A|\} \\ T_2 = \{\ell : |B|\} \\ B^\circ \qquad x_0 : A <: y_0 : B \;\boxed{\leadsto J} \end{array}}{x : \{\ell : A\} <: y : \{\ell : B\} \;\boxed{\leadsto S_{13}}}$$

S-Split
$$\frac{\begin{array}{c} B_1 \lhd B \rhd B_2 \\ x : A <: y_1 : B_1 \;\boxed{\leadsto J_1} \\ x : A <: y_2 : B_2 \;\boxed{\leadsto J_2} \\ y_1 : B_1 \rhd z : B \lhd y_2 : B_2 \;\boxed{\leadsto J_3} \end{array}}{x : A <: z : B \;\boxed{\leadsto S_{14}}}$$

S-AndL
$$\frac{C^\circ \qquad x : A <: y : C \;\boxed{\leadsto J}}{x : A \,\&\, B <: y : C \;\boxed{\leadsto J}}$$

S-AndR
$$\frac{C^\circ \qquad x : B <: y : C \;\boxed{\leadsto J}}{x : A \,\&\, B <: y : C \;\boxed{\leadsto J}}$$

```
/* S10 */
y[T] = x[T];

/* S11 */
y[T2] = p => {
  var x2 = x[T1](p);
  var y2 = {};  J2;
  return y2;
};
```

```
/* S12 */
y[T2] = X => {
  var x2 = x[T1](X);
  var y2 = {};  J2;
  return y2;
};

/* S13 */
y[T2] = () => {
  var x0 = x[T1]();
```

```
  var y0 = {};  J;
  return y0;
}

/* S14 */
var y1 = {};  J1;
var y2 = {};  J2;
J3;
```

$$\boxed{x : A \ \triangleright \ z : C \ \triangleleft \ y : B \ \boxed{\rightsquigarrow \ J}}$$ *(Coercive merging)*

M-ARROW
$$T = \overrightarrow{|B|}$$
$$T_1 = \overrightarrow{|B_1|} \qquad T_2 = \overrightarrow{|B_2|}$$

M-AND
$$\frac{T_1 = |A| \qquad T_2 = |B|}{x : A \ \triangleright \ z : A \,\&\, B \ \triangleleft \ y : B \ \boxed{\rightsquigarrow \ S_{15}}}$$

$$\frac{y_1 : B_1 \ \triangleright \ y : B \ \triangleleft \ y_2 : B_2 \ \boxed{\rightsquigarrow \ J}}{x_1 : A \to B_1 \ \triangleright \ z : A \to B \ \triangleleft \ x_2 : A \to B_2 \ \boxed{\rightsquigarrow \ S_{16}}}$$

M-ALL
$$T = |B|^{\forall}$$
$$T_1 = |B_1|^{\forall} \qquad T_2 = |B_2|^{\forall}$$
$$\frac{y_1 : B_1 \ \triangleright \ y : B \ \triangleleft \ y_2 : B_2 \ \boxed{\rightsquigarrow \ J}}{x_1 : \forall X * A.\ B_1 \ \triangleright \ z : \forall X * A.\ B \ \triangleleft \ x_2 : \forall X * A.\ B_2 \ \boxed{\rightsquigarrow \ S_{17}}}$$

M-RCD
$$T = \{\ell : |A|\}$$
$$T_1 = \{\ell : |A_1|\}$$
$$T_2 = \{\ell : |A_2|\}$$
$$\frac{y_1 : A_1 \ \triangleright \ y : A \ \triangleleft \ y_2 : A_2 \ \boxed{\rightsquigarrow \ J}}{x_1 : \{\ell : A_1\} \ \triangleright \ z : \{\ell : A\} \ \triangleleft \ x_2 : \{\ell : A_2\} \ \boxed{\rightsquigarrow \ S_{18}}}$$

```
/* S15 */
Object.assign(z, x, y);

/* S16 */
z[T] = p => {
  var y1 = x1[T1](p);
  var y2 = x2[T2](p);
  var y = {};   J;
  return y;
};
```

```
/* S17 */
z[T] = X => {
  var y1 = x1[T1](X);
  var y2 = x2[T2](X);
  var y = {};   J;
  return y;
};
```

```
/* S18 */
z[T] = () => {
  var y1 = x1[T1]();
  var y2 = x2[T2]();
  var y = {};   J;
  return y;
};
```