# Compiling from $\mathsf{F}_i^+$ to JavaScript

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## Syntax of $\mathsf{F}_i^+$

| | |
|---|---|
| Types | $A, B, C ::= \top \mid \bot \mid \mathbb{B} \mid X \mid A \to B \mid \forall X{*}A.\ B \mid \{\ell : A\} \mid A \,\&\, B$ |
| Type indices | $T ::= \mathbb{B} \mid \overrightarrow{T} \mid T^{\forall} \mid \{\ell : T\} \mid T_1 \,\&\, T_2$ |
| Expressions | $e ::= \{\} \mid b \mid x \mid \mathbf{fix}\, x{:}A.\ e \mid \lambda x{:}A.\ e{:}B \mid e_1\, e_2 \mid \Lambda X{*}A.\ e{:}B \mid e\, A \mid \{\ell = e\} \mid e.\ell \mid e_1 \mathbin{,,} e_2 \mid e : A$ |
| Values | $v ::= \{\} \mid b \mid \lambda x{:}A.\ e{:}B \mid \Lambda X{*}A.\ e{:}B \mid \{\ell = v\} \mid v_1 \mathbin{,,} v_2$ |

$\boxed{|A| = T}$ *(Type translation)*

$$|\mathbb{B}| = \mathbb{B} \qquad |X| = \mathbf{atoi}(X) \qquad |\forall X{*}A.\ B| = |B|^{\forall} \qquad |A \to B| = \overrightarrow{|B|} \qquad |\{\ell : A\}| = \{\ell : |A|\}$$

$$\frac{A_{k_1} < A_{k_2} < \cdots < A_{k_m} \qquad \neg \rceil A_k \lceil}{|A_1 \& A_2 \& \cdots \& A_n| = |A_{k_1}| \& |A_{k_2}| \& \cdots \& |A_{k_m}|}$$

$\boxed{\rceil A \lceil}$ *(Top-like types)*

$$\textsc{TL-Top}\ \frac{}{\rceil \top \lceil} \qquad \textsc{TL-And}\ \frac{\rceil A \lceil \qquad \rceil B \lceil}{\rceil A \,\&\, B \lceil} \qquad \textsc{TL-Arrow}\ \frac{\rceil B \lceil}{\rceil A \to B \lceil} \qquad \textsc{TL-All}\ \frac{\rceil B \lceil}{\rceil \forall X{*}A.\ B \lceil} \qquad \textsc{TL-Rcd}\ \frac{\rceil A \lceil}{\rceil \{\ell : A\} \lceil}$$

$\boxed{A^\circ}$ *(Ordinary types)*

$$\textsc{O-Top}\ \frac{}{\top^\circ} \qquad \textsc{O-Bot}\ \frac{}{\bot^\circ} \qquad \textsc{O-Base}\ \frac{}{\mathbb{B}^\circ} \qquad \textsc{O-Var}\ \frac{}{X^\circ} \qquad \textsc{O-Arrow}\ \frac{B^\circ}{(A \to B)^\circ} \qquad \textsc{O-All}\ \frac{B^\circ}{(\forall X{*}A.\ B)^\circ} \qquad \textsc{O-Rcd}\ \frac{A^\circ}{\{\ell : A\}^\circ}$$

$\boxed{\Gamma \vdash e \Leftrightarrow A \rightsquigarrow J \mid z^{\pm}}$ *(Type-directed compilation)*

J-Gen
$$\frac{\Gamma \vdash e \Leftrightarrow A \rightsquigarrow J \mid z^-}{\Gamma \vdash e \Leftrightarrow A \rightsquigarrow S_0 \mid z^+}$$

J-Top
$$\frac{}{\Gamma \vdash \{\} \Rightarrow \top \rightsquigarrow \varnothing \mid z^-}$$

J-TopAbs
$$\frac{\rceil B \lceil}{\Gamma \vdash \lambda x\!:\!A.\ e\!:\!B \Rightarrow A \to B \rightsquigarrow \varnothing \mid z^-}$$

J-TopTAbs
$$\frac{\rceil B \lceil}{\Gamma \vdash \Lambda X * A.\ e\!:\!B \Rightarrow \forall X * A.\ B \rightsquigarrow \varnothing \mid z^-}$$

J-TopRcd
$$\frac{\Gamma \vdash e \Rightarrow A \qquad \rceil A \lceil}{\Gamma \vdash \{\ell = e\} \Rightarrow \{\ell : A\} \rightsquigarrow \varnothing \mid z^-}$$

J-Base
$$\frac{T = |\mathbb{B}|}{\Gamma \vdash b \Rightarrow \mathbb{B} \rightsquigarrow S_1 \mid z^-}$$

J-Var
$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \rightsquigarrow S_2 \mid z^-}$$

J-VarGen
$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \rightsquigarrow S_3 \mid z^+}$$

J-App
$$\frac{\begin{array}{c}\Gamma \vdash e_1 \Rightarrow A \rightsquigarrow J_1 \mid x^+ \\ A \rhd B \to C \\ \Gamma \vdash e_2 \Leftarrow B \rightsquigarrow J_2 \mid y^+ \\ x : A \bullet y_0 \rightsquigarrow J_3 \mid z\end{array}}{\Gamma \vdash e_1\ e_2 \Rightarrow C \rightsquigarrow S_6 \mid z^-}$$

J-Fix
$$\frac{\Gamma, x : A \vdash e \Leftarrow A \rightsquigarrow J \mid z^-}{\Gamma \vdash \mathbf{fix}\, x\!:\!A.\ e \Rightarrow A \rightsquigarrow S_4 \mid z^-}$$

J-Abs
$$\frac{\begin{array}{c}T = |\overrightarrow{B}| \\ \Gamma, x : A \vdash e \Leftarrow B \rightsquigarrow J \mid y^+\end{array}}{\Gamma \vdash \lambda x\!:\!A.\ e\!:\!B \Rightarrow A \to B \rightsquigarrow S_5 \mid z^-}$$

J-TApp
$$\frac{\begin{array}{c}\Gamma \vdash e \Rightarrow B \rightsquigarrow J_1 \mid y^+ \\ B \rhd \forall X * C_1.\ C_2 \\ \Gamma \vdash A * C_1 \qquad Ts = \mathbf{itoa}\,|A| \\ y : B \bullet Ts \rightsquigarrow J_2 \mid z\end{array}}{\Gamma \vdash e\ A \Rightarrow C_2[X \mapsto A] \rightsquigarrow J_1; J_2 \mid z^-}$$

J-TAbs
$$\frac{\begin{array}{c}T = |B|^{\forall} \\ \Gamma, X * A \vdash e \Leftarrow B \rightsquigarrow J \mid y^+\end{array}}{\Gamma \vdash \Lambda X * A.\ e\!:\!B \Rightarrow \forall X * A.\ B \rightsquigarrow S_7 \mid z^-}$$

J-Rcd
$$\frac{\begin{array}{c}T = \{\ell : |A|\} \\ \Gamma \vdash e \Rightarrow A \rightsquigarrow J \mid y^+\end{array}}{\Gamma \vdash \{\ell = e\} \Rightarrow \{\ell : A\} \rightsquigarrow S_8 \mid z^-}$$

J-Proj
$$\frac{\begin{array}{c}\Gamma \vdash e \Rightarrow A \rightsquigarrow J_1 \mid y^+ \\ A \rhd \{\ell : B\} \\ y : A \bullet \ell \rightsquigarrow J_2 \mid z\end{array}}{\Gamma \vdash e.\ell \Rightarrow B \rightsquigarrow J_1; J_2 \mid z^-}$$

J-Merge
$$\frac{\begin{array}{c}\Gamma \vdash e_1 \Rightarrow A \rightsquigarrow J_1 \mid z^- \\ \Gamma \vdash e_2 \Rightarrow B \rightsquigarrow J_2 \mid z^- \\ \Gamma \vdash A * B\end{array}}{\Gamma \vdash e_1\ ,,\ e_2 \Rightarrow A \,\&\, B \rightsquigarrow J_1; J_2 \mid z^-}$$

J-Anno
$$\frac{\Gamma \vdash e \Leftarrow A \rightsquigarrow J \mid z^-}{\Gamma \vdash e : A \Rightarrow A \rightsquigarrow J \mid z^-}$$

J-AnnoGen
$$\frac{\Gamma \vdash e \Leftarrow A \rightsquigarrow J \mid z^+}{\Gamma \vdash e : A \Rightarrow A \rightsquigarrow J \mid z^+}$$

J-Sub
$$\frac{\begin{array}{c}\Gamma \vdash e \Rightarrow A \rightsquigarrow J_1 \mid x^+ \\ x : A <: y : B \rightsquigarrow J_2\end{array}}{\Gamma \vdash e \Leftarrow B \rightsquigarrow J_1; J_2 \mid y^-}$$

J-SubEq
$$\frac{\Gamma \vdash e \Rightarrow A \rightsquigarrow J \mid z^-}{\Gamma \vdash e \Leftarrow A \rightsquigarrow J \mid z^-}$$

J-SubEqGen
$$\frac{\Gamma \vdash e \Rightarrow A \rightsquigarrow J \mid z^+}{\Gamma \vdash e \Leftarrow A \rightsquigarrow J \mid z^+}$$

```
/* S0 */
var z = {};  J;

/* S1 */
z[T] = b;

/* S2 */
Object.assign(z, x());

/* S3 */
var z = x();
```

```
/* S4 */
var x = () => z;  J;

/* S5 */
z[T] = x => {
  J;  return y;
};

/* S6 */
J1;
var y0 = () => {
  J2; return y;
};
```

```
};
J3;

/* S7 */
z[T] = X => {
  J;  return y;
};

/* S8 */
z[T] = () => {
  J;  return y;
};
```

$x : A \bullet \mathit{arg} \rightsquigarrow J \mid z$ *(Distributive application)*

A-Top
$$\frac{\rceil A \lceil}{x : A \bullet \mathit{arg} \rightsquigarrow \varnothing \mid z}$$

A-Arrow
$$\frac{T = |\overrightarrow{B}|}{x : A \to B \bullet y \rightsquigarrow S_9 \mid z}$$

A-All
$$\frac{T = |B|^{\forall}}{x : \forall X * A.\ B \bullet \mathit{Ts} \rightsquigarrow S_9' \mid z}$$

A-Rcd
$$\frac{T = \{\ell : |A|\}}{x : \{\ell : A\} \bullet \ell \rightsquigarrow S_9'' \mid z}$$

A-And
$$\frac{x : A \bullet \mathit{arg} \rightsquigarrow J_1 \mid z \qquad x : B \bullet \mathit{arg} \rightsquigarrow J_2 \mid z}{x : A \,\&\, B \bullet \mathit{arg} \rightsquigarrow J_1; J_2 \mid z}$$

```
/* S9 */
Object.assign(z, x[T](y));
```

```
/* S9' */
Object.assign(z, x[T](Ts));
```

```
/* S9'' */
Object.assign(z, x[T]());
```

$\boxed{x : A <: y : B \rightsquigarrow J}$ *(Coercive subtyping)*

S-Top
$$\frac{B^\circ \qquad \rceil B \lceil}{x : A <: y : B \rightsquigarrow \varnothing}$$

S-Base
$$\frac{T = |\mathbb{B}|}{x : \mathbb{B} <: y : \mathbb{B} \rightsquigarrow S_{10}}$$

S-Var
$$\frac{}{x : X <: y : X \rightsquigarrow S'_{10}}$$

S-Arrow
$$\frac{\begin{array}{c} T_1 = \overrightarrow{|A_2|} \\ T_2 = \overrightarrow{|B_2|} \qquad B_2{}^\circ \\ x_1 : B_1 <: y_1 : A_1 \rightsquigarrow J_1 \\ x_2 : A_2 <: y_2 : B_2 \rightsquigarrow J_2 \end{array}}{x : A_1 \to A_2 <: y : B_1 \to B_2 \rightsquigarrow S_{11}}$$

S-All
$$\frac{\begin{array}{c} T_1 = |A_2|^\forall \qquad T_2 = |B_2|^\forall \\ B_2{}^\circ \qquad B_1 <: A_1 \\ x_0 : A_2 <: y_0 : B_2 \rightsquigarrow J \end{array}}{x : \forall X * A_1.\ A_2 <: y : \forall X * B_1.\ B_2 \rightsquigarrow S_{12}}$$

S-Rcd
$$\frac{\begin{array}{c} T_1 = \{\ell : |A|\} \\ T_2 = \{\ell : |B|\} \\ B^\circ \qquad x_0 : A <: y_0 : B \rightsquigarrow J \end{array}}{x : \{\ell : A\} <: y : \{\ell : B\} \rightsquigarrow S_{13}}$$

S-AndL
$$\frac{C^\circ \qquad x : A <: y : C \rightsquigarrow J}{x : A \,\&\, B <: y : C \rightsquigarrow J}$$

S-AndR
$$\frac{C^\circ \qquad x : B <: y : C \rightsquigarrow J}{x : A \,\&\, B <: y : C \rightsquigarrow J}$$

S-Split
$$\frac{\begin{array}{c} B_1 \lhd B \rhd B_2 \\ x : A <: y_1 : B_1 \rightsquigarrow J_1 \\ x : A <: y_2 : B_2 \rightsquigarrow J_2 \\ y_1 : B_1 \rhd z : B \lhd y_2 : B_2 \rightsquigarrow J_3 \end{array}}{x : A <: z : B \rightsquigarrow S_{14}}$$

```
/* S10 */
y[T] = x[T];

/* S10' */
for (var T of X) {
  y[T] = x[T];
}

/* S11 */
y[T2] = p => {
  var x2 = x[T1](() => {
    var x1 = p();
    var y1 = {};  J1;
    return y1;
  });
  var y2 = {};  J2;
  return y2;
};

/* S12 */
y[T2] = X => {
  var x0 = x[T1](X);
  var y0 = {};  J;
  return y2;
};

/* S13 */
y[T2] = () => {
  var x0 = x[T1]();
  var y0 = {};  J;
  return y0;
}

/* S14 */
var y1 = {};  J1;
var y2 = {};  J2;
J3;
```

$$\boxed{x : A \ \triangleright \ z : C \ \triangleleft \ y : B \rightsquigarrow J}$$ *(Coercive merging)*

M-And

$$\frac{}{x : A \ \triangleright \ z : A \,\&\, B \ \triangleleft \ y : B \rightsquigarrow S_{15}}$$

M-Arrow

$$\frac{T = |\overrightarrow{B}| \quad T_1 = |\overrightarrow{B_1}| \quad T_2 = |\overrightarrow{B_2}| \quad y_1 : B_1 \ \triangleright \ y : B \ \triangleleft \ y_2 : B_2 \rightsquigarrow J}{x_1 : A \to B_1 \ \triangleright \ z : A \to B \ \triangleleft \ x_2 : A \to B_2 \rightsquigarrow S_{16}}$$

M-All

$$\frac{T = |B|^{\forall} \quad T_1 = |B_1|^{\forall} \quad T_2 = |B_2|^{\forall} \quad y_1 : B_1 \ \triangleright \ y : B \ \triangleleft \ y_2 : B_2 \rightsquigarrow J}{x_1 : \forall X * A.\ B_1 \ \triangleright \ z : \forall X * A.\ B \ \triangleleft \ x_2 : \forall X * A.\ B_2 \rightsquigarrow S_{17}}$$

M-Rcd

$$\frac{T = \{\ell : |A|\} \quad T_1 = \{\ell : |A_1|\} \quad T_2 = \{\ell : |A_2|\} \quad y_1 : A_1 \ \triangleright \ y : A \ \triangleleft \ y_2 : A_2 \rightsquigarrow J}{x_1 : \{\ell : A_1\} \ \triangleright \ z : \{\ell : A\} \ \triangleleft \ x_2 : \{\ell : A_2\} \rightsquigarrow S_{18}}$$

```
/* S15 */
Object.assign(z, x, y);
```

```
/* S16 */
z[T] = p => {
  var y1 = x1[T1](p);
  var y2 = x2[T2](p);
  var y = {};  J;
  return y;
};
```

```
/* S17 */
z[T] = X => {
  var y1 = x1[T1](X);
  var y2 = x2[T2](X);
  var y = {};  J;
  return y;
};
```

```
/* S18 */
z[T] = () => {
  var y1 = x1[T1]();
  var y2 = x2[T2]();
  var y = {};  J;
  return y;
};
```