Compiling from F_i^+ to JavaScript

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Syntax of F_i^+

Types	$A, B, C ::= \top \mid \bot \mid \mathbb{B} \mid X \mid A \to B \mid \forall X * A.\ B \mid \{\ell : A\} \mid A \,\&\, B$
Type indices	$\tau ::= \mathbb{B} \mid \overrightarrow{\tau} \mid \tau^{\forall} \mid \{\ell : \tau\} \mid \tau_1 \,\&\, \tau_2$
Expressions	$e ::= \{\} \mid b \mid x \mid \mathbf{fix}\ x\!:\!A.\ e \mid \lambda x\!:\!A.\ e\!:\!B \mid e_1\ e_2 \mid \Lambda X * A.\ e\!:\!B \mid e\ A \mid \{\ell = e\} \mid e.\ell$
	$\mid e_1 \,,, e_2 \mid e : A$
Values	$v ::= \{\} \mid b \mid \lambda x\!:\!A.\ e\!:\!B \mid \Lambda X * A.\ e\!:\!B \mid \{\ell = v\} \mid v_1 \,,, v_2$

$\boxed{|A| = \tau}$ *(Type translation)*

$$|\mathbb{B}| = \mathbb{B} \qquad |X| = X \qquad |\forall X * A.\ B| = |B|^{\forall} \qquad |A \to B| = \overrightarrow{|B|} \qquad |\{\ell : A\}| = \{\ell : |A|\} \qquad |A \,\&\, B| = |A| \,\&\, |B|$$

$\boxed{\rceil A \lceil}$ *(Top-like types)*

$$\frac{}{\rceil \top \lceil}\ \text{TL-Top} \qquad \frac{\rceil A \lceil \quad \rceil B \lceil}{\rceil A \,\&\, B \lceil}\ \text{TL-And} \qquad \frac{\rceil B \lceil}{\rceil A \to B \lceil}\ \text{TL-Arrow} \qquad \frac{\rceil B \lceil}{\rceil \forall X * A.\ B \lceil}\ \text{TL-All} \qquad \frac{\rceil A \lceil}{\rceil \{\ell : A\} \lceil}\ \text{TL-Rcd}$$

$\boxed{A^{\circ}}$ *(Ordinary types)*

$$\frac{}{\top^{\circ}}\ \text{O-Top} \qquad \frac{}{\bot^{\circ}}\ \text{O-Bot} \qquad \frac{}{\mathbb{B}^{\circ}}\ \text{O-Base} \qquad \frac{}{X^{\circ}}\ \text{O-Var} \qquad \frac{B^{\circ}}{(A \to B)^{\circ}}\ \text{O-Arrow} \qquad \frac{B^{\circ}}{(\forall X * A.\ B)^{\circ}}\ \text{O-All} \qquad \frac{A^{\circ}}{\{\ell : A\}^{\circ}}\ \text{O-Rcd}$$

$$\boxed{\Gamma \vdash e \Leftrightarrow A \;\boxed{\rightsquigarrow z \textbf{ in } J}} \hspace{4cm} \textit{(Type-directed compilation)}$$

J-Top
$$\frac{}{\Gamma \vdash \{\} \Rightarrow \top \;\boxed{\rightsquigarrow z \textbf{ in } \varnothing}}$$

J-TopAbs
$$\frac{\lceil B \rceil}{\Gamma \vdash \lambda x{:}A.\, e{:}B \Rightarrow A \to B \;\boxed{\rightsquigarrow z \textbf{ in } \varnothing}}$$

J-TopTAbs
$$\frac{\lceil B \rceil}{\Gamma \vdash \Lambda X{*}A.\, e{:}B \Rightarrow \forall X{*}A.\, B \;\boxed{\rightsquigarrow z \textbf{ in } \varnothing}}$$

J-TopRcd
$$\frac{\Gamma \vdash e \Rightarrow A \qquad \lceil A \rceil}{\Gamma \vdash \{\ell = e\} \Rightarrow \{\ell : A\} \;\boxed{\rightsquigarrow z \textbf{ in } \varnothing}}$$

J-Base
$$\frac{\tau = |\mathbb{B}|}{\Gamma \vdash b \Rightarrow \mathbb{B} \;\boxed{\rightsquigarrow z \textbf{ in } S_1}}$$

J-Var
$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \;\boxed{\rightsquigarrow z \textbf{ in } S_2}}$$

J-Fix
$$\frac{\Gamma, x : A \vdash e \Leftarrow A \;\boxed{\rightsquigarrow x \textbf{ in } J}}{\Gamma \vdash \textbf{fix}\, x{:}A.\, e \Rightarrow A \;\boxed{\rightsquigarrow z \textbf{ in } S_3}}$$

J-Abs
$$\frac{\tau = \overrightarrow{|B|} \qquad \Gamma, x : A \vdash e \Leftarrow B \;\boxed{\rightsquigarrow y \textbf{ in } J}}{\Gamma \vdash \lambda x{:}A.\, e{:}B \Rightarrow A \to B \;\boxed{\rightsquigarrow z \textbf{ in } S_4}}$$

J-App
$$\frac{\Gamma \vdash e_1 \Rightarrow A \;\boxed{\rightsquigarrow x \textbf{ in } J_1} \quad A \rhd B \to C \quad \Gamma \vdash e_2 \Leftarrow B \;\boxed{\rightsquigarrow y \textbf{ in } J_2} \quad \boxed{A \;\rhd\; x \bullet y \;\hookrightarrow\; z \textbf{ with } J_3}}{\Gamma \vdash e_1\, e_2 \Rightarrow C \;\boxed{\rightsquigarrow z \textbf{ in } S_5}}$$

J-TAbs
$$\frac{\tau = |B|^{\forall} \qquad \Gamma, X{*}A \vdash e \Leftarrow B \;\boxed{\rightsquigarrow y \textbf{ in } J}}{\Gamma \vdash \Lambda X{*}A.\, e{:}B \Rightarrow \forall X{*}A.\, B \;\boxed{\rightsquigarrow z \textbf{ in } S_6}}$$

J-TApp
$$\frac{\Gamma \vdash e \Rightarrow B \;\boxed{\rightsquigarrow y \textbf{ in } J_1} \quad B \rhd \forall X{*}C_1.\, C_2 \quad \Gamma \vdash A * C_1 \quad \boxed{B \;\rhd\; y \bullet A \;\hookrightarrow\; z \textbf{ with } J_2}}{\Gamma \vdash e\, A \Rightarrow C_2[X \mapsto A] \;\boxed{\rightsquigarrow z \textbf{ in } S_7}}$$

J-Rcd
$$\frac{\tau = \{\ell : |A|\} \qquad \Gamma \vdash e \Rightarrow A \;\boxed{\rightsquigarrow y \textbf{ in } J}}{\Gamma \vdash \{\ell = e\} \Rightarrow \{\ell : A\} \;\boxed{\rightsquigarrow z \textbf{ in } S_8}}$$

J-Proj
$$\frac{\Gamma \vdash e \Rightarrow A \;\boxed{\rightsquigarrow y \textbf{ in } J_1} \quad A \rhd \{\ell : B\} \quad \boxed{A \;\rhd\; y \bullet \ell \;\hookrightarrow\; z \textbf{ with } J_2}}{\Gamma \vdash e.\ell \Rightarrow B \;\boxed{\rightsquigarrow z \textbf{ in } S_9}}$$

J-Merge
$$\frac{\Gamma \vdash e_1 \Rightarrow A \;\boxed{\rightsquigarrow z \textbf{ in } J_1} \quad \Gamma \vdash e_2 \Rightarrow B \;\boxed{\rightsquigarrow z \textbf{ in } J_2} \quad \Gamma \vdash A * B}{\Gamma \vdash e_1 \,,, e_2 \Rightarrow A \,\&\, B \;\boxed{\rightsquigarrow z \textbf{ in } S_{10}}}$$

J-Anno
$$\frac{\Gamma \vdash e \Leftarrow A \;\boxed{\rightsquigarrow z \textbf{ in } J}}{\Gamma \vdash e : A \Rightarrow A \;\boxed{\rightsquigarrow z \textbf{ in } J}}$$

J-Sub
$$\frac{\Gamma \vdash e \Rightarrow A \;\boxed{\rightsquigarrow x \textbf{ in } J_1} \quad A <: B \;\boxed{\rightsquigarrow x \mapsto y \textbf{ with } J_2}}{\Gamma \vdash e \Leftarrow B \;\boxed{\rightsquigarrow y \textbf{ in } S_{11}}}$$

```
/* S1 */
z[t] = b;

/* S2 */
Object.assign(z, x);
```

```
/* S3 */
var f = x => { J };
Object.assign(z,
  new function() {
    f(this)
  }
);
```

```
/* S4 */
z[t] = x => {
  var y = {};
  J;
  return y;
};
```

```
/* S5 */
var x = {};
J1;
var y = {};
J2;
J3;

/* S6 */
z[t] = X => {
  var y = {};
  J;
  return y;
};
```

```
/* S7 */
var y = {};
J1;
J2;

/* S8 */
z[t] = () => {
  var y = {};
  J;
  return y;
};
```

```
/* S9 */
var y = {};
J1;
J2;

/* S10 */
J1;
J2;

/* S11 */
var x = {};
J1;
J2;
```

$$\boxed{A \;\triangleright\; x \bullet arg \;\hookrightarrow\; z \textbf{ with } J}$$
(Distributive application)

A-Top
$$\frac{\rceil A\lceil}{A \;\triangleright\; x \bullet arg \;\hookrightarrow\; z \textbf{ with } \varnothing}$$

A-Arrow
$$\frac{\tau = \overrightarrow{|B|}}{A \to B \;\triangleright\; x \bullet y \;\hookrightarrow\; z \textbf{ with } S_{12}}$$

A-All
$$\frac{\tau_1 = |B|^{\forall} \qquad \tau_2 = |C|}{\forall X * A.\ B \;\triangleright\; x \bullet C \;\hookrightarrow\; z \textbf{ with } S_{13}}$$

A-Rcd
$$\frac{\tau = \{\ell : |A|\}}{\{\ell : A\} \;\triangleright\; x \bullet \ell \;\hookrightarrow\; z \textbf{ with } S_{14}}$$

A-And
$$\frac{\begin{array}{c} A \;\triangleright\; x \bullet arg \;\hookrightarrow\; z \textbf{ with } J_1 \\ B \;\triangleright\; x \bullet arg \;\hookrightarrow\; z \textbf{ with } J_2 \end{array}}{A \,\&\, B \;\triangleright\; x \bullet arg \;\hookrightarrow\; z \textbf{ with } S_{15}}$$

```
/* S12 */
Object.assign(z, x[t](y));

/* S13 */
```

```
Object.assign(z, x[t1](t2));

/* S14 */
Object.assign(z, x[t]());
```

```
/* S15 */
J1;
J2;
```

$A <: B \;\boxed{\rightsquigarrow x \mapsto y \textbf{ with } J}$ *(Coercive subtyping)*

S-Top
$$\frac{B^\circ \qquad \rceil B \lceil}{A <: B \;\boxed{\rightsquigarrow x \mapsto y \textbf{ with } \varnothing}}$$

S-Base
$$\frac{\tau = |\mathbb{B}|}{\mathbb{B} <: \mathbb{B} \;\boxed{\rightsquigarrow x \mapsto y \textbf{ with } S_{16}}}$$

S-Arrow
$$\frac{\tau_1 = \overrightarrow{|A_2|} \qquad \tau_2 = \overrightarrow{|B_2|} \qquad B_2^\circ \qquad B_1 <: A_1 \qquad A_2 <: B_2 \;\boxed{\rightsquigarrow x_2 \mapsto y_2 \textbf{ with } J_2}}{A_1 \to A_2 <: B_1 \to B_2 \;\boxed{\rightsquigarrow x \mapsto y \textbf{ with } S_{17}}}$$

S-All
$$\frac{\tau_1 = |A_2|^\forall \qquad \tau_2 = |B_2|^\forall \qquad B_2^\circ \qquad B_1 <: A_1 \qquad A_2 <: B_2 \;\boxed{\rightsquigarrow x_2 \mapsto y_2 \textbf{ with } J_2}}{\forall X * A_1.\, A_2 <: \forall X * B_1.\, B_2 \;\boxed{\rightsquigarrow x \mapsto y \textbf{ with } S_{18}}}$$

S-Rcd
$$\frac{\tau_1 = \{\ell : |A|\} \qquad \tau_2 = \{\ell : |B|\} \qquad B^\circ \qquad A <: B \;\boxed{\rightsquigarrow x_0 \mapsto y_0 \textbf{ with } J}}{\{\ell : A\} <: \{\ell : B\} \;\boxed{\rightsquigarrow x \mapsto y \textbf{ with } S_{19}}}$$

S-AndL
$$\frac{C^\circ \qquad A <: C \;\boxed{\rightsquigarrow x \mapsto y \textbf{ with } J}}{A \,\&\, B <: C \;\boxed{\rightsquigarrow x \mapsto y \textbf{ with } J}}$$

S-AndR
$$\frac{C^\circ \qquad B <: C \;\boxed{\rightsquigarrow x \mapsto y \textbf{ with } J}}{A \,\&\, B <: C \;\boxed{\rightsquigarrow x \mapsto y \textbf{ with } J}}$$

S-Split
$$\frac{B_1 \triangleleft B \triangleright B_2 \qquad A <: B_1 \;\boxed{\rightsquigarrow x \mapsto y_1 \textbf{ with } J_1} \qquad A <: B_2 \;\boxed{\rightsquigarrow x \mapsto y_2 \textbf{ with } J_2} \qquad \boxed{y_1 : B_1 \;\triangleright\; z : B \;\triangleleft\; y_2 : B_2 \textbf{ with } J_3}}{A <: B \;\boxed{\rightsquigarrow x \mapsto z \textbf{ with } S_{20}}}$$

```
/* S16 */
y[t] = x[t];

/* S17 */
y[t2] = p => {
  var x2 = x[t1](p);
  var y2 = {};
  J2;
  return y2;
};
```

```
/* S18 */
y[t2] = X => {
  var x2 = x[t1](X);
  var y2 = {};
  J2;
  return y2;
};

/* S19 */
y[t2] = () => {
  var x0 = x[t1]();
```

```
  var y0 = {};
  J;
  return y0;
}

/* S20 */
var y1 = {};
J1;
var y2 = {};
J2;
J3;
```

$$\boxed{x : A \;\triangleright\; z : C \;\triangleleft\; y : B \;\textbf{with}\; J}$$ 　　　　　　　　　　　　　　　　　　　　　　　　　　　 *(Coercive merging)*

M-Arrow

$$\dfrac{\begin{array}{c} \tau = \overrightarrow{|B|} \\ \tau_1 = \overrightarrow{|B_1|} \qquad \tau_2 = \overrightarrow{|B_2|} \\ y_1 : B_1 \;\triangleright\; y : B \;\triangleleft\; y_2 : B_2 \;\textbf{with}\; J \end{array}}{x_1 : A \to B_1 \;\triangleright\; z : A \to B \;\triangleleft\; x_2 : A \to B_2 \;\textbf{with}\; S_{22}}$$

M-And

$$\dfrac{\tau_1 = |A| \qquad \tau_2 = |B|}{x : A \;\triangleright\; z : A\,\&\,B \;\triangleleft\; y : B \;\textbf{with}\; S_{21}}$$

M-All

$$\dfrac{\begin{array}{c} \tau = |B|^{\forall} \\ \tau_1 = |B_1|^{\forall} \qquad \tau_2 = |B_2|^{\forall} \\ y_1 : B_1 \;\triangleright\; y : B \;\triangleleft\; y_2 : B_2 \;\textbf{with}\; J \end{array}}{x_1 : \forall X*A.\; B_1 \;\triangleright\; z : \forall X*A.\; B \;\triangleleft\; x_2 : \forall X*A.\; B_2 \;\textbf{with}\; S_{23}}$$

M-Rcd

$$\dfrac{\begin{array}{c} \tau = \{\ell : |A|\} \\ \tau_1 = \{\ell : |A_1|\} \\ \tau_2 = \{\ell : |A_2|\} \\ y_1 : A_1 \;\triangleright\; y : A \;\triangleleft\; y_2 : A_2 \;\textbf{with}\; J \end{array}}{x_1 : \{\ell : A_1\} \;\triangleright\; z : \{\ell : A\} \;\triangleleft\; x_2 : \{\ell : A_2\} \;\textbf{with}\; S_{24}}$$

```
/* S21 */
Object.assign(z, x, y);

/* S22 */
z[t] = p => {
  var y1 = x1[t1](p);
  var y2 = x2[t2](p);
  var y = {};
  J;
  return y;
};
```

```
/* S23 */
z[t] = X => {
  var y1 = x1[t1](X);
  var y2 = x2[t2](X);
  var y = {};
  J;
  return y;
};
```

```
/* S24 */
var y1 = x1[t1]();
var y2 = x2[t2]();
var y = {};
J;
z[t] = y;
```