Compiling from F_i^+ to JavaScript

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September 7, 2022

Syntax of F_i^+

$$
\begin{array}{llrl}
\text{Types} & A, B, C & ::= & \top \mid \bot \mid \mathbb{B} \mid X \mid A \to B \mid \forall X {*} A.\ B \mid \{\ell : A\} \mid A \,\&\, B \\
\text{Type indices} & T & ::= & \mathbb{B} \mid \overrightarrow{T} \mid T^{\forall} \mid \{\ell : T\} \mid T_1 \,\&\, T_2 \\
\text{Expressions} & e & ::= & \{\} \mid b \mid x \mid \textbf{fix}\, x : A.\ e \mid \lambda x : A.\ e : B \mid e_1\ e_2 \mid \Lambda X {*} A.\ e : B \mid e\ A \mid \{\ell = e\} \mid e.\ell \\
 & & & \mid e_1 \,,,\, e_2 \mid e : A \\
\text{Values} & v & ::= & \{\} \mid b \mid \lambda x : A.\ e : B \mid \Lambda X {*} A.\ e : B \mid \{\ell = v\} \mid v_1 \,,,\, v_2
\end{array}
$$

$\boxed{|A| = T}$ *(Type translation)*

$$
|\mathbb{B}| = \mathbb{B} \qquad |X| = X \qquad |\forall X {*} A.\ B| = |B|^{\forall} \qquad |A \to B| = \overrightarrow{|B|} \qquad |\{\ell : A\}| = \{\ell : |A|\} \qquad |A \,\&\, B| = |A| \,\&\, |B|
$$

$\boxed{\rceil A \lceil}$ *(Top-like types)*

$$
\textsc{TL-Top}\ \frac{}{\rceil \top \lceil} \qquad
\textsc{TL-And}\ \frac{\rceil A \lceil \quad \rceil B \lceil}{\rceil A \,\&\, B \lceil} \qquad
\textsc{TL-Arrow}\ \frac{\rceil B \lceil}{\rceil A \to B \lceil} \qquad
\textsc{TL-All}\ \frac{\rceil B \lceil}{\rceil \forall X {*} A.\ B \lceil} \qquad
\textsc{TL-Rcd}\ \frac{\rceil A \lceil}{\rceil \{\ell : A\} \lceil}
$$

$\boxed{A^{\circ}}$ *(Ordinary types)*

$$
\textsc{O-Top}\ \frac{}{\top^{\circ}} \qquad
\textsc{O-Bot}\ \frac{}{\bot^{\circ}} \qquad
\textsc{O-Base}\ \frac{}{\mathbb{B}^{\circ}} \qquad
\textsc{O-Var}\ \frac{}{X^{\circ}} \qquad
\textsc{O-Arrow}\ \frac{B^{\circ}}{(A \to B)^{\circ}} \qquad
\textsc{O-All}\ \frac{B^{\circ}}{(\forall X {*} A.\ B)^{\circ}} \qquad
\textsc{O-Rcd}\ \frac{A^{\circ}}{\{\ell : A\}^{\circ}}
$$

$\boxed{\Gamma \vdash e \Leftrightarrow A \rightsquigarrow J \mid z^{\pm}}$ *(Type-directed compilation)*

$$\frac{\Gamma \vdash e \Leftrightarrow A \rightsquigarrow J \mid z^{-}}{\Gamma \vdash e \Leftrightarrow A \rightsquigarrow S_0 \mid z^{+}}\ \textsc{J-Gen}$$

$$\frac{}{\Gamma \vdash \{\} \Rightarrow \top \rightsquigarrow \varnothing \mid z^{-}}\ \textsc{J-Top}$$

$$\frac{\rceil B \lceil}{\Gamma \vdash \lambda x\!:\!A.\ e\!:\!B \Rightarrow A \to B \rightsquigarrow \varnothing \mid z^{-}}\ \textsc{J-TopAbs}$$

$$\frac{\rceil B \lceil}{\Gamma \vdash \Lambda X \ast A.\ e\!:\!B \Rightarrow \forall X \ast A.\ B \rightsquigarrow \varnothing \mid z^{-}}\ \textsc{J-TopTAbs}$$

$$\frac{\Gamma \vdash e \Rightarrow A \qquad \rceil A \lceil}{\Gamma \vdash \{\ell = e\} \Rightarrow \{\ell : A\} \rightsquigarrow \varnothing \mid z^{-}}\ \textsc{J-TopRcd}$$

$$\frac{T = |\mathbb{B}|}{\Gamma \vdash b \Rightarrow \mathbb{B} \rightsquigarrow S_1 \mid z^{-}}\ \textsc{J-Base}$$

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \rightsquigarrow S_2 \mid z^{-}}\ \textsc{J-Var}$$

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \rightsquigarrow \varnothing \mid x^{+}}\ \textsc{J-VarGen}$$

$$\frac{\Gamma, \mathit{this} : A \vdash e \Leftarrow A \rightsquigarrow J \mid \mathit{this}^{-}}{\Gamma \vdash \mathbf{fix}\ \mathit{this}\!:\!A.\ e \Rightarrow A \rightsquigarrow S_3 \mid z^{-}}\ \textsc{J-Fix}$$

$$\frac{\begin{array}{c} T = |\overrightarrow{B}| \\ \Gamma, x : A \vdash e \Leftarrow B \rightsquigarrow J \mid y^{+} \end{array}}{\Gamma \vdash \lambda x\!:\!A.\ e\!:\!B \Rightarrow A \to B \rightsquigarrow S_4 \mid z^{-}}\ \textsc{J-Abs}$$

$$\frac{\begin{array}{c} \Gamma \vdash e_1 \Rightarrow A \rightsquigarrow J_1 \mid x^{+} \\ A \rhd B \to C \\ \Gamma \vdash e_2 \Leftarrow B \rightsquigarrow J_2 \mid y^{+} \\ x : A \bullet y \rightsquigarrow J_3 \mid z \end{array}}{\Gamma \vdash e_1\ e_2 \Rightarrow C \rightsquigarrow J_1; J_2; J_3 \mid z^{-}}\ \textsc{J-App}$$

$$\frac{\begin{array}{c} T = |B|^{\forall} \\ \Gamma, X \ast A \vdash e \Leftarrow B \rightsquigarrow J \mid y^{+} \end{array}}{\Gamma \vdash \Lambda X \ast A.\ e\!:\!B \Rightarrow \forall X \ast A.\ B \rightsquigarrow S_5 \mid z^{-}}\ \textsc{J-TAbs}$$

$$\frac{\begin{array}{c} \Gamma \vdash e \Rightarrow B \rightsquigarrow J_1 \mid y^{+} \\ B \rhd \forall X \ast C_1.\ C_2 \\ \Gamma \vdash A \ast C_1 \\ y : B \bullet A \rightsquigarrow J_2 \mid z \end{array}}{\Gamma \vdash e\ A \Rightarrow C_2[X \mapsto A] \rightsquigarrow J_1; J_2 \mid z^{-}}\ \textsc{J-TApp}$$

$$\frac{\begin{array}{c} T = \{\ell : |A|\} \\ \Gamma \vdash e \Rightarrow A \rightsquigarrow J \mid y^{+} \end{array}}{\Gamma \vdash \{\ell = e\} \Rightarrow \{\ell : A\} \rightsquigarrow S_6 \mid z^{-}}\ \textsc{J-Rcd}$$

$$\frac{\begin{array}{c} \Gamma \vdash e \Rightarrow A \rightsquigarrow J_1 \mid y^{+} \\ A \rhd \{\ell : B\} \\ y : A \bullet \ell \rightsquigarrow J_2 \mid z \end{array}}{\Gamma \vdash e.\ell \Rightarrow B \rightsquigarrow J_1; J_2 \mid z^{-}}\ \textsc{J-Proj}$$

$$\frac{\begin{array}{c} \Gamma \vdash e_1 \Rightarrow A \rightsquigarrow J_1 \mid z^{-} \\ \Gamma \vdash e_2 \Rightarrow B \rightsquigarrow J_2 \mid z^{-} \\ \Gamma \vdash A \ast B \end{array}}{\Gamma \vdash e_1 \,,,\, e_2 \Rightarrow A \,\&\, B \rightsquigarrow J_1; J_2 \mid z^{-}}\ \textsc{J-Merge}$$

$$\frac{\Gamma \vdash e \Leftarrow A \rightsquigarrow J \mid z^{-}}{\Gamma \vdash e : A \Rightarrow A \rightsquigarrow J \mid z^{-}}\ \textsc{J-Anno}$$

$$\frac{\Gamma \vdash e \Leftarrow A \rightsquigarrow J \mid z^{+}}{\Gamma \vdash e : A \Rightarrow A \rightsquigarrow J \mid z^{+}}\ \textsc{J-AnnoGen}$$

$$\frac{\begin{array}{c} \Gamma \vdash e \Rightarrow A \rightsquigarrow J_1 \mid x^{+} \\ x : A <: y : B \rightsquigarrow J_2 \end{array}}{\Gamma \vdash e \Leftarrow B \rightsquigarrow J_1; J_2 \mid y^{-}}\ \textsc{J-Sub}$$

$$\frac{\Gamma \vdash e \Rightarrow A \rightsquigarrow J \mid z^{-}}{\Gamma \vdash e \Leftarrow A \rightsquigarrow J \mid z^{-}}\ \textsc{J-SubEq}$$

J-SubEqGen

$$\frac{\Gamma \vdash e \Rightarrow A \;\; \rightsquigarrow J \mid z^{+}}{\Gamma \vdash e \Leftarrow A \;\; \rightsquigarrow J \mid z^{+}}$$

```
/* S0 */
var z = {};  J;

/* S1 */
z[T] = b;

/* S2 */
Object.assign(z, x);
```

```
/* S3 */
Object.assign(z,
  new function() { J }
);

/* S4 */
z[T] = x => {
  J;  return y;
};
```

```
/* S5 */
z[T] = X => {
  J;  return y;
};

/* S6 */
z[T] = () => {
  J;  return y;
};
```

$\boxed{x : A \bullet \mathit{arg} \;\; \rightsquigarrow \;\; J \mid z}$ *(Distributive application)*

A-Top

$$\frac{\rceil A \lceil}{x : A \bullet \mathit{arg} \;\; \rightsquigarrow \;\; \varnothing \mid z}$$

A-Arrow

$$\frac{T = \overrightarrow{|B|}}{x : A \to B \bullet y \;\; \rightsquigarrow \;\; S_7 \mid z}$$

A-All

$$\frac{T_1 = |B|^{\forall} \qquad T_2 = |C|}{x : \forall X * A.\ B \bullet C \;\; \rightsquigarrow \;\; S_8 \mid z}$$

A-Rcd

$$\frac{T = \{\ell : |A|\}}{x : \{\ell : A\} \bullet \ell \;\; \rightsquigarrow \;\; S_9 \mid z}$$

A-And

$$\frac{x : A \bullet \mathit{arg} \;\; \rightsquigarrow \;\; J_1 \mid z \qquad x : B \bullet \mathit{arg} \;\; \rightsquigarrow \;\; J_2 \mid z}{x : A \,\&\, B \bullet \mathit{arg} \;\; \rightsquigarrow \;\; J_1; J_2 \mid z}$$

```
/* S7 */
Object.assign(z, x[T](y));
```

```
/* S8 */
Object.assign(z, x[T1](T2));
```

```
/* S9 */
Object.assign(z, x[T]());
```

$\boxed{x : A <: y : B \rightsquigarrow J}$ *(Coercive subtyping)*

S-Top

$$\frac{B^{\circ} \qquad \rceil B \lceil}{x : A <: y : B \rightsquigarrow \varnothing}$$

S-Base

$$\frac{T = |\mathbb{B}|}{x : \mathbb{B} <: y : \mathbb{B} \rightsquigarrow S_{10}}$$

S-Arrow

$$\frac{\begin{array}{c} T_1 = \overrightarrow{|A_2|} \qquad T_2 = \overrightarrow{|B_2|} \\ B_2{}^{\circ} \qquad B_1 <: A_1 \\ x_2 : A_2 <: y_2 : B_2 \rightsquigarrow J_2 \end{array}}{x : A_1 \to A_2 <: y : B_1 \to B_2 \rightsquigarrow S_{11}}$$

S-All

$$\frac{\begin{array}{c} T_1 = |A_2|^{\forall} \qquad T_2 = |B_2|^{\forall} \\ B_2{}^{\circ} \qquad B_1 <: A_1 \\ x_2 : A_2 <: y_2 : B_2 \rightsquigarrow J_2 \end{array}}{x : \forall X * A_1.\ A_2 <: y : \forall X * B_1.\ B_2 \rightsquigarrow S_{12}}$$

S-Rcd

$$\frac{\begin{array}{c} T_1 = \{\ell : |A|\} \\ T_2 = \{\ell : |B|\} \\ B^{\circ} \qquad x_0 : A <: y_0 : B \rightsquigarrow J \end{array}}{x : \{\ell : A\} <: y : \{\ell : B\} \rightsquigarrow S_{13}}$$

S-AndL

$$\frac{C^{\circ} \qquad x : A <: y : C \rightsquigarrow J}{x : A \,\&\, B <: y : C \rightsquigarrow J}$$

S-AndR

$$\frac{C^{\circ} \qquad x : B <: y : C \rightsquigarrow J}{x : A \,\&\, B <: y : C \rightsquigarrow J}$$

S-Split

$$\frac{\begin{array}{c} B_1 \lhd B \rhd B_2 \\ x : A <: y_1 : B_1 \rightsquigarrow J_1 \\ x : A <: y_2 : B_2 \rightsquigarrow J_2 \\ y_1 : B_1 \rhd z : B \lhd y_2 : B_2 \rightsquigarrow J_3 \end{array}}{x : A <: z : B \rightsquigarrow S_{14}}$$

```
/* S10 */
y[T] = x[T];

/* S11 */
y[T2] = p => {
  var x2 = x[T1](p);
  var y2 = {};  J2;
  return y2;
};

/* S12 */
y[T2] = X => {
  var x2 = x[T1](X);
  var y2 = {};  J2;
  return y2;
};

/* S13 */
y[T2] = () => {
  var x0 = x[T1]();
  var y0 = {};  J;
  return y0;
}

/* S14 */
var y1 = {};  J1;
var y2 = {};  J2;
J3;
```

$$\boxed{x : A \;\rhd\; z : C \;\lhd\; y : B \;\colorbox{lightgray}{$\rightsquigarrow\; J$}}$$ *(Coercive merging)*

M-And
$$\frac{T_1 = |A| \qquad T_2 = |B|}{x : A \;\rhd\; z : A \,\&\, B \;\lhd\; y : B \;\colorbox{lightgray}{$\rightsquigarrow\; S_{15}$}}$$

M-Arrow
$$\frac{\begin{array}{c} T = \overrightarrow{|B|} \\ T_1 = \overrightarrow{|B_1|} \qquad T_2 = \overrightarrow{|B_2|} \\ y_1 : B_1 \;\rhd\; y : B \;\lhd\; y_2 : B_2 \;\colorbox{lightgray}{$\rightsquigarrow\; J$} \end{array}}{x_1 : A \to B_1 \;\rhd\; z : A \to B \;\lhd\; x_2 : A \to B_2 \;\colorbox{lightgray}{$\rightsquigarrow\; S_{16}$}}$$

M-All
$$\frac{\begin{array}{c} T = |B|^{\forall} \\ T_1 = |B_1|^{\forall} \qquad T_2 = |B_2|^{\forall} \\ y_1 : B_1 \;\rhd\; y : B \;\lhd\; y_2 : B_2 \;\colorbox{lightgray}{$\rightsquigarrow\; J$} \end{array}}{x_1 : \forall X * A.\, B_1 \;\rhd\; z : \forall X * A.\, B \;\lhd\; x_2 : \forall X * A.\, B_2 \;\colorbox{lightgray}{$\rightsquigarrow\; S_{17}$}}$$

M-Rcd
$$\frac{\begin{array}{c} T = \{\ell : |A|\} \\ T_1 = \{\ell : |A_1|\} \\ T_2 = \{\ell : |A_2|\} \\ y_1 : A_1 \;\rhd\; y : A \;\lhd\; y_2 : A_2 \;\colorbox{lightgray}{$\rightsquigarrow\; J$} \end{array}}{x_1 : \{\ell : A_1\} \;\rhd\; z : \{\ell : A\} \;\lhd\; x_2 : \{\ell : A_2\} \;\colorbox{lightgray}{$\rightsquigarrow\; S_{18}$}}$$

```
/* S15 */
Object.assign(z, x, y);

/* S16 */
z[T] = p => {
  var y1 = x1[T1](p);
  var y2 = x2[T2](p);
  var y = {};  J;
  return y;
};
```

```
/* S17 */
z[T] = X => {
  var y1 = x1[T1](X);
  var y2 = x2[T2](X);
  var y = {};  J;
  return y;
};
```

```
/* S18 */
z[T] = () => {
  var y1 = x1[T1]();
  var y2 = x2[T2]();
  var y = {};  J;
  return y;
};
```