

Compiling from F_i^+ to JavaScript

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Syntax of F_i^+

Types	$A, B, C ::= \top \mid \perp \mid \mathbb{B} \mid X \mid A \rightarrow B \mid \forall X * A. B \mid \{\ell : A\} \mid A \& B$
Type indices	$\tau ::= \mathbb{B} \mid \vec{\tau} \mid \tau^\forall \mid \{\ell : \tau\} \mid \tau_1 \& \tau_2$
Expressions	$e ::= \{\} \mid b \mid x \mid \mathbf{fix} \, x : A. e \mid \lambda x : A. e : B \mid e_1 \, e_2 \mid \Lambda X * A. e : B \mid e \, A \mid \{\ell = e\} \mid e.\ell$ $\mid e_1 \, , \, e_2 \mid e : A$
Values	$v ::= \{\} \mid b \mid \lambda x : A. e : B \mid \Lambda X * A. e : B \mid \{\ell = v\} \mid v_1 \, , \, v_2$

$$\boxed{|A| = \tau}$$

(Type translation)

$$|\mathbb{B}| = \mathbb{B} \quad |X| = X \quad |\forall X * A. B| = |B|^\forall \quad |A \rightarrow B| = |\vec{B}| \quad |\{\ell : A\}| = \{\ell : |A|\} \quad |A \& B| = |A| \& |B|$$

$$\boxed{\lceil A \rceil}$$

(Top-like types)

$$\begin{array}{c} \text{TL-TOP} \\ \frac{}{\lceil \top \rceil} \end{array} \quad \begin{array}{c} \text{TL-AND} \\ \frac{\lceil A \rceil \quad \lceil B \rceil}{\lceil A \& B \rceil} \end{array} \quad \begin{array}{c} \text{TL-ARROW} \\ \frac{\lceil B \rceil}{\lceil A \rightarrow B \rceil} \end{array} \quad \begin{array}{c} \text{TL-ALL} \\ \frac{\lceil B \rceil}{\lceil \forall X * A. B \rceil} \end{array} \quad \begin{array}{c} \text{TL-RCD} \\ \frac{}{\lceil \{\ell : A\} \rceil} \end{array}$$

$$\boxed{A^\circ}$$

(Ordinary types)

$$\begin{array}{c} \text{O-TOP} \\ \overline{\top^\circ} \end{array} \quad \begin{array}{c} \text{O-BOT} \\ \overline{\perp^\circ} \end{array} \quad \begin{array}{c} \text{O-BASE} \\ \overline{\mathbb{B}^\circ} \end{array} \quad \begin{array}{c} \text{O-VAR} \\ \overline{X^\circ} \end{array} \quad \begin{array}{c} \text{O-ARROW} \\ \frac{B^\circ}{(A \rightarrow B)^\circ} \end{array} \quad \begin{array}{c} \text{O-ALL} \\ \frac{B^\circ}{(\forall X * A. B)^\circ} \end{array} \quad \begin{array}{c} \text{O-RCD} \\ \frac{A^\circ}{\{\ell : A\}^\circ} \end{array}$$

$$\boxed{\Gamma \vdash e \Leftarrow A \rightsquigarrow z \text{ in } J}$$

(Type-directed compilation)

$$\begin{array}{c}
\text{J-TOP} \\
\hline
\Gamma \vdash \{\} \Rightarrow \top \rightsquigarrow z \text{ in } \emptyset
\end{array}
\quad
\begin{array}{c}
\text{J-TOPABS} \\
\hline
\frac{\lceil B \rceil}{\Gamma \vdash \lambda x:A. e:B \Rightarrow A \rightarrow B \rightsquigarrow z \text{ in } \emptyset}
\end{array}$$

$$\begin{array}{c}
\text{J-TOPTABS} \\
\hline
\frac{\lceil B \rceil}{\Gamma \vdash \Lambda X * A. e:B \Rightarrow \forall X * A. B \rightsquigarrow z \text{ in } \emptyset}
\end{array}
\quad
\begin{array}{c}
\text{J-TOPRCD} \\
\hline
\frac{\Gamma \vdash e \Rightarrow A \quad \lceil A \rceil}{\Gamma \vdash \{\ell = e\} \Rightarrow \{\ell : A\} \rightsquigarrow z \text{ in } \emptyset}
\end{array}$$

$$\begin{array}{c}
\text{J-BASE} \\
\hline
\frac{\tau = |\mathbb{B}|}{\Gamma \vdash b \Rightarrow \mathbb{B} \rightsquigarrow z \text{ in } S_1}
\end{array}
\quad
\begin{array}{c}
\text{J-VAR} \\
\hline
\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \rightsquigarrow z \text{ in } S_2}
\end{array}
\quad
\begin{array}{c}
\text{J-FIX} \\
\hline
\frac{\Gamma, x : A \vdash e \Leftarrow A \rightsquigarrow x \text{ in } J}{\Gamma \vdash \text{fix } x:A. e \Rightarrow A \rightsquigarrow z \text{ in } S_3}$$

$$\begin{array}{c}
\text{J-APP} \\
\hline
\frac{\Gamma \vdash e_1 \Rightarrow A \rightsquigarrow x \text{ in } J_1 \quad A \triangleright B \rightarrow C \quad \Gamma \vdash e_2 \Leftarrow B \rightsquigarrow y \text{ in } J_2 \quad A \triangleright x \bullet y \hookrightarrow z \text{ with } J_3}{\Gamma \vdash e_1 e_2 \Rightarrow C \rightsquigarrow z \text{ in } S_5}$$

$$\begin{array}{c}
\text{J-ABS} \\
\hline
\frac{\tau = \overrightarrow{|B|} \quad \Gamma, x : A \vdash e \Leftarrow B \rightsquigarrow y \text{ in } J}{\Gamma \vdash \lambda x:A. e:B \Rightarrow A \rightarrow B \rightsquigarrow z \text{ in } S_4}$$

$$\begin{array}{c}
\text{J-TABS} \\
\hline
\frac{\tau = |B|^\forall \quad \Gamma, X * A \vdash e \Leftarrow B \rightsquigarrow y \text{ in } J}{\Gamma \vdash \Lambda X * A. e:B \Rightarrow \forall X * A. B \rightsquigarrow z \text{ in } S_6}$$

$$\begin{array}{c}
\text{J-TAPP} \\
\hline
\frac{\Gamma \vdash e \Rightarrow B \rightsquigarrow y \text{ in } J_1 \quad B \triangleright \forall X * C_1. C_2 \quad \Gamma \vdash A * C_1 \quad B \triangleright y \bullet A \hookrightarrow z \text{ with } J_2}{\Gamma \vdash e A \Rightarrow C_2[X \mapsto A] \rightsquigarrow z \text{ in } S_7}$$

$$\begin{array}{c}
\text{J-RCD} \\
\hline
\frac{\tau = \{\ell : |A|\} \quad \Gamma \vdash e \Rightarrow A \rightsquigarrow y \text{ in } J}{\Gamma \vdash \{\ell = e\} \Rightarrow \{\ell : A\} \rightsquigarrow z \text{ in } S_8}$$

$$\begin{array}{c}
\text{J-PROJ} \\
\hline
\frac{\Gamma \vdash e \Rightarrow A \rightsquigarrow y \text{ in } J_1 \quad A \triangleright \{\ell : B\} \quad A \triangleright y \bullet \ell \hookrightarrow z \text{ with } J_2}{\Gamma \vdash e.\ell \Rightarrow B \rightsquigarrow z \text{ in } S_9}$$

$$\begin{array}{c}
\text{J-MERGE} \\
\hline
\frac{\Gamma \vdash e_1 \Rightarrow A \rightsquigarrow z \text{ in } J_1 \quad \Gamma \vdash e_2 \Rightarrow B \rightsquigarrow z \text{ in } J_2 \quad \Gamma \vdash A * B}{\Gamma \vdash e_1 ,, e_2 \Rightarrow A \& B \rightsquigarrow z \text{ in } S_{10}}$$

$$\begin{array}{c}
\text{J-ANNO} \\
\hline
\frac{\Gamma \vdash e \Leftarrow A \rightsquigarrow z \text{ in } J}{\Gamma \vdash e : A \Rightarrow A \rightsquigarrow z \text{ in } J}$$

$$\begin{array}{c}
\text{J-SUB} \\
\hline
\frac{\Gamma \vdash e \Rightarrow A \rightsquigarrow x \text{ in } J_1 \quad A <: B \rightsquigarrow x \mapsto y \text{ with } J_2}{\Gamma \vdash e \Leftarrow B \rightsquigarrow y \text{ in } S_{11}}$$

```

/* S1 */
z[t] = b;

/* S2 */
Object.assign(z, x);

```

```

/* S3 */
var f = x => { J };
Object.assign(z,
  new function() {
    f(this)
  }
);

```

```

/* S4 */
z[t] = x => {
  var y = {};
  J;
  return y;
};

```

```

/* S5 */
var x = {};
J1;
var y = {};
J2;
J3;

/* S6 */
z[t] = X => {
  var y = {};
  J;
  return y;
};

/* S7 */
var y = {};
J1;
J2;

/* S8 */
var y = {};
J;
z[t] = y;

/* S9 */
var y = {};

J1;
J2;

/* S10 */
J1;
J2;

/* S11 */
var x = {};
J1;
J2;

```

$A \triangleright x \bullet \text{arg} \hookrightarrow z \text{ with } J$ *(Distributive application)*

$$\begin{array}{c}
\text{A-TOP} \\
\frac{\lceil A \rceil}{A \triangleright x \bullet \text{arg} \hookrightarrow z \text{ with } \emptyset} \\
\\
\text{A-ALL} \\
\frac{\tau_1 = |B|^\forall \quad \tau_2 = |C|}{\forall X * A. B \triangleright x \bullet C \hookrightarrow z \text{ with } S_{13}} \\
\\
\text{A-AND} \\
\frac{A \triangleright x \bullet \text{arg} \hookrightarrow z \text{ with } J_1 \quad B \triangleright x \bullet \text{arg} \hookrightarrow z \text{ with } J_2}{A \& B \triangleright x \bullet \text{arg} \hookrightarrow z \text{ with } S_{15}} \\
\\
\text{A-ARROW} \\
\frac{\tau = \overrightarrow{|B|}}{A \rightarrow B \triangleright x \bullet y \hookrightarrow z \text{ with } S_{12}} \\
\\
\text{A-RCD} \\
\frac{\tau = \{\ell : |A|\}}{\{\ell : A\} \triangleright x \bullet \ell \hookrightarrow z \text{ with } S_{14}}
\end{array}$$

```

/* S12 */
Object.assign(z, x[t](y));

/* S13 */
Object.assign(z, x[t1](t2));

/* S14 */
Object.assign(z, x[t]);

/* S15 */
J1;
J2;

```

$$A <: B \quad \rightsquigarrow x \mapsto y \textbf{ with } J$$

(Coercive subtyping)

$$\text{S-TOP} \quad \frac{B^\circ \quad \lceil B \rceil}{A <: B \quad \rightsquigarrow x \mapsto y \textbf{ with } \emptyset}$$

$$\text{S-BASE} \quad \frac{\tau = |\mathbb{B}|}{\mathbb{B} <: \mathbb{B} \quad \rightsquigarrow x \mapsto y \textbf{ with } S_{16}}$$

$$\text{S-ARROW} \quad \frac{\tau_1 = \overrightarrow{|A_2|} \quad \tau_2 = \overrightarrow{|B_2|} \quad B_2^\circ \quad B_1 <: A_1 \quad A_2 <: B_2 \quad \rightsquigarrow x_2 \mapsto y_2 \textbf{ with } J_2}{A_1 \rightarrow A_2 <: B_1 \rightarrow B_2 \quad \rightsquigarrow x \mapsto y \textbf{ with } S_{17}}$$

$$\text{S-ALL} \quad \frac{\tau_1 = |A_2|^\forall \quad \tau_2 = |B_2|^\forall \quad B_2^\circ \quad B_1 <: A_1 \quad A_2 <: B_2 \quad \rightsquigarrow x_2 \mapsto y_2 \textbf{ with } J_2}{\forall X * A_1. A_2 <: \forall X * B_1. B_2 \quad \rightsquigarrow x \mapsto y \textbf{ with } S_{18}}$$

$$\text{S-RCD} \quad \frac{\tau_1 = \{\ell : |A|\} \quad \tau_2 = \{\ell : |B|\} \quad B^\circ \quad A <: B \quad \rightsquigarrow x_0 \mapsto y_0 \textbf{ with } J}{\{\ell : A\} <: \{\ell : B\} \quad \rightsquigarrow x \mapsto y \textbf{ with } S_{19}}$$

$$\text{S-ANDL} \quad \frac{C^\circ \quad A <: C \quad \rightsquigarrow x \mapsto y \textbf{ with } J}{A \& B <: C \quad \rightsquigarrow x \mapsto y \textbf{ with } J}$$

$$\text{S-ANDR} \quad \frac{C^\circ \quad B <: C \quad \rightsquigarrow x \mapsto y \textbf{ with } J}{A \& B <: C \quad \rightsquigarrow x \mapsto y \textbf{ with } J}$$

$$\text{S-SPLIT} \quad \frac{B_1 \triangleleft B \triangleright B_2 \quad A <: B_1 \quad \rightsquigarrow x \mapsto y_1 \textbf{ with } J_1 \quad A <: B_2 \quad \rightsquigarrow x \mapsto y_2 \textbf{ with } J_2 \quad y_1 : B_1 \triangleright z : B \triangleleft y_2 : B_2 \textbf{ with } J_3}{A <: B \quad \rightsquigarrow x \mapsto z \textbf{ with } S_{20}}$$

```
/* S16 */
y[t] = x[t];

/* S17 */
y[t2] = p => {
  var x2 = x[t1](p);
  var y2 = {};
  J2;
  return y2;
};
```

```
/* S18 */
y[t2] = X => {
  var x2 = x[t1](X);
  var y2 = {};
  J2;
  return y2;
};

/* S19 */
var x0 = x[t1];
var y0 = {};
```

```
J;
y[t2] = y0;

/* S20 */
var y1 = {};
J1;
var y2 = {};
J2;
J3;
```

$$\boxed{x : A \triangleright z : C \triangleleft y : B \text{ with } J}$$

(Coercive merging)

M-ARROW

$$\text{M-AND} \quad \frac{\tau_1 = |A| \quad \tau_2 = |B|}{x : A \triangleright z : A \& B \triangleleft y : B \text{ with } S_{21}}$$

$$\frac{\tau = \overrightarrow{|B|} \quad \tau_1 = \overrightarrow{|B_1|} \quad \tau_2 = \overrightarrow{|B_2|} \quad y_1 : B_1 \triangleright y : B \triangleleft y_2 : B_2 \text{ with } J}{x_1 : A \rightarrow B_1 \triangleright z : A \rightarrow B \triangleleft x_2 : A \rightarrow B_2 \text{ with } S_{22}}$$

M-ALL

$$\frac{\tau = |B|^\forall \quad \tau_1 = |B_1|^\forall \quad \tau_2 = |B_2|^\forall \quad y_1 : B_1 \triangleright y : B \triangleleft y_2 : B_2 \text{ with } J}{x_1 : \forall X * A. B_1 \triangleright z : \forall X * A. B \triangleleft x_2 : \forall X * A. B_2 \text{ with } S_{23}}$$

M-RCD

$$\frac{\tau = \{\ell : |A|\} \quad \tau_1 = \{\ell : |A_1|\} \quad \tau_2 = \{\ell : |A_2|\} \quad y_1 : A_1 \triangleright y : A \triangleleft y_2 : A_2 \text{ with } J}{x_1 : \{\ell : A_1\} \triangleright z : \{\ell : A\} \triangleleft x_2 : \{\ell : A_2\} \text{ with } S_{24}}$$

/* S21 */

Object.assign(z, x, y);

/* S22 */

```
z[t] = p => {
  var y1 = x1[t1](p);
  var y2 = x2[t2](p);
  var y = {};
  J;
  return y;
};
```

/* S23 */

```
z[t] = X => {
  var y1 = x1[t1](X);
  var y2 = x2[t2](X);
  var y = {};
  J;
  return y;
};
```

/* S24 */

```
var y1 = x1[t1];
var y2 = x2[t2];
var y = {};
J;
z[t] = y;
```