# 1 Syntax of $\lambda_i$

| | |
|---|---|
| Types | $A, B, C ::= \top \mid \mathbb{B} \mid A \to B \mid A \,\&\, B$ |
| Expressions | $e ::= () \mid b \mid x \mid \textbf{fix}\, x : A.\ e \mid \lambda x : A.\ e : B \mid e_1\, e_2 \mid e_1\, ,\, e_2 \mid e : A$ |
| Values | $v ::= () \mid b \mid \lambda x : A.\ e : B \mid v_1\, ,\, v_2$ |

# 2 Syntax of MiniJS

| | |
|---|---|
| Type indices | $\tau ::= \mathbb{B} \mid \overrightarrow{\tau} \mid \tau_1 \,\&\, \tau_2$ |
| Expressions | $\epsilon ::= \alpha \mid b \mid x \mid \textbf{fix}\, x.\ \epsilon \mid \lambda x.\ \epsilon \mid \epsilon_1 \,\$\, \epsilon_2 \mid \epsilon_1 \uplus \epsilon_2 \mid co\ \epsilon$ |
| Values | $\alpha ::= \{\overline{\tau_i \longmapsto \epsilon_i}\}$ |
| Coercion | $co ::= \textbf{identity} \mid \textbf{clear} \mid \textbf{becard}_\tau\ co_1\ co_2 \mid \Pi_\tau \mid co_1 \diamond co_2 \mid co_1 \circ co_2$ |

$\boxed{|A|}$ *(Type translation)*

$$|\mathbb{B}| = \mathbb{B} \qquad |A \to B| = \overrightarrow{|B|} \qquad \frac{|A| \leq |B|}{|A \,\&\, B| = |A| \,\&\, |B|} \qquad \frac{|A| > |B|}{|A \,\&\, B| = |B| \,\&\, |A|}$$

# 3 Coercion

$\boxed{A <: B \ \rightsquigarrow co}$ *(Coercive subtyping)*

$$\textsc{CS-refl}\ \frac{}{A <: A \ \rightsquigarrow \textbf{identity}} \qquad \textsc{CS-top}\ \frac{\rceil B \lceil}{A <: B \ \rightsquigarrow \textbf{clear}} \qquad \textsc{CS-arrow}\ \frac{B_1 <: A_1 \ \rightsquigarrow co_1 \quad A_2 <: B_2 \ \rightsquigarrow co_2}{A_1 \to A_2 <: B_1 \to B_2 \ \rightsquigarrow \textbf{becard}_{|B_2|}\ co_1\ co_2}$$

$$\textsc{CS-andLL}\ \frac{A <: C \ \rightsquigarrow co}{A \,\&\, B <: C \ \rightsquigarrow co \circ \Pi_{|A|}} \qquad \textsc{CS-andLR}\ \frac{B <: C \ \rightsquigarrow co}{A \,\&\, B <: C \ \rightsquigarrow co \circ \Pi_{|B|}} \qquad \textsc{CS-andR}\ \frac{A <: B \ \rightsquigarrow co_1 \quad A <: C \ \rightsquigarrow co_2}{A <: B \,\&\, C \ \rightsquigarrow co_1 \diamond co_2}$$

$\boxed{co\ \epsilon \Downarrow \alpha}$ *(Coercion evaluation)*

$$\textsc{CE-identity}\ \frac{\epsilon \Downarrow \alpha}{\textbf{identity}\ \epsilon \Downarrow \alpha} \qquad \textsc{CE-clear}\ \frac{}{\textbf{clear}\ \epsilon \Downarrow \{\,\}} \qquad \textsc{CE-becard}\ \frac{}{(\textbf{becard}_\tau\ co_1\ co_2)\ \epsilon \Downarrow \{\overrightarrow{\tau} \longmapsto \lambda x.\ co_2\ (\epsilon \,\$\, co_1\ x)\}}$$

$$\textsc{CE-project}\ \frac{\epsilon \Downarrow \{\overline{\tau_i \longmapsto \epsilon_i}; \tau \longmapsto \epsilon'; \overline{\tau_j \longmapsto \epsilon_j}\}}{\Pi_\tau\ \epsilon \Downarrow \{\tau \longmapsto \epsilon'\}} \qquad \textsc{CE-append}\ \frac{co_1\ \epsilon \Downarrow \alpha_1 \quad co_2\ \epsilon \Downarrow \alpha_2}{(co_1 \diamond co_2)\ \epsilon \Downarrow \alpha_1 \uplus \alpha_2} \qquad \textsc{CE-compose}\ \frac{co_1\ \epsilon \Downarrow \alpha_1 \quad co_2\ \alpha_1 \Downarrow \alpha_2}{(co_2 \circ co_1)\ \epsilon \Downarrow \alpha_2}$$

# 4 Semantics

$\boxed{\Gamma \vdash e \Leftrightarrow A \; \rightsquigarrow \epsilon}$ *(Type-directed compilation)*

$$\textsc{T-top}\quad \frac{}{\Gamma \vdash () \Rightarrow \top \; \rightsquigarrow \{\,\}} \qquad \textsc{T-base}\quad \frac{}{\Gamma \vdash b \Rightarrow \mathbb{B} \; \rightsquigarrow \{\mathbb{B} \longmapsto b\}} \qquad \textsc{T-var}\quad \frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \; \rightsquigarrow x}$$

$$\textsc{T-fix}\quad \frac{\Gamma, x : A \vdash e \Leftarrow A \; \rightsquigarrow \epsilon}{\Gamma \vdash \mathbf{fix}\, x : A.\ e \Rightarrow A \; \rightsquigarrow \mathbf{fix}\, x.\ \epsilon} \qquad \textsc{T-abs}\quad \frac{\neg\rceil B \lceil \qquad \Gamma, x : A \vdash e \Leftarrow B \; \rightsquigarrow \epsilon}{\Gamma \vdash \lambda x : A.\ e : B \Rightarrow A \to B \; \rightsquigarrow \{\overrightarrow{|B|} \longmapsto \lambda x.\ \epsilon\}}$$

$$\textsc{T-absTop}\quad \frac{\rceil B \lceil}{\Gamma \vdash \lambda x : A.\ e : B \Rightarrow A \to B \; \rightsquigarrow \{\,\}} \qquad \textsc{T-app}\quad \frac{\Gamma \vdash e_1 \Rightarrow A \to B \; \rightsquigarrow \epsilon_1 \qquad \Gamma \vdash e_2 \Leftarrow A \; \rightsquigarrow \epsilon_2}{\Gamma \vdash e_1\, e_2 \Rightarrow B \; \rightsquigarrow \epsilon_1 \,\$\, \epsilon_2} \qquad \textsc{T-appTop}\quad \frac{\Gamma \vdash e_1 \Rightarrow \top \; \rightsquigarrow \{\,\}}{\Gamma \vdash e_1\, e_2 \Rightarrow \top \; \rightsquigarrow \{\,\}}$$

$$\textsc{T-merge}\quad \frac{\Gamma \vdash e_1 \Rightarrow A \; \rightsquigarrow \epsilon_1 \qquad \Gamma \vdash e_2 \Rightarrow B \; \rightsquigarrow \epsilon_2 \qquad A * B}{\Gamma \vdash e_1 \,,\, e_2 \Rightarrow A \,\&\, B \; \rightsquigarrow \epsilon_1 \uplus \epsilon_2} \qquad \textsc{T-anno}\quad \frac{\Gamma \vdash e \Leftarrow A \; \rightsquigarrow \epsilon}{\Gamma \vdash e : A \Rightarrow A \; \rightsquigarrow \epsilon} \qquad \textsc{T-sub}\quad \frac{\Gamma \vdash e \Rightarrow A \; \rightsquigarrow \epsilon \qquad A <: B \; \rightsquigarrow co}{\Gamma \vdash e \Leftarrow B \; \rightsquigarrow co\ \epsilon}$$

$\boxed{\epsilon \Downarrow \alpha}$ *(Big-step operational semantics)*

$$\textsc{E-fix}\quad \frac{\epsilon[x \mapsto \mathbf{fix}\, x.\ \epsilon] \Downarrow \alpha}{\mathbf{fix}\, x.\ \epsilon \Downarrow \alpha} \qquad \textsc{E-app}\quad \frac{\epsilon_1 \Downarrow \{\overrightarrow{\tau} \longmapsto \lambda x.\ \epsilon_1'\} \qquad \epsilon_2 \Downarrow \alpha_2 \qquad \epsilon_1'[x \mapsto \alpha_2] \Downarrow \alpha}{\epsilon_1 \,\$\, \epsilon_2 \Downarrow \alpha} \qquad \textsc{E-appTop}\quad \frac{\epsilon_1 \Downarrow \{\,\}}{\epsilon_1 \,\$\, \epsilon_2 \Downarrow \{\,\}}$$

$$\textsc{E-merge}\quad \frac{\epsilon_1 \Downarrow \{\overline{\tau_i \longmapsto \epsilon_i}\} \qquad \epsilon_2 \Downarrow \{\overline{\tau_j \longmapsto \epsilon_j}\}}{\epsilon_1 \uplus \epsilon_2 \Downarrow \{\overline{\tau_i \longmapsto \epsilon_i}; \overline{\tau_j \longmapsto \epsilon_j}\}}$$