

Compiling from F_i^+ to JavaScript

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October 19, 2022

Syntax of F_i^+

Types	$A, B, C ::= \top \mid \perp \mid \mathbb{B} \mid X \mid A \rightarrow B \mid \forall X * A. B \mid \{\ell : A\} \mid A \& B$
Type indices	$T ::= \mathbb{B} \mid \vec{T} \mid T^\forall \mid \{\ell : T\} \mid T_1 \& T_2$
Expressions	$e ::= \{\} \mid b \mid x \mid \mathbf{fix} \ x : A. e \mid \lambda x : A. e : B \mid e_1 \ e_2 \mid \Lambda X * A. e : B \mid e \ A \mid \{\ell = e\} \mid e.\ell$ $\mid e_1 \ , \ e_2 \mid e : A$
Values	$v ::= \{\} \mid b \mid \lambda x : A. e : B \mid \Lambda X * A. e : B \mid \{\ell = v\} \mid v_1 \ , \ v_2$

$$\boxed{|A| = T} \quad (Type\ translation)$$

$$|\mathbb{B}| = \mathbb{B} \quad |X| = \mathbf{atoi}(X) \quad |\forall X * A. B| = |B|^\forall \quad |A \rightarrow B| = |\vec{B}| \quad |\{\ell : A\}| = \{\ell : |A|\}$$

$$\frac{A_{k_1} < A_{k_2} < \dots < A_{k_m} \quad \neg |A_k|}{|A_1 \& A_2 \& \dots \& A_n| = |A_{k_1}| \& |A_{k_2}| \& \dots \& |A_{k_m}|}$$

$$\boxed{\lceil A \rceil} \quad (Top-like\ types)$$

$$\begin{array}{c} \text{TL-TOP} \\ \frac{}{\lceil \top \rceil} \end{array} \quad \begin{array}{c} \text{TL-AND} \\ \frac{\lceil A \rceil \quad \lceil B \rceil}{\lceil A \& B \rceil} \end{array} \quad \begin{array}{c} \text{TL-ARROW} \\ \frac{}{\lceil A \rightarrow B \rceil} \end{array} \quad \begin{array}{c} \text{TL-ALL} \\ \frac{}{\lceil \forall X * A. B \rceil} \end{array} \quad \begin{array}{c} \text{TL-RCD} \\ \frac{}{\lceil \{\ell : A\} \rceil} \end{array}$$

$$\boxed{A^\circ} \quad (Ordinary\ types)$$

$$\begin{array}{c} \text{O-TOP} \\ \overline{\top^\circ} \end{array} \quad \begin{array}{c} \text{O-BOT} \\ \overline{\perp^\circ} \end{array} \quad \begin{array}{c} \text{O-BASE} \\ \overline{\mathbb{B}^\circ} \end{array} \quad \begin{array}{c} \text{O-VAR} \\ \overline{X^\circ} \end{array} \quad \begin{array}{c} \text{O-ARROW} \\ \overline{B^\circ} \\ (A \rightarrow B)^\circ \end{array} \quad \begin{array}{c} \text{O-ALL} \\ \overline{B^\circ} \\ (\forall X * A. B)^\circ \end{array} \quad \begin{array}{c} \text{O-RCD} \\ \overline{A^\circ} \\ \{\ell : A\}^\circ \end{array}$$

$$\boxed{\Gamma \vdash e \Leftrightarrow A \quad \rightsquigarrow J \mid z^\pm}$$

(Type-directed compilation)

<p>J-GEN</p> $\frac{\Gamma \vdash e \Leftrightarrow A \quad \rightsquigarrow J \mid z^-}{\Gamma \vdash e \Leftrightarrow A \quad \rightsquigarrow S_0 \mid z^+}$	<p>J-TOP</p> $\frac{}{\Gamma \vdash \{\} \Rightarrow \top \quad \rightsquigarrow \emptyset \mid z^-}$	<p>J-TOPABS</p> $\frac{\lceil B \rceil}{\Gamma \vdash \lambda x:A. e:B \Rightarrow A \rightarrow B \quad \rightsquigarrow \emptyset \mid z^-}$		
<p>J-TOPTABS</p> $\frac{\lceil B \rceil}{\Gamma \vdash \Lambda X * A. e:B \Rightarrow \forall X * A. B \quad \rightsquigarrow \emptyset \mid z^-}$	<p>J-TOPRCD</p> $\frac{\Gamma \vdash e \Rightarrow A \quad \lceil A \rceil}{\Gamma \vdash \{\ell = e\} \Rightarrow \{\ell : A\} \quad \rightsquigarrow \emptyset \mid z^-}$			
<p>J-BASE</p> $\frac{T = \mathbb{B} }{\Gamma \vdash b \Rightarrow \mathbb{B} \quad \rightsquigarrow S_1 \mid z^-}$	<p>J-VAR</p> $\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \quad \rightsquigarrow S_2 \mid z^-}$	<p>J-VARGEN</p> $\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \quad \rightsquigarrow S_3 \mid z^+}$		
<p>J-FIX</p> $\frac{\Gamma, x : A \vdash e \Leftarrow A \quad \rightsquigarrow J \mid z^-}{\Gamma \vdash \mathbf{fix} x:A. e \Rightarrow A \quad \rightsquigarrow S_4 \mid z^-}$	<p>J-ABS</p> $\frac{T = \overrightarrow{ B }}{\Gamma, x : A \vdash e \Leftarrow B \quad \rightsquigarrow J \mid y^+}$	<p>J-APP</p> $\frac{\Gamma \vdash e_1 \Rightarrow A \quad \rightsquigarrow J_1 \mid x^+ \quad A \triangleright B \rightarrow C \quad \Gamma \vdash e_2 \Leftarrow B \quad \rightsquigarrow J_2 \mid y^+ \quad x : A \bullet y_0 \rightsquigarrow J_3 \mid z}{\Gamma \vdash e_1 e_2 \Rightarrow C \quad \rightsquigarrow S_6 \mid z^-}$		
<p>J-TABS</p> $\frac{T = B ^\forall \quad \Gamma, X * A \vdash e \Leftarrow B \quad \rightsquigarrow J \mid y^+}{\Gamma \vdash \Lambda X * A. e:B \Rightarrow \forall X * A. B \quad \rightsquigarrow S_7 \mid z^-}$		<p>J-TAPP</p> $\frac{\Gamma \vdash e \Rightarrow B \quad \rightsquigarrow J_1 \mid y^+ \quad B \triangleright \forall X * C_1. C_2 \quad \Gamma \vdash A * C_1 \quad Ts = \mathbf{itoa} A \quad y : B \bullet Ts \rightsquigarrow J_2 \mid z}{\Gamma \vdash e A \Rightarrow C_2[X \mapsto A] \quad \rightsquigarrow J_1; J_2 \mid z^-}$		
<p>J-RCD</p> $\frac{T = \{\ell : A \} \quad \Gamma \vdash e \Rightarrow A \quad \rightsquigarrow J \mid y^+}{\Gamma \vdash \{\ell = e\} \Rightarrow \{\ell : A\} \quad \rightsquigarrow S_8 \mid z^-}$		<p>J-PROJ</p> $\frac{\Gamma \vdash e \Rightarrow A \quad \rightsquigarrow J_1 \mid y^+ \quad A \triangleright \{\ell : B\} \quad y : A \bullet \ell \rightsquigarrow J_2 \mid z}{\Gamma \vdash e.\ell \Rightarrow B \quad \rightsquigarrow J_1; J_2 \mid z^-}$		
<p>J-MERGE</p> $\frac{\Gamma \vdash e_1 \Rightarrow A \quad \rightsquigarrow J_1 \mid z^- \quad \Gamma \vdash e_2 \Rightarrow B \quad \rightsquigarrow J_2 \mid z^-}{\Gamma \vdash A * B}$ $\frac{}{\Gamma \vdash e_1 ,, e_2 \Rightarrow A \& B \quad \rightsquigarrow J_1; J_2 \mid z^-}$			<p>J-ANNO</p> $\frac{\Gamma \vdash e \Leftarrow A \quad \rightsquigarrow J \mid z^-}{\Gamma \vdash e : A \Rightarrow A \quad \rightsquigarrow J \mid z^-}$	<p>J-ANNOGEN</p> $\frac{\Gamma \vdash e \Leftarrow A \quad \rightsquigarrow J \mid z^+}{\Gamma \vdash e : A \Rightarrow A \quad \rightsquigarrow J \mid z^+}$
<p>J-SUB</p> $\frac{\Gamma \vdash e \Rightarrow A \quad \rightsquigarrow J_1 \mid x^+ \quad x : A <: y : B \rightsquigarrow J_2}{\Gamma \vdash e \Leftarrow B \quad \rightsquigarrow J_1; J_2 \mid y^-}$	<p>J-SUBEQ</p> $\frac{\Gamma \vdash e \Rightarrow A \quad \rightsquigarrow J \mid z^-}{\Gamma \vdash e \Leftarrow A \quad \rightsquigarrow J \mid z^-}$	<p>J-SUBEQGEN</p> $\frac{\Gamma \vdash e \Rightarrow A \quad \rightsquigarrow J \mid z^+}{\Gamma \vdash e \Leftarrow A \quad \rightsquigarrow J \mid z^+}$		

```

/* S0 */
var z = {}; J;

/* S1 */
z[T] = b;

/* S2 */
Object.assign(z, x());

/* S3 */
var z = x();

/* S4 */
var x = () => z; J;

/* S5 */
z[T] = x => {
  J; return y;
};

/* S6 */
J1;
var y0 = () => {
  J2; return y;
};

};
J3;

/* S7 */
z[T] = X => {
  J; return y;
};

/* S8 */
z[T] = () => {
  J; return y;
};

```

$x : A \bullet \arg \rightsquigarrow J \mid z$

(Distributive application)

$$\frac{\text{A-TOP}}{\frac{}{\lceil A \rceil} \mid z} \rightsquigarrow \emptyset \mid z$$

$$\frac{\text{A-ARROW}}{x : A \rightarrow B \bullet y \mid z} \rightsquigarrow S_9 \mid z$$

$$\frac{\text{A-ALL}}{x : \forall X * A. B \bullet Ts \mid z} \rightsquigarrow S'_9 \mid z$$

$$\frac{\text{A-RCD}}{x : \{\ell : A\} \bullet \ell \mid z} \rightsquigarrow S''_9 \mid z$$

$$\frac{\text{A-AND}}{x : A \& B \bullet \arg \mid z} \rightsquigarrow J_1; J_2 \mid z$$

```

/* S9 */
Object.assign(z, x[T](y));

/* S9' */
Object.assign(z, x[T](Ts));

/* S9'' */
Object.assign(z, x[T]());

```

$$\boxed{x : A <: y : B \rightsquigarrow J}$$

(Coercive subtyping)

$$\frac{\text{S-TOP} \quad B^\circ \quad \lceil B \rceil}{x : A <: y : B \rightsquigarrow \emptyset}$$

$$\frac{\text{S-BASE} \quad T = |\mathbb{B}|}{x : \mathbb{B} <: y : \mathbb{B} \rightsquigarrow S_{10}}$$

$$\frac{\text{S-VAR}}{x : X <: y : X \rightsquigarrow S'_{10}}$$

$$\frac{\text{S-ARROW} \quad \begin{array}{c} T_1 = \overrightarrow{|A_2|} \quad T_2 = \overrightarrow{|B_2|} \\ B_2^\circ \quad B_1 <: A_1 \\ x_2 : A_2 <: y_2 : B_2 \rightsquigarrow J_2 \end{array}}{x : A_1 \rightarrow A_2 <: y : B_1 \rightarrow B_2 \rightsquigarrow S_{11}}$$

$$\frac{\text{S-ALL} \quad \begin{array}{c} T_1 = |A_2|^\forall \quad T_2 = |B_2|^\forall \\ B_2^\circ \quad B_1 <: A_1 \\ x_2 : A_2 <: y_2 : B_2 \rightsquigarrow J_2 \end{array}}{x : \forall X * A_1. A_2 <: y : \forall X * B_1. B_2 \rightsquigarrow S_{12}}$$

$$\frac{\text{S-RCD} \quad \begin{array}{c} T_1 = \{\ell : |A|\} \\ T_2 = \{\ell : |B|\} \\ B^\circ \quad x_0 : A <: y_0 : B \rightsquigarrow J \end{array}}{x : \{\ell : A\} <: y : \{\ell : B\} \rightsquigarrow S_{13}}$$

$$\frac{\text{S-ANDL} \quad \begin{array}{c} C^\circ \quad x : A <: y : C \rightsquigarrow J \end{array}}{x : A \& B <: y : C \rightsquigarrow J}$$

$$\frac{\text{S-ANDR} \quad \begin{array}{c} C^\circ \quad x : B <: y : C \rightsquigarrow J \end{array}}{x : A \& B <: y : C \rightsquigarrow J}$$

$$\frac{\text{S-SPLIT} \quad \begin{array}{c} B_1 \triangleleft B \triangleright B_2 \\ x : A <: y_1 : B_1 \rightsquigarrow J_1 \\ x : A <: y_2 : B_2 \rightsquigarrow J_2 \\ y_1 : B_1 \triangleright z : B \triangleleft y_2 : B_2 \rightsquigarrow J_3 \end{array}}{x : A <: z : B \rightsquigarrow S_{14}}$$

```
/* S10 */
y[T] = x[T];

/* S10' */
for (var T of X) {
  y[T] = x[T];
}

/* S11 */
y[T2] = p => {
  var x2 = x[T1](p);
```

```
var y2 = {}; J2;
return y2;
};

/* S12 */
y[T2] = X => {
  var x2 = x[T1](X);
  var y2 = {}; J2;
  return y2;
};
```

```
/* S13 */
y[T2] = () => {
  var x0 = x[T1]();
  var y0 = {}; J;
  return y0;
}

/* S14 */
var y1 = {}; J1;
var y2 = {}; J2;
J3;
```

$$\boxed{x : A \triangleright z : C \triangleleft y : B \rightsquigarrow J}$$

(Coercive merging)

$$\begin{array}{c} \text{M-AND} \\ \hline x : A \triangleright z : A \& B \triangleleft y : B \rightsquigarrow S_{15} \end{array} \quad \begin{array}{c} \text{M-ARROW} \\ \hline \begin{array}{c} T = \overrightarrow{|B|} \\ T_1 = \overrightarrow{|B_1|} \quad T_2 = \overrightarrow{|B_2|} \\ y_1 : B_1 \triangleright y : B \triangleleft y_2 : B_2 \rightsquigarrow J \end{array} \\ x_1 : A \rightarrow B_1 \triangleright z : A \rightarrow B \triangleleft x_2 : A \rightarrow B_2 \rightsquigarrow S_{16} \end{array}$$

$$\begin{array}{c} \text{M-ALL} \\ \hline \begin{array}{c} T = |B|^\forall \\ T_1 = |B_1|^\forall \quad T_2 = |B_2|^\forall \\ y_1 : B_1 \triangleright y : B \triangleleft y_2 : B_2 \rightsquigarrow J \end{array} \\ x_1 : \forall X * A. B_1 \triangleright z : \forall X * A. B \triangleleft x_2 : \forall X * A. B_2 \rightsquigarrow S_{17} \end{array}$$

$$\begin{array}{c} \text{M-RCD} \\ \hline \begin{array}{c} T = \{\ell : |A|\} \\ T_1 = \{\ell : |A_1|\} \\ T_2 = \{\ell : |A_2|\} \\ y_1 : A_1 \triangleright y : A \triangleleft y_2 : A_2 \rightsquigarrow J \end{array} \\ x_1 : \{\ell : A_1\} \triangleright z : \{\ell : A\} \triangleleft x_2 : \{\ell : A_2\} \rightsquigarrow S_{18} \end{array}$$

```
/* S15 */
Object.assign(z, x, y);
```

```
/* S16 */
z[T] = p => {
  var y1 = x1[T1](p);
  var y2 = x2[T2](p);
  var y = {}; J;
  return y;
};
```

```
/* S17 */
z[T] = X => {
  var y1 = x1[T1](X);
  var y2 = x2[T2](X);
  var y = {}; J;
  return y;
};
```

```
/* S18 */
z[T] = () => {
  var y1 = x1[T1]();
  var y2 = x2[T2]();
  var y = {}; J;
  return y;
};
```