

# Compiling from $F_i^+$ to JavaScript

Yaozhu Sun

August 22, 2022

## Syntax of $F_i^+$

Types	$A, B, C ::= \top \mid \perp \mid \mathbb{B} \mid X \mid A \rightarrow B \mid \forall X * A. B \mid \{\ell : A\} \mid A \& B$
Type indices	$\tau ::= \mathbb{B} \mid \vec{\tau} \mid \tau^\forall \mid \{\ell : \tau\} \mid \tau_1 \& \tau_2$
Expressions	$e ::= \{\} \mid b \mid x \mid \mathbf{fix} \ e \mid \lambda x : A. e : B \mid e_1 \ e_2 \mid \Lambda X * A. e : B \mid e \ A \mid \{\ell = e\} \mid e.\ell$ $\mid e_1 \ , \ e_2 \mid e : A$
Values	$v ::= \{\} \mid b \mid \lambda x : A. e : B \mid \Lambda X * A. e : B \mid \{\ell = v\} \mid v_1 \ , \ v_2$

$$\boxed{|A| = \tau}$$

(Type translation)

$$|\mathbb{B}| = \mathbb{B} \quad |X| = X \quad |\forall X * A. B| = |B|^\forall \quad |A \rightarrow B| = |\vec{B}| \quad |\{\ell : A\}| = \{\ell : |A|\} \quad |A \& B| = |A| \& |B|$$

$$\boxed{\lceil A \rceil}$$

(Top-like types)

$$\begin{array}{ccccc} \text{TL-TOP} & \text{TL-AND} & \text{TL-ARROW} & \text{TL-ALL} & \text{TL-RCD} \\ \frac{}{\lceil \top \rceil} & \frac{\lceil A \rceil \quad \lceil B \rceil}{\lceil A \& B \rceil} & \frac{\lceil B \rceil}{\lceil A \rightarrow B \rceil} & \frac{\lceil B \rceil}{\lceil \forall X * A. B \rceil} & \frac{\lceil A \rceil}{\lceil \{\ell : A\} \rceil} \end{array}$$

$$\boxed{A^\circ}$$

(Ordinary types)

$$\begin{array}{ccccccc} \text{O-TOP} & \text{O-BOT} & \text{O-BASE} & \text{O-VAR} & \text{O-ARROW} & \text{O-ALL} & \text{O-RCD} \\ \frac{}{\top^\circ} & \frac{}{\perp^\circ} & \frac{}{\mathbb{B}^\circ} & \frac{}{X^\circ} & \frac{B^\circ}{(A \rightarrow B)^\circ} & \frac{B^\circ}{(\forall X * A. B)^\circ} & \frac{A^\circ}{\{\ell : A\}^\circ} \end{array}$$

$$\boxed{\Gamma \vdash e \Leftrightarrow A \rightsquigarrow z \text{ in } J}$$

(Type-directed compilation)

$$\begin{array}{c}
\text{J-TOP} \\
\frac{}{\Gamma \vdash \{\} \Rightarrow \top \rightsquigarrow z \text{ in } \emptyset} \\
\\
\text{J-TOPABS} \\
\frac{}{\Gamma \vdash \lambda x:A. e:B \Rightarrow A \rightarrow B \rightsquigarrow z \text{ in } \emptyset} \\
\\
\text{J-TOPTABS} \\
\frac{}{\Gamma \vdash \Lambda X * A. e:B \Rightarrow \forall X * A. B \rightsquigarrow z \text{ in } \emptyset} \\
\\
\text{J-TOPRCD} \\
\frac{}{\Gamma \vdash \{\ell = e\} \Rightarrow \{\ell : A\} \rightsquigarrow z \text{ in } \emptyset} \\
\\
\text{J-FIX} \\
\frac{\tau = \overrightarrow{|A|}}{\Gamma \vdash e \Rightarrow A \rightarrow A \rightsquigarrow y \text{ in } J} \\
\\
\text{J-BASE} \\
\frac{\tau = |\mathbb{B}|}{\Gamma \vdash b \Rightarrow \mathbb{B} \rightsquigarrow z \text{ in } S_1} \\
\\
\text{J-VAR} \\
\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \rightsquigarrow z \text{ in } S_2} \\
\\
\text{J-APP} \\
\frac{\Gamma \vdash e_1 \Rightarrow A \rightsquigarrow x \text{ in } J_1 \quad A \triangleright B \rightarrow C \quad \Gamma \vdash e_2 \Leftarrow B \rightsquigarrow y \text{ in } J_2 \quad A \triangleright x \bullet y \hookrightarrow z \text{ with } J_3}{\Gamma \vdash e_1 e_2 \Rightarrow C \rightsquigarrow z \text{ in } S_5} \\
\\
\text{J-TAPP} \\
\frac{\Gamma \vdash e \Rightarrow B \rightsquigarrow y \text{ in } J_1 \quad B \triangleright \forall X * C_1. C_2 \quad \Gamma \vdash A * C_1 \quad B \triangleright y \bullet A \hookrightarrow z \text{ with } J_2}{\Gamma \vdash e A \Rightarrow C_2[X \mapsto A] \rightsquigarrow z \text{ in } S_7} \\
\\
\text{J-ABS} \\
\frac{\tau = \overrightarrow{|B|} \quad \Gamma, x : A \vdash e \Leftarrow B \rightsquigarrow y \text{ in } J}{\Gamma \vdash \lambda x:A. e:B \Rightarrow A \rightarrow B \rightsquigarrow z \text{ in } S_4} \\
\\
\text{J-TABS} \\
\frac{\tau = |B|^\forall \quad \Gamma, X * A \vdash e \Leftarrow B \rightsquigarrow y \text{ in } J}{\Gamma \vdash \Lambda X * A. e:B \Rightarrow \forall X * A. B \rightsquigarrow z \text{ in } S_6} \\
\\
\text{J-RCD} \\
\frac{\tau = \{\ell : |A|\} \quad \Gamma \vdash e \Rightarrow A \rightsquigarrow y \text{ in } J}{\Gamma \vdash \{\ell = e\} \Rightarrow \{\ell : A\} \rightsquigarrow z \text{ in } S_8} \\
\\
\text{J-PROJ} \\
\frac{\Gamma \vdash e \Rightarrow A \rightsquigarrow y \text{ in } J_1 \quad A \triangleright \{\ell : B\} \quad A \triangleright y \bullet \ell \hookrightarrow z \text{ with } J_2}{\Gamma \vdash e.\ell \Rightarrow B \rightsquigarrow z \text{ in } S_9} \\
\\
\text{J-MERGE} \\
\frac{\Gamma \vdash e_1 \Rightarrow A \rightsquigarrow z \text{ in } J_1 \quad \Gamma \vdash e_2 \Rightarrow B \rightsquigarrow z \text{ in } J_2 \quad \Gamma \vdash A * B}{\Gamma \vdash e_1 \text{ ,, } e_2 \Rightarrow A \& B \rightsquigarrow z \text{ in } S_{10}} \\
\\
\text{J-SUB} \\
\frac{\Gamma \vdash e \Rightarrow A \rightsquigarrow x \text{ in } J_1 \quad A <: B \rightsquigarrow x \mapsto y \text{ with } J_2}{\Gamma \vdash e \Leftarrow B \rightsquigarrow y \text{ in } S_{11}} \\
\\
\text{J-ANNO} \\
\frac{\Gamma \vdash e \Leftarrow A \rightsquigarrow z \text{ in } J}{\Gamma \vdash e : A \Rightarrow A \rightsquigarrow z \text{ in } J}
\end{array}$$

```

/* S1 */
z[t] = b;

/* S2 */
Object.assign(z, x);

/* S3 */

Object.assign(z,
  new function() {
    var y = {};
    J;
    y[t](this, this);
  }
);

/* S4 */
z[t] = (x, y) => { J };

/* S5 */
var x = {};
J1;

```

```

var y = {};
J2;
J3;

/* S6 */
z[t] = X => {
  var y = {};
  J;
  return y;
};

/* S7 */

```

```

var y = {};
J1;
J2;

/* S8 */
z[t] = () => {
  var y = {};
  J;
  return y;
};

/* S9 */

```

```

var y = {};
J1;
J2;

/* S10 */
J1;
J2;

/* S11 */
var x = {};
J1;
J2;

```

$$\boxed{A \triangleright x \bullet \text{arg} \hookrightarrow z \textbf{with } J}$$

(Distributive application)

$$\begin{array}{c}
\text{A-TOP} \\
\frac{\lceil A \rceil}{A \triangleright x \bullet \text{arg} \hookrightarrow z \textbf{with } \emptyset} \\
\\
\text{A-ALL} \\
\frac{\tau_1 = |B|^\forall \quad \tau_2 = |C|}{\forall X * A. B \triangleright x \bullet C \hookrightarrow z \textbf{with } S_{13}} \\
\\
\text{A-AND} \\
\frac{A \triangleright x \bullet \text{arg} \hookrightarrow z \textbf{with } J_1 \quad B \triangleright x \bullet \text{arg} \hookrightarrow z \textbf{with } J_2}{A \& B \triangleright x \bullet \text{arg} \hookrightarrow z \textbf{with } S_{15}} \\
\\
\text{A-ARROW} \\
\frac{\tau = \overrightarrow{|B|}}{A \rightarrow B \triangleright x \bullet y \hookrightarrow z \textbf{with } S_{12}} \\
\\
\text{A-RCD} \\
\frac{\tau = \{\ell : |A|\}}{\{\ell : A\} \triangleright x \bullet \ell \hookrightarrow z \textbf{with } S_{14}}
\end{array}$$

```

/* S12 */
x[t](y, z);

```

```

/* S13 */
Object.assign(z, x[t1](t2)); /* S15 */
J1;
/* S14 */
J2;
Object.assign(z, x[t]());

```

$$A <: B \quad \rightsquigarrow x \mapsto y \textbf{ with } J$$

(Coercive subtyping)

$$\frac{\text{S-TOP} \quad B^\circ \quad \lceil B \rceil}{A <: B \quad \rightsquigarrow x \mapsto y \textbf{ with } \emptyset}$$

$$\frac{\text{S-BASE} \quad \tau = |\mathbb{B}|}{\mathbb{B} <: \mathbb{B} \quad \rightsquigarrow x \mapsto y \textbf{ with } S_{16}}$$

$$\frac{\text{S-ARROW} \quad \begin{array}{c} \tau_1 = \overrightarrow{|A_2|} \\ \tau_2 = \overrightarrow{|B_2|} \end{array} \quad \begin{array}{c} B_2^\circ \quad B_1 <: A_1 \\ A_2 <: B_2 \quad \rightsquigarrow x_2 \mapsto y_2 \textbf{ with } J_2 \end{array}}{A_1 \rightarrow A_2 <: B_1 \rightarrow B_2 \quad \rightsquigarrow x \mapsto y \textbf{ with } S_{17}}$$

$$\frac{\text{S-ALL} \quad \begin{array}{c} \tau_1 = |A_2|^\forall \quad \tau_2 = |B_2|^\forall \\ B_2^\circ \quad B_1 <: A_1 \\ A_2 <: B_2 \quad \rightsquigarrow x_2 \mapsto y_2 \textbf{ with } J_2 \end{array}}{\forall X * A_1. A_2 <: \forall X * B_1. B_2 \quad \rightsquigarrow x \mapsto y \textbf{ with } S_{18}}$$

$$\frac{\text{S-RCD} \quad \begin{array}{c} \tau_1 = \{\ell : |A|\} \\ \tau_2 = \{\ell : |B|\} \quad B^\circ \\ A <: B \quad \rightsquigarrow x_0 \mapsto y_0 \textbf{ with } J \end{array}}{\{\ell : A\} <: \{\ell : B\} \quad \rightsquigarrow x \mapsto y \textbf{ with } S_{19}}$$

$$\frac{\text{S-ANDL} \quad C^\circ \quad A <: C \quad \rightsquigarrow x \mapsto y \textbf{ with } J}{A \& B <: C \quad \rightsquigarrow x \mapsto y \textbf{ with } J}$$

$$\frac{\text{S-ANDR} \quad C^\circ \quad B <: C \quad \rightsquigarrow x \mapsto y \textbf{ with } J}{A \& B <: C \quad \rightsquigarrow x \mapsto y \textbf{ with } J}$$

$$\frac{\text{S-SPLIT} \quad \begin{array}{c} B_1 \triangleleft B \triangleright B_2 \\ A <: B_1 \quad \rightsquigarrow x \mapsto y_1 \textbf{ with } J_1 \\ A <: B_2 \quad \rightsquigarrow x \mapsto y_2 \textbf{ with } J_2 \\ y_1 : B_1 \triangleright z : B \triangleleft y_2 : B_2 \textbf{ with } J_3 \end{array}}{A <: B \quad \rightsquigarrow x \mapsto z \textbf{ with } S_{20}}$$

```
/* S16 */
y[t] = x[t];

/* S17 */
y[t2] = p => {
  var x2 = x[t1](p);
  var y2 = {};
  J2;
  return y2;
};
```

```
/* S18 */
y[t2] = X => {
  var x2 = x[t1](X);
  var y2 = {};
  J2;
  return y2;
};

/* S19 */
y[t2] = () => {
  var x0 = x[t1]();
```

```
var y0 = {};
J;
return y0;
}

/* S20 */
var y1 = {};
J1;
var y2 = {};
J2;
J3;
```

$$\boxed{x : A \triangleright z : C \triangleleft y : B \textbf{ with } J}$$

(Coercive merging)

M-ARROW

$$\begin{array}{c} \text{M-AND} \\ \frac{\tau_1 = |A| \quad \tau_2 = |B|}{x : A \triangleright z : A \& B \triangleleft y : B \textbf{ with } S_{21}} \end{array} \quad \frac{\begin{array}{c} \tau = \overrightarrow{|B|} \\ \tau_1 = \overrightarrow{|B_1|} \quad \tau_2 = \overrightarrow{|B_2|} \\ y_1 : B_1 \triangleright y : B \triangleleft y_2 : B_2 \textbf{ with } J \end{array}}{x_1 : A \rightarrow B_1 \triangleright z : A \rightarrow B \triangleleft x_2 : A \rightarrow B_2 \textbf{ with } S_{22}}$$

M-ALL

$$\frac{\begin{array}{c} \tau = |B|^\forall \\ \tau_1 = |B_1|^\forall \quad \tau_2 = |B_2|^\forall \\ y_1 : B_1 \triangleright y : B \triangleleft y_2 : B_2 \textbf{ with } J \end{array}}{x_1 : \forall X * A. B_1 \triangleright z : \forall X * A. B \triangleleft x_2 : \forall X * A. B_2 \textbf{ with } S_{23}}$$

M-RCD

$$\frac{\begin{array}{c} \tau = \{\ell : |A|\} \\ \tau_1 = \{\ell : |A_1|\} \\ \tau_2 = \{\ell : |A_2|\} \\ y_1 : A_1 \triangleright y : A \triangleleft y_2 : A_2 \textbf{ with } J \end{array}}{x_1 : \{\ell : A_1\} \triangleright z : \{\ell : A\} \triangleleft x_2 : \{\ell : A_2\} \textbf{ with } S_{24}}$$

/\* S21 \*/

Object.assign(z, x, y);

/\* S22 \*/

```
z[t] = p => {
  var y1 = x1[t1](p);
  var y2 = x2[t2](p);
  var y = {};
  J;
  return y;
};
```

/\* S23 \*/

```
z[t] = X => {
  var y1 = x1[t1](X);
  var y2 = x2[t2](X);
  var y = {};
  J;
  return y;
};
```

/\* S24 \*/

```
z[t] = () => {
  var y1 = x1[t1]();
  var y2 = x2[t2]();
  var y = {};
  J;
  return y;
};
```