## Syntax of $\lambda_i^+$

| | |
|---|---|
| Types | $A, B, C ::= \top \mid \mathbb{B} \mid A \to B \mid A \,\&\, B$ |
| Type indices | $\tau ::= \mathbb{B} \mid \overrightarrow{\tau} \mid \tau_1 \,\&\, \tau_2$ |
| Expressions | $e ::= () \mid b \mid x \mid \mathbf{fix}\, x : A.\ e \mid \lambda x : A.\ e : B \mid e_1\ e_2 \mid e_1 \,,,\, e_2 \mid e : A$ |
| Values | $v ::= () \mid b \mid \lambda x : A.\ e : B \mid v_1 \,,,\, v_2$ |

$\boxed{|A| = \tau}$ *(Type translation)*

$$|\mathbb{B}| = \mathbb{B} \qquad |A \to B| = \overrightarrow{|B|} \qquad |A \,\&\, B| = |A| \,\&\, |B|$$

## Compiling from $\lambda_i^+$ to JavaScript

$\boxed{\Gamma \vdash e \Leftrightarrow A \rightsquigarrow z \textbf{ in } J}$ *(Type-directed compilation)*

J-Top
$$\frac{}{\Gamma \vdash () \Rightarrow \top \rightsquigarrow z \textbf{ in } S_1}$$

J-TopAbs
$$\frac{\rceil B \lceil}{\Gamma \vdash \lambda x : A.\ e : B \Rightarrow A \to B \rightsquigarrow z \textbf{ in } S_1}$$

J-Base
$$\frac{\tau = |\mathbb{B}|}{\Gamma \vdash b \Rightarrow \mathbb{B} \rightsquigarrow z \textbf{ in } S_2}$$

J-Var
$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \rightsquigarrow x \textbf{ in } \varnothing}$$

J-Fix
$$\frac{\Gamma, x : A \vdash e \Leftarrow A \rightsquigarrow y \textbf{ in } J}{\Gamma \vdash \mathbf{fix}\, x : A.\ e \Rightarrow A \rightsquigarrow x \textbf{ in } S_3}$$

J-Abs
$$\frac{\tau = \overrightarrow{|B|} \qquad \Gamma, x : A \vdash e \Leftarrow B \rightsquigarrow y \textbf{ in } J}{\Gamma \vdash \lambda x : A.\ e : B \Rightarrow A \to B \rightsquigarrow z \textbf{ in } S_4}$$

J-App
$$\frac{\Gamma \vdash e_1 \Rightarrow A \rightsquigarrow x \textbf{ in } J_1 \qquad A \triangleright B \to C \qquad \Gamma \vdash e_2 \Leftarrow B \rightsquigarrow y \textbf{ in } J_2 \qquad A \ \triangleright\ x \bullet y \hookrightarrow z \textbf{ with } J_3}{\Gamma \vdash e_1\ e_2 \Rightarrow C \rightsquigarrow z \textbf{ in } S_5}$$

J-Merge
$$\frac{\Gamma \vdash e_1 \Rightarrow A \rightsquigarrow x \textbf{ in } J_1 \qquad \Gamma \vdash e_2 \Rightarrow B \rightsquigarrow y \textbf{ in } J_2 \qquad A * B}{\Gamma \vdash e_1 \,,,\, e_2 \Rightarrow A \,\&\, B \rightsquigarrow z \textbf{ in } S_6}$$

J-Anno
$$\frac{\Gamma \vdash e \Leftarrow A \rightsquigarrow x \textbf{ in } J}{\Gamma \vdash e : A \Rightarrow A \rightsquigarrow x \textbf{ in } J}$$

J-Sub
$$\frac{\Gamma \vdash e \Rightarrow A \rightsquigarrow x \textbf{ in } J_1 \qquad A <: B \rightsquigarrow x \mapsto y \textbf{ with } J_2}{\Gamma \vdash e \Leftarrow B \rightsquigarrow y \textbf{ in } S_7}$$

```
/* S1 */
var z = {};

/* S2 */
var z = {};
z.<t> = b;

/* S3 */
var x = (() => {
  J;
  return y;
})();
```

```
/* S4 */
var z = {};
z.<t> = x => {
  J;
  return y;
};

/* S5 */
J1;
J2;
J3;
```

```
/* S6 */
J1;
J2;
var z = {...x, ...y};

/* S7 */
J1;
var y = {};
J2;
```

$\boxed{A \ \triangleright \ x \bullet y \ \hookrightarrow \ z \textbf{ with } J}$ *(Distributive application)*

A-TopArrow
$$\frac{\rceil A \lceil}{A \ \triangleright \ x \bullet y \ \hookrightarrow \ z \textbf{ with } S_8}$$

```
/* S8 */
var z = {};
```

A-AndArrow
$$\frac{\tau_1 = |\overrightarrow{A_2}| \qquad \tau_2 = |\overrightarrow{B_2}| \qquad A \ \triangleright \ x \bullet y \ \hookrightarrow \ z_1 \textbf{ with } J_1 \qquad B \ \triangleright \ x \bullet y \ \hookrightarrow \ z_2 \textbf{ with } J_2}{A \,\&\, B \ \triangleright \ x \bullet y \ \hookrightarrow \ z \textbf{ with } S_9}$$

```
/* S9 */
J1;
J2;
var z = {...z1, ...z2};
```

A-Arrow
$$\frac{\tau = |\overrightarrow{B}|}{A \to B \ \triangleright \ x \bullet y \ \hookrightarrow \ z \textbf{ with } S_{10}}$$

```
/* S10 */
var z = x.<t>(y);
```

$\boxed{A <: B \ \rightsquigarrow x \mapsto y \textbf{ with } J}$ *(Coercive subtyping)*

S-Top
$$\frac{B^\circ \qquad \rceil B \lceil}{A <: B \ \rightsquigarrow x \mapsto y \textbf{ with } \varnothing}$$

S-Base
$$\frac{\tau = |\mathbb{B}|}{\mathbb{B} <: \mathbb{B} \ \rightsquigarrow x \mapsto y \textbf{ with } S_{11}}$$

S-Arrow
$$\frac{\tau_1 = |\overrightarrow{A_2}| \qquad \tau_2 = |\overrightarrow{B_2}| \qquad B_2{}^\circ \qquad B_1 <: A_1 \ \rightsquigarrow x_1 \mapsto y_1 \textbf{ with } J_1 \qquad A_2 <: B_2 \ \rightsquigarrow x_2 \mapsto y_2 \textbf{ with } J_2}{A_1 \to A_2 <: B_1 \to B_2 \ \rightsquigarrow x \mapsto y \textbf{ with } S_{12}}$$

S-AndL
$$\frac{C^\circ \qquad A <: C \ \rightsquigarrow x \mapsto y \textbf{ with } J}{A \,\&\, B <: C \ \rightsquigarrow x \mapsto y \textbf{ with } J}$$

S-AndR
$$\frac{C^\circ \qquad B <: C \ \rightsquigarrow x \mapsto y \textbf{ with } J}{A \,\&\, B <: C \ \rightsquigarrow x \mapsto y \textbf{ with } J}$$

S-Split
$$\frac{B_1 \lhd B \rhd B_2 \qquad A <: B_1 \ \rightsquigarrow x \mapsto y_1 \textbf{ with } J_1 \qquad A <: B_2 \ \rightsquigarrow x \mapsto y_2 \textbf{ with } J_2 \qquad y_1 : B_1 \ \triangleright \ z : B \ \triangleleft \ y_2 : B_2 \textbf{ with } J_3}{A <: B \ \rightsquigarrow x \mapsto z \textbf{ with } S_{13}}$$

```
/* S11 */
y.<t> = x.<t>;
```

```
/* S12 */
y.<t2> = p => {
  var x2 = x.<t1>(p);
  var y2 = {};
  J2;
  return y2;
};
```

```
/* S13 */
var y1 = {};
J1;
var y2 = {};
J2;
J3;
```

$\boxed{x : A \ \triangleright \ z : C \ \triangleleft \ y : B \textbf{ with } J}$ *(Coercive merging)*

M-And
$$\frac{\tau_1 = |A| \qquad \tau_2 = |B|}{x : A \ \triangleright \ z : A \,\&\, B \ \triangleleft \ y : B \textbf{ with } S_{14}}$$

M-Arrow
$$\frac{\tau = |\overrightarrow{B}| \qquad \tau_1 = |\overrightarrow{B_1}| \qquad \tau_2 = |\overrightarrow{B_2}| \qquad y_1 : B_1 \ \triangleright \ y : B \ \triangleleft \ y_2 : B_2 \textbf{ with } J}{x_1 : A \to B_1 \ \triangleright \ z : A \to B \ \triangleleft \ x_2 : A \to B_2 \textbf{ with } S_{15}}$$

```
/* S14 */
z = {...x, ...y};
```

```
/* S15 */
z.<t> = p => {
  var y1 = x1.<t1>(p);
  var y2 = x2.<t2>(p);
  var y = {};
  J;
  return y;
};
```