Compiling from F_i^+ to JavaScript
(simplified scheme with call-by-value)

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Syntax of F_i^+

Types	$A, B, C ::= \top \mid \bot \mid \mathbb{B} \mid X \mid A \to B \mid \forall X{*}A.\ B \mid \{\ell : A\} \mid A \,\&\, B$
Type indices	$T ::= \mathbb{B} \mid \overrightarrow{T} \mid T^{\forall} \mid \{\ell : T\} \mid T_1 \,\&\, T_2$
Expressions	$e ::= \{\} \mid b \mid x \mid \mathbf{fix}\, x{:}A.\ e \mid \lambda x{:}A.\ e{:}B \mid e_1\ e_2 \mid \Lambda X{*}A.\ e{:}B \mid e\ A \mid \{\ell = e\} \mid e.\ell$ $\mid e_1 \,,,\, e_2 \mid e : A$
Values	$v ::= \{\} \mid b \mid \lambda x{:}A.\ e{:}B \mid \Lambda X{*}A.\ e{:}B \mid \{\ell = v\} \mid v_1 \,,,\, v_2$

$\boxed{|A| = T}$ *(Type translation)*

$$|\mathbb{B}| = \mathbb{B} \qquad |X| = \mathbf{atoi}(X) \qquad |A \to B| = \overrightarrow{|B|} \qquad |\forall X{*}A.\ B| = |B|^{\forall} \qquad |\{\ell : A\}| = \{\ell : |A|\}$$

$$\frac{A_{k_1} < A_{k_2} < \cdots < A_{k_m} \qquad \neg \rceil A_k \lceil}{|A_1 \& A_2 \& \cdots \& A_n| = |A_{k_1}| \& |A_{k_2}| \& \cdots \& |A_{k_m}|}$$

$\boxed{\rceil A \lceil}$ *(Top-like types)*

$$\textsc{TL-Top}\ \frac{}{\rceil \top \lceil} \qquad \textsc{TL-And}\ \frac{\rceil A \lceil \qquad \rceil B \lceil}{\rceil A \,\&\, B \lceil} \qquad \textsc{TL-Arrow}\ \frac{\rceil B \lceil}{\rceil A \to B \lceil} \qquad \textsc{TL-All}\ \frac{\rceil B \lceil}{\rceil \forall X{*}A.\ B \lceil} \qquad \textsc{TL-Rcd}\ \frac{\rceil A \lceil}{\rceil \{\ell : A\} \lceil}$$

$\boxed{A^{\circ}}$ *(Ordinary types)*

$$\textsc{O-Top}\ \frac{}{\top^{\circ}} \qquad \textsc{O-Bot}\ \frac{}{\bot^{\circ}} \qquad \textsc{O-Base}\ \frac{}{\mathbb{B}^{\circ}} \qquad \textsc{O-Var}\ \frac{}{X^{\circ}} \qquad \textsc{O-Arrow}\ \frac{B^{\circ}}{(A \to B)^{\circ}} \qquad \textsc{O-All}\ \frac{B^{\circ}}{(\forall X{*}A.\ B)^{\circ}} \qquad \textsc{O-Rcd}\ \frac{A^{\circ}}{\{\ell : A\}^{\circ}}$$

$\boxed{\Gamma \vdash e \Leftrightarrow A \;\rightsquigarrow J \mid z^{\pm}}$ *(Type-directed compilation)*

$$\textsc{J-Gen}\quad \frac{\Gamma \vdash e \Leftrightarrow A \;\rightsquigarrow J \mid z^{-}}{\Gamma \vdash e \Leftrightarrow A \;\rightsquigarrow \mathsf{code} \mid z^{+}}$$

$$\textsc{J-Top}\quad \frac{}{\Gamma \vdash \{\} \Rightarrow \top \;\rightsquigarrow \varnothing \mid z^{-}}$$

$$\textsc{J-TopAbs}\quad \frac{\rceil B \lceil}{\Gamma \vdash \lambda x\!:\!A.\ e\!:\!B \Rightarrow A \rightarrow B \;\rightsquigarrow \varnothing \mid z^{-}}$$

$$\textsc{J-TopTAbs}\quad \frac{\rceil B \lceil}{\Gamma \vdash \Lambda X * A.\ e\!:\!B \Rightarrow \forall X * A.\ B \;\rightsquigarrow \varnothing \mid z^{-}}$$

$$\textsc{J-TopRcd}\quad \frac{\Gamma \vdash e \Rightarrow A \qquad \rceil A \lceil}{\Gamma \vdash \{\ell = e\} \Rightarrow \{\ell : A\} \;\rightsquigarrow \varnothing \mid z^{-}}$$

$$\textsc{J-Base}\quad \frac{T = |\mathbb{B}|}{\Gamma \vdash b \Rightarrow \mathbb{B} \;\rightsquigarrow \mathsf{code} \mid z^{-}}$$

$$\textsc{J-Var}\quad \frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \;\rightsquigarrow \mathsf{code} \mid z^{-}}$$

$$\textsc{J-VarGen}\quad \frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \;\rightsquigarrow \varnothing \mid x^{+}}$$

$$\textsc{J-Fix}\quad \frac{\Gamma, x : A \vdash e \Leftarrow A \;\rightsquigarrow J \mid z^{-}}{\Gamma \vdash \mathbf{fix}\, x\!:\!A.\ e \Rightarrow A \;\rightsquigarrow \mathsf{code} \mid z^{-}}$$

$$\textsc{J-Abs}\quad \frac{T = |\overrightarrow{B}| \qquad \Gamma, x : A \vdash e \Leftarrow B \;\rightsquigarrow J \mid y^{-}}{\Gamma \vdash \lambda x\!:\!A.\ e\!:\!B \Rightarrow A \rightarrow B \;\rightsquigarrow \mathsf{code} \mid z^{-}}$$

$$\textsc{J-App}\quad \frac{\Gamma \vdash e_1 \Rightarrow A \;\rightsquigarrow J_1 \mid x^{+} \qquad \Gamma \vdash e_2 \Rightarrow B \;\rightsquigarrow J_2 \mid y^{+} \qquad \Gamma \vdash x : A \bullet y : B \;\rightsquigarrow J_3 \mid z : C}{\Gamma \vdash e_1\, e_2 \Rightarrow C \;\rightsquigarrow J_1; J_2; J_3 \mid z^{-}}$$

$$\textsc{J-TAbs}\quad \frac{T = |B|^{\forall} \qquad \Gamma, X * A \vdash e \Leftarrow B \;\rightsquigarrow J \mid y^{-}}{\Gamma \vdash \Lambda X * A.\ e\!:\!B \Rightarrow \forall X * A.\ B \;\rightsquigarrow \mathsf{code} \mid z^{-}}$$

$$\textsc{J-TApp}\quad \frac{\Gamma \vdash e \Rightarrow B \;\rightsquigarrow J_1 \mid y^{+} \qquad \Gamma \vdash y : B \bullet A \;\rightsquigarrow J_2 \mid z : C}{\Gamma \vdash e\, A \Rightarrow C \;\rightsquigarrow J_1; J_2 \mid z^{-}}$$

$$\textsc{J-Rcd}\quad \frac{T = \{\ell : |A|\} \qquad \Gamma \vdash e \Rightarrow A \;\rightsquigarrow J \mid y^{+}}{\Gamma \vdash \{\ell = e\} \Rightarrow \{\ell : A\} \;\rightsquigarrow \mathsf{code} \mid z^{-}}$$

$$\textsc{J-Proj}\quad \frac{\Gamma \vdash e \Rightarrow A \;\rightsquigarrow J_1 \mid y^{+} \qquad y : A \bullet \{\ell\} \;\rightsquigarrow J_2 \mid z : B}{\Gamma \vdash e.\ell \Rightarrow B \;\rightsquigarrow J_1; J_2 \mid z^{-}}$$

$$\textsc{J-Merge}\quad \frac{\Gamma \vdash e_1 \Rightarrow A \;\rightsquigarrow J_1 \mid z^{-} \qquad \Gamma \vdash e_2 \Rightarrow B \;\rightsquigarrow J_2 \mid z^{-} \qquad \Gamma \vdash A * B}{\Gamma \vdash e_1 \,,,\, e_2 \Rightarrow A \,\&\, B \;\rightsquigarrow J_1; J_2 \mid z^{-}}$$

$$\textsc{J-Anno}\quad \frac{\Gamma \vdash e \Leftarrow A \;\rightsquigarrow J \mid z^{-}}{\Gamma \vdash e : A \Rightarrow A \;\rightsquigarrow J \mid z^{-}}$$

$$\textsc{J-AnnoGen}\quad \frac{\Gamma \vdash e \Leftarrow A \;\rightsquigarrow J \mid z^{+}}{\Gamma \vdash e : A \Rightarrow A \;\rightsquigarrow J \mid z^{+}}$$

$$\textsc{J-Sub}\quad \frac{\Gamma \vdash e \Rightarrow A \;\rightsquigarrow J_1 \mid x^{+} \qquad x : A <:^{+} y : B \;\rightsquigarrow J_2}{\Gamma \vdash e \Leftarrow B \;\rightsquigarrow J_1; J_2 \mid y^{-}}$$

$$\textsc{J-SubEqGen}\quad \frac{\Gamma \vdash e \Rightarrow A \;\rightsquigarrow J \mid z^{+}}{\Gamma \vdash e \Leftarrow A \;\rightsquigarrow J \mid z^{+}}$$

$$\textsc{J-Def}\quad \frac{\Gamma \vdash e_1 \Rightarrow A \;\rightsquigarrow J_1 \mid x^{-} \qquad \Gamma, x : A \vdash e_2 \Rightarrow B \;\rightsquigarrow J_2 \mid z^{-}}{\Gamma \vdash x = e1; e_2 \Rightarrow B \;\rightsquigarrow \mathsf{code} \mid z^{-}}$$

```
/* J-Gen */
var z = {};  J;

/* J-Base */
z[T] = b;

/* J-Var */
Object.assign(z, x);
```

```
/* J-Fix */
var x = z;  J;

/* J-Abs */
z[T] = (x, y) => { J };

/* J-TAbs */
z[T] = (X, y) => { J };
```

```
/* J-Rcd */
J;
z[T] = y;

/* J-Def */
export var x = {};
J1;
J2;
```

$\boxed{\Gamma \vdash x : A \bullet p \;\rightsquigarrow\; J \mid z : B}$ *(Distributive application)*

A-Top
$$\frac{\rceil A \lceil}{\Gamma \vdash x : A \bullet p \;\rightsquigarrow\; \varnothing \mid z : \top}$$

A-Arrow
$$\frac{T = |\overrightarrow{B}| \qquad y : C <:^{+} y_0 : A \;\rightsquigarrow\; J}{\Gamma \vdash x : A \to B \bullet y : C \;\rightsquigarrow\; \mathsf{code} \mid z : B}$$

A-All
$$\frac{\Gamma \vdash A * C \qquad T = |B|^{\forall} \qquad Ts = \mathbf{itoa}\,|C|}{\Gamma \vdash x : \forall X{*}A.\, B \bullet C \;\rightsquigarrow\; \mathsf{code} \mid z : B[X \mapsto C]}$$

A-And
$$\frac{\Gamma \vdash x : A \bullet p \;\rightsquigarrow\; J_1 \mid z : A' \qquad \Gamma \vdash x : B \bullet p \;\rightsquigarrow\; J_2 \mid z : B'}{\Gamma \vdash x : A \,\&\, B \bullet p \;\rightsquigarrow\; J_1; J_2 \mid z : A' \,\&\, B'}$$

```
/* A-Arrow */
var y0 = {};  J;
x[T](y0, z);
```

```
/* A-All */
x[T](Ts, z);
```

$\boxed{x : A \bullet \{\ell\} \;\rightsquigarrow\; J \mid z : B}$ *(Distributive projection)*

P-Top
$$\frac{\rceil A \lceil}{x : A \bullet \{\ell\} \;\rightsquigarrow\; \varnothing \mid z : \top}$$

P-RcdEq
$$\frac{T = \{\ell : |A|\}}{x : \{\ell : A\} \bullet \{\ell\} \;\rightsquigarrow\; \mathsf{code} \mid z : A}$$

P-RcdNeq
$$\frac{\ell_1 \neq \ell_2 \qquad T = \{\ell : |A|\}}{x : \{\ell_1 : A\} \bullet \{\ell_2\} \;\rightsquigarrow\; \varnothing \mid z : \top}$$

P-And
$$\frac{x : A \bullet \{\ell\} \;\rightsquigarrow\; J_1 \mid z : A' \qquad x : B \bullet \{\ell\} \;\rightsquigarrow\; J_2 \mid z : B'}{x : A \,\&\, B \bullet \{\ell\} \;\rightsquigarrow\; J_1; J_2 \mid z : A' \,\&\, B'}$$

```
/* P-Rcd */
Object.assign(z, x[T]);
```

$\boxed{x : A <:^{\pm} y : B \rightsquigarrow J}$ *(Coercive subtyping)*

$$\textsc{S-Top}\quad \frac{B^{\circ} \qquad \rceil B \lceil}{x : A <:^{\pm} y : B \rightsquigarrow \varnothing}$$

$$\textsc{S-Bot}\quad \frac{T = |A| \qquad A^{\circ}}{x : \bot <:^{\pm} y : A \rightsquigarrow \mathsf{code}}$$

$$\textsc{S-Eq}\quad \frac{}{x : A <:^{+} y : A \rightsquigarrow \mathsf{code}}$$

$$\textsc{S-Base}\quad \frac{T = |\mathbb{B}|}{x : \mathbb{B} <:^{\pm} y : \mathbb{B} \rightsquigarrow \mathsf{code}}$$

$$\textsc{S-Var}\quad \frac{}{x : X <:^{\pm} y : X \rightsquigarrow \mathsf{code}}$$

$$\textsc{S-Arrow}\quad \frac{\begin{array}{c} T_1 = \overrightarrow{|A_2|} \\ T_2 = \overrightarrow{|B_2|} \qquad B_2{}^{\circ} \\ x_1 : B_1 <:^{+} y_1 : A_1 \rightsquigarrow J_1 \\ x_2 : A_2 <:^{+} y_2 : B_2 \rightsquigarrow J_2 \end{array}}{x : A_1 \to A_2 <:^{\pm} y : B_1 \to B_2 \rightsquigarrow \mathsf{code}}$$

$$\textsc{S-All}\quad \frac{\begin{array}{c} T_1 = |A_2|^{\forall} \qquad T_2 = |B_2|^{\forall} \\ B_2{}^{\circ} \qquad B_1 <: A_1 \\ x_0 : A_2 <:^{+} y_0 : B_2 \rightsquigarrow J \end{array}}{x : \forall X {*} A_1.\ A_2 <:^{\pm} y : \forall X {*} B_1.\ B_2 \rightsquigarrow \mathsf{code}}$$

$$\textsc{S-Rcd}\quad \frac{\begin{array}{c} T_1 = \{\ell : |A|\} \\ T_2 = \{\ell : |B|\} \\ B^{\circ} \qquad x_0 : A <:^{+} y_0 : B \rightsquigarrow J \end{array}}{x : \{\ell : A\} <:^{\pm} y : \{\ell : B\} \rightsquigarrow \mathsf{code}}$$

$$\textsc{S-AndL}\quad \frac{C^{\circ} \qquad x : A <:^{-} y : C \rightsquigarrow J}{x : A \,\&\, B <:^{\pm} y : C \rightsquigarrow J}$$

$$\textsc{S-AndR}\quad \frac{C^{\circ} \qquad x : B <:^{-} y : C \rightsquigarrow J}{x : A \,\&\, B <:^{\pm} y : C \rightsquigarrow J}$$

$$\textsc{S-Split}\quad \frac{\begin{array}{c} B_1 \lhd B \rhd B_2 \\ y_1 : B_1 \rhd z : B \lhd y_2 : B_2 \rightsquigarrow J_3 \\ x : A <:^{\pm} y_1 : B_1 \rightsquigarrow J_1 \\ x : A <:^{\pm} y_2 : B_2 \rightsquigarrow J_2 \end{array}}{x : A <:^{\pm} z : B \rightsquigarrow \mathsf{code}}$$

```
/* S-Bot */
y[T] = null;

/* S-Eq */
Object.assign(y, x);

/* S-Base */
y[T] = x[T];

/* S-Var */
for (var T of X) {
  y[T] = x[T];
}

/* S-Arrow */
y[T2] = (x1, y2) => {
  var y1 = {};  J1;
  var x2 = {};
  x[T1](y1, x2);
  J2;
};

/* S-All */
y[T2] = (X, y0) => {
  var x0 = {};
  x[T1](X, x0);
  J;
};

/* S-Rcd */
var x0 = x[T1];
var y0 = {};  J;
y[T2] = y0;

/* S-Split */
var y1 = {}; // if y1 != z
var y2 = {}; // if y2 != z
J1;  J2;  J3;
```

$\boxed{x : A \ \rhd\ z : C \ \lhd\ y : B \ \leadsto\ J}$ *(Coercive merging)*

M-And

$$\frac{}{z : A \ \rhd\ z : A \,\&\, B \ \lhd\ z : B \ \leadsto\ \varnothing}$$

M-Arrow

$$\frac{\begin{array}{c} T = |\overrightarrow{B}| \\ T_1 = |\overrightarrow{B_1}| \qquad T_2 = |\overrightarrow{B_2}| \\ y_1 : B_1 \ \rhd\ y : B \ \lhd\ y_2 : B_2 \ \leadsto\ J \end{array}}{x_1 : A \to B_1 \ \rhd\ z : A \to B \ \lhd\ x_2 : A \to B_2 \ \leadsto\ \mathtt{code}}$$

M-All

$$\frac{\begin{array}{c} T = |B|^{\forall} \\ T_1 = |B_1|^{\forall} \qquad T_2 = |B_2|^{\forall} \\ y_1 : B_1 \ \rhd\ y : B \ \lhd\ y_2 : B_2 \ \leadsto\ J \end{array}}{x_1 : \forall X * A.\ B_1 \ \rhd\ z : \forall X * A.\ B \ \lhd\ x_2 : \forall X * A.\ B_2 \ \leadsto\ \mathtt{code}}$$

M-Rcd

$$\frac{\begin{array}{c} T = \{\ell : |A|\} \\ T_1 = \{\ell : |A_1|\} \\ T_2 = \{\ell : |A_2|\} \\ y_1 : A_1 \ \rhd\ y : A \ \lhd\ y_2 : A_2 \ \leadsto\ J \end{array}}{x_1 : \{\ell : A_1\} \ \rhd\ z : \{\ell : A\} \ \lhd\ x_2 : \{\ell : A_2\} \ \leadsto\ \mathtt{code}}$$

```
/* M-Arrow */
z[T] = (p, y) => {
  var y1 = {}; // if y1 != y
  var y2 = {}; // if y2 != y
  x1[T1](p, y1);
  x2[T2](p, y2);
  J;
};
```

```
/* M-All */
z[T] = (X, y) => {
  var y1 = {}; // if y1 != y
  var y2 = {}; // if y2 != y
  x1[T1](X, y1);
  x2[T2](X, y2);
  J;
};
```

```
/* M-Rcd */
var y = {};
var y1 = {}; // if y1 != y
var y2 = {}; // if y2 != y
Object.assign(y1, x1[T1]);
Object.assign(y2, x2[T2]);
J;
z[T] = y;
```