Compiling from F_i^+ to JavaScript

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Syntax of F_i^+

Types	$A, B, C ::= \top \mid \bot \mid \mathbb{B} \mid X \mid A \to B \mid \forall X * A.\ B \mid \{\ell : A\} \mid A \,\&\, B$
Type indices	$T ::= \mathbb{B} \mid \overrightarrow{T} \mid T^\forall \mid \{\ell : T\} \mid T_1 \,\&\, T_2$
Expressions	$e ::= \{\} \mid b \mid x \mid \mathbf{fix}\ x\!:\!A.\ e \mid \lambda x\!:\!A.\ e\!:\!B \mid e_1\ e_2 \mid \Lambda X * A.\ e\!:\!B \mid e\ A \mid \{\ell = e\} \mid e.\ell$
	$\mid e_1 \,,,\, e_2 \mid e : A$
Values	$v ::= \{\} \mid b \mid \lambda x\!:\!A.\ e\!:\!B \mid \Lambda X * A.\ e\!:\!B \mid \{\ell = v\} \mid v_1 \,,,\, v_2$

$\boxed{|A| = T}$ *(Type translation)*

$$|\mathbb{B}| = \mathbb{B} \qquad |X| = X \qquad |\forall X * A.\ B| = |B|^\forall \qquad |A \to B| = \overrightarrow{|B|} \qquad |\{\ell : A\}| = \{\ell : |A|\} \qquad |A \,\&\, B| = |A| \,\&\, |B|$$

$\boxed{\rceil A \lceil}$ *(Top-like types)*

$$\frac{}{\rceil \top \lceil}\ \text{TL-Top} \qquad \frac{\rceil A \lceil \qquad \rceil B \lceil}{\rceil A \,\&\, B \lceil}\ \text{TL-And} \qquad \frac{\rceil B \lceil}{\rceil A \to B \lceil}\ \text{TL-Arrow} \qquad \frac{\rceil B \lceil}{\rceil \forall X * A.\ B \lceil}\ \text{TL-All} \qquad \frac{\rceil A \lceil}{\rceil \{\ell : A\} \lceil}\ \text{TL-Rcd}$$

$\boxed{A^\circ}$ *(Ordinary types)*

$$\frac{}{\top^\circ}\ \text{O-Top} \qquad \frac{}{\bot^\circ}\ \text{O-Bot} \qquad \frac{}{\mathbb{B}^\circ}\ \text{O-Base} \qquad \frac{}{X^\circ}\ \text{O-Var} \qquad \frac{B^\circ}{(A \to B)^\circ}\ \text{O-Arrow} \qquad \frac{B^\circ}{(\forall X * A.\ B)^\circ}\ \text{O-All} \qquad \frac{A^\circ}{\{\ell : A\}^\circ}\ \text{O-Rcd}$$

$$\boxed{\Gamma \vdash e \Leftrightarrow A \;\boxed{\leadsto J \mid z^{\pm}}} \hspace{5cm} \textit{(Type-directed compilation)}$$

J-Gen
$$\frac{\Gamma \vdash e \Leftrightarrow A \;\boxed{\leadsto J \mid z^{-}}}{\Gamma \vdash e \Leftrightarrow A \;\boxed{\leadsto S_0 \mid z^{+}}}$$

J-Top
$$\frac{}{\Gamma \vdash \{\} \Rightarrow \top \;\boxed{\leadsto \varnothing \mid z^{-}}}$$

J-TopAbs
$$\frac{\rceil B \lceil}{\Gamma \vdash \lambda x\!:\!A.\, e\!:\!B \Rightarrow A \rightarrow B \;\boxed{\leadsto \varnothing \mid z^{-}}}$$

J-TopTAbs
$$\frac{\rceil B \lceil}{\Gamma \vdash \Lambda X * A.\, e\!:\!B \Rightarrow \forall X * A.\, B \;\boxed{\leadsto \varnothing \mid z^{-}}}$$

J-TopRcd
$$\frac{\Gamma \vdash e \Rightarrow A \qquad \rceil A \lceil}{\Gamma \vdash \{\ell = e\} \Rightarrow \{\ell : A\} \;\boxed{\leadsto \varnothing \mid z^{-}}}$$

J-Base
$$\frac{T = |\mathbb{B}|}{\Gamma \vdash b \Rightarrow \mathbb{B} \;\boxed{\leadsto S_1 \mid z^{-}}}$$

J-Var
$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \;\boxed{\leadsto S_2 \mid z^{-}}}$$

J-VarGen
$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \;\boxed{\leadsto S_3 \mid z^{+}}}$$

J-Fix
$$\frac{\Gamma, x : A \vdash e \Leftarrow A \;\boxed{\leadsto J \mid y^{-}}}{\Gamma \vdash \mathbf{fix}\, x\!:\!A.\, e \Rightarrow A \;\boxed{\leadsto S_4 \mid z^{-}}}$$

J-Abs
$$\frac{T = \overrightarrow{|B|} \qquad \Gamma, x : A \vdash e \Leftarrow B \;\boxed{\leadsto J \mid y^{+}}}{\Gamma \vdash \lambda x\!:\!A.\, e\!:\!B \Rightarrow A \rightarrow B \;\boxed{\leadsto S_5 \mid z^{-}}}$$

J-App
$$\frac{\begin{array}{c}\Gamma \vdash e_1 \Rightarrow A \;\boxed{\leadsto J_1 \mid x^{+}} \\ A \rhd B \rightarrow C \\ \Gamma \vdash e_2 \Leftarrow B \;\boxed{\leadsto J_2 \mid y^{+}} \\ x : A \bullet y_0 \;\boxed{\leadsto J_3 \mid z}\end{array}}{\Gamma \vdash e_1\, e_2 \Rightarrow C \;\boxed{\leadsto S_6 \mid z^{-}}}$$

J-TAbs
$$\frac{T = |B|^{\forall} \qquad \Gamma, X * A \vdash e \Leftarrow B \;\boxed{\leadsto J \mid y^{+}}}{\Gamma \vdash \Lambda X * A.\, e\!:\!B \Rightarrow \forall X * A.\, B \;\boxed{\leadsto S_7 \mid z^{-}}}$$

J-TApp
$$\frac{\begin{array}{c}\Gamma \vdash e \Rightarrow B \;\boxed{\leadsto J_1 \mid y^{+}} \\ B \rhd \forall X * C_1.\, C_2 \\ \Gamma \vdash A * C_1 \\ y : B \bullet A \;\boxed{\leadsto J_2 \mid z}\end{array}}{\Gamma \vdash e\, A \Rightarrow C_2[X \mapsto A] \;\boxed{\leadsto J_1 ; J_2 \mid z^{-}}}$$

J-Rcd
$$\frac{T = \{\ell : |A|\} \qquad \Gamma \vdash e \Rightarrow A \;\boxed{\leadsto J \mid y^{+}}}{\Gamma \vdash \{\ell = e\} \Rightarrow \{\ell : A\} \;\boxed{\leadsto S_8 \mid z^{-}}}$$

J-Proj
$$\frac{\begin{array}{c}\Gamma \vdash e \Rightarrow A \;\boxed{\leadsto J_1 \mid y^{+}} \\ A \rhd \{\ell : B\} \\ y : A \bullet \ell \;\boxed{\leadsto J_2 \mid z}\end{array}}{\Gamma \vdash e.\ell \Rightarrow B \;\boxed{\leadsto J_1 ; J_2 \mid z^{-}}}$$

J-Merge
$$\frac{\begin{array}{c}\Gamma \vdash e_1 \Rightarrow A \;\boxed{\leadsto J_1 \mid z^{-}} \\ \Gamma \vdash e_2 \Rightarrow B \;\boxed{\leadsto J_2 \mid z^{-}} \\ \Gamma \vdash A * B\end{array}}{\Gamma \vdash e_1 \,,, e_2 \Rightarrow A \,\&\, B \;\boxed{\leadsto J_1 ; J_2 \mid z^{-}}}$$

J-Anno
$$\frac{\Gamma \vdash e \Leftarrow A \;\boxed{\leadsto J \mid z^{-}}}{\Gamma \vdash e : A \Rightarrow A \;\boxed{\leadsto J \mid z^{-}}}$$

J-AnnoGen
$$\frac{\Gamma \vdash e \Leftarrow A \;\boxed{\leadsto J \mid z^{+}}}{\Gamma \vdash e : A \Rightarrow A \;\boxed{\leadsto J \mid z^{+}}}$$

J-Sub
$$\frac{\begin{array}{c}\Gamma \vdash e \Rightarrow A \;\boxed{\leadsto J_1 \mid x^{+}} \\ x : A <: y : B \;\boxed{\leadsto J_2}\end{array}}{\Gamma \vdash e \Leftarrow B \;\boxed{\leadsto J_1 ; J_2 \mid y^{-}}}$$

J-SubEq
$$\frac{\Gamma \vdash e \Rightarrow A \;\boxed{\leadsto J \mid z^{-}}}{\Gamma \vdash e \Leftarrow A \;\boxed{\leadsto J \mid z^{-}}}$$

J-SubEqGen
$$\frac{\Gamma \vdash e \Rightarrow A \;\boxed{\leadsto J \mid z^{+}}}{\Gamma \vdash e \Leftarrow A \;\boxed{\leadsto J \mid z^{+}}}$$

```
/* S0 */                  /* S4 */                    var y0 = () => {
var z = {};   J;          Object.assign(z,              J2; return y;
                            new function() {          };
                              var y = this;           J3;
/* S1 */                      var x = () => y;
z[T] = b;                     J;                      /* S7 */
                            }                         z[T] = X => {
                          );                            J;   return y;
/* S2 */                                              };
Object.assign(z, x());    /* S5 */
                          z[T] = x => {               /* S8 */
                            J;   return y;            z[T] = () => {
/* S3 */                  };                            J;   return y;
var z = x();                                          };
                          /* S6 */
                          J1;
```

$$\boxed{x : A \bullet arg \;\boxed{\leadsto J \mid z}}$$ $\hfill$ *(Distributive application)*

$$
\frac{\lceil A \rceil}{x : A \bullet arg \;\boxed{\leadsto \varnothing \mid z}} \; \text{A-Top}
\qquad
\frac{T = \overrightarrow{|B|}}{x : A \to B \bullet y \;\boxed{\leadsto S_9 \mid z}} \; \text{A-Arrow}
\qquad
\frac{T_1 = |B|^{\forall} \qquad T_2 = |C|}{x : \forall X * A.\, B \bullet C \;\boxed{\leadsto S_9' \mid z}} \; \text{A-All}
$$

$$
\frac{T = \{\ell : |A|\}}{x : \{\ell : A\} \bullet \ell \;\boxed{\leadsto S_9'' \mid z}} \; \text{A-Rcd}
\qquad
\frac{\begin{array}{c} x : A \bullet arg \;\boxed{\leadsto J_1 \mid z} \\ x : B \bullet arg \;\boxed{\leadsto J_2 \mid z} \end{array}}{x : A \,\&\, B \bullet arg \;\boxed{\leadsto J_1 ; J_2 \mid z}} \; \text{A-And}
$$

```
/* S9 */                      /* S9' */                     /* S9'' */
Object.assign(z, x[T](y));    Object.assign(z, x[T1](T2));   Object.assign(z, x[T]());
```

$$\boxed{x : A <: y : B \;\leadsto J}$$ (Coercive subtyping)

S-Arrow
$$T_1 = \overrightarrow{|A_2|} \qquad T_2 = \overrightarrow{|B_2|}$$
$$B_2{}^\circ \qquad B_1 <: A_1$$
$$\frac{x_2 : A_2 <: y_2 : B_2 \;\leadsto J_2}{x : A_1 \to A_2 <: y : B_1 \to B_2 \;\leadsto S_{11}}$$

S-Top
$$\frac{B^\circ \qquad \rceil B \lceil}{x : A <: y : B \;\leadsto \varnothing}$$

S-Base
$$\frac{T = |\mathbb{B}|}{x : \mathbb{B} <: y : \mathbb{B} \;\leadsto S_{10}}$$

S-All
$$T_1 = |A_2|^\forall \qquad T_2 = |B_2|^\forall$$
$$B_2{}^\circ \qquad B_1 <: A_1$$
$$\frac{x_2 : A_2 <: y_2 : B_2 \;\leadsto J_2}{x : \forall X * A_1.\, A_2 <: y : \forall X * B_1.\, B_2 \;\leadsto S_{12}}$$

S-Rcd
$$T_1 = \{\ell : |A|\}$$
$$T_2 = \{\ell : |B|\}$$
$$\frac{B^\circ \qquad x_0 : A <: y_0 : B \;\leadsto J}{x : \{\ell : A\} <: y : \{\ell : B\} \;\leadsto S_{13}}$$

S-AndL
$$\frac{C^\circ \qquad x : A <: y : C \;\leadsto J}{x : A \,\&\, B <: y : C \;\leadsto J}$$

S-AndR
$$\frac{C^\circ \qquad x : B <: y : C \;\leadsto J}{x : A \,\&\, B <: y : C \;\leadsto J}$$

S-Split
$$B_1 \lhd B \rhd B_2$$
$$x : A <: y_1 : B_1 \;\leadsto J_1$$
$$x : A <: y_2 : B_2 \;\leadsto J_2$$
$$\frac{y_1 : B_1 \rhd z : B \lhd y_2 : B_2 \;\leadsto J_3}{x : A <: z : B \;\leadsto S_{14}}$$

```
/* S10 */
y[T] = x[T];

/* S11 */
y[T2] = p => {
  var x2 = x[T1](p);
  var y2 = {};   J2;
  return y2;
};
```

```
/* S12 */
y[T2] = X => {
  var x2 = x[T1](X);
  var y2 = {};   J2;
  return y2;
};

/* S13 */
y[T2] = () => {
  var x0 = x[T1]();
```

```
  var y0 = {};   J;
  return y0;
}

/* S14 */
var y1 = {};   J1;
var y2 = {};   J2;
J3;
```

$$\boxed{x : A \;\triangleright\; z : C \;\triangleleft\; y : B \;\rightsquigarrow\; J}$$ (Coercive merging)

M-ARROW
$$\frac{\begin{array}{c} T = \overrightarrow{|B|} \\ T_1 = \overrightarrow{|B_1|} \qquad T_2 = \overrightarrow{|B_2|} \\ y_1 : B_1 \;\triangleright\; y : B \;\triangleleft\; y_2 : B_2 \;\boxed{\rightsquigarrow\; J} \end{array}}{x_1 : A \to B_1 \;\triangleright\; z : A \to B \;\triangleleft\; x_2 : A \to B_2 \;\boxed{\rightsquigarrow\; S_{16}}}$$

M-AND
$$\frac{T_1 = |A| \qquad T_2 = |B|}{x : A \;\triangleright\; z : A \,\&\, B \;\triangleleft\; y : B \;\boxed{\rightsquigarrow\; S_{15}}}$$

M-ALL
$$\frac{\begin{array}{c} T = |B|^\forall \\ T_1 = |B_1|^\forall \qquad T_2 = |B_2|^\forall \\ y_1 : B_1 \;\triangleright\; y : B \;\triangleleft\; y_2 : B_2 \;\boxed{\rightsquigarrow\; J} \end{array}}{x_1 : \forall X * A.\, B_1 \;\triangleright\; z : \forall X * A.\, B \;\triangleleft\; x_2 : \forall X * A.\, B_2 \;\boxed{\rightsquigarrow\; S_{17}}}$$

M-RCD
$$\frac{\begin{array}{c} T = \{\ell : |A|\} \\ T_1 = \{\ell : |A_1|\} \\ T_2 = \{\ell : |A_2|\} \\ y_1 : A_1 \;\triangleright\; y : A \;\triangleleft\; y_2 : A_2 \;\boxed{\rightsquigarrow\; J} \end{array}}{x_1 : \{\ell : A_1\} \;\triangleright\; z : \{\ell : A\} \;\triangleleft\; x_2 : \{\ell : A_2\} \;\boxed{\rightsquigarrow\; S_{18}}}$$

```
/* S15 */
Object.assign(z, x, y);

/* S16 */
z[T] = p => {
  var y1 = x1[T1](p);
  var y2 = x2[T2](p);
  var y = {};   J;
  return y;
};
```

```
/* S17 */
z[T] = X => {
  var y1 = x1[T1](X);
  var y2 = x2[T2](X);
  var y = {};   J;
  return y;
};
```

```
/* S18 */
z[T] = () => {
  var y1 = x1[T1]();
  var y2 = x2[T2]();
  var y = {};   J;
  return y;
};
```