EE23BTECH11042 - Khusinadha Naik*

Exercise 9.3

29. If A and G be A.M. and G.M., respectively between two positive numbers, prove that the numbers are A $\pm\sqrt{(A+G)(A-G)}$.

Ans.

Parameter	Value	Description
$x_1(n)$	$(x_1(0)+nd)\,u(n)$	AP series
$x_2(n)$	$(x_2(0)\cdot r^n)\,u(n)$	GP series
$x_1(0), x_2(0)$	a	First number
$x_1(2), x_2(2)$	b	Second number
$x_1(1)$	$(x_1(0)+d)\,u(n)$	A.M.(A)
$x_2(1)$	$(x_1(0)\cdot r)\,u(n)$	G.M.(B)

TABLE I
Input parameters table

From Table I

$$x_1(2) = x_2(2) \tag{1}$$
$$(x_1(0)+2d)\,u(n) = \left(x_1(0)\,r^2\right)u(n) \tag{2}$$
$$2d = x_1(0)\left(r^2-1\right) \tag{3}$$

Now the two numbers are

$$(a,b) = (x_1(0)\,u(n), (x_1(0)+d)\,u(n)) \tag{4}$$
$$= (x_1(0)+d\pm 2d)\,u(n) \tag{5}$$
$$= \left(x_1(0)+d\pm\sqrt{d^2}\right)u(n) \tag{6}$$
$$= (x_1(0)+d)\,u(n)\pm \sqrt{2x_1(0)\,d-2x_1(0)\,d+d^2}u(n) \tag{7}$$

Substituting (3) in (7):

$$(a,b) = (x_1(0)+d)\,u(n)\pm \sqrt{2x_1(0)d+x_1(0)x_1(0)(1-r^2)+d^2}u(n) \tag{8}$$
$$= (x_1(0)+d)\,u(n)\pm \sqrt{x_1(0)^2+d^2+2x_1(0)d-x_1(0)^2r^2}u(n) \tag{9}$$
$$= (x_1(0)+d)\,u(n)\pm \sqrt{((x_1(0)+d)\,u(n))^2-((x_1(0)\,r)\,u(n))^2} \tag{10}$$

Comparing (10) , Table I

$$(a,b) = A\pm\sqrt{A^2-G^2} \tag{11}$$
$$\implies (a,b) = A\pm\sqrt{(A+G)(A-G)} \tag{12}$$

$$u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{1-z^{-1}} \quad ,|z|>1 \tag{13}$$
$$nu(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}}{(1-z^{-1})^2} \quad ,|z|>1 \tag{14}$$
$$a^n u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{1-az^{-1}} \quad ,|z|>a \tag{15}$$

From (13) , (14)

$$x_1(z) = \frac{x(0)}{1-z^{-1}}+\frac{dz^{-1}}{(1-z^{-1})^2} \quad ,|z|>1 \tag{16}$$

From (15)

$$x_2(z) = \frac{x(0)}{1-rz^{-1}} \quad ,|z|>r \tag{17}$$