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EE23BTECH11042 - Khusinadha Naik*

Exercise 9.3

29. If A and G be A.M. and G.M., respectively between two positive numbers, prove that the numbers are A $\pm \sqrt{(A+G)(A-G)}$.

Ans.

$(a,b) = A \pm \sqrt{A^2 - G^2}$	(11)
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Comparing (??), Table ??

$$\implies$$
 $(a,b) = A \pm \sqrt{(A+G)(A-G)}$ (12)

Parameter	Value	Description	
$x_1(n)$	$\left(x_{1}\left(0\right)+nd\right)u\left(n\right)$	AP series	
$x_2(n)$	$(x_2(0)\cdot r^n)u(n)$	GP series	
$x_1(0), x_2(0)$	a	First number	
$x_1(2), x_2(2)$	b	Second number	
$x_1(1)$	$\left(x_{1}\left(0\right)+d\right)u\left(n\right)$	A.M.(A)	
$x_2(1)$	$(x_1(0)\cdot r)u(n)$	G.M.(<i>B</i>)	
TABLE I			

INPUT PARAMETERS TABLE

From Table ??

$$x_1(2) = x_2(2) \tag{1}$$

$$(x_1(0) + 2d) u(n) = (x_1(0) r^2) u(n)$$
 (2)

$$2d = x_1(0)(r^2 - 1) \tag{3}$$

Now the two numbers are

$$(a,b) = (x_1(0) u(n), (x_1(0) + d) u(n))$$
 (4)

$$= (x_1(0) + d \pm 2d) u(n)$$
 (5)

$$= \left(x_1\left(0\right) + d \pm \sqrt{d^2}\right) u\left(n\right) \tag{6}$$

$$= (x_1(0) + d) u(n) \pm$$

$$\sqrt{2x_1(0)d - 2x_1(0)d + d^2}u(n)$$
 (7)

Substituting (??) in (??):

$$(a,b) = (x_1(0) + d) u(n) \pm \sqrt{2x_1(0)d + x_1(0)x_1(0)(1 - r^2) + d^2} u(n)$$

$$= (x_1(0) + d) u(n) \pm \sqrt{x_1(0)^2 + d^2 + 2x_1(0)d - x_1(0)^2 r^2} u(n)$$
(9)
$$= (x_1(0) + d) u(n) \pm \sqrt{((x_1(0) + d) u(n))^2 - ((x_1(0) r) u(n))^2}$$

$$(10)$$