EE23BTECH11042 - Khusinadha Naik*

26. A causal, discrete time system is described by the difference equation $y[n] = 0.5y[n-1] + x[n]$, for all n, where $y[n]$ denotes the output sequence and $x[n]$ denotes the input sequence. Which of the following statements is/are TRUE?

(GATE 2023 BM)

(a) The system has an impulse response described by $0.5^n u[-n]$ where $u[n]$ is the unit step sequence.
(b) The system is stable in the bounded input, bounded output sense.
(c) The system has an infinite number of non-zero samples in its impulse response
(d) The system has a finite number of non-zero samples in its impulse response.

Ans.

Parameter	Value	Description
$x[n]$	?	Input Sequence
$y[n]$	?	Output Sequence

TABLE I
INPUT PARAMETERS TABLE

$$y[n] = 0.5y[n-1] + x[n] \tag{1}$$

Taking Z-Transform

$$Y(z) = 0.5z^{-1}Y(z) + X(z) \tag{2}$$

$$\implies \frac{Y(z)}{X(z)} = \frac{1}{1-0.5z^{-1}} = H(z) \tag{3}$$

If $x[n]$ is impulse input

$$\implies Y(z) = H(z) = \frac{1}{1-0.5z^{-1}} \tag{4}$$

From (4) pole lies at $z = 0.5$

$$a^n u(n) \overset{Z}{\leftrightarrow} \frac{1}{1-az^{-1}} \quad , |z| > a \tag{5}$$

From (4) , (5)

$$h[n] = 0.5^n u[n] \quad , |z| > 0.5 \tag{6}$$

Plotting $h[n]$ vs n

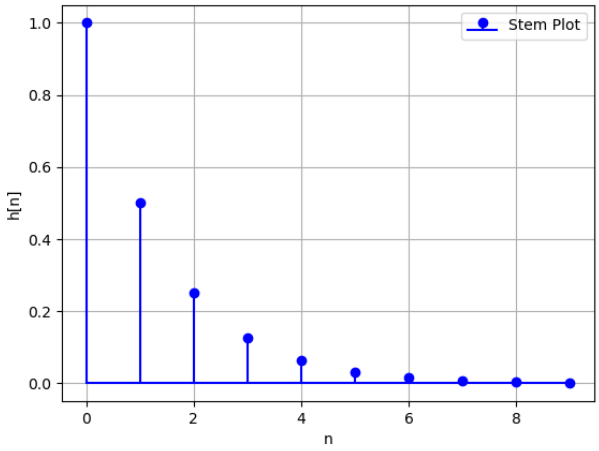

Fig. 1. Plot of $h[n]$ vs n

1) From (6) , (a) is wrong
2) As pole lies within unit circle (b) is true
3) From (6) and Fig. 1 ,(c) is true and hence
4) (d) is false