EE23BTECH11042 - Khusinadha Naik*

26. A causal, discrete time system is described by the difference equation $y[n] = 0.5y[n-1] + x[n]$, for all n, where $y[n]$ denotes the output sequence and $x[n]$ denotes the input sequence. Which of the following statements is/are TRUE?

(GATE 2023 BM)

(a) The system has an impulse response described by $0.5^n u[-n]$ where $u[n]$ is the unit step sequence.
(b) The system is stable in the bounded input, bounded output sense.
(c) The system has an infinite number of non-zero samples in its impulse response
(d) The system has a finite number of non-zero samples in its impulse response.

Ans.

Parameter	Value	Description
$x[n]$	?	Input Sequence
$y[n]$	?	Output Sequence

TABLE I
INPUT PARAMETERS TABLE

$$y[n] = 0.5y[n-1] + x[n] \tag{1}$$

Taking Z-Transform

$$Y(Z) = 0.5Z^{-1}Y(Z) + X(Z) \tag{2}$$

$$\implies \frac{Y(Z)}{X(Z)} = \frac{1}{1-0.5z^{-1}} = H(Z) \tag{3}$$

If $x[n]$ is impulse input

$$X(Z) = 1 \tag{4}$$

$$\implies Y(Z) = H(Z) = \frac{1}{1-0.5z^{-1}} \tag{5}$$

Taking inverse Z-Transform

$$y[n] = h[n] = 0.5^n u[n] \tag{6}$$

Hence (a) is wrong,

$y[n]$ is a non-zero infinite G.P. hence (c) holds true and (d) is false

Plotting $y[n]$ vs n

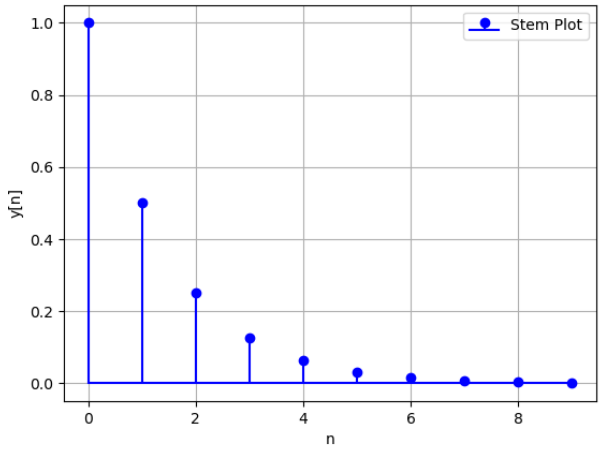

Fig. 1. Plot of $y[n]$ vs n

To check BIBO stability we'll check absolute convergence of values for $h[n]$

$$= \sum_{n=-\infty}^{+\infty} |\, h[n] \,| \tag{7}$$

$$= \sum_{n=-\infty}^{+\infty} |\, 0.5^n u[n] \,| \tag{8}$$

It's a infinite G.P. with a = 1 , r = 0.5

$$\sum_{n=-\infty}^{+\infty} |\, 0.5^n u[n] \,| = \frac{a}{1-r} \tag{9}$$

$$= \frac{1}{1-0.5} = 2 \tag{10}$$

As it is a finite value it is BIBO stable so (b) holds true.