數學思通

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SIR model

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - \gamma$$

$$\frac{dR}{dt} = \gamma I$$

S : the susceptibles who are capable of catching the disease and becoming infected.

I : the infectives who have the disease and can transmit it.

R : the removed class consisting of the individuals who are recovered with immunity or dead due to the disease.

β : the infection transmission rate.

γ : the rate of recovery.

N : total population, $N = S + I + R$.

t : per day.

Definition (R_0, basic reproduction number)**.**

$$R_0 = \frac{\beta N}{\gamma}.$$

0.1 求鑽石公主號和 2003 香港 SARS 的 R_0，引用論文中的 γ。

方法 1. 利用差分方程的方法估計 β。

$$\frac{dS}{dt} \cong \frac{\Delta S}{\Delta t} = \frac{S(t+1) - S(t)}{(t+1) - t}$$

$$= S(t+1) - S(t) = -\beta S(t) I(t)$$

$$\Rightarrow \beta = \frac{S(t) - S(t+1)}{S(t)I(t)}.$$

方法 2. 利用 Logistic function curve fitting 估計 β。

$P(t)$: the size of a population at time t.

K : carrying capacity.

r : intrinsic growth rate.

C : constant.

$$\frac{dP}{dt} = P(t)(r - \frac{r}{K}P(t))$$

$$\Rightarrow P(t) = \frac{K}{1 + Ce^{-rt}}.$$

0.2 觀察 R_0

T : 病毒平均傳播天數。

$I_c(m)$: m 天的累積病例數。

$$I_c(m) = 1 + R_0 + R_0^2 + ... + R_0^n = \frac{R_0^{n+1} - 1}{R_0 - 1} \text{ , } n = \frac{m}{T}.$$

$$S \xrightarrow{\beta SI} I \xrightarrow{\gamma I} R$$