數學思通

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Theorem (SIR model)**.**

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - \gamma$$

$$\frac{dR}{dt} = \gamma I$$

S *: the susceptibles who are capable of catching the disease and becoming infected.*

I *: the infectives who have the disease and can transmit it.*

R *: the removed class consisting of the individuals who are recovered with immunity or dead due to the disease.*

β *: the infection transmission rate.*

γ *: the rate of recovery.*

N *: total population,* $N = S + I + R$*.*

Problem (鑽石公主號、2003 香港 SARS 求感染率)**.**

$$\frac{dS}{dt} \cong \frac{\Delta S}{\Delta t} = \Delta S$$

$$= S_{t+1} - S_t = -\beta_t S_t I_t$$

$$\Rightarrow \beta_t = \frac{S_t - S_{t+1}}{S_t I_t}$$

$S_t = N - I_t - R_t$.

Problem (R_0, basic reproduction number)**.**

$$R_0 = \frac{\beta N}{\gamma}$$

設平均 T 天病毒傳播出去，N 天後之累積病例數為 R_0 之等比級數*:*

$$1 + R_0 + R_0^2 + ... + R_0^n = \frac{R_0^{n+1} - 1}{R_0 - 1},$$

$n = \frac{N}{T}$.

Problem (Logistic curve fitting idea)**.**

$$\begin{aligned}
\frac{dIC}{dt} &= \beta_t S_t I_t \\
&= \beta_t (N - It - R_t) I_t \\
&= \beta_t N (1 - \frac{IC_t}{N}) I_t \\
&\Rightarrow IC_t = \frac{N}{1 + (N-1)e^{-\beta_t N t}},
\end{aligned}$$

$IC_t = I_t + R_t$.