Iris Tutorial

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Preparation for this tutorial

- Clone the tutorial lecture material
  `https://iris-project.org/tutorial-popl21`
- Follow README to install Iris

What is Iris?

Language-independent higher-order separation logic with simple foundations for verifying fine-grained concurrent programs in Coq.

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- **Higher-order separation logic:** Supports modular reasoning about higher-order stateful programs
- **Fine-grained concurrent programs:** Programs that use low-level synchronization primitives for more parallelism
- **Language-independent:** Parameterized by the language
- **Simple foundations:** Small, "canonical" set of primitive rules
- **Coq:** Provides practical support for machine-checked proof

The versatility of Iris

Iris has been used to formalize many projects, ranging from program logics to logical relations to program proofs.

- RustBelt
- Perennial
- Many other examples

RustBelt: formalizing the Rust type system

Rust is a safe systems programming language with a sophisticated type system based on **ownership**, **borrowing**, and **lifetimes**.

- Safety of high-level Rust code relies on safe encapsulation of **unsafe code** in the lower layers.
- RustBelt uses Iris to build a **logical relation** for the Rust type system, formalizing the invariants encoded by the types.
- Borrowing and lifetimes are formalized by the lifetime logic, which puts Iris' flexibility to the test.
- RustBelt is able to verify the safety of `Mutex` and other Rust standard library abstractions.

Perennial: logic for crash-safety reasoning

Storage systems need proofs of correctness both under failures (due to kernel panic or disconnecting disk) and normal execution.

- Perennial uses Iris to build a variant of Hoare logic with a **crash condition** that holds at all intermediate points, even on failure.
- Iris gives the Perennial logic the flexibility to combine concurrency and failure reasoning.
- Perennial is built on top of a custom language which models the executable code written in **Go**.

Many other diverse projects using Iris

- Concurrent Search Templates uses Iris to prove some data structures correct
- Aneris is a program logic for distributed systems built using Iris
- Scala Step-by-Step formalizes soundness of the Scala type system using Iris to handle step-indexing
- Hazel is a sequential separation logic for effect handlers that uses Iris to handle recursive predicates (at this POPL 2021, first session on Friday!)

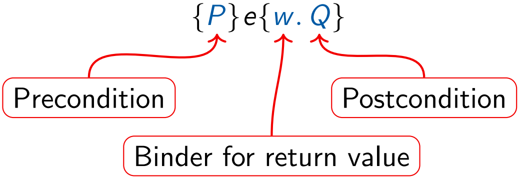
Outline

- **The Iris story, Part 1:** Working with invariants and ghost state
- **The Iris story, Part 2:** Modeling ghost state via “PCMs”
- **Iris in Coq:** The Interactive Proof Mode (IPM), live demo
- **Hands-on Iris:** Work on the exercises (we will be available for help throughout the conference) `https://iris-project.org/tutorial-popl21/`

The Iris story, Part 1:
Working with invariants and ghost state

Hoare triples

Hoare triples for partial program correctness:



$$\{P\}\,e\,\{w.\,Q\}$$

Precondition

Binder for return value

Postcondition

If the initial state satisfies P, then:

- e does not get stuck/crash
- if e terminates with value v, the final state satisfies $Q[v/w]$

Separation logic [O'Hearn, Reynolds, Yang]

The points-to connective $x \mapsto v$

- provides the knowledge that location x has value v, and
- provides exclusive ownership of x

Separating conjunction $P * Q$: the state consists of *disjoint parts* satisfying P and Q

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Example:

$$\{x \mapsto v_1 * y \mapsto v_2\}\mathit{swap}(x, y)\{w.\, w = () \wedge x \mapsto v_2 * y \mapsto v_1\}$$

the $*$ ensures that x and y are different

Concurrent separation logic [O'Hearn, Brookes]

The *par* rule:

$$\frac{\{P_1\}e_1\{Q_1\} \qquad \{P_2\}e_2\{Q_2\}}{\{P_1 * P_2\}e_1||e_2\{Q_1 * Q_2\}}$$

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For example:

$$\begin{array}{c}
\{x \mapsto 4 * y \mapsto 6\} \\
x := !\,x + 2 \;\Bigg\|\; y := !\,y + 2 \\
\{x \mapsto 6 * y \mapsto 8\}
\end{array}$$

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Works great for concurrent programs without shared memory: concurrent quick sort, . . .

What about shared state/racy programs?

A classic problem:

$$\texttt{let}\, x = \texttt{ref}(0)\, \texttt{in}$$

$$\begin{array}{l|l} \texttt{fetchandadd}(x, 2) & \texttt{fetchandadd}(x, 2) \end{array}$$

$$!\, x$$

Where $\texttt{fetchandadd}(x, y)$ is the atomic version of $x := !\, x + y$.

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Problem: can only give ownership of x to one thread

Invariants

The invariant assertion $\boxed{R}$ expresses that R is maintained as an invariant on the state

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Invariant opening:

$$\frac{\{R * P\}\, e\, \{R * Q\} \qquad e \text{ atomic}}{\left\{\boxed{R} * P\right\}\, e\, \left\{\boxed{R} * Q\right\}}$$

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Invariant duplication: $\boxed{R} \vdash \boxed{R} * \boxed{R}$

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Invariants in action

Let us consider a simpler problem first:

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$\texttt{let}\, x = \texttt{ref}(0)\, \texttt{in}$

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$\{n.\, \mathit{even}(n)\}$

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Problem: still cannot prove it returns 4

Ghost variables

Consider the invariant:

$$\boxed{\exists n.\, x \mapsto n * \ldots}$$

How to avoid **information loss** due to existential quantification?

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Consider the invariant:

$$\boxed{\exists n_1, n_2.\, x \mapsto (n_1 + n_2) * \gamma_1 \hookrightarrow_{\bullet} n_1 * \gamma_2 \hookrightarrow_{\bullet} n_2}$$

How to avoid **information loss** due to existential quantification?

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Ghost variables come in "entangled" pairs:

$$\underbrace{\gamma \hookrightarrow_{\bullet} n}_{\substack{\text{in the invariant} \\ \text{(“\textbf{authoritative}”)}}} \quad * \quad \underbrace{\gamma \hookrightarrow_{\circ} n}_{\substack{\text{in the Hoare triple} \\ \text{(“\textbf{fragment}”)}}}$$

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When you own both parts you obtain that the values are equal and can update both parts:

$$\gamma \hookrightarrow_\bullet n * \gamma \hookrightarrow_\circ m \quad \Rightarrow \quad n = m$$
$$\gamma \hookrightarrow_\bullet n * \gamma \hookrightarrow_\circ m \quad \mathrel{\Rrightarrow\!\!\!\!\ast} \quad \gamma \hookrightarrow_\bullet n' * \gamma \hookrightarrow_\circ n'$$

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\quad \{\gamma_1 \hookrightarrow_{\circ} 0 * x \mapsto (n_1 + n_2) * \gamma_1 \hookrightarrow_{\bullet} n_1 * \gamma_2 \hookrightarrow_{\bullet} n_2\} & \\
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\end{array}$$

$\{\gamma_1 \hookrightarrow_{\circ} 2 * \gamma_2 \hookrightarrow_{\circ} 2\}$

$!\,x$

$\{n.\ n = 4\}$

Ghost variables in action

$\{\mathsf{True}\}$

$\texttt{let}\, x = \texttt{ref}(0)\, \texttt{in}$

$\{x \mapsto 0\}$

$\{x \mapsto 0 * \gamma_1 \hookrightarrow_\bullet 0 * \gamma_1 \hookrightarrow_\circ 0 * \gamma_2 \hookrightarrow_\bullet 0 * \gamma_2 \hookrightarrow_\circ 0\}$

allocate $\boxed{\exists n_1, n_2.\, x \mapsto n_1 + n_2 * \gamma_1 \hookrightarrow_\bullet n_1 * \gamma_2 \hookrightarrow_\bullet n_2}$

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$\texttt{fetchandadd}(x, 2)$

$\{\gamma_1 \hookrightarrow_\circ 2\}$

$\|$

$\{\gamma_2 \hookrightarrow_\circ 0\}$

$\texttt{fetchandadd}(x, 2)$

$\{\gamma_2 \hookrightarrow_\circ 2\}$

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$\quad \texttt{fetchandadd}(x, 2)$

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$\|$

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$\{\gamma_2 \hookrightarrow_\circ 2\}$

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$!\, x$

$\{n.\, n = 4\}$

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$\{n.\ n = 4\}$

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$\texttt{let}\ x = \texttt{ref}(0)\ \texttt{in}$

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$\quad\texttt{fetchandadd}(x, 2)$

$\quad\{\gamma_1 \hookrightarrow_\circ 0 * x \mapsto (2 + n_2) * \gamma_1 \hookrightarrow_\bullet 0 * \gamma_2 \hookrightarrow_\bullet n_2\}$

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$\{\gamma_1 \hookrightarrow_\circ 2\}$

$\|$

$\{\gamma_2 \hookrightarrow_\circ 0\}$

$\quad\{\ldots\}$

$\quad\texttt{fetchandadd}(x, 2)$

$\quad\{\ldots\}$

$\{\gamma_2 \hookrightarrow_\circ 2\}$

$\{\gamma_1 \hookrightarrow_\circ 2 * \gamma_2 \hookrightarrow_\circ 2\}$

$!\,x$

$\{n.\ n = 4\}$

Ghost variables in action

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$\texttt{let}\ x = \texttt{ref}(0)\ \texttt{in}$

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$\texttt{fetchandadd}(x, 2)$

$\{\gamma_1 \hookrightarrow_\circ 0 * x \mapsto (2 + n_2) * \gamma_1 \hookrightarrow_\bullet 0 * \gamma_2 \hookrightarrow_\bullet n_2\}$

$\{\gamma_1 \hookrightarrow_\circ 2 * x \mapsto (2 + n_2) * \gamma_1 \hookrightarrow_\bullet 2 * \gamma_2 \hookrightarrow_\bullet n_2\}$

$\{\gamma_1 \hookrightarrow_\circ 2\}$

$\|$

$\{\gamma_2 \hookrightarrow_\circ 0\}$

$\{\dots\}$

$\texttt{fetchandadd}(x, 2)$

$\{\dots\}$

$\{\gamma_2 \hookrightarrow_\circ 2\}$

$\{\gamma_1 \hookrightarrow_\circ 2 * \gamma_2 \hookrightarrow_\circ 2\}$

$\{\gamma_1 \hookrightarrow_\circ 2 * \gamma_2 \hookrightarrow_\circ 2 * x \mapsto (n_1 + n_2) * \gamma_1 \hookrightarrow_\bullet n_1 * \gamma_2 \hookrightarrow_\bullet n_2\}$

$!\, x$

$\{n.\ n = 4\}$

Ghost variables in action

$\{\mathsf{True}\}$

$\texttt{let}\, x = \texttt{ref}(0)\ \texttt{in}$

$\{x \mapsto 0\}$

$\{x \mapsto 0 * \gamma_1 \hookrightarrow_{\bullet} 0 * \gamma_1 \hookrightarrow_{\circ} 0 * \gamma_2 \hookrightarrow_{\bullet} 0 * \gamma_2 \hookrightarrow_{\circ} 0\}$

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$\texttt{fetchandadd}(x, 2)$

$\{\gamma_1 \hookrightarrow_{\circ} 0 * x \mapsto (2 + n_2) * \gamma_1 \hookrightarrow_{\bullet} 0 * \gamma_2 \hookrightarrow_{\bullet} n_2\}$

$\{\gamma_1 \hookrightarrow_{\circ} 2 * x \mapsto (2 + n_2) * \gamma_1 \hookrightarrow_{\bullet} 2 * \gamma_2 \hookrightarrow_{\bullet} n_2\}$

$\{\gamma_1 \hookrightarrow_{\circ} 2\}$

$\|$

$\{\gamma_2 \hookrightarrow_{\circ} 0\}$

$\{\ldots\}$

$\texttt{fetchandadd}(x, 2)$

$\{\ldots\}$

$\{\gamma_2 \hookrightarrow_{\circ} 2\}$

$\{\gamma_1 \hookrightarrow_{\circ} 2 * \gamma_2 \hookrightarrow_{\circ} 2\}$

$\{\gamma_1 \hookrightarrow_{\circ} 2 * \gamma_2 \hookrightarrow_{\circ} 2 * x \mapsto (n_1 + n_2) * \gamma_1 \hookrightarrow_{\bullet} n_1 * \gamma_2 \hookrightarrow_{\bullet} n_2\}$

$!\, x$

$\{n.\ n = 4\}$

Ghost variables in action

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$\texttt{let}\ x = \texttt{ref}(0)\ \texttt{in}$

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$\quad \{\gamma_1 \hookrightarrow_\circ 2 * \gamma_2 \hookrightarrow_\circ 2 * x \mapsto 4 * \gamma_1 \hookrightarrow_\bullet 2 * \gamma_2 \hookrightarrow_\bullet 2\}$

$\quad !\,x$

$\{n.\ n = 4\}$

Ghost variables in action

$\{\mathsf{True}\}$

$\texttt{let}\, x = \texttt{ref}(0)\, \texttt{in}$

$\{x \mapsto 0\}$

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$\{\gamma_1 \hookrightarrow_\circ 2 * \gamma_2 \hookrightarrow_\circ 2\}$

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$\quad \{\gamma_1 \hookrightarrow_\circ 2 * \gamma_2 \hookrightarrow_\circ 2 * {\color{red} x \mapsto 4} * \gamma_1 \hookrightarrow_\bullet 2 * \gamma_2 \hookrightarrow_\bullet 2\}$

$\quad !\, x$

$\{n.\, n = 4\}$

Ghost variables in action

$$\{\mathsf{True}\}$$

$$\texttt{let}\; x = \texttt{ref}(0)\; \texttt{in}$$

$$\{x \mapsto 0\}$$

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$$\quad \{\gamma_1 \hookrightarrow_\circ 2 * \gamma_2 \hookrightarrow_\circ 2 * x \mapsto 4 * \gamma_1 \hookrightarrow_\bullet 2 * \gamma_2 \hookrightarrow_\bullet 2\}$$

$$\quad !\, x$$

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$$\{\mathsf{True}\}$$

$$\texttt{let}\, x = \texttt{ref}(0)\, \texttt{in}$$

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$$\quad !\, x$$

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$$\{n.\, n = 4\}$$

The Iris story, Part 2:

Modeling ghost state via “PCMs”

Mechanisms for concurrent reasoning

We have seen so far:

- Invariants $\boxed{R}^{\mathcal{N}}$
- Ghost variables $\gamma \hookrightarrow_{\bullet} n$ and $\gamma \hookrightarrow_{\circ} n$

You may also have heard of:

- Fractional permissions $a \mapsto_{\pi} v$
- State-transition systems and monotonic state

How can we make sure we have all the mechanisms we will need?

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How can we make sure we have all the mechanisms we will need?
The Iris story: these mechanisms can be **encoded** using a simple mechanism of *ghost resource ownership*

Resource algebras (RAs): A generalization of PCMs

Resource algebra (RA) with carrier M:

- Composition $(\cdot) : M \to M \to M$
- Validity predicate $\mathcal{V} \subseteq M$

Satisfying:

$$a \cdot b = b \cdot a \qquad a \cdot (b \cdot c) = (a \cdot b) \cdot c \qquad (a \cdot b) \in \mathcal{V} \Rightarrow a \in \mathcal{V}$$

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Iris provides $\boxed{a : M}^{\gamma}$ expressing ownership of an element a of resource algebra M (with name γ)

Ghost variables revisited

Resource algebra for ghost variables:

$$M \triangleq \bullet\, n \mid \circ\, n \mid \bullet\!\!\circ\, n \mid \bot$$

$$\mathcal{V} \triangleq \{a \neq \bot \mid a \in M\}$$

$$\bullet\, n \cdot \circ\, n' = \circ\, n' \cdot \bullet\, n \triangleq \begin{cases} \bullet\!\!\circ\, n & \text{if } n = n' \\ \bot & \text{otherwise} \end{cases}$$

$$\text{other combinations} \triangleq \bot$$

Ghost variables revisited

Resource algebra for ghost variables:

$$
\begin{aligned}
M &\triangleq \bullet\, n \mid \circ\, n \mid \bullet\!\circ\, n \mid \bot \\
\mathcal{V} &\triangleq \{a \neq \bot \mid a \in M\} \\
\bullet\, n \cdot \circ\, n' = \circ\, n' \cdot \bullet\, n &\triangleq \begin{cases} \bullet\!\circ\, n & \text{if } n = n' \\ \bot & \text{otherwise} \end{cases} \\
\text{other combinations} &\triangleq \bot
\end{aligned}
$$

And define:

$$\gamma \hookrightarrow_{\bullet} n \triangleq \boxed{\bullet\, n}^{\gamma} \qquad\qquad \gamma \hookrightarrow_{\circ} n \triangleq \boxed{\circ\, n}^{\gamma}$$

Ghost resource laws

Iris provides general laws for ghost resources:

$$a \in \mathcal{V} \Rrightarrow \exists \gamma.\, \boxed{a}^{\gamma} \qquad \boxed{a \cdot b}^{\gamma} \Leftrightarrow \boxed{a}^{\gamma} * \boxed{b}^{\gamma} \qquad \boxed{a}^{\gamma} \Rightarrow \mathcal{V}(a)$$

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The ghost variable laws follow from these:

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Remember:

$$\bullet\, n \cdot \circ\, n' = \circ\, n' \cdot \bullet\, n \triangleq \begin{cases} \bullet\!\circ\, n & \text{if } n = n' \\ \bot & \text{otherwise} \end{cases}$$

Ghost resource laws

Iris provides general laws for ghost resources:

$$a \in \mathcal{V} \Rrightarrow\!\!\!\!\ast\; \exists \gamma.\, \boxed{a}^{\gamma} \qquad \boxed{a \cdot b}^{\gamma} \Leftrightarrow \boxed{a}^{\gamma} * \boxed{b}^{\gamma} \qquad \boxed{a}^{\gamma} \Rightarrow \mathcal{V}(a)$$

The ghost variable laws follow from these:

$$\text{True} \Rrightarrow\!\!\!\!\ast\; \exists \gamma.\, \boxed{\bullet\!\!\circ\, n}^{\gamma} \Rrightarrow\!\!\!\!\ast\; \exists \gamma.\, \boxed{\bullet\, n}^{\gamma} * \boxed{\circ\, n}^{\gamma}$$

$$\boxed{\bullet\, n}^{\gamma} * \boxed{\circ\, m}^{\gamma} \Rightarrow n = m$$

Remember:

$$\bullet\, n \cdot \circ\, n' = \circ\, n' \cdot \bullet\, n \triangleq \begin{cases} \bullet\!\!\circ\, n & \text{if } n = n' \\ \bot & \text{otherwise} \end{cases}$$

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Updating resources

Resources can be *updated* using frame-preserving updates:

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Key idea: a resource can be updated if the update does not invalidate the resources of concurrently-running threads

$$\begin{array}{ccccccccc}
\text{Thread 1} & & \text{Thread 2} & & \ldots & & \text{Thread n} & \\
a & \cdot & a_2 & \cdot & \ldots & \cdot & a_n & \in \mathcal{V} \\
\rotatebox{-90}{$\rightsquigarrow$} & & & & & & & \\
b & \cdot & a_2 & \cdot & \ldots & \cdot & a_n & \in \mathcal{V}
\end{array}$$

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For ghost variables:

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$\bullet\!\circ\, n \cdot a_{\mathrm{f}} = \bot$, so the premise holds vacuously.

Generalizing to a library of RA combinators

Iris comes with a library of useful RA combinators

- $\textsc{Auth}(M)$: Generalizes the $\bullet$, $\circ$, $\bullet\!\circ$ construction over an arbitrary RA M – we call it the "authoritative" RA.
- $\textsc{Excl}(X)$: The "exclusive" RA, whose valid elements are the elements of X, and where composition is always undefined.
- $\textsc{Frac}$: The RA for fractions in $(0, 1]$ with addition.
- The expected RA liftings of products, sums, etc.

Using these combinators, we can easily construct the necessary models of many desired forms of ghost state:

- Ghost variables from this talk: $\textsc{Auth}\ (\textsc{Excl}\ \textsc{Nat})$
- Fractional ghost variables: $\textsc{Auth}\ (\textsc{Frac} \times \textsc{Nat}_{+})$

Iris in Coq:
The Interactive Proof Mode (IPM)

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Goal of IPM: reasoning in Iris in the same style as reasoning in Coq

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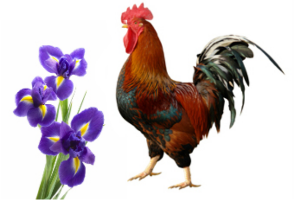
Features of IPM:

- Extends Coq with spatial contexts for Iris
- Tactics for introduction and elimination of all connectives of Iris
- Implemented entirely using reflection, type classes and Ltac (no OCaml plugin needed)



What's next?

- Exercises for this tutorial, in Coq
- Iris from the Ground Up
- Iris Lecture Notes