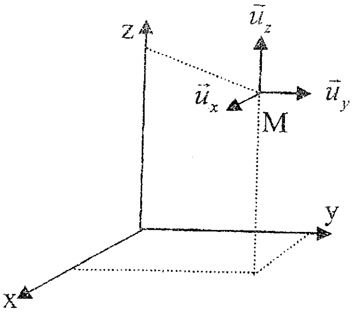
Licence et masters

1 LES SYTEMES de COORDONNEES

1. Cartésiennes

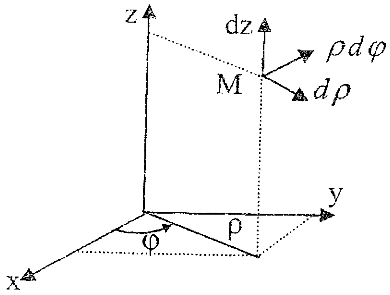


M(x, y, z)

$$M\left|\begin{array}{l}x\\y\\z\end{array}\right. \quad \overrightarrow{dS}\left|\begin{array}{l}\overrightarrow{dx}=dx\,\vec{u}_x\\ \overrightarrow{dy}=dy\,\vec{u}_y\\ \overrightarrow{dz}=dz\,\vec{u}_z\end{array}\right.$$

trièdre $\left(\vec{u}_x,\vec{u}_y,\vec{u}_z\right)$ *direct*

2. Cylindriques

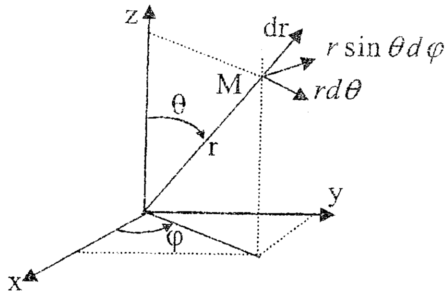


M(ρ, φ, z)

$$M\left|\begin{array}{l}x=\rho\cos\varphi\\y=\rho\sin\varphi\\z=z\end{array}\right. \quad \overrightarrow{dS}\left|\begin{array}{l}\overrightarrow{d\rho}=d\rho\,\vec{u}_\rho\\ \overrightarrow{\rho d\varphi}=d\varphi\,\vec{u}_\varphi\\ \overrightarrow{dz}=dz\,\vec{u}_z\end{array}\right.$$

trièdre $\left(\vec{u}_\rho,\vec{u}_\varphi,\vec{u}_z\right)$ *direct*

3. Sphériques



M(r, θ, φ)

$$M\left|\begin{array}{l}x=r\sin\theta\cos\varphi\\y=r\sin\theta\sin\varphi\\z=r\cos\theta\end{array}\right. \quad \overrightarrow{dS}\left|\begin{array}{l}\overrightarrow{rd\theta}=d\theta\,\vec{u}_\theta\\ r\sin\theta\overrightarrow{d\varphi}=d\varphi\,\vec{u}_\varphi\\ \overrightarrow{dr}=dr\,\vec{u}_r\end{array}\right.$$

trièdre $\left(\vec{u}_\theta,\vec{u}_\varphi,\vec{u}_r\right)$ *direct*

$$\left|\begin{array}{ll}\vec{u}_x=\vec{u}_\rho\cos\varphi-\vec{u}_\varphi\sin\varphi & \text{C1}\\ \vec{u}_y=\vec{u}_\rho\sin\varphi+\vec{u}_\varphi\cos\varphi & \text{C2}\\ \vec{u}_z=\vec{u}_z & \text{C3}\end{array}\right.$$

$$\left|\begin{array}{ll}\vec{u}_\rho=\vec{u}_x\cos\varphi+\vec{u}_y\sin\varphi & \text{C4}\\ \vec{u}_\varphi=-\vec{u}_x\sin\varphi+\vec{u}_y\cos\varphi & \text{C5}\\ \vec{u}_z=\vec{u}_z & \text{C6}\end{array}\right.$$

$$\left|\begin{array}{ll}\vec{u}_x=\vec{u}_r\sin\theta\cos\varphi+\vec{u}_\theta\cos\theta\cos\varphi-\vec{u}_\varphi\sin\varphi & \text{C7}\\ \vec{u}_y=\vec{u}_r\sin\theta\sin\varphi+\vec{u}_\theta\cos\theta\sin\varphi+\vec{u}_\varphi\cos\varphi & \text{C8}\\ \vec{u}_z=\vec{u}_r\cos\theta-\vec{u}_\theta\sin\theta & \text{C9}\end{array}\right.$$

$$\left|\begin{array}{ll}\vec{u}_r=\vec{u}_x\sin\theta\cos\varphi+\vec{u}_y\sin\theta\sin\varphi+\vec{u}_z\cos\theta & \text{C10}\\ \vec{u}_\theta=\vec{u}_x\cos\theta\cos\varphi+\vec{u}_y\cos\theta\sin\varphi-\vec{u}_z\sin\theta & \text{C11}\\ \vec{u}_\varphi=-\vec{u}_x\sin\varphi+\vec{u}_y\cos\varphi & \text{C12}\end{array}\right.$$

2 IDENTITES VECTORIELLES

Vecteurs $\vec{A},\vec{B},\vec{C},\vec{G},\vec{F},\vec{H}$

Scalaires f, g

$\nabla g=\overrightarrow{grad}\;g=\vec{\nabla}g$

$\nabla\vec{G}=div\;\vec{G}=\vec{\nabla}\vec{G}$

$\vec{\nabla}\wedge\vec{G}=\overrightarrow{rot}\;\vec{G}=\nabla\times\vec{G}$

I - 1 $\quad\vec{A}\wedge\left(\vec{B}\wedge\vec{C}\right)=\left(\vec{A}.\vec{C}\right)\vec{B}-\left(\vec{A}.\vec{B}\right)\vec{C}$

I - 2 $\quad\left(\vec{A}\wedge\vec{B}\right)\wedge\vec{C}=\left(\vec{C}.\vec{A}\right)\vec{B}-\left(\vec{C}.\vec{B}\right)\vec{A}$

I - 3 $\quad\nabla.\left(\vec{\nabla}\wedge\vec{G}\right)=0$

I - 4 $\quad\vec{\nabla}\wedge\nabla g=0$

I - 5 $\quad\nabla.\nabla g=\nabla^2 g=\Delta g$

I - 6 $\quad\nabla\left(f+g\right)=\nabla f+\nabla g$

I - 7 $\quad\nabla\left(\vec{F}+\vec{G}\right)=\nabla.\vec{F}+\nabla.\vec{G}$

I - 8 $\quad\nabla\left(fg\right)=g\nabla f+f\nabla g$

I - 9 $\quad\nabla\left(f\,\vec{G}\right)=\vec{G}\left(\nabla.f\right)+f\left(\nabla.\vec{G}\right)$

I - 10 $\quad\vec{\nabla}\wedge\left(f\,\vec{G}\right)=\left(\nabla f\right)\wedge\vec{G}+f\left(\vec{\nabla}\wedge\vec{G}\right)$

I - 11 $\quad\vec{\nabla}\wedge\left(\vec{\nabla}\wedge\vec{G}\right)=\nabla\left(\nabla.\vec{G}\right)-\nabla^2\vec{G}$ $\longleftarrow$ $\overrightarrow{rot}\,\overrightarrow{rot}\,\vec{G}=\overrightarrow{grad}\,div\,\vec{G}-\Delta\vec{G}$

I - 12 $\quad\nabla^2\vec{G}=\vec{u}_x\nabla^2 G_x+\vec{u}_y\nabla^2 G_y+\vec{u}_z\nabla^2 G_z$

I - 13 $\quad\vec{\nabla}.\left(\vec{F}\wedge\vec{G}\right)=\vec{G}.\left(\vec{\nabla}\wedge\vec{F}\right)-\vec{F}.\left(\vec{\nabla}\wedge\vec{G}\right)$

I - 14 $\quad\vec{F}.\left(\vec{G}\wedge\vec{H}\right)=\vec{G}.\left(\vec{H}\wedge\vec{F}\right)-\vec{H}.\left(\vec{F}\wedge\vec{G}\right)$

I - 15 $\quad\vec{\nabla}\wedge\left(\vec{F}\wedge\vec{G}\right)=\vec{F}.\left(\nabla.\vec{G}\right)-\vec{G}.\left(\nabla.\vec{F}\right)+\left(\vec{G}.\vec{\nabla}\right)\vec{F}-\left(\vec{F}.\vec{\nabla}\right)\vec{G}$

I -16 $\quad\vec{\nabla}.\left(\vec{F}.\vec{G}\right)=\left(\vec{F}.\vec{\nabla}\right)\vec{G}+\left(\vec{G}.\vec{\nabla}\right)\vec{F}+\vec{F}\wedge\left(\vec{\nabla}\wedge\vec{G}\right)+\vec{G}\wedge\left(\vec{\nabla}\wedge\vec{F}\right)$

I - 17 $\quad\iiint_v\nabla.\vec{G}\,dv=\iint_S\vec{G}.\vec{n}\,dS$ théorème de la divergence v fermé par S

I - 18 $\quad\iint_S\left(\vec{\nabla}\wedge\vec{G}\right).\,\vec{n}\,dS=\oint_\ell\vec{G}.\overrightarrow{d\ell}$ théorème de Stokes contour ℓ de la surface S