Licence et masters

3 OPERATEURS VECTORIELS DIFFERENTIELS

1 Cartésiens

$\overrightarrow{\text{grad}}$ O.1 $$\nabla g = \frac{\partial g}{\partial x}\vec{u}_x + \frac{\partial g}{\partial y}\vec{u}_y + \frac{\partial g}{\partial z}\vec{u}_z \quad = \overrightarrow{\text{grad}}\, g$$

div O.2 $$\nabla \vec{G} = \frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y} + \frac{\partial G_z}{\partial z} \quad = \text{div}\, \vec{G}$$

$\overrightarrow{\text{rot}}$ O.3 $$\vec{\nabla} \wedge \vec{G} = \left(\frac{\partial G_z}{\partial y} - \frac{\partial G_y}{\partial z}\right)\vec{u}_x + \left(\frac{\partial G_x}{\partial z} - \frac{\partial G_z}{\partial x}\right)\vec{u}_y + \left(\frac{\partial G_y}{\partial x} - \frac{\partial G_x}{\partial y}\right)\vec{u}_z \quad = \overrightarrow{\text{rot}}\, \vec{G}$$

Δ O.4 $$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} + \frac{\partial^2 g}{\partial z^2} \quad = \Delta g$$

2 Cylindriques

O.5 $$\nabla g = \frac{\partial g}{\partial \rho}\vec{u}_\rho + \frac{1}{\rho}\frac{\partial g}{\partial \varphi}\vec{u}_\varphi + \frac{\partial g}{\partial z}\vec{u}_z$$

O.6 $$\nabla \vec{G} = \frac{1}{\rho}\frac{\partial(\rho G_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial G_\varphi}{\partial \varphi} + \frac{\partial Gz}{\partial z}$$

O.7 $$\vec{\nabla} \wedge \vec{G} = \left(\frac{1}{\rho}\frac{\partial G_z}{\partial \varphi} - \frac{\partial G_\varphi}{\partial z}\right)\vec{u}_\rho + \left(\frac{\partial G_\rho}{\partial z} - \frac{\partial G_z}{\partial \rho}\right)\vec{u}_\varphi + \frac{1}{\rho}\left(\frac{\partial(\rho G_\varphi)}{\partial \rho} - \frac{\partial G_\rho}{\partial \varphi}\right)\vec{u}_z$$

O.8 $$\nabla^2 g = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial g}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 g}{\partial \varphi^2} + \frac{\partial^2 g}{\partial z^2}$$

3 Sphériques

O.9 $$\nabla g = \frac{\partial g}{\partial r}\vec{u}_r + \frac{1}{r}\frac{\partial g}{\partial \theta}\vec{u}_\theta + \frac{1}{r\sin\theta}\frac{\partial g}{\partial \varphi}\vec{u}_\varphi$$

O.10 $$\nabla \vec{G} = \frac{1}{r^2}\frac{\partial(r^2 G_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(G_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial G\varphi}{\partial \varphi}$$

O.11 $$\vec{\nabla} \wedge \vec{G} = \frac{1}{r\sin\theta}\left(\frac{\partial(G_\varphi \sin\theta)}{\partial \theta} - \frac{\partial G_\theta}{\partial \varphi}\right)\vec{u}_r$$
$$+\left(\frac{1}{r}\frac{1}{\sin\theta}\frac{\partial G_r}{\partial \varphi} - \frac{1}{r}\frac{\partial(r\,G_\varphi)}{\partial r}\right)\vec{u}_\theta$$
$$+\frac{1}{r}\left(\frac{\partial(rG_\theta)}{\partial r} - \frac{\partial G_r}{\partial \theta}\right)\vec{u}_\varphi$$

O.12 $$\nabla^2 g = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial g}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial g}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 g}{\partial \varphi^2}$$