A Comparison of Selected Methods of Inverse Kinematics for Robot Manipulators

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July 14, 2022

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Introduction

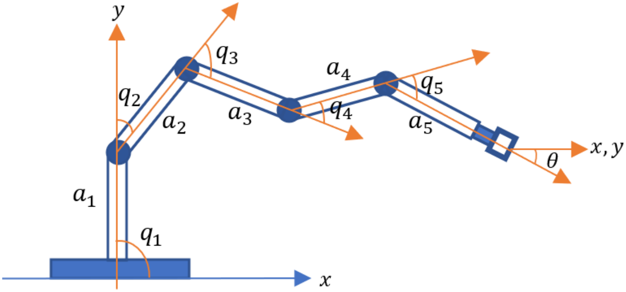


Figure: 5-DOF Manipulator Model

Objectives

- Solving the inverse kinematics problem using the Jacobi matrix methods based on the Newton-Gauss iterative formula.
- Implementation and testing of selected methods using different tasks.
- Evaluation of performance, and comparison of the methods.

Inverse Kinematic Methods

1. Analytical Method
2. Cyclic Coordinate Descent (CCD)
3. Dual Quaternions
4. Jacobian Transpose Method
5. Jacobian Pseudo-Inverse Method
6. Modified Levenberg-Marquardt Method

Jacobian Transpose Method

Definition

$$\boldsymbol{q}_{i+1} = \boldsymbol{q}_i + \xi \cdot \boldsymbol{J}^T(\boldsymbol{q}_i)(\boldsymbol{x}_f - \boldsymbol{k}(\boldsymbol{q}_i)) \tag{1}$$

Jacobian Pseudo-Inverse Method

Definition

$$\boldsymbol{q}_{i+1} = \boldsymbol{q}_i + \xi \cdot \boldsymbol{J}^{\#}(\boldsymbol{q}_i)(\boldsymbol{x}_f - \boldsymbol{k}(\boldsymbol{q}_i)) \tag{2}$$

$$\boldsymbol{J}^{\#}(\boldsymbol{q}_i) = \boldsymbol{J}^T(\boldsymbol{q}_i).(\boldsymbol{J}(\boldsymbol{q}_i)\boldsymbol{J}^T(\boldsymbol{q}_i))^{-1} = \boldsymbol{J}^T(\boldsymbol{q}_i).(\boldsymbol{M}(\boldsymbol{q}_i))^{-1} \tag{3}$$

Modified Levenberg-Marquardt Method

Definition

$$\boldsymbol{q}_{i+1} = \boldsymbol{q}_i + \xi \cdot \boldsymbol{J}^T(\boldsymbol{q}_i)(diag(\boldsymbol{M}(\boldsymbol{q}_i))^{-1})(\boldsymbol{x}_f - \boldsymbol{k}(\boldsymbol{q}_i)) \tag{4}$$

$$\boldsymbol{M}(\boldsymbol{q}_i) = (\boldsymbol{J}(\boldsymbol{q}_i)\boldsymbol{J}^T(\boldsymbol{q}_i))^{-1} \tag{5}$$

Kinematics and Jacobian Matrix

Definition

forward kinematics of the n-dof planar pendulum:

$$\boldsymbol{k}(\boldsymbol{q}_i) = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} C_1(\boldsymbol{q}_i) \\ S_1(\boldsymbol{q}_i) \end{bmatrix} \tag{6}$$

$(m \times n)$ Jacobian matrix:

$$\boldsymbol{J}(\boldsymbol{q}_i) = \frac{\partial \boldsymbol{k}(\boldsymbol{q}_i)}{\partial \boldsymbol{q}_i} = \begin{bmatrix} -S_1(\boldsymbol{q}_i) & -S_2(\boldsymbol{q}_i) & \dots & -S_n(\boldsymbol{q}_i) \\ C_1(\boldsymbol{q}_i) & C_2(\boldsymbol{q}_i) & \dots & C_n(\boldsymbol{q}_i) \end{bmatrix} \tag{7}$$

Algorithm Implementation

Initialization:
Select method: Jacobian Transpose, Jacobian Pseudo-Inverse or Modified Levenberg-Marquardt
$x_f \longrightarrow$ Goal point
$\epsilon \longrightarrow$ Error margin (tolerance)
$q_0 \longrightarrow$ Initial configuration
$a_{i:n} \longrightarrow$ Manipulator arm lengths

```
procedure MATRIX MANIPULATIONS(J(q_i)^T, J(q_i)^#, diag(M(q_i))^{-1}.I)
    tic                                                   ▷ Start timer
    for i = 1 : ∞ do
        calculate q_{i+1}, toc    ▷ Calculate next configuration using previous configuration, stop timer
        if (‖ x_f − k(q_i) ‖) ≤ ε then
            break;                ▷ Stop iteration when tolerance condition is met
        end if
    end for
    d = |(x2−x1)(y1−y0)−(x1−x0)(y2−y1)| / sqrt((x2−x1)^2+(y2−y1)^2)
                                  ▷ Calculate distance between points and straight line segment
    α_i = arccos(⟨w1, w2⟩ / (‖w1‖·‖w2‖)), w1 = x_{i+1} − x_i,   w2 = x_f − x_i
                                  ▷ Calculate angle quality measure
    Display results               ▷ Print t, number of iterations, k_f and q_f
    plots                         ▷ Display plots
end procedure
```

$$d = \frac{|(x_2-x_1)(y_1-y_0)-(x_1-x_0)(y_2-y_1)|}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}$$

$$\alpha_i = \arccos\left(\frac{\langle w_1, w_2\rangle}{\|w_1\|\cdot\|w_2\|}\right), \quad w_1 = x_{i+1} - x_i, \quad w_2 = x_f - x_i$$

Statement of Problem

Definition

$\boldsymbol{x} = \boldsymbol{k}(\boldsymbol{q}), \boldsymbol{x} \in \mathbb{X} \subset \mathbb{SE}$

$\boldsymbol{n} \in \{2, 3, ...\}$: Planar pendula of **n** DOF

$\boldsymbol{q}_0 = (q_1, ..., q_n)^T, q \in \mathbb{Q}$

$\boldsymbol{x}_f = (x, y)^T$: Goal point

$\boldsymbol{a}_n$: Manipulator arm length for $\boldsymbol{n}^{th}$ DOF

$\xi = 0.01$: Regulation Constant

$\epsilon = 0.01$: Error Margin

$\boldsymbol{m} = 2$

$dim(\boldsymbol{q}) = n \geq m = dim(\boldsymbol{x})$

Tasks simulated

Task 1: $2 - DOF, \boldsymbol{x}_f = (3, -2)^T, \;\; \boldsymbol{q}_0 = (15, -45)^T$

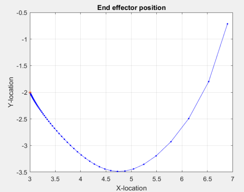
Task 2: $3 - DOF, \boldsymbol{x}_f = (-2, -3)^T, \;\; \boldsymbol{q}_0 = (30, -10, 45)^T$

Task 3: $4 - DOF, \boldsymbol{x}_f = (1, 3.6)^T, \;\; \boldsymbol{q}_0 = (0, -50, -10, -75)^T$

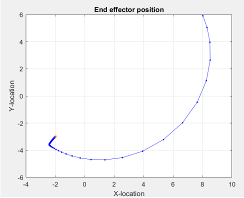
Task 4: $5 - DOF, \boldsymbol{x}_f = (4, 0)^T, \;\; \boldsymbol{q}_0 = (-60, 30, 0, 19, -55)^T$

Link Lengths: $a_1 = 4, \; a_2 = 3.5, \; a_3 = 3, \; a_4 = 3, \; a_5 = 3$

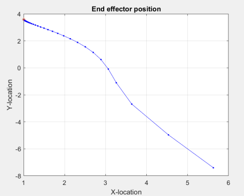
Jacobian Transpose Method



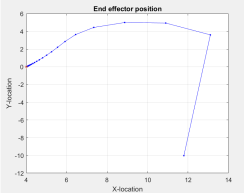
(a) Task 1: x-y position



(b) Task 2: x-y position

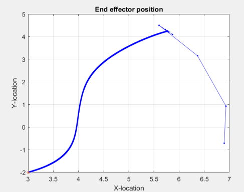


(c) Task 3: x-y position

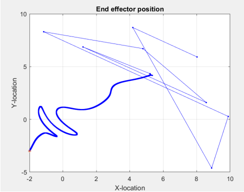


(d) Task 4: x-y position

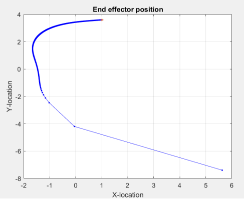
Modified Levenberg-Marquardt Method



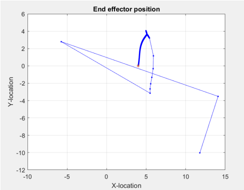
(a) Task 1: x-y position



(b) Task 2: x-y position

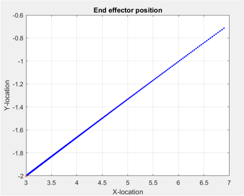


(c) Task 3: x-y position

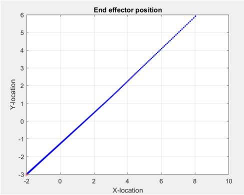


(d) Task 4: x-y position

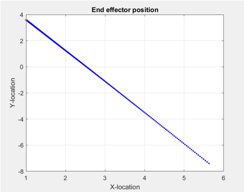
Jacobian Pseudo-Inverse Method



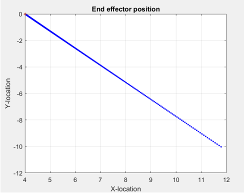
(a) Task 1: x-y position



(b) Task 2: x-y position



(c) Task 3: x-y position



(d) Task 4: x-y position

Execution Time

Table: Average Execution Time for Tasks 1 to 4:

Method	JPI	JT	MLM
Time (secs)	140.9075	5.3850	486.2975

Number of Iterations

Table: Average Number of Iterations for Tasks 1 to 4:

Method	JPI	JT	MLM
No. of Iteration	684	111	3,136

Distance from Straight Line Segment

Table: Average sum of distances from straight for Tasks 1 to 4:

Method	JPI	JT	MLM
Average (m)	0.0011535	0.1616535	0.4978198

Performance Characteristics

Table: Characteristics of the Jacobi methods discussed

Method	**Speed/Time**	**No. of iteration**	**Convergence**	**Straight Line**
JPI	—	—	Best	Best
JT	Fastest	Least	—	—
MLM	Takes much time	More iterations	least	least

Conclusion

- The number of iterations and execution time are not reliable for comparing the efficiency of these algorithms.
- The Levenberg-Marquardt method looses convergence at some point during the iteration process.
- Though completes the iterations faster and at less time, the Jacobian transpose method is still not the most efficient.
- A rare situation occurs when the Jacobian Transpose method runs into a continuous loop of iterations without convergence.
- the pseudo-inverse method is more effective and efficient in tracing a straight line to the goal point. Thus, this algorithm appears to be more intelligent compared to the other two.
- However, conclusions from the carried-out simulations should be treated with a due caution as they are based only on a few simulations and for arbitrary set of parameters.

Contributions

- Evaluation of Inverse Kinematics methods to determine optimal performance and efficiency.
- Algorithm and chart flow development, and implementation in MATLAB environment.
- Objective comparison of results based on data set obtained from algorithm implementation and simulation.

Thank You