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Computer Architecture Homework 6

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Instructions:

Homework 6 is due in May. 6, covers the content of caches and float-points, please refer to the lecture slides. You can print it out and write on it, and scan it into a pdf, or you can take photos or write Latex if you want, just remember: you must create a **PDF** and upload to the **Gradescope**, please assign the questions properly on Gradescope, otherwise you will lose 25% of points.

Question Set 1. Direct Mapped Cache

[30 points] In a 16-bit byte-addresses machine, the clock frequency is 2GHz. We have a cache with properties as follows:

1. Cache size is 4 KiB;
2. Block size is 8 Bytes;
3. Cache hit time is 2 cycles;
4. Cache miss penalty is 200 cycles;

1-A. What the width of each field of following address bit assignment:

TAG: 4	Set index: 9	Block offset: 3

Please show the procedure that your solutions derive from.

Answer 6pt + Analysis 4pt

offset width $= \log_2(\text{Block Size}) = \log_2 8 = 3$

Since direct mapped, $\# \text{Sets} = \dfrac{\text{Cache size}}{\text{Block Size}} = 512$

index width $= \log_2(\#\text{Sets}) = \log_2 512 = 9$

tag width $=$ address width $-$ offset width $-$ index width

$= 16 - 9 - 3 = 4$

1-B. We will access the data of addresses as follows. Fill in the blanks. It is about the index, tag (in decimal) and whether there is a hit or miss. If there is a miss, then give what type is the miss (either compulsory or replace). (Here we define replace as either conflict or capacity that causes a miss.)

Addresses (serially access)	Tag/Index	Hit, Compulsory or Replace
0x0000	0x0/0x0	Compulsory
0x0004	0x0/0x0	Hit
0x0008	0x0/0x1	Compulsory
0x000c	0x0/0x1	Hit
0x1000	0x1/0x0	Replace
0x1004	0x1/0x0	Hit
0x1008	0x1/0x1	Replace
0x100c	0x1/0x1	Hit
0x0000	0x0/0x0	Replace
0x0004	0x0/0x0	Hit

1-C. Calculations. (Step-by-step, worth 50% pts)

1-C-i: Miss rate: (4 pt.)

$$\text{Miss rate} = \frac{\text{Misses}}{\text{Accesses}} = \frac{5}{10} = \frac{1}{2} = 50\%$$

1-C-ii: AMAT (ns): (3 pt.)

$$\text{AMAT} = \text{Hit time} + \text{Miss rate} \cdot \text{Miss penalty}$$
$$= \frac{2\text{ Cycles}}{2\text{GHz}} + 50\% \cdot \frac{200\text{ Cycles}}{2\text{GHz}} = 51\text{ ns}$$

1-C-iii: AMAT if we don't have this cache (ns): (3 pt.)

$$\text{AMAT} = 1 \cdot \text{Miss penalty}$$
$$= 1 \cdot \frac{200\text{ Cycles}}{2\text{GHz}} = 100\text{ ns}$$

Question Set 2. Two-Way Set Associative Cache

From QS 1. We change the block size to 4 Bytes and implemented a two-way set associative cache. The parameters are shown as follows:

1. Cache size is 4 KiB;
2. 16-bit byte-addresses machine;
3. Block size is 4 Bytes;

2-A. What is the width of each field of following address bit assignment? :

TAG: 5	Set index: 9	Block offset: 2

Please show the procedure that your solutions derive from.

Answer 6pt + Analysis 4pt

offset width $= \log_2(\text{Block size}) = \log_2 4 = 2$

$$\# \text{Sets} = \frac{\text{Cache size}}{\text{Block size} \cdot \text{associativity}} = 512$$

index width $= \log_2(\#\text{Sets}) = \log_2 512 = 9$

tag with = address width − offset width − index width = 5

2-B. We will access the data of the addresses as follows. Fill in the blanks. It is about the index, tag (in decimal) and whether there is a hit or miss. If there is a miss, then give what type is the miss (either compulsory or replace). Use LRU policy to decide which block to replace. (Here we define replace as either conflict or capacity that causes a miss.)

Addresses (serially access)	Tag/Index	Hit, Compulsory or Replace
0x0000	0x0 / 0x0	Compulsory
0x0004	0x0 / 0x1	Compulsory
0x0008	0x0 / 0x2	Compulsory
0x000c	0x0 / 0x3	Compulsory
0x1000	0x2 / 0x0	Compulsory
0x1004	0x2 / 0x1	Compulsory
0x0000	0x0 / 0x0	Hit
0x2000	0x4 / 0x0	Replace
0x0000	0x0 / 0x0	Hit
0x1000	0x2 / 0x0	Replace

2-C. Calculations.

2-C-i. Miss rate: (5 pt.)

$$\text{Miss rate} = \frac{\text{Misses}}{\text{Accesses}} = \frac{8}{10} = 80\%$$

2-C-ii. Assume the new cache miss time is 300 cycles and hit time is 3 cycles. Calculate the AMAT in ns. Round to the nearest tenth. (5 pt.)

$$\text{AMAT} = \text{Hit time} + \text{Miss rate} \cdot \text{Miss penalty}$$
$$= \frac{3\,\text{Cycles}}{2\text{GHz}} + 80\% \cdot \frac{300\,\text{Cycles}}{2\text{GHz}}$$
$$= 121.5\ \text{ns}$$

Question Set 3. Floating Point Numbers

We consider the IEEE 32-bit floating point representation except with a 6-bit exponent (bias of 31) and a denorm implicit exponent of -30.

3-A. Convert `-95.2` to that form. In hexadecimal.

$$(-95.2)_{10} = (-1011111.\dot{0}01\dot{1})_2 = (-1.011111\dot{0}01\dot{1})_2 \times (2^6)_{10}$$
$$(6+31)_{10} = (37)_{10} = (100101)_2$$
$$\Rightarrow \text{Binary}: (1\ 100101\ 0111110011001100110011001)_2$$
$$\Rightarrow \text{Hexadecimal}: 0\text{xCAF99999}$$

3-B. Convert floating point number `0x49480000` from binary to decimal, specify infinities as `+inf` and `-inf`, and not a number as `NaN`.

$$0\text{x}49480000 = (0\ 100100\ 1010010000000000000000000)_2$$
$$(100100)_2 - (31)_{10} = (36-31)_{10} = (5)_{10}$$
$$\Rightarrow (1.101001)_2 \times (2^5)_{10} = (110100.1)_2$$
$$= (52.5)_{10}$$

3-C. What is the next smallest positive number larger than 2 that can be represented completely? (Not every number can be represented perfectly using floating point. For example, $\frac{1}{3}$ can only be approximated with floating point). Please explain why.

$$(2)_{10} = (1.0)_2 \times (2^1)_{10} \qquad (1+31)_{10} = (100000)_2$$

The next smallest positive number is:

$$(0\ 100000\ 0000000000000000000000001)_2$$
$$= (1.0000000000000000000000001)_2 \times (2^1)_{10}$$
$$= 2\frac{1}{2^{24}}$$

The precision of this representation constrains that the smallest significant is only one 1 at the 25th (last) bit, if the 1 is placed behind the 25th bit, it will be truncated.

3-D. What's the smallest positive value it can represent that is not a denorm? Leave your answer as a power of 2. Please explain why.

Since not a denorm, $(000001)_2 - (31)_{10} = (-30)_{10}$

The smallest one must have all zeroes in its significand

Thus $(0\ 000001\ 0000000000000000000000000)_2$

$= (1.0 \times 2^{-30})_{10} = (2^{-30})_{10}$

3-E. What's the smallest positive value it can represent? Leave your answer as a power of 2. Please explain why.

The smallest positive number is a denorm number, meaning the implicit leading 1 is ignored and it only has one 1 at the end of significand

$\Rightarrow (0\ 000000\ 0000000000000000000000001)_2$

$= (0.0000000000000000000000001)_2 \times (2^{-30})_{10}$

$= (2^{-55})_{10}$