Appendix
Proof of Theorem 1

We begin with the analysis of push operations. Define $\tau_{push}^{1\sim l}$ as the timestamp when $push^1$, $push^2$, ... , $push^l$ (written as $push^{1\sim l}$ as a whole) all finish. Obviously, there has

$$\tau_{push}^{1\sim l} = \max_{1\leq i\leq l}(\tau_{push}^{i}). \tag{1}$$

Claim 1. Mercury obtains the minimum $\tau_{push}^{1\sim l}, (1 \leq l \leq L)$.

Mercury prioritizes $push_m$ over $push_n$ when $m < n$, for all $1 \leq m, n \leq L$. Hence, $push^{1\sim l}$ are scheduled before $push^{l+1\sim L}$. Given the assumption that the preemption action happens immediately, Mercury obtains minimum $\tau_{push}^{1\sim l}$.

We next investigate the scheduling priority of pull operations. Let $\tau_{pull}^{1\sim l}$ be the timestamps when $pull^1$, $pull^2$, ... , $pull^l$ (denoted by $pull^{1\sim l}$ as a whole) all finish. Then, there exists

$$\tau_{pull}^{1\sim l} = \max_{1\leq i\leq l}(\tau_{pull}^{i}). \tag{2}$$

Lemma 1. Mercury obtains minimum the $\tau_{pull}^{1\sim l}, (1 \leq l \leq L)$.

Proof. For any feasible policy of scheduling pull operations, τ_{pull}^{l} can only be the same as or later than τ_{push}^{l}. Mercury obtains $\tau_{pull}^{l} = \tau_{push}^{l}$, which means $\tau_{pull}^{1\sim l} = \tau_{push}^{1\sim l}$. Then Lemma 1 is proved based on Claim 1.

We then move on to τ_{fp}^{l} calculated in Section 5.1, and repeated as below

$$\tau_{fp}^{l} = \begin{cases} \tau_{pull}^{l} + t_{upd}^{l} + t_{fp}^{l}, & l = 1; \\ \max\{\tau_{pull}^{l}, \tau_{fp}^{l-1}\} + t_{upd}^{l} + t_{fp}^{l}, & 2 \leq l \leq L. \end{cases} \tag{3}$$

We will show that the above equation can be represented equivalently as follows

$$\tau_{fp}^{l} = \begin{cases} \tau_{pull}^{l} + t_{upd}^{l} + t_{fp}^{l}, & l = 1; \\ \max\{\tau_{pull}^{1\sim l}, \tau_{fp}^{l-1}\} + t_{upd}^{l} + t_{fp}^{l}, & 2 \leq l \leq L. \end{cases} \tag{4}$$

First of all, fp^l relies on $pull^l$ and fp^{l-1}, and fp^{l-1} relies on $pull^{l-1}$ and fp^{l-2}. Based on their recurrence relations, fp^l can not be executed before the accomplishment of $pull^1$, $pull^2$, ... , $pull^l$. Thus we have

$$\tau_{pull}^{1\sim l} < \tau_{fp}^{l}, 1 \leq l \leq L. \tag{5}$$

Case 1: $\tau_{pull}^{l} \leq \tau_{pull}^{1\sim l-1}$ $(2 \leq l \leq L)$. There exists

$$\tau_{pull}^{1\sim l} = \max\{\tau_{pull}^{1\sim l-1}, \tau_{pull}^{l}\} = \tau_{pull}^{1\sim l-1}. \tag{6}$$

With Equation (5) and (6), we have

$$\begin{cases} \tau_{pull}^{l} \leq \tau_{pull}^{1\sim l-1} < \tau_{fp}^{l-1}, \\ \tau_{pull}^{1\sim l} = \tau_{pull}^{1\sim l-1} < \tau_{fp}^{l-1}. \end{cases} \tag{7}$$

Equation (3) and (4) are equivalent by eliminating *max* using Equation (7).

Case 2: $\tau_{pull}^{l} > \tau_{pull}^{1\sim l-1}$ $(2 \leq l \leq L)$. There has

$$\tau_{pull}^{1\sim l} = \max\{\tau_{pull}^{1\sim l-1}, \tau_{pull}^{l}\} = \tau_{pull}^{l}. \tag{8}$$

Then Equation (3) and (4) are equivalent accordingly.

Lemma 2. Mercury obtains minimum $\tau_{fp}^{l}, (1 \leq l \leq L)$.

Proof. It can be proved by induction on l using Equation (4). When $l = 1$, substituting τ_{pull}^{l} by $\tau_{pull}^{1\sim l}$ in Equation (4), we have $\tau_{fp}^{l} = \tau_{pull}^{1\sim l} + t_{upd}^{l} + t_{fp}^{l}$. Then, considering Lemma 1, Mercury obtains minimum τ_{fp}^{l} when $l = 1$. Next we assume Lemma 2 holds true for $l = n - 1$, which means Mercury obtains minimum τ_{fp}^{n-1}. We know Mercury obtains minimum $\tau_{pull}^{1\sim n}$ from Lemma 1. We have $\tau_{fp}^{n} = \max\{\tau_{pull}^{1\sim n}, \tau_{fp}^{n-1}\} + t_{upd}^{n} + t_{fp}^{n}$ from Equation (4). Therefore, Mercury obtains the minimum τ_{fp}^{n}, i.e., Lemma 2 holds true for $l = n$.

Given $t_{iter} = \tau_{fp}^{L}$, Mercury obtains the minimum t_{iter}, thus completing the proof of Theorem 1. ■