Team Contest Reference

Universität zu Lübeck

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1 Mathematische Algorithmen

1.1 Primzahlen

Für Primzahlen gilt immer (aber nicht nur für Primzahlen)

$$a^p \equiv a \mod p \quad \text{bzw.} \quad a^{p-1} \equiv 1 \mod p.$$

1.1.1 Sieb des Eratosthenes

```
static boolean[] sieve(int until) {
  boolean[] a = new boolean[until + 1];
  Arrays.fill(a, true);
  for (int i = 2; i < Math.sqrt(a.length); i++) {
    if (a[i]) {
      for (int j = i * i; j < a.length; j += i) a[j] = false;
    }
  }
  return a; // a[i] == true, iff. i is prime. a[0] is ignored
}
```

1.1.2 Primzahlentest

```
static boolean isPrim(int p) {
  if (p < 2 || p > 2 && p % 2 == 0) return false;
  for (int i = 3; i <= Math.sqrt(p); i += 2)
    if (p % i == 0) return false;
  return true;
}
```

1.2 Binomial Koeffizient

```
static int[][] mem = new int[MAX_N][(MAX_N + 1) / 2];
static int binoCo(int n, int k) {
  if (k < 0 || k > n) return 0;
  if (2 * k > n) binoCo(n, n - k);
  if (mem[n][k] > 0) return mem[n][k];
  int ret = 1;
  for (int i = 1; i <= k; i++) {
    ret *= n - k + i;
    ret /= i;
    mem[n][i] = ret;
  }
  return ret;
}
```

2 Mathematisch Formeln und Gesetze

2.1 Catalan

$C_n = \frac{1}{n+1}\binom{2n}{n} = \prod_{k=2}^{n}(n+k)/k$
$C_{n+1} = \frac{4n+2}{n+2}C_n = \sum_{k=0}^{n} C_k C_{n-k}$

2.2 kgV und ggT

$$ggT(n,m) \cdot kgV(m,n) = |m \cdot n|$$

2.3 Kreuzprodukt

$$\vec{a} \times \vec{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$$

2.4 Orthogonale Projektion

r_0 : Ortsvektor; u : Richtungsvektor; n : Normalenvektor
$P_g(\vec{x}) = \vec{r_0} + \frac{(\vec{x}-\vec{r_0})\cdot\vec{u}}{\vec{u}\cdot\vec{u}}\,\vec{u}$
$P_g(\vec{x}) = \vec{x} - \frac{(\vec{x}-\vec{r_0})\cdot\vec{n}}{\vec{n}\cdot\vec{n}}\,\vec{n}$(nur 2D bzw. 3D auf Ebene)

2.5 Geradenschnittpunkt

$g_1 : ax + by = c;\ g_2 : px + qx = r;\ \Rightarrow \vec{p} = \frac{1}{aq-bp}\begin{pmatrix} x = cq - br \\ y = ar - cp \end{pmatrix}$

$g_1 : \vec{p} = \begin{pmatrix} r_x \\ r_y \end{pmatrix} + s\begin{pmatrix} s_x \\ s_y \end{pmatrix}\ g_2 : \vec{p} = \begin{pmatrix} q_x \\ q_y \end{pmatrix} + t\begin{pmatrix} t_x \\ t_y \end{pmatrix}\ w_x = (r_x - q_x), w_y = (r_y - q_y)$
$\Rightarrow D = (s_x t_y - t_x s_y)\ D_s = (t_x w_y - t_y w_x)\ D_t = (s_y w_x - s_x w_y)\ s = D_s/D, t = D_t/D$

2.6 Dreicksfläche

$F = \sqrt{s(s-a)(s-b)(s-c)};\ s = \frac{a+b+c}{2}$

2.7 Kombinatorik

	mit ZL	ohne ZL
Variationen	n^k	$\frac{n!}{(n-k)!}$
Kombinationen	$\binom{n}{k} = \binom{n}{n-k} = \frac{n!}{k!(n-k)!}$	$\binom{n+k-1}{k} = \binom{n+k-1}{n-1}$

2.8 Modulare Arithmetik

Bedeutung der größten gemeinsamen Teiler:

$$d = \text{ggT}(a, b) = as + bt$$

Verwendung zu Berechnung des inversen Elements b zu a bezüglich einer Restklassengruppe n (a und n müssen teilerfremd sein):

$$ab \equiv 1 \mod n \quad \Leftrightarrow \quad s \equiv b \mod n \quad \text{für } 1 = \text{ggT}(a, n)$$

2.8.1 Erweiterter Euklidischer Algorithmus

```
static int[] eea(int a, int b) {
  int[] dst = new int[3];
  if (b == 0) {
    dst[0] = a;
    dst[1] = 1;
    return dst; // a, 1, 0
  }
  dst = eea(b, a % b);
  int tmp = dst[2];
  dst[2] = dst[1] - ((a / b) * dst[2]);
  dst[1] = tmp;
  return dst;
}
```

3 Datenstukturen

3.1 Fenwick Tree (Binary Indexed Tree)

```
class FenwickTree {
  private int[] values;
  private int n;
  public FenwickTree(int n) {
    this.n = n;
    values = new int[n];
  }
  public int get(int i) { //get value of i
    int x = values[0];
    while (i > 0) {
      x += values[i];
      i -= i & -i; }
    return x;
  }
  public void add(int i, int x) { // add x to interval [i,n]
    if (i == 0) values[0] += x;
    else {
      while (i < n) {
        values[i] += x;
        i += i & -i; }
    }
  }
}
```

4 Graphenalgorithmen

4.1 Topologische Sortierung

```
static List<Integer> topoSort(Map<Integer, List<Integer>> edges,
    Map<Integer, List<Integer>> revedges) {
  Queue<Integer> q = new LinkedList<Integer>();
  List<Integer> ret = new LinkedList<Integer>();
  Map<Integer, Integer> indeg = new HashMap<Integer, Integer>();
  for (int v : revedges.keySet()) {
    indeg.put(v, revedges.get(v).size());
    if (revedges.get(v).size() == 0)
      q.add(v);
  }
  while (!q.isEmpty()) {
    int tmp = q.poll();
    ret.add(tmp);
    for (int dest : edges.get(tmp)) {
      indeg.put(dest, indeg.get(dest) - 1);
      if (indeg.get(dest) == 0)
        q.add(dest);
    }
  }
  return ret;
}
```

4.2 Prim (Minimum Spanning Tree)

```
#define WHITE 0
#define BLACK 1
#define INF INT_MAX

int baum( int **matrix, int N){
  int i, sum = 0;

  int color[N];
  int dist[N];

    // markiere alle Knoten ausser 0 als unbesucht
  color[0] = BLACK;
  for( i=1; i<N; i++){
    color[i] = WHITE;
    dist[i] = INF;
  }

    // berechne den Rand
  for( i=1; i<N; i++){
        if( dist[i] > matrix[i][nextIndex]){
            dist[i] = matrix[i][nextIndex];
        }
    }

  while( 1){
    int nextDist = INF, nextIndex = -1;

    /* Den naechsten Knoten waehlen */
    for(i=0; i<N; i++){
      if( color[i] != WHITE) continue;

      if( dist[i] < nextDist){
        nextDist = dist[i];
        nextIndex = i;
      }
    }

    /* Abbruchbedingung*/
    if( nextIndex == -1) break;

    /* Knoten in MST aufnehmen */
    color[nextIndex] = RED;
    sum += nextDist;

    /* naechste kuerzeste Distanzen berechnen */
    for( i=0; i<N; i++){
            if( i == nextIndex || color[i] == BLACK ) continue;

            if( dist[i] > matrix[i][nextIndex]){
                dist[i] = matrix[i][nextIndex];
            }
    }
  }

  return sum;
}
```

5 Geometrische Algorithmen

5.1 Graham Scan (Convex Hull)

```
static List<P> graham(List<P> l) {
  if (l.size() < 3)
    return l;
```

```
  P temp = l.get(0);
  for (P p : l)
    if (temp.y > p.y || temp.y == p.y && temp.x > p.x)
      temp = p;
  final P start = temp; // min y (then leftmost)

  Collections.sort(l, new Comparator<P>() {
    public int compare(P o1, P o2) {
      if (new Double(Math.atan2(o1.y - start.y, o1.x - start.x)) // same angle
          .compareTo(Math.atan2(o2.y - start.y, o2.x - start.x)) == 0)
        return new Double(Math.sqrt((o1.x - start.x)
            * (o1.x - start.x) + (o1.y - start.y)
            * (o1.y - start.y))).compareTo((o2.x - start.x)
            * (o2.x - start.x) + (o2.y - start.y)
            * (o2.y - start.y)); // use distance
      return new Double(Math.atan2(o1.y - start.y, o1.x - start.x))
          .compareTo(Math.atan2(o2.y - start.y, o2.x - start.x));
    }
  });
  Stack<P> s = new Stack<P>();
  s.add(start);
  s.add(l.get(1));
  for (int i = 2; i < l.size(); i++) {
    while (s.size() >= 2
        && ccw(s.get(s.size() - 2), s.get(s.size() - 1), l.get(i)) <= 0)
      s.pop();
    s.push(l.get(i));
  }
  return s;
}

// turn is counter-clockwise if > 0; collinear if = 0; clockwise else
static double ccw(P p1, P p2, P p3) {
  return (p2.x - p1.x) * (p3.y - p1.y) - (p2.y - p1.y) * (p3.x - p1.x);
}

public static class P {
  double x, y;

  P(double x, double y) {
    this.x = x;
    this.y = y;
  }
  // polar coordinates (not used)
  // double r() { return Math.sqrt(x * x + y * y); }
  // double d() { return Math.atan2(y, x); }
}
```

5.2 Punkt in Polygon

```
  /**
   * -1: A->R schneidet BC (ausser unterer Endpunkt)
   *  0: A auf BC
   * +1: sonst
   */
  public static int KreuzProdTest(double ax, double ay, double bx, double by,
      double cx, double cy) {
    if (ay == by && by == cy) {
      if ((bx <= ax && ax <= cx) || (cx <= ax && ax <= bx))
        return 0;
      else
        return +1;
    }
    if(by>cy){double tmpx=bx;double tmpy=by; bx=cx;by=cy;cx=tmpx;cy=tmpy;}
    if(ay==by && ax==bx) return 0;
    if(ay<=by || ay>cy) return +1;
    double delta = (bx-ax)*(cy-ay)-(by-ay)*(cx-ax);
    if(delta>0)return -1; else if(delta<0)return +1;else return 0;
  }
  /**
   * Input: P[i] (x[i],y[i]); P[0]:=P[n]
   * -1: Q ausserhalb Polygon
   *  0: Q auf Polygon
   * +1: Q innerhalt des Polygons
   */
  public static int PunktInPoly(double[] x,double[] y, double qx,double qy){
    int n = x.length - 1;
    int t = -1;
    for (int i = 0; i <= n - 1; i++) {
      t = t * KreuzProdTest(qx, qy, x[i], y[i], x[i + 1], y[i + 1]);
    }
    return t;
  }
```

6 Verschiedenes

6.1 Potenzmenge

```
static <T> Iterator<List<T>> powerSet(final List<T> l) {
  return new Iterator<List<T>>() {
    int i; // careful: i becomes 2^l.size()
```

```
    public boolean hasNext() {
      return i < (1 << l.size());
    }
    public List<T> next() {
      Vector<T> temp = new Vector<T>();
      for (int j = 0; j < l.size(); j++)
        if (((i >>> j) & 1) == 1)
          temp.add(l.get(j));
      i++;
      return temp;
    }
    public void remove() {}
    };
  }
```

6.2 LongestCommonSubsequence

```
#include <iostream>
#include <vector>
#include <string>
#include <sstream>
#include <algorithm>
#include <iterator>
using namespace std;

#define MAX(a,b) (a > b) ? a : b

string X,Y;
vector< vector<int> > c(101, vector<int>(101,0));
int m,n,ctr;

int LCS()
{
     m = X.length(),n=Y.length();

    c.resize(m+1);
  for(int i = 0; i<n+1; i++) {
    c[i].resize(n+1);
    c[i][0] = 0;
  }

     int i,j;

     for (i=0;i<=m;i++)
         for (j=0;j<=n;j++)
             c[i][j]=0;

     for (i=1;i<=m;i++)
         for (j=1;j<=n;j++)
         {
             if (X[i-1]==Y[j-1])
                c[i][j]=c[i-1][j-1]+1;
             else
                 c[i][j]=max(c[i][j-1],c[i-1][j]);
         }
     return c[m][n];
}
/** Print a songle LCS */
void printLCS(int i,int j)
{
    if (i==0 || j==0)
       return;
    if (X[i-1]==Y[j-1])
    {
       printLCS(i-1,j-1);
       cout<<X[i-1];
    }
    else if (c[i][j]==c[i-1][j])
         printLCS(i-1,j);
    else
        printLCS(i,j-1);
}

int main()
{
    while(cin>>X>>Y)
    {
  cout << "Length: " << LCS() << endl;
        printLCS(m,n);
        cout<<endl ;
    }
}
```

6.3 LongestCommonSubstring

```
  private static List<String> longestCommonSubstring(String S1, String S2)
  {
    List<String> ret = new ArrayList<String>();
    List<Integer> idx  =new ArrayList<Integer>();
      int Start = 0;
      int Max = 0;
      for (int i = 0; i < S1.length(); i++)
```

```
      {
          for (int j = 0; j < S2.length(); j++)
          {
              int x = 0;
              while (S1.charAt(i + x) == S2.charAt(j + x))
              {
                  x++;
                  if (((i + x) >= S1.length()) || ((j + x) >= S2.length())) break;
              }
              if (x > Max)
              {
                  Max = x;
                Start = i;
                idx.clear();
                idx.add(Start);
              } else if(x==Max){
                Start = i;
                idx.add(Start);
              }
           }
      }
      HashSet<String> set = new HashSet<String>(idx.size(),1f);
      for(Integer start : idx){
        String substr = S1.substring(start,start+Max);
        if(!set.contains(substr)){
          ret.add(substr);
          set.add(substr);
        }
      }
      Collections.sort(ret);
      //return S1.substring(Start, (Start + Max));
      return ret;
  }
```

6.4 LongestIncreasingSubsequence

```
#include <vector>
using namespace std;

/** finde LIS in O(n log k)
 *a: Sequenz (in)
 *b: LIS (out)
 */
void find_lis(vector<int> &a, vector<int> &b)
{
  vector<int> p(a.size());
  int u, v;
  if (a.empty()) return;
  b.push_back(0);

  for (size_t i = 1; i < a.size(); i++)
        {
        // ist naechstes Element a[i] groesser als letztes der aktuelle LIS
    // a[b.back()], fuege es (Index) an "b" an.
    if (a[b.back()] < a[i]) {
      p[i] = b.back();
      b.push_back(i);
      continue;
    }

        // finde kleinstes El. in LIS (index in b) welches gerade groesser als a[i] ist
        // binaere suche |b|<=k  =>  O(log k)
    for (u = 0, v = b.size()-1; u < v;)
        {
      int c = (u + v) / 2;
      if (a[b[c]] < a[i]) u=c+1; else v=c;
    }

        // aktualisiere b falls neuer Wert kleiner als vorheriger kleinerer Wert
    if (a[i] < a[b[u]])
                {
      if (u > 0) p[i] = b[u-1];
      b[u] = i;
    }
  }

  for (u = b.size(), v = b.back(); u--; v = p[v]) b[u] = v;
}

#include <cstdio>
int main()
{
  int a[] = { 1, 9, 3, 8, 11, 4, 5, 6, 4, 19, 7, 1, 7 };
  vector<int> seq(a, a+sizeof(a)/sizeof(a[0])); // seq : Eingabesequent
  vector<int> lis;                              // lis : Index Vektor fuer LIS
    find_lis(seq, lis);
     //Sequenz ausgeben:
  for (size_t i = 0; i < lis.size(); i++)
    printf("%d␣", seq[lis[i]]);
        printf("\n");

  return 0;
```

```
}
```

6.5 Permutation & Sequenzen

```
import java.util.Scanner;

public class PermsAndSequ {
  public static void main(String[] args) {
    Scanner sc = new Scanner(System.in);
    int n;
    while ((n = sc.nextInt()) != 0) {
      int k = sc.nextInt();
      Sequences(n, k);
      Permutations(n);
    }

  }

  public static void Sequences(int n, int k) {
    int[] x = new int[k];
    for (int i = 0; i < k; i++)
      x[i] = 1;
    Print(x);
    while (true) {
      boolean lastX = true;
      for (int i = 0; i < k; i++)
        if (x[i] != n) {
          lastX = false;
          break;
        }
      if (lastX)
        break;
      int p = k - 1;
      while (!(x[p] < n))
        p--;
      x[p] = x[p] + 1;
      for (int i = p + 1; i < k; i++)
        x[i] = 1;
      Print(x);
    }
  }

  public static void Permutations(int n) {
    int[] x = new int[n];
    for (int i = 0; i < n; i++)
      x[i] = i + 1;
    Print(x);
    while (true) {
      boolean lastX = true;
      for (int i = 0; i < n - 1; i++)
        if (x[i] < x[i + 1]) {
          lastX = false;
          break;
        }
      if (lastX)
        break;
      int k = n - 1 - 1;

      while (x[k] > x[k + 1])
        k--;
      int t = k + 1;

      while (t < (n - 1) && x[t + 1] > x[k])
        t++;

      int tmp = x[k];
      x[k] = x[t];
      x[t] = tmp;
      // reverse x[k+1] ... x[n-1]

      for (int i = 0; i <= ((n - 1) - (k + 1)) / 2; i++) {
        tmp = x[k + 1 + i];
        x[k + 1 + i] = x[n - 1 - i];
        x[n - 1 - i] = tmp;

      }
      Print(x);
    }
  }

  public static void Print(int[] x) {
    for (int i = 0; i < x.length; i++)
      System.out.print(x[i] + "␣");
    System.out.println("");
  }

}
```