SDM4 in R: Probability Models (Chapter 16)

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Introduction and background

This document is intended to help describe how to undertake analyses introduced as examples in the Fourth Edition of *Stats: Data and Models* (2014) by De Veaux, Velleman, and Bock. More information about the book can be found at http://wps.aw.com/aw_deveaux_stats_series. This file as well as the associated R Markdown reproducible analysis source file used to create it can be found at http://www.amherst.edu/~nhorton/sdm4.

This work leverages initiatives undertaken by Project MOSAIC (http://www.mosaic-web.org), an NSF-funded effort to improve the teaching of statistics, calculus, science and computing in the undergraduate curriculum. In particular, we utilize the mosaic package, which was written to simplify the use of R for introductory statistics courses. A short summary of the R needed to teach introductory statistics can be found in the mosaic package vignettes (http://cran.r-project.org/web/packages/mosaic).

Chapter 16: Probability Models

Section 16.1: Bernoulli models

We can replicate the calculation on page 413:

```
library(mosaic); library(readr); options(digits=3)
dbinom(0, size=1, prob=0.2) # Probability that you don't get Hope's picture

## [1] 0.8

dbinom(1, size=1, prob=0.2) # Probability that you do

## [1] 0.2

dgeom(1, prob=0.2) # Probability that you get it on the second try P(X=1)

## [1] 0.16

# Note that the geometric in R doesn't include the final success!
```

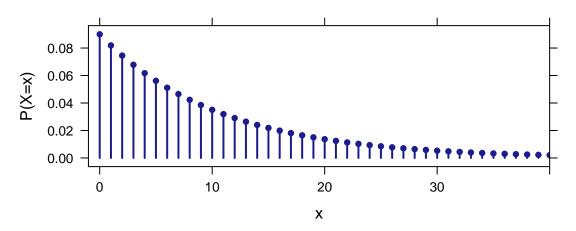
Section 16.2: The Geometric model

We can replicate the example from page 414:

```
p <- 0.09; p
## [1] 0.09
```

```
plotDist("geom", p, xlim=c(-1, 40), xlab="x", ylab="P(X=x)", main="Geometric distribution")
```

Geometric distribution



1/p # true expected value

[1] 11.1

```
mean(~ rgeom(20000, prob=p)) # simulation to check
```

[1] 10.2

and the Think/Show/Tell on pages 415-416:

```
p <- 0.06
dgeom(0, prob=p)</pre>
```

[1] 0.06

```
dgeom(1, prob=p)
```

[1] 0.0564

```
dgeom(2, prob=p)
```

[1] 0.053

```
dgeom(3, prob=p)
```

[1] 0.0498

```
pgeom(3, prob=p)
```

[1] 0.219

Section 16.3: The Binomial model

Let's replicate the example on page 417:

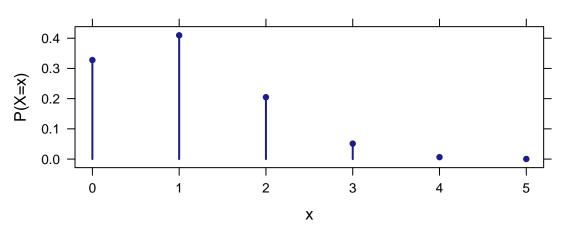
```
p <- 0.2; p # back to Hope Solo example

## [1] 0.2

n <- 5
plotDist("binom", params=c(size=n, prob=p), xlim=c(-0.2, 5.2),</pre>
```

Binomial distribution

xlab="x", ylab="P(X=x)", main="Binomial distribution")



```
n*p # true expected value
```

[1] 1

```
n*p*(1-p) # true variance
```

[1] 0.8

```
mysamp <- rbinom(20000, size=n, prob=p) # simulation to check
mean(~ mysamp)</pre>
```

[1] 0.994

```
var(~ mysamp)
```

[1] 0.786

```
dbinom(2, size=5, prob=p)
```

[1] 0.205

or the SPAM example on page 418:

```
dbinom(1, size=25, prob=0.09)
dbinom(2, size=25, prob=0.09)
sum(dbinom(1:2, size=25, prob=0.09))
```

Section 16.4: Normal approximation to the binomial

1800

Let's replicate the example on page 419:

```
p <- 0.06; n <- 32000
plotDist("binom", params=list(n, p), xlab="x", ylab="P(X=x)")

0.008

0.006

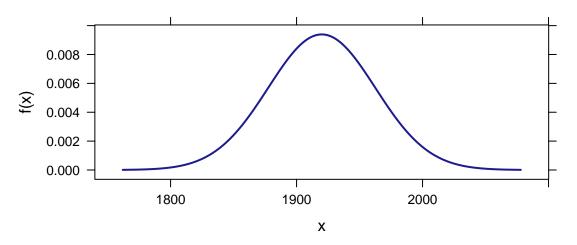
0.002

0.000
```

```
ex <- n*p; varx <- n*p*(1-p)
plotDist("norm", params=list(ex, sqrt(varx)), xlab="x", ylab="f(x)")</pre>
```

Χ

2000



1900

```
pbinom(1849, n, p)
```

[1] 0.0479

```
pnorm(1849, ex, sqrt(varx))
```

[1] 0.0473

Your parents had to use the normal approximation. The good news is that you don't have to if you use R.

Section 16.5: The continuity correction

Let's replicate the example on page 421 to demonstrate that we don't need the continuity correction if we calculate the exact distribution of the binomial.

```
p <- 0.2; n <- 50
dbinom(10, n, p)
```

[1] 0.14

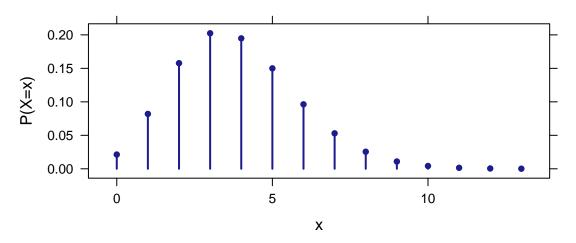
```
\texttt{pnorm}(\texttt{10.5}, \texttt{n*p}, \texttt{sqrt}(\texttt{n*p*(1-p)})) - \texttt{pnorm}(\texttt{9.5}, \texttt{n*p}, \texttt{sqrt}(\texttt{n*p*(1-p)})) \ \# \ \textit{continuity correction}(\texttt{n*p*(1-p)})) \ \# \ \textit{continuity}(\texttt{n*p*(1-p)})) \ \# \ \textit{continuity}(\texttt{n*p*(1-p)}) \ \# \ \textit{continuity}(\texttt{n*p*(1-p)})) \ \# \ \textit{continuity}(\texttt{n*p*(1-p)}) \ \# \ \textit{continuity}(\texttt{n*p*(1-p)})) \ \# \ \textit{continuity}(\texttt{n*p*(1-p)}) \ \# \ \textit{continuity}(\texttt{n*p*(1-p)}) \ \# \ \textit{continuity}(\texttt{n*p*(1-p)})) \ \# \ \textit{continuity}(\texttt{n*p*(1-p)}) \ \# \ \textit{continuity}(\texttt{n*p*(1-
```

[1] 0.14

Section 16.6: The Poisson model

Let's replicate the example on page 424:

```
lambda <- 35000*0.00011
plotDist("pois", lambda, xlab="x", ylab="P(X=x)")</pre>
```



```
1 - ppois(7, lambda)
```

[1] 0.0427

Section 16.7: Uniform and exponential

Let's replicate the example on page 426 for the uniform distribution:

```
x <- runif(20000, min=0, max=20)
mean(~ x)</pre>
```

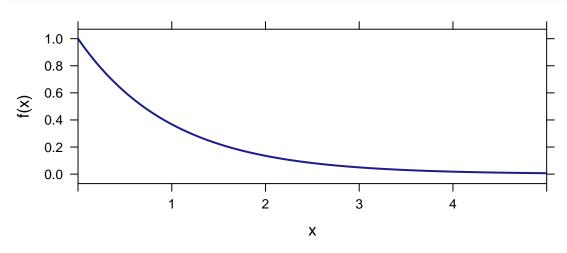
[1] 10

sd(~ x)

[1] 5.77

and on page 427 for the exponential distribution:

plotDist("exp", 1, xlim=c(0, 5), xlab="x", ylab="f(x)")



pexp(1/3, rate=4) # rate is 1/E[X]

[1] 0.736