Computer Architecture

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Exercise Sheet 2

Exercise sheet 2 – Arithmetic and Digital Logic

1) Use the Booth algorithm to multiply 23 (multiplicand) by 29 (multiplier), where each number is represented using 6 bits.

2) The following numbers use the IEEE 32-bit floating-point format. What is the equivalent decimal value?
   a) 1 10000011 11000000000000000000000
   b) 0 01111110 10100000000000000000000
   c) 0 10000000 11000000000000000000000

3) Consider a floating-point format with 8 bits for the biased exponent and 23 bits for the significand. Show the bit pattern for the following numbers in this format:
   a) -720
   b) 0.645

4) Any floating-point representation used in a computer can represent only certain real numbers exactly; all others must be approximated. If A' is the stored value approximating the real value A, then the relative error, r, is expressed as $r = \frac{A-A'}{A}$. Represent the decimal quantity 0.4 in the following floating-point format: exponent: biased, 4 bits; significand, 7 bits. What is the relative error?

5) One of the most serious errors in computer calculations occurs when two nearly equal numbers are subtracted. Consider $A = 0.22288$ and $B = 0.22211$. The computer truncates all values to four decimal digits. Thus $A' = 0.2228$ and $B' = 0.2221$.
   a) What are the relative errors for A' and B'?
   b) What is the relative error for $C' = A' - B'$?

6) Show how the following floating-point calculations are performed (where significands are truncated to 4 decimal digits). Show the results in normalized form.
   a) $7.286 \cdot 10^2 + 7.847 \cdot 10^2$

b) $3.314 \cdot 10^{1} + 8.227 \cdot 10^{-2}$
c) $9.784 \cdot 10^{-3} - 4.666 \cdot 10^{-3}$
d) $9.844 \cdot 10^{-3} - 8.233 \cdot 10^{-4}$
e) $8.954 \cdot 10^{1} \times 1.324 \cdot 10^{0}$
f) $9.633 \cdot 10^{2} \div 5.554 \cdot 10^{4}$

7) Construct a truth table for the following Boolean Expressions:
   a) $ABC + \acute{A}\acute{B}\acute{C}$
   b) $ABC + A\acute{B}\acute{C} + \acute{A}\acute{B}\acute{C}$
   c) $A(B\acute{C} + \acute{B}C)$
   d) $(A + B)(A + C)(\acute{A} + \acute{B})$

8) A combinational circuit is used to control a seven-segment display of decimal digits, as shown in the figure. The circuit has four inputs, which provide the four-bit code used in packed decimal representation. The seven outputs define which segments will be activated to display a given decimal digit. Note that some combinations of inputs and outputs are not needed.
   a) Develop a truth table for this circuit.
   b) Express the truth table in SOP form.
   c) Express the truth table in POS form.
   d) Provide a simplified expression.