Question 4.1.

Since S is defined by $f(\vec{x})=0$. Any level curve $\vec{r}(t) = (x(t), y(t), z(t))$ defined on S.

satisfies $\frac{d}{dt} f(\vec{r}(t)) = 0$.

Given point $\vec{p} = (x(t), y(t), z(t))$, this can be re-written as.

$$\frac{d}{dt} f(\vec{r}(t)) = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial f}{\partial z} \cdot \frac{dz}{dt} = 0$$

In vector form:

$$\begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix} \cdot \begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{pmatrix} = 0.$$

where $\nabla f(\vec{p}) = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix}$ and $\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{pmatrix}$ is the tangent vector of $\vec{r}(t)$.

Since the dot product is 0. we have shown that the gradient is perpendicular to the tangent of any curve that lies on the surface. Therefore, by definition, the normal of S at $\vec{p}$ is proportional to $\nabla f(\vec{p})$.

Question 4.2.

Given point x, We want to solve for $c(x)$ in the equation

$$W(x) B c(x) = W(x) f$$

where $W(x)$, B, $c(x)$ and f are defined in ex2 slides.

Normal equations:

$$B^T (W(x))^2 B c(x) = B^T (W(x))^2 f \Rightarrow c(x) = H^{-1} B^T (W(x))^2 f$$

x^2

Evaluate $f(x)$: where $H = B^T (W(x))^2 B$.

$$f(x) = b^T(x)\, c(x)$$

$$= b^T(x) H^{-1} B^T (W(x))^2 f \qquad \text{where } H = B^T (W(x))^2 B.$$

$$f'(x) = (b^T)'(x) H^{-1} B^T (W(x))^2 f + b^T(x) H^{-1} B^T ((W(x))^2)' f$$
$$- b^T(x) H^{-1} H' H^{-1} B^T (W(x))^2 f.$$

where $H' = B^T ((W(x))^2)' B$

and $((W(x))^2)' = 2 W(x) (W(x))'$.

as $W(x)$ is diagonal.