Euler Angles to Rotation Matrix

Given ψ, θ, ϕ are Euler Angles.

$$R_x(\psi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{bmatrix}.$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}.$$

$$R_z(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Assume rotation around x first, then y, then z. Rotation matrix is given by:

$$R = R_z(\phi)\, R_y(\theta)\, R_x(\psi).$$

Rotation Matrix to Quaternion.

Given a rotation matrix R.

Compute the unit eigenvector for the eigenvalue 1 for this matrix

call it $\hat{u} = (u_1, u_2, u_3)$. This is the axis of rotation.

By $tr(R) = 2\cos(\theta) + 1$, solve for θ, the angle of rotation.

So the corresponding quaternion,

$$q = [\sin(\tfrac{1}{2}\theta)\hat{u}, \cos(\tfrac{1}{2}\theta)]^T$$

Quaternion + Translation to Dual Quaternion.

Given rotation quaternion q_r. and translation vector $\vec{v}.=(x,y,z)$

The dual quaternion describing the rotation and translation is

$$q = q_r + q_d \varepsilon.$$

$$q_r = r \qquad q_d = \frac{1}{2} t r.$$

where $t = (0, x, y, z)$.

r is the unit quaternion representing the rotation.