Whitman
SCHOOL *of* MANAGEMENT
SYRACUSE UNIVERSITY

MBC 638
DATA ANALYSIS AND DECISION MAKING

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CONFIDENCE INTERVAL for the difference in population means
Chapter 8

Today: Confidence interval for $\mu_1 - \mu_2$

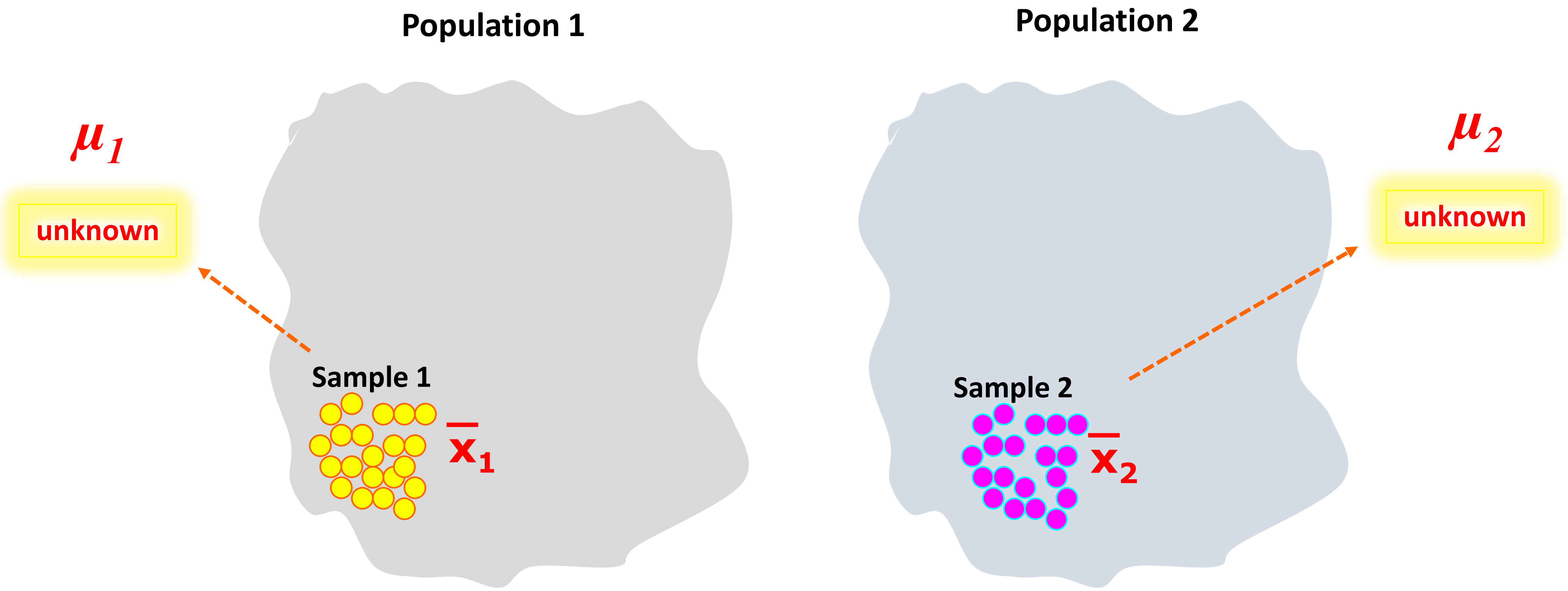
- Independent samples

- Dependent samples

Confidence interval for $\mu_1 - \mu_2$

- … will help you answer questions such as:
  - Do Toyota cars cost higher in California than New York?
  - Have the NY city apartment prices risen significantly since last year?
  - Do TV advertisements help sell your product better?
  - Is there a gender difference in the preferences regarding a product?
  - Does having an MBA degree help increase salary? Job satisfaction?
- To answer such questions, we would need data from 2 samples taken from 2 different populations.

- We use information from 2 observed samples to conclude how μ_1 and μ_2 compare.
- Assumption: The two samples are **independent**. (We will relax this assumption later.)



Recall:

- $\bar{X}$ is a **point estimate** of μ
- $\bar{X}$ is a **random variable**
- μ is the **parameter** that we want to estimate

$(1-\alpha)\%$ C.I. for μ is: $\bar{x} \pm z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}$

Standard deviation of $\bar{X}$

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Standard deviation of $\overline{X}$

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \cdot \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

By the same logic, to construct a confidence interval for $\boldsymbol{\mu_1 - \mu_2}$:

- $\overline{x}_1 - \overline{x}_2$ is the **point estimate**
- $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ is the **standard deviation** of $\overline{x}_1 - \overline{x}_2$ (the two samples are independent)

Note:
Whenever sample sizes are small, we always assume **bell-shaped** populations for X1 and X2

- Case 1: **σ_1, σ_2 are known**: Use Z

$$\left(\bar{x}_1-\bar{x}_2\right) \pm z_{\alpha/2} \cdot \sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

- Case 2: **σ_1, σ_2 are unknown, $n_1,n_2 \geq 30$** : Use ***t*** (exact), d.f.=n_1+n_2-2 or Z (approximation)

$$\left(\bar{x}_1-\bar{x}_2\right) \pm t_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$$

or

$$\left(\bar{x}_1-\bar{x}_2\right) \pm z_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$$

- Case 3: **σ_1, σ_2 are unknown, $n_1,n_2<30$** : Use ***t*** , d.f.=n_1+n_2-2

$$\left(\bar{x}_1-\bar{x}_2\right) \pm t_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$$

Examples

Example

Periodically, Merrill Lynch customers are asked to evaluate Merrill Lynch financial consultants and services (*2000 Merrill Lynch Client Satisfaction Survey*) . Higher ratings on the client satisfaction survey indicate better service, with 7 the maximum service rating.

Independent samples of service ratings for 2 financial consultants—Ms. Chen and Mr. Wang are summarized here. Ms. Chen has 10 years of experience, whereas Mr. Wang has 1 year of experience.

	Ms. Chen	Mr. Wang
# reviews collected	56	40
Mean	6.82	6.25
Standard deviation	0.64	0.75

At 95% confidence level, determine whether consultants with more experience provide better service.

Confidence interval for $\mu_1 - \mu_2$ in Excel

Example SAT Verbal.xlsx

Today: Confidence interval for $\mu_1 - \mu_2$

- Independent samples
- Dependent samples

What if the samples are **not independent**?

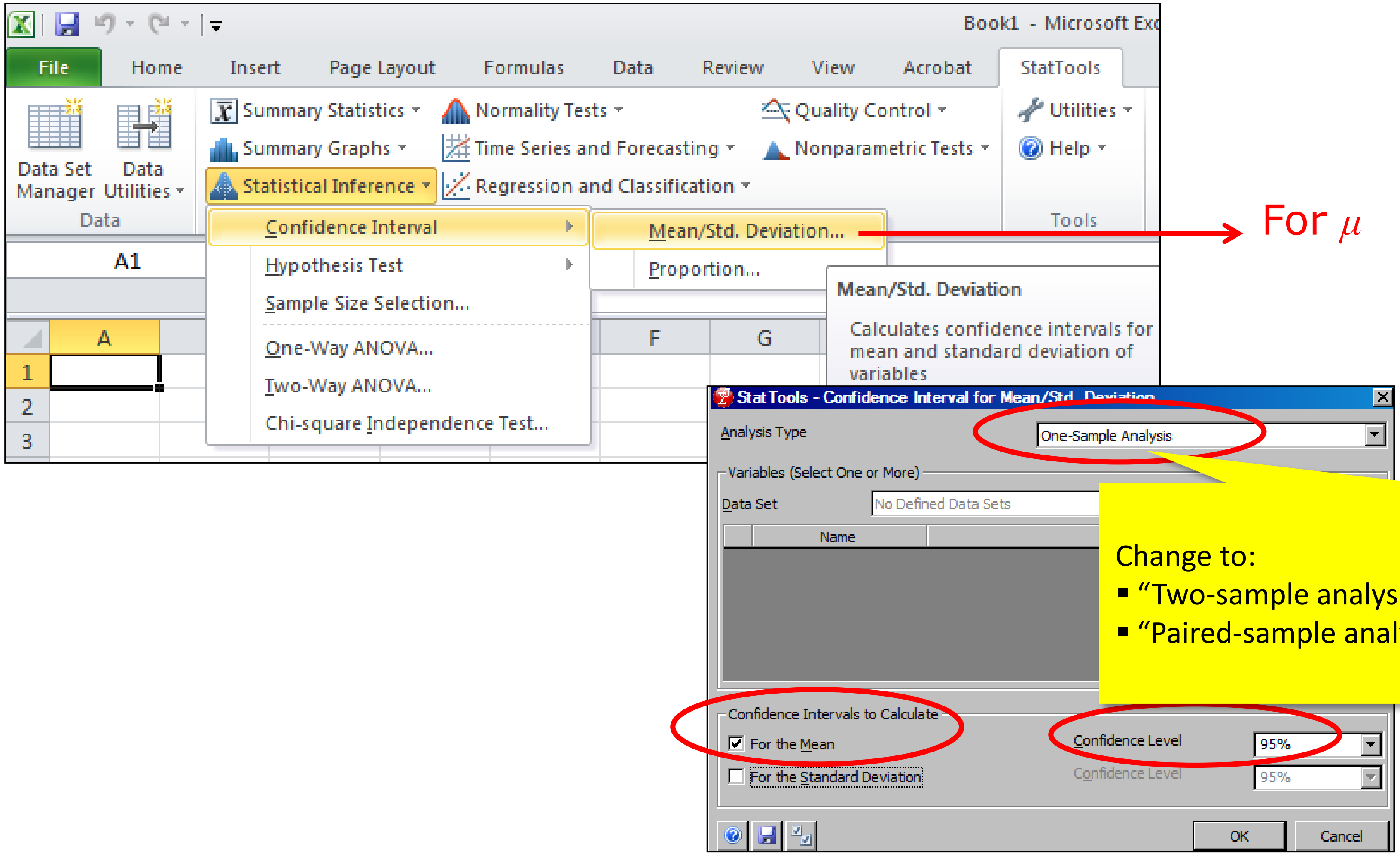
Paired samples

Example Sales Presentation Ratings.xlsx

Example Task Times.xlsx

Excel

StatTools → Statistical Inference → Confidence Interval → Mean



For μ

Change to:

- "Two-sample analysis" for independent samples
- "Paired-sample analysis" for paired (matched) samples

Today: Confidence interval for $\mu_1 - \mu_2$

- Independent samples ✓
- Dependent (paired) samples ✓