

Lab 7

problem 1: 1, 2, 4, 4, 5, 6, 9, 11, 12, 12, 17

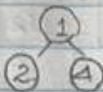
Insert 1.



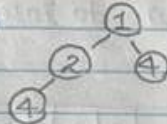
Insert 2



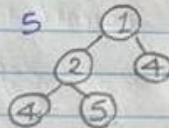
Insert 4



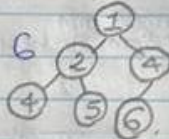
Insert 4



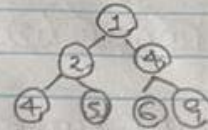
Insert 5



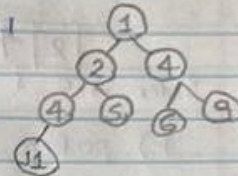
Insert 6



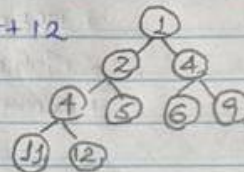
Insert 9



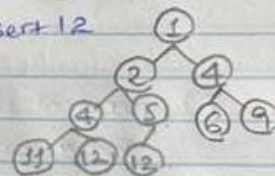
Insert 11



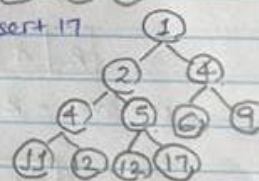
Insert 12



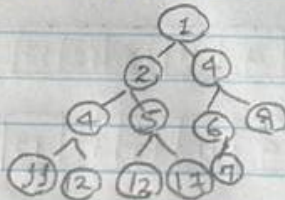
Insert 12



Insert 17

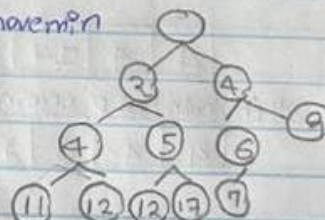


B) Insert 7

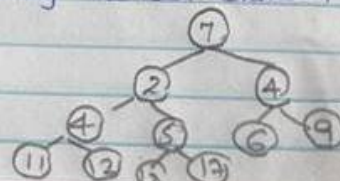


C) Show remove min

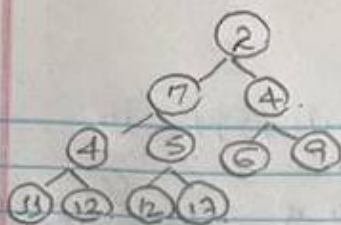
- remove min



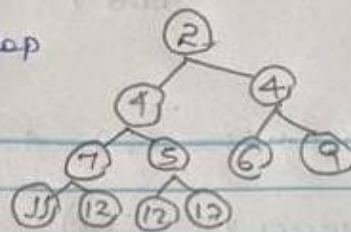
- bring the last element to pos min



- do down heap.



→ downheap



D) Represent in an Array

2	4	4	7	5	6	9	11	12	12	17
index → 0	1	2	3	4	5	6	7	8	9	10

E) perform Array based insertion to insert 14

* left child → $2i+1$

* right child → $2i+2$

* parent node → $\frac{(i-1)}{2}$

0	1	2	3	4	5	6	7	8	9	10	11
2	4	4	7	5	6	9	11	12	12	17	14

$$\text{parent}(14) = \frac{(11-1)}{2} = 5$$

6 is smaller so ✓

F) perform Array-based ^{remove min} heap sort on the input array A

2	4	4	7	5	6	9	11	12	12	17	14
---	---	---	---	---	---	---	----	----	----	----	----

remove - 2

14	4	4	7	5	6	9	11	12	12	17
----	---	---	---	---	---	---	----	----	----	----

then do downheap

4	4	14	7	5	6	9	11	12	12	17
---	---	----	---	---	---	---	----	----	----	----

then downheap again

4	4	6	7	5	14	9	11	12	12	17
---	---	---	---	---	----	---	----	----	----	----

problem 2: sort: [1, 4, 3, 9, 12, 2, 4]

phase I: Heapification

1) 1 | 4 3 9 12 2 4

2) 4 | 1 3 9 12 2 4 \rightarrow upheap

3) 4 | 1 3 | 9 12 2 4

4) 4 | 1 3 9 | 12 2 4 \rightarrow upheap x 2 9 4 3 1 | 12 2 4

5) 9 4 | 3 1 12 | 2 4 \rightarrow upheap x 2 12 9 3 1 4 | 2 4

6) 12 9 3 1 4 2 | 4 \rightarrow No upheap

7) 12 9 3 1 4 2 4 \rightarrow upheap 12 9 4 1 4 2 3

phase II: remove max

12

9 4 1 4 2 3 \rightarrow 3 9 4 1 4 2 |

down heap

9 3 4 1 4 2 | \rightarrow down heap 9 4 4 1 3 2 | 12

9

4 4 1 3 2 | 12 \rightarrow 2 4 4 1 3 | 12

down heap

4 2 4 1 3 | 12 \rightarrow down heap 4 3 4 1 2 | 9 12

4

3 4 1 2 | 9 12 \rightarrow 2 3 4 1 | 9 12

down heap

4 3 2 1 | 9 12 \rightarrow 4 3 2 1 | 4 9 12

4

3 2 1 | 4 9 12 \rightarrow 1 3 2 | 4 9 12

down heap

3 1 2 | 4 9 12 \rightarrow 3 1 2 | 4 4 9 12

3 $\boxed{} \mid 1 \ 2 \mid 4 \ 4 \ 9 \ 12 \rightarrow 2 \ 1 \mid 4 \ 4 \ 9 \ 12$
 $2 \ 1 \mid 3 \ 4 \ 4 \ 9 \ 12$

2 $\boxed{} \mid 1 \mid 3 \ 4 \ 4 \ 9 \ 12 \rightarrow 1 \mid 3 \ 4 \ 4 \ 9 \ 12$
 $1 \mid 2 \ 3 \ 4 \ 4 \ 9 \ 12$

1 $\boxed{} \mid 2 \ 3 \ 4 \ 4 \ 9 \ 12 \rightarrow$ No element to swap with
 $1 \ 2 \ 3 \ 4 \ 4 \ 9 \ 12 \rightarrow$ sorted



problem 3 Carryout B+H on the input sequence.

11, 5, 2, 3, 17, 24, 1.

$k = 11$

$A_1 = [5, 2, 3]$ $A_2 = [17, 24, 11]$

B+H ($A_1 = [5, 2, 3]$)

$k = 5$

$A_1 = [2]$ $A_2 = [3]$

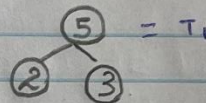
B+H ($A_1 = [2]$)

$k = 2$ $A_1 = \emptyset$ (2)

B+H ($A_2 = [3]$)

$k = 3$ $A_2 = \emptyset$ (3)

→ take the 2 nodes and the key



Now B+H ($A_2 = [17, 24, 11]$)

$k = 17$

$A_1 = 24$ $A_2 = 1$

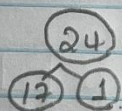
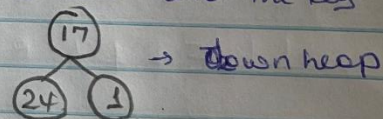
B+H ($A_1 = [24]$)

$k = 24$ $A_1 = \emptyset$ (24)

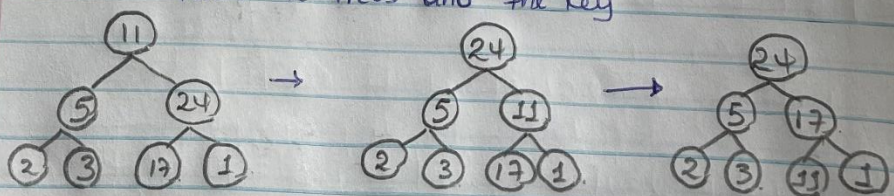
B+H ($A_2 = [1]$)

$k = 1$ $A_2 = \emptyset$ (1)

→ take 2 nodes and the key



take 2 trees and the key



problem 4: draw Max heap using keys [1, 21]

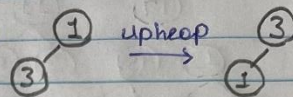
our keys will be

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21

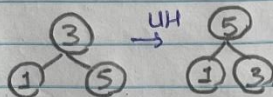
Insert 1



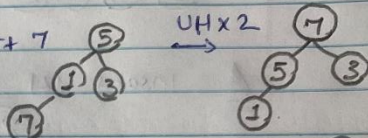
Insert 3



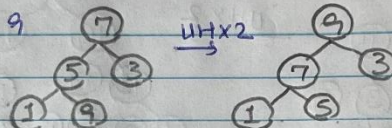
Insert 5



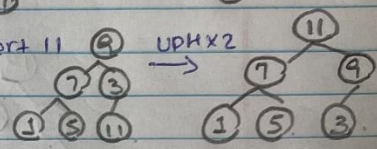
Insert 7



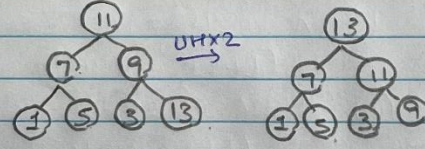
Insert 9



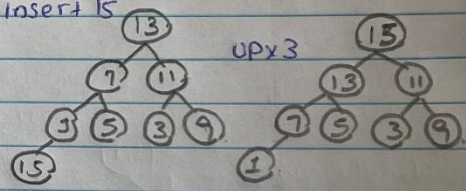
Insert 11



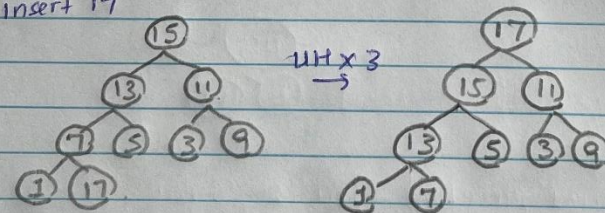
Insert 13



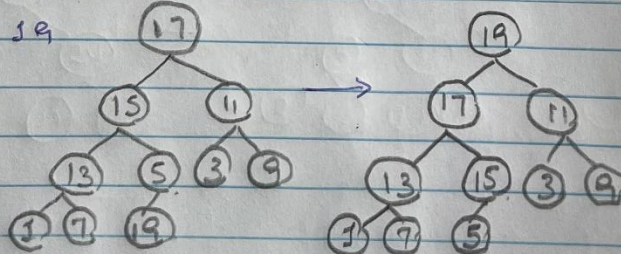
Insert 15



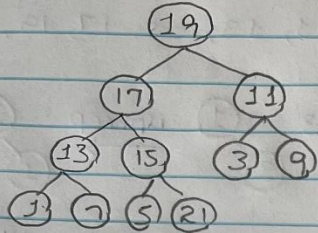
Insert 17



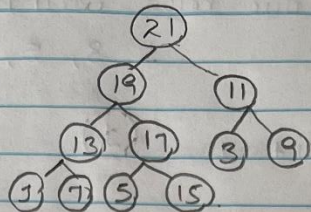
Insert 19



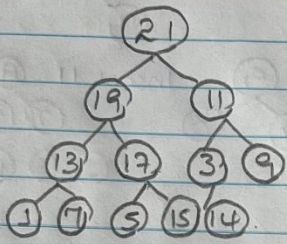
insert 21



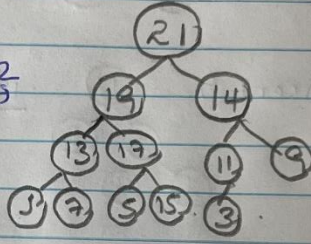
UHx3



Insert 14



UHx2



problem 5: Optimization B+H

10, 3, 7, 6, 4, 12, 5, 2, 12, 10, 1

$n = 11$ am m ms

step: make $\frac{n+1}{2}$ single item heap tree

$$\frac{11+1}{2} = \frac{12}{2} = 6$$

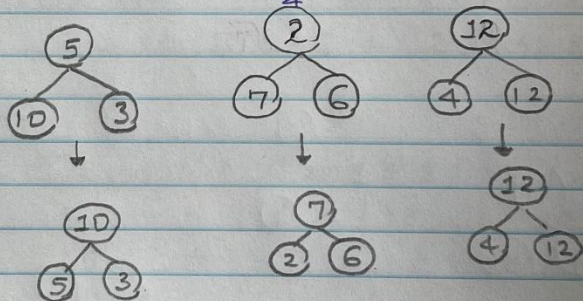
10, 3, 7, 6, 4, 12

$\frac{n-1}{2}$ - reserve

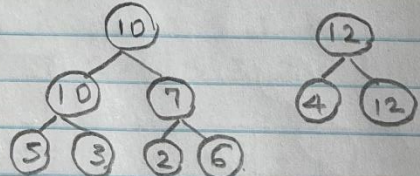
5, 2, 12, 10, 1

10 3 7 6 4 12

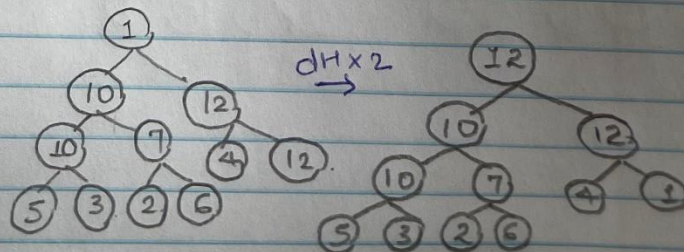
pull next $\frac{n+1}{4} = \frac{11+1}{4} = 3$



pull next $\frac{n+1}{8} = \frac{11+1}{8} = 1$



finally



the array obtained from this is

12	10	12	10	7	4	1	5	3	2	6
----	----	----	----	---	---	---	---	---	---	---

phase II remove max

12

10	12	10	7	4	1	5	3	2	6
----	----	----	---	---	---	---	---	---	---

6 10 12 10 7 4 1 5 3 2 |

12 10 6 10 7 4 1 5 3 2 |

12 10 7 10 6 4 1 5 3 2 | 12

12

10	7	10	6	4	1	5	3	2	12
----	---	----	---	---	---	---	---	---	----

downheapx3 2 10 7 10 6 4 1 5 3 | 12

10 10 7 5 6 4 1 2 3 | 12 12

10

10	7	5	6	4	1	2	3	12	12
----	---	---	---	---	---	---	---	----	----

downheapx3 3 10 7 5 6 4 1 2 | 12 12

10 6 7 5 3 4 1 2 | 10 12 12

10

6	7	5	3	4	1	2	10	12	12
---	---	---	---	---	---	---	----	----	----

2 6 7 5 3 4 1 | 10 12 12

7 6 4 5 3 2 1 | 10 10 12 12

7

6	4	5	3	2	1	10	10	12	12
---	---	---	---	---	---	----	----	----	----

1 6 4 5 3 2 | 10 10 12 12

6 5 4 1 3 2 | 7 10 10 12 12

6

5	4	1	3	2	7	10	10	12	12
---	---	---	---	---	---	----	----	----	----

2 5 4 1 3 | 7 10 10 12 12

5 3 4 1 2 | 6 7 10 10 12 12

5

☐ 3 4 1 2 | 6 7 10 10 12 12
 2 3 4 1 | 6 7 10 10 12 12
 4 3 2 1 | 5 6 7 10 10 12 12

4

☐ 3 2 1 | 5 6 7 10 10 12 12
 1 3 2 | 5 6 7 10 10 12 12
 3 1 2 | 4 5 6 7 10 10 12 12

3

☐ 1 2 | 4 5 6 7 10 10 12 12
 2 1 | 4 5 6 7 10 10 12 12
 2 1 | 3 4 5 6 7 10 10 12 12

2

☐ 1 | 3 4 5 6 7 10 10 12 12
 1 | 3 4 5 6 7 10 10 12 12
 1 | 2 3 4 5 6 7 10 10 12 12

1

☐ 1 2 3 4 5 6 7 10 10 12 12

finally, 1 2 3 4 5 6 7 10 10 12 12