CMSC 250

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Introduction

This is the lecture notes scribed by me. If you find any mistakes in the notes please email me at `lzhang34@terpmail.umd.edu`.

These notes are taken from Paul Kline's CMSC250 course and are an abridged version of notes I take during lectures. I am not a particularly heavy notetaker, so do note a strong possibility of some missing content in here if I deemed it not needed to be noted. For that reason, I would recommend you only use this as supplemental notes in addition to your own. An excellent source are the in-depth notes from Justin, which I believe are the teaching notes the course is based upon https://www.math.umd.edu/ immortal/CMSC250/

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Chapter 1

Discrete Structures Intro Statements

Definition 1.0.1: Statements

Declarative sentence with truth value

- Non subjective, contains defined true or false (Never both)

- Logical, non meaningless. Statements mean something

- Does not need to be verifiable: Ex.

  - There is exactly 12 people sneezing at this second
  - Aliens exist

Example 1.0.1 (Statement Examples)

- $2 + 2 = 4$

- $2 + 2 = 0$

- The sky is blue

Note above how both $2 + 2 = 4$ and $2 + 2 = 0$ are both valid statements despite them contradicting each other. Both have a defined true-false value and thus can both be perfectly valid as a statement. $2 + 2 = 4$ is always true, and $2 + 2 = 0$ is always false, no question about it.

Example 1.0.2 (Non-Statement Examples)

- $x > 30$

- CMSC250 is hard

The first line is not a statement as $x > 30$ is subjective. What is x? if x is 40 then the statement is true, but if x is 0 then the statement is false. Thus there is no defined true-false value. Until x is defined, it is not a statement.

Similarly the second non-statement is false. According to what metric is CMSC250 hard? Some people will agree or disagree, making it subjective and thus not a statement. We can easily fix the two above into statements however with a few quick word changes

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Example 1.0.3 (Fixed Statements)

- $x = 50 : x > 30$

> • I think CMSC250 is hard

These are now both valid statements, as with a defined x value, we know x will always be greater than 30. Similarly we defined the second statement by adjusting it from a vague subjective thought, to a statement of my opinion. A statement of my opinion is always true or false.

1.1   Statement Notation

Statements are notated at all times with a lowercase letter, I.E. $s =$ the sky is blue. Additionally there are various special symbols that can be used to modify statements and their relation to other statements

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Definition 1.1.1: Logical Operators

- $\neg$ or $\sim$ symbolizes Negation (Not)

- $\wedge$ symbolizes Conjunction (And)

- $\vee$ symbolizes Disjunction (Or)

- $\oplus$ symbolizes mutually exclusive and/or (XOR)

    - *Note: We will not use XOR as it can be expressed with a combination of conjunctions & and disjunctions $\vee$

    - Ex: $(p \wedge \neg q) \vee (\neg p \wedge q)$

    - p and not q or not p and q

Example 1.1.1 (Statement Examples)

Take these two statements:
s : French fries are green
q : The sky is blue

- Conjunction Example: s $\wedge$ q - False

- Disjunction Example: s $\vee$ q - True

- Not example: $\neg$ s - True

Like more traditional math, Statement Symbols have a defined order of operations in a sense that notation matters. Read left to right, $\neg$ always takes priority. Following this, $\wedge$ takes priority over $\vee$ in the absence of ().

Question 1: Statement Ordering

Notate the following: Today I have a discussion or lecture and a presentation

- p: I have a discussion

- q: I have a lecture

- r: I have a presentation

Now consider two possible notations:

- (p $\vee$ q) $\wedge$ r

- p $\vee$ (q $\wedge$ r)

Both can be true, so we should use () to dictate what we mean, however in this case since $\wedge$ takes priority, B is correct. () is always optimal though

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Now consider this notation:

- x ∧ q $\implies$ r ∨ z ⇔ t

This is (x ∧ q) $\implies$ ((r ∨ z) ⇔ t) as precedence is right associative

Edge cases of statements. Not all statements are obviously a statement

Example 1.1.2 (Unusual Statements)

- Let x be any number between 20 and 50. $x > 30$ This is a statement, but is false as it is saying x is always greater than 30, but that is not true. However it is still a statement as x is defined

- This statement is false. This is not a statement as there is a paradox, and cannot have a true false value

Definition 1.1.2: Standard Conventions

- True = 1

- False = 0

- We will use these numerical values over t and f

1.2   Truth Tables

Example 1.2.1 (Truth Table)

Simple Example:

p	¬ p
0	1
1	0

Another Example:

p	q	p∧q
0	0	0
0	1	0
1	0	0
1	1	1

And another:

p	q	p∨q
0	0	0
0	1	1
1	0	1
1	1	1

We always want to follow this ordering of truth tables values. Always go smallest to largest. It's binary but you can think of it as combining them together. Lets look at the last example, merging both rows we get $00 < 01 < 10 < 11$.

For variable order we try and go alphabetical when listing them.

Truth table lengths are exponential based upon the number of variables.

- 1 var = 2 rows

- 2 var = 4 rows

- 3 var = 8 rows

- n var = 2^n rows

Definition 1.2.1: Contradiction

A contradiction is a statement form that is always false regardless of the truth values of the individual statements substituted for its statement variables. AKA it always evaluates out to 0.

The truth table for p ∧ q ¬ p

p	¬ p	p ∧ ¬p
0	1	0
1	0	0

Definition 1.2.2: Tautology

A tautology is a statement that is always true regardless of the truth values of the individual statements substituted for its statement variables. AKA it always evaluates out to 1

p	¬ p	p ∨ ¬p
0	1	1
1	0	1

Definition 1.2.3: Equivalence

Two statements are called logically equivalent if, and only if, they have identical truth values for each possible substitution of statements for their statement variables. Symbolized with: p ≡ q.
Basically, if the two statements have the same truth tables results for same input values, they are logically equivalent

Example 1.2.2 (Proving XOR)

Using truth tables we can construct a truth table for XOR.Any truth table that evaluates out to 0 1 1 0 is logically equivalent to XOR

p	q	¬p	¬q	$(p \wedge \neg q)$	$(\neg p \wedge q)$	$(p \wedge \neg q) \vee (\neg p \wedge q)$
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	0	0	0	0

1.3 Conditionals and Implications

Definition 1.3.1: Implication Definition in English

We can construct implications from statements:
If I am in 250 lecture then it is Tuesday or Thursday
It is Tuesday or Thursday if I am in 250 lecture
I am in 250 lecture implies it is Tuesday or Thursday

- p: I am in 250 lecture

- q: It is Tuesday or Thursday

p $\implies$ q

In Lecture	Tuesday or Thursday	If I am in lecture then it is Tues/Thurs
NO	NO	YES
NO	YES	YES
YES	NO	NO
YES	YES	YES

Note how the above is not causal, it does not mean that I have to be in lecture to make it Tuesday or Thursday. It can be Tues/Thurs even if I'm not in lecture. It simply implies that I can only be in lecture on Tues/Thurs.

Thus the converse implication of q $\implies$ p is not always true.

Question 2: Evaluating Inverse, Contrapositive

How about the inverse? If $p \implies q$ then is $\neg p \implies \neg q$?
No. If I'm not in lecture it doesn't mean it's not Tues or Thurs. (I might be sick) Thus neither the inverse nor converse always hold

What about the contrapositive?
Given $p \implies q$ then is $\neg q \implies \neg p$?
Yes. If it is not Tues/Thurs then it implies I am not in lecture, as lecture is only on Tues/Thurs. Thus, only the contrapositive always holds

Definition 1.3.2: Converse, Inverse, Contrapositive

In summary...

- Converse - Not always true

- Inverse - Not always true

- Contrapositive - Always true

Note:-

If the hypothesis is false, then $p \implies q$ is vacuously true
If I go to Mars, then I bring 12 pink elephants.
I go to mars = 0
Vacuously true. Anything I say will be true because the initial hypothesis (I go to mars) is false.