MGSC 660

Mathematical & Statistical Foundations for Analytics

Contributors

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Group Assignment

Question 1: Freemark Abbey Winery case

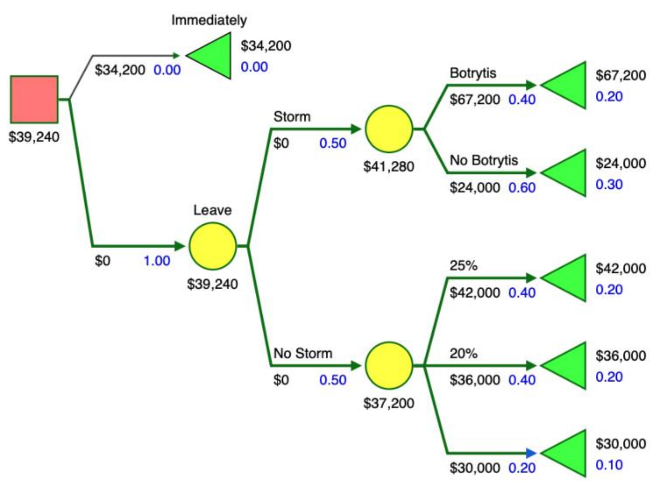
Assuming Mr. Jaeger chooses to harvest the Riesling grapes before the storm arrives, how much money will he make?

Revenue = $2.85*1000*12 = $34200
Hence, if Mr. Jaeger chooses to harvest the Riesling papers before the storm arrives, he will make $34, 200.

Assuming Mr. Jaeger chooses to leave the grapes on the vine, what is the probability that the grapes will end up with botrytis, and how much money will he make if that occurs?

Probability = 0.4*0.5 = 0.2
Revenue = $8*1000*12*0.7 = $67200
The probability that the grapes will end up with botrytis is 0.2, and he will gain revenue of
$67, 200 if that occurs.

Taking account of all the various possibilities, what should Mr. Jaeger do?



Based on the above decision tree, it is recommended that Mr.Jaeger should leave the grapes and harvest after storm as it will generate higher revenue of $39,240.

How much should Mr. Jaeger be willing to pay to learn whether the storm really will hit the Napa Valley?

(Conditions can be find in the above decision tree. Cf. *Appendix 1*).
Assume the storm will hit the Valley:
Storm-hit Revenue = 0.4 * $67200 + 0.6 * $24000 = $41280

Assume the storm will not hit the Valley:
Storm-not-hit Revenue = 0.4 * $42000 +0.4 * $36000 + 0.2 * $30000 = $37200

Revenue generated by knowing the storm will come:
((0.5 * $41280) + (0.5 * $37200)) - $39240 = $0

Therefore, Mr. Jaeger should pay $0 to learn whether the storm really will hit the Napa Valley.

How much should Mr. Jaeger be willing to pay to learn whether botrytis would form if the storm were to hit the Napa Valley?

Assume botrytis would form and the storm would strike, Mr.Jaeger can choose whether he want to leave the grapes after the storm that will form the botrytis, or immediately harvest the grapes:
Revenue = 67200*0.4 + 34200*0.6 = $47400

Assume Mr.Jaeger doesn't have the information on either storm or botrytis:
Revenue = 39240

The value of the information = 47400 - 39240 = $8160

$$\boldsymbol{Confidence\ Interval\ (CI)\ Estimate\ =\ Point\ Estimate\ \pm\ Margin\ of\ Error}$$

- Upper Confidence Limit = 0.6750 + 0.0290 = 0.7040
- Lower Confidence Limit = 0.6750 + 0.0290 = 0.6460

Question 2: Revenue management on Air Canada Flight 660

N.B.: The presented figures for means and standard deviations are derived from a randomized simulation conducted in Excel. It's important to note that due to the nature of the simulation, the numbers continuously update. The values provided in the responses correspond to those displayed in the attached screenshot and represent a single possible valid outcome. This clarification ensures the accuracy of the reported results.

Construct a simulation model in Excel (or in Python) to simulate the following policy: protect 30 seats exclusively for full-fare customers. Simulate for 1000 times. What are the mean and standard deviation of the revenue? Calculate the 95% confidence interval for the mean. What do you think about the policy?

Details on "Possible Full-Fare" and "Possible Discount" Computations:
Initially, we generated random numbers for the number of full-fare tickets and subsequently established a lower limit of 30 (min). We followed a similar approach for discount seats as well.

After conducting 1000 iterations of the proposed simulation involving the protection of 30 seats exclusively for full-fare customers, the resulting values for revenue mean and standard deviation are the following:

- Expected value of revenue: $63,701.60
- Standard deviation of revenue: $6,155.41

Confidence Level
With 95% confidence, we can say that the true population mean is likely to fall within the range of \$63,320.09 and \$64,083.11. This means that if we were to take many random samples and calculate the confidence interval for each, we would expect about 95% of these intervals to contain the true population mean in other words the true revenue.

In simpler terms, we are certain that the actual average value of the entire population lies somewhere between \$63,320.09 and \$64,083.11, based on the information from the sample.

Optimal Policy Interpretation
Within the problem's context, while simulating the protective policy that allocates exclusive full-fare tickets to 30 customers, **an average of approximately 4.79 vacant seats per flight** was observed. This suggests potential room for enhancement in flight occupancy. Yet, further experimentation with various policies is necessary to ascertain the most effective number of seats to be safeguarded for optimal outcomes.

Appendix 2.1 displays the simulation results in **Excel**.

Use Data Table function in Excel (or create a function in Python) to optimize the mean of the revenue by varying the protection level for full-fare customers between 20 and 40. Report the mean and standard deviation of the revenue corresponding to the optimal policy. Calculate the 95% confidence interval for the mean of the revenue corresponding the optimal policy.

We used Excel's Data Table function to find the best protection level for full-fare customers, ranging from 20 to 40, that maximized the average revenue. The resulting average revenue and its standard deviation for the optimal policy are as follows:

- Expected value of revenue: \$ 64,112.80
- Standard deviation of revenue: \$ 5724.86

Find the 95% confidence interval for the probability of optimal revenue greater than or equal to the mean revenue that corresponds to protection level of 30 seats (i.e., the mean revenue that you found in part a).

After performing simulations to find possible full-fare and discount seats, we computed the optimal revenue and compared this average revenue for each of 1000 iterations with mean revenue that corresponds to protection level of 30 seats, i.e., \$ 63,856.40.

We found that the optimal revenue was greater than mean revenue in 675 instances, resulting in probability of 0.6750.

Confidence level estimate can be calculated as:

$$\boldsymbol{Probability\ (Optimal\ Revenue\ \geq\ Mean\ Revenue\ for\ 30\ seats)\ =\ 675/1000\ =\ 0.6750}$$

Then, we compute the standard deviation using the below formula

$$\boldsymbol{Standard\ Error\ =\ \sqrt{p}(\ 1-p)\ /\ \sqrt{n}}$$

Sample standard deviation = SQRT ((0.6750) * (1-0.6750) / 1000) which gives us **0.0148**

Next, we compute Margin of Error (e) for 95% confidence level

$$\boldsymbol{Margin\ of\ Error\ (e)\ =\ Z-score\ *\ Stamdard\ Error}$$

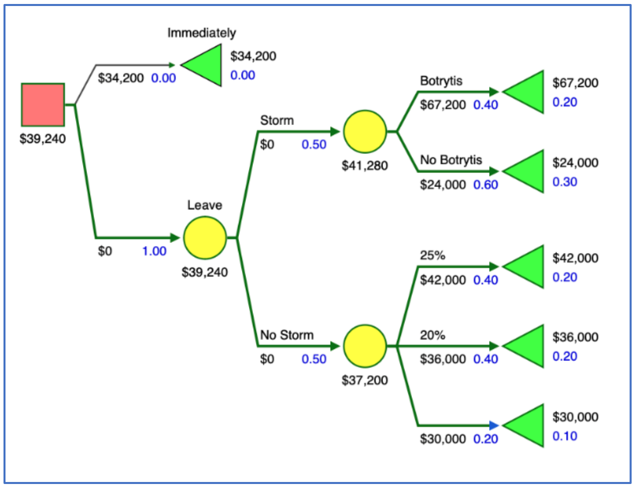
$$e = -\ NORM.S.INV(0.025) * (\sigma / (n)^{1/2})$$

$$e = 1.96 * (0.0148)$$

$$e = 0.0290$$

Appendix

Appendix 1: Decision Tree Rationale



Appendix 2.1: Excel Simulation Results: Impact of Reserving 30 Exclusive Seats for Full-Fare Customers.

	Full fare	Discount
Revenue per ticket	$ 1,000.00	$ 400.00
Mean	$ 30.00	$ 120.00
Std Dev	$ 10.00	$ 20.00
Reserve	30	95

Total Capacity	125
Number of full-fare tickets to sell	90
Expected Profit	$ 63,701.60
Standard Deviation	$ 6,155.41
Positive Z-score	1.96
Negative Z-score	-1.96
Upper band	$ 64,083.11
Lower band	$ 63,320.09
Average non-assigned/empty seats	4.797

Appendix 2.2: Excel Simulation Results: Impact of varying the protection level for full-fare customers between 20 and 40.

Protection level for full-fare customers	$ 64,046.00	$ 5,761.34
20	$ 63,830.00	$ 5,844.11
21	$ 63,529.40	$ 6,139.40
22	$ 64,112.80	$ 5,724.86
23	$ 63,822.80	$ 6,072.85
24	$ 63,882.00	$ 5,800.33
25	$ 63,956.00	$ 5,747.64
26	$ 63,761.80	$ 6,033.10
27	$ 63,686.80	$ 5,961.10
28	$ 63,732.80	$ 6,114.78
29	$ 63,903.20	$ 5,826.71
30	$ 63,895.20	$ 5,968.99
31	$ 63,740.80	$ 6,043.68
32	$ 63,775.40	$ 5,995.95
33	$ 63,751.40	$ 6,237.80
34	$ 63,827.80	$ 5,843.36
35	$ 63,854.40	$ 5,847.50
36	$ 63,683.40	$ 6,041.62
37	$ 63,706.00	$ 6,030.44
38	$ 64,067.40	$ 5,586.08
39	$ 63,793.20	$ 5,785.44
40	$ 63,869.80	$ 5,899.70