Univ Gustave Eiffel - Cosys / Grettia

Reinforcement Learning and Optimal Control - Master 2 SIA

Monte Carlo Methods

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- Monte Carlo methods can thus be incremental in an episode-by-episode sense, but not in a step-by-step (online) sense.
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- As in DP, we will see Policy evaluation, policy improvement, and then Policy iterations and GPI.

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- The average should converge to the expected value.
- First-visit MC method : estimates $v_\pi(s)$ as the average of the returns following first visits to s.
- Every-visit MC method : averages the returns following all visits to s.

First-visit MC method

First-visit MC prediction, for estimating $V \approx v_\pi$

Input: a policy π to be evaluated

Initialize:

$V(s) \in \mathbb{R}$, arbitrarily, for all $s \in \mathcal{S}$

$Returns(s) \leftarrow$ an empty list, for all $s \in \mathcal{S}$

Loop forever (for each episode):

Generate an episode following π: $S_0, A_0, R_1, S_1, A_1, R_2, \ldots, S_{T-1}, A_{T-1}, R_T$

$G \leftarrow 0$

Loop for each step of episode, $t = T-1, T-2, \ldots, 0$:

$G \leftarrow \gamma G + R_{t+1}$

Unless S_t appears in $S_0, S_1, \ldots, S_{t-1}$:

Append G to $Returns(S_t)$

$V(S_t) \leftarrow \text{average}(Returns(S_t))$

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- Unlike DP, in Monte Carlo methods, the estimates for each state are independent, i.e. the estimate for one state does not build upon the estimate of any other state.
- The computational expense of estimating the value of a single state is independent of the number of states.
- This can make Monte Carlo methods particularly attractive when one requires the value of only one or a subset of states.

Monte Carlo Estimation of Action Values

- We consider the policy evaluation problem for action values (estiamtion of $q_\pi(s, a)$.
- We are interested here in visits to a state-action pair rather than to a state.
- A state-action pair (s, a) is said to be visited in an episode if ever the state s is visited and action a is taken in it.
- We assume that the episodes start in a state-action pair, and that every pair has a nonzero probability of being selected as the start.
- This guarantees that all state-action pairs will be visited an infinite number of times in the limit of an infinite number of episodes.
- This is called the assumption of exploring starts.
- The most common alternative approach to assuring that all state-action pairs are encountered is to consider only policies that are stochastic with a nonzero probability of selecting all actions in each state.

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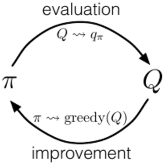

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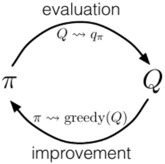

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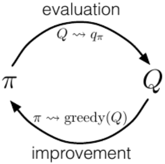

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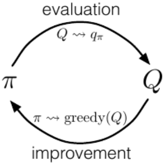

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- Policy evaluation is done as descriobed above : many episodes are experienced, with the approximate action-value function approaching the true function asymptotically.
- Policy improvement is done by making the policy greedy with respect to the current value function.

$$\pi(s) \doteq \arg\max_a q(s,a).$$

Monte Carlo Exploring Starts

Monte Carlo ES (Exploring Starts), for estimating $\pi \approx \pi_*$

Initialize:

$\pi(s) \in \mathcal{A}(s)$ (arbitrarily), for all $s \in \mathcal{S}$

$Q(s,a) \in \mathbb{R}$ (arbitrarily), for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$

$Returns(s,a) \leftarrow$ empty list, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$

Loop forever (for each episode):

Choose $S_0 \in \mathcal{S}$, $A_0 \in \mathcal{A}(S_0)$ randomly such that all pairs have probability > 0

Generate an episode from S_0, A_0, following π: $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$

$G \leftarrow 0$

Loop for each step of episode, $t = T-1, T-2, \ldots, 0$:

$G \leftarrow \gamma G + R_{t+1}$

Unless the pair S_t, A_t appears in $S_0, A_0, S_1, A_1 \ldots, S_{t-1}, A_{t-1}$:

Append G to $Returns(S_t, A_t)$

$Q(S_t, A_t) \leftarrow \text{average}(Returns(S_t, A_t))$

$\pi(S_t) \leftarrow \arg\max_a Q(S_t, a)$

Monte Carlo Control without Exploring Starts

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- Among ε-soft policies, ε-greedy policies are in some sense those that are closest to greedy.
- Indeed, with a ε-greedy policy :
  - all nongreedy actions are given the minimal probability of selection, $\frac{\varepsilon}{|\mathcal{A}(s)|}$,
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- GPI does not require that the policy be taken all the way to a greedy policy, only that it be moved toward a greedy policy.
- We propose here that the policy will be moved only to an ε-greedy.
- We know that for any ε-soft policy, π, any ε-greedy policy with respect to q_π is guaranteed to be better than or equal to π (proof below).

Monte Carlo Control without Exploring Starts

For any ε-soft policy, π, any ε-greedy policy with respect to q_π is guaranteed to be better than or equal to π :

$$
\begin{aligned}
q_\pi(s,\pi'(s)) &= \sum_a \pi'(a|s) q_\pi(s,a) \\
&= \frac{\varepsilon}{|\mathcal{A}(s)|} \sum_a q_\pi(s,a) \; + \; (1-\varepsilon) \max_a q_\pi(s,a) \\
&\geq \frac{\varepsilon}{|\mathcal{A}(s)|} \sum_a q_\pi(s,a) \; + \; (1-\varepsilon) \sum_a \frac{\pi(a|s) - \frac{\varepsilon}{|\mathcal{A}(s)|}}{1-\varepsilon} q_\pi(s,a) \\
&= \frac{\varepsilon}{|\mathcal{A}(s)|} \sum_a q_\pi(s,a) \; - \; \frac{\varepsilon}{|\mathcal{A}(s)|} \sum_a q_\pi(s,a) \; + \; \sum_a \pi(a|s) q_\pi(s,a) \\
&= v_\pi(s).
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Therefore $\pi' \geq \pi$, $\quad$ *i.e.*$\forall s \in \mathcal{S}, v_{\pi'}(s) \geq v_\pi(s)$.

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Therefore $\pi' \geq \pi, \quad i.e. \forall s \in \mathcal{S}, v_{\pi'}(s) \geq v_\pi(s)$.

Indeed, we know also that equality can hold only when both π and π' are optimal among the ε-soft policies.

Monte Carlo Control without Exploring Starts

On-policy first-visit MC control (for ε-soft policies), estimates $\pi \approx \pi_*$

Algorithm parameter: small $\varepsilon > 0$

Initialize:

$\quad \pi \leftarrow$ an arbitrary ε-soft policy

$\quad Q(s,a) \in \mathbb{R}$ (arbitrarily), for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$

$\quad Returns(s,a) \leftarrow$ empty list, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$

Repeat forever (for each episode):

$\quad$ Generate an episode following π: $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$

$\quad G \leftarrow 0$

$\quad$ Loop for each step of episode, $t = T-1, T-2, \ldots, 0$:

$\qquad G \leftarrow \gamma G + R_{t+1}$

$\qquad$ Unless the pair S_t, A_t appears in $S_0, A_0, S_1, A_1 \ldots, S_{t-1}, A_{t-1}$:

$\qquad\quad$ Append G to $Returns(S_t, A_t)$

$\qquad\quad Q(S_t, A_t) \leftarrow \text{average}(Returns(S_t, A_t))$

$\qquad\quad A^* \leftarrow \arg\max_a Q(S_t, a)$ (with ties broken arbitrarily)

$\qquad\quad$ For all $a \in \mathcal{A}(S_t)$:

$$\pi(a|S_t) \leftarrow \begin{cases} 1 - \varepsilon + \varepsilon/|\mathcal{A}(S_t)| & \text{if } a = A^* \\ \varepsilon/|\mathcal{A}(S_t)| & \text{if } a \neq A^* \end{cases}$$

Now we achieve the best policy among the ε-soft policies, but we have eliminated the assumption of exploring starts.

On-policy and off-policy learning

Off-policy learning considers two policies :

- Target policy : the one being learned and becomes the optimal policy.
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On-policy learning considers only one policy used to generate beahivior and is learned and becomes the optimal pilicy.

With off-policy methods, because the data is not due to the target policy, we have greater variance, and slow convergence.

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- Importance Sampling : a general technique for estimating expected values under one distribution given samples from another.

Off-policy Prediction via Importance Sampling

- Suppose we like to estimate v_π or q_π for a given target policy π,
- but we only have episodes following a given behavior policy b, $b \neq \pi$.
- Assuption of coverage : In order to use episodes from b to estimate values for π, we require that every action taken under π is also taken, at least occasionally, under b.
- Assuption of coverage : $\pi(a \mid s) > 0 \Rightarrow b(a \mid s) > 0$.
- Importance Sampling : a general technique for estimating expected values under one distribution given samples from another.
- importance-sampling ratio : We weight returns according to the relative probability of their trajectories occurring under the target and behavior policies.

Off-policy Prediction via Importance Sampling

- From a starting state S_t :

$$
\begin{aligned}
&\Pr\{A_t, S_{t+1}, A_{t+1}, \ldots, S_T \mid S_t, A_{t:T-1} \sim \pi\} \\
&\quad = \pi(A_t|S_t)p(S_{t+1}|S_t, A_t)\pi(A_{t+1}|S_{t+1}) \cdots p(S_T|S_{T-1}, A_{T-1}) \\
&\quad = \prod_{k=t}^{T-1} \pi(A_k|S_k)p(S_{k+1}|S_k, A_k),
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- The importance sampling ration is :

$$\rho_{t:T-1} \doteq \frac{\prod_{k=t}^{T-1} \pi(A_k|S_k)p(S_{k+1}|S_k, A_k)}{\prod_{k=t}^{T-1} b(A_k|S_k)p(S_{k+1}|S_k, A_k)} = \prod_{k=t}^{T-1} \frac{\pi(A_k|S_k)}{b(A_k|S_k)}.$$

which depends only on the two policies and the sequence, not on the MDP.

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  which depends only on the two policies and the sequence, not on the MDP.

- The expected returns under π are :

$$
\mathbb{E}[\rho_{t:T-1} G_t \mid S_t = s] = v_\pi(s).
$$

Off-policy Prediction via Importance Sampling

- $\mathcal{T}(s)$: includes time steps that were first visits to s within their episodes.
- $T(t)$: the first time of termination following time t.
- G_t : the return after t up through $T(t)$.

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(unbiased with unbounded variance)

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- Given a sequance of returns $G_1, G_2, \dots, G_{n-1}$ all starting in the same state.

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Off-policy MC Policy Evaluation

Off-policy MC prediction (policy evaluation) for estimating $Q \approx q_\pi$

Input: an arbitrary target policy π
Initialize, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$:
    $Q(s,a) \in \mathbb{R}$ (arbitrarily)
    $C(s,a) \leftarrow 0$

Loop forever (for each episode):
    $b \leftarrow$ any policy with coverage of π
    Generate an episode following b: $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$
    $G \leftarrow 0$
    $W \leftarrow 1$
    Loop for each step of episode, $t = T-1, T-2, \ldots, 0$, while $W \neq 0$:
        $G \leftarrow \gamma G + R_{t+1}$
        $C(S_t, A_t) \leftarrow C(S_t, A_t) + W$
        $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \frac{W}{C(S_t,A_t)} [G - Q(S_t, A_t)]$
        $W \leftarrow W \frac{\pi(A_t|S_t)}{b(A_t|S_t)}$

Off-policy Monte Carlo Control

- Advanatage : the target policy may be deterministic (e.g., greedy), while the behavior policy can continue to sample all possible actions.

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Off-policy Monte Carlo Control

- Advanatage : the target policy may be deterministic (e.g., greedy), while the behavior policy can continue to sample all possible actions.
- In off policy methods, we follow the behavior policy while learning about and improving the target policy.
- It is required that the behavior policy has a nonzero probability of selecting all actions that might be selected by the target policy (assumption of coverage).
- To explore all possibilities, we require that the behavior policy be soft (i.e., that it select all actions in all states with nonzero probability).

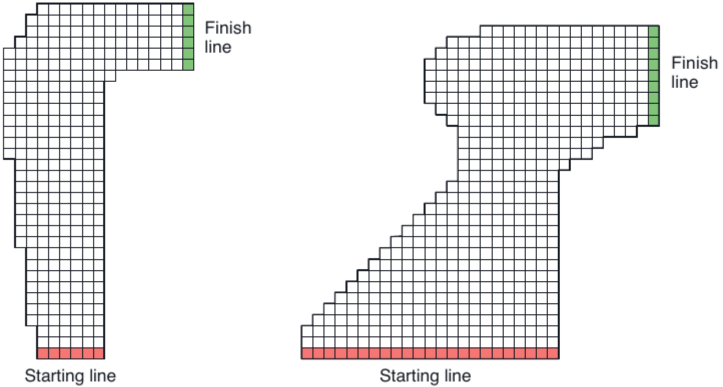
Off-policy Monte Carlo Control

Off-policy MC control, for estimating $\pi \approx \pi_*$

Initialize, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$:
- $Q(s,a) \in \mathbb{R}$ (arbitrarily)
- $C(s,a) \leftarrow 0$
- $\pi(s) \leftarrow \arg\max_a Q(s,a)$ (with ties broken consistently)

Loop forever (for each episode):
- $b \leftarrow$ any soft policy
- Generate an episode using b: $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$
- $G \leftarrow 0$
- $W \leftarrow 1$
- Loop for each step of episode, $t = T-1, T-2, \ldots, 0$:
  - $G \leftarrow \gamma G + R_{t+1}$
  - $C(S_t, A_t) \leftarrow C(S_t, A_t) + W$
  - $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \frac{W}{C(S_t, A_t)} [G - Q(S_t, A_t)]$
  - $\pi(S_t) \leftarrow \arg\max_a Q(S_t, a)$ (with ties broken consistently)
  - If $A_t \neq \pi(S_t)$ then exit inner Loop (proceed to next episode)
  - $W \leftarrow W \frac{1}{b(A_t|S_t)}$

Racetrack exercise



Racetrack exercise

Exercise 5.12: Racetrack (programming) Consider driving a race car around a turn like those shown in Figure 5.5. You want to go as fast as possible, but not so fast as to run off the track. In our simplified racetrack, the car is at one of a discrete set of grid positions, the cells in the diagram. The velocity is also discrete, a number of grid cells moved horizontally and vertically per time step. The actions are increments to the velocity components. Each may be changed by $+1$, -1, or 0 in each step, for a total of nine (3×3) actions. Both velocity components are restricted to be nonnegative and less than 5, and they cannot both be zero except at the starting line. Each episode begins in one of the randomly selected start states with both velocity components zero and ends when the car crosses the finish line. The rewards are -1 for each step until the car crosses the finish line. If the car hits the track boundary, it is moved back to a random position on the starting line, both velocity components are reduced to zero, and the episode continues. Before updating the car's location at each time step, check to see if the projected path of the car intersects the track boundary. If it intersects the finish line, the episode ends; if it intersects anywhere else, the car is considered to have hit the track boundary and is sent back to the starting line. To make the task more challenging, with probability 0.1 at each time step the velocity increments are both zero, independently of the intended increments. Apply a Monte Carlo control method to this task to compute the optimal policy from each starting state. Exhibit several trajectories following the optimal policy (but turn the noise off for these trajectories). □

Thank you !