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Use of Logistic, Gompertz and Richards functions for fitting normal and malformed mango panicle growth data

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Logistic, Gompertz and Richards functions were used for examining their suitability in fitting growth data of malformed and normal mango panicles. The functional analysis of data indicated that Richards function was a suitable model for summarising panicle growth data. The model was found to be superior to logistic and Gompertz because of its greater flexibility. The study revealed that Richards function can be successfully used for simulating panicle growth under different treatments and conditions.

Keywords: Logistic, Gompertz, Richards functions, growth, malformed, mango panicles, functional analysis

Introduction

Many attempts have been made to simulate limited growth curves by mathematical functions either accounting for their form through certain growth process or to obtain any relatively simple equation, which contain essence of numerical data. Limited attempts have been made to explain the growth of mango panicle through equations, which are used for describing growth. Mango panicles follow asymptotic growth curve, which can be fitted by using monomolecular, logistic and Gompertz equations (Rajan and Majumder, 1995). Sukhvibul *et al.* (1999) used the regression model y= $a + b/(1+exp[-(x-c)/d])$ for describing the effect of temperature on panicle growth. However Richards growth function, which is being used in biological studies for explaining growth of the plant parts and whole plants (Causton *et al.*, 1978), has not been used for modelling growth of mango plant parts. Usefulness of this function for modelling growth of different plant species and for several process and has been established (Pienaar and Turnbull, 1973; Moore *et al.*, 1988, Lalancette *et al.*, 1988)

The background of this biological equation has been described in detail (Richard, 1969, Causton *et al.*, 1978). The present paper deals with the evaluation of the sigmoid models including Richards function for studying growth pattern of malformed and normal panicles.

Materials and methods

The linear growth of Amrapali mango panicle was measured in terms of length of the main axis. Observations were recorded on the tagged panicles at three days interval, starting from the bud-burst stage. The main axis was measured from the juncture of shoot and panicle on the twigs examined for protuberances and axilary buds for the selection of normal and malformed panicles (Rajan, 1987). Finally the panicles were marked as malformed and normal when they attained about 3 cm growth. Ten panicles, each in normal and malformed category, were selected for recording growth data. Mean growth data of ten panicles was used for fitting the models. Initial estimates of the parameters were obtained as suggested by Draper and Smith (1981) and the estimates of nonlinear parameters were obtained by applying Marquard algorithm (Marquard, 1963). Cubic spline was used for interpolating the data for generating mean growth data at 5 days interval for malformed and normal panicles.

Following equations were used for fitting the models

$$l = \frac{a}{(1+be^{-ct})}, \quad \frac{dl}{dt} = \frac{abce^{-ct}}{(1+be^{-ct})^2}$$

and estimating growth rate of panicle

Logistic

Where, l= length of panicle main axis at the time t,

$$l = ae^{-e^{(b-ct)}}, \quad \frac{dl}{dt} = cae^{(b-ct)}.e^{-(b-ct)}$$

a is the measure of final size, b and c are the constants

Gompertz

Where l= length of panicle main axis at the time t, a

$$l = \frac{a}{(1+e^{(b-ct)})^{(1/m)}}, \quad \frac{dl}{dt} = \frac{ace^{(b-ct)}}{m(1+e^{(b-ct)})^{(\frac{1+m}{m})}}$$

is the measure of final size, b and c are the constants

Richards

Where, l= length of panicle main axis at the time t, a is the measure of final size, b, c and m are the constants. The constant m lie in the range of $-1 \leq m \leq \infty$ but $m \neq 0$. dl/dt is the absolute growth rate.

Results and discussion

All the three sigmoid growth functions, *viz.*, logistic, Gompertz and Richards, used for fitting panicle growth data were found to be appropriate for both the cases *i.e.* normal and malformed panicles. However, Richards function provided a better fit in terms of standard error and correlation coefficient values (Table 1). Richards function was found superior to explain the data of both types of panicles.

The Richards function yielded the parameters *a*, *b*, *c*, and *m* of which *a* and *m* can be considered biologically meaningful. Parameter *a*, gives the asymptotic maximum size of the panicle and *m* describes the shape of the growth curve. As the value of *m* increases there is an increasing duration of panicle growth when its growth rate is almost constant implying approximate exponential growth.

Table 1. Parameter estimates of growth models for mango panicles

Model	*a*	*b*	*c*	*m*	*SE*	*r*
Normal panicle						
Richard	34.0127	15.1053	0.5284	4.1301	1.0716	0.9979
Logistic	36.2971	79.8551	0.1868		1.6194	0.9947
Gompertz	36.4857	3.0319	0.1476		1.7692	0.9936
Malformed panicle						
Richard	20.0989	14.8010	0.4772	5.1611	0.9093	0.9952
Logistic	22.9187	24.1705	0.1280		1.3548	0.9882
Gompertz	31.7133	1.2940	0.0500		2.0780	0.9721

Table 2. Estimated values of panicle growth and absolute growth rate estimated by different models

Days	Normal panicle							Malformed panicle						
		Panicle growth estimates			Growth rate (cm/day)				Panicle growth estimates			Growth rate(cm/day)		
	PL	L	G	R	L	G	R	PL	L	G	R	L	G	R
0	0.8516	0.4489	0.0000	0.8776	0.0828	0.0000	0.1123	1.5373	0.9105	0.8265	1.1421	0.1119	0.1507	0.1056
5	2.2483	1.1210	0.0018	1.6639	0.2030	0.0026	0.2129	1.8776	1.6677	1.8517	1.8134	0.1980	0.2630	0.1677
10	3.2754	2.7231	0.3187	3.1546	0.4706	0.2230	0.4036	2.1326	2.9695	3.4708	2.8793	0.3309	0.3839	0.2662
15	4.5000	6.2102	3.7831	5.9801	0.9617	1.2654	0.7646	3.4110	5.0462	5.6615	4.5713	0.5038	0.4877	0.4225
20	10.0296	12.5004	12.3463	11.3112	1.5311	1.9744	1.4319	6.2176	7.9934	8.2879	7.2516	0.6665	0.5560	0.6671
25	23.0000	20.7646	21.7337	20.7840	1.6600	1.6617	2.3115	13.2200	11.5494	11.1518	11.4031	0.7335	0.5827	0.9979
30	29.9631	28.0519	28.4809	30.9668	1.1905	1.0411	1.2728	16.6369	15.0882	14.0519	16.6744	0.6600	0.5718	0.9539
35	33.2774	32.5390	32.4111	33.7409	0.6294	0.5664	0.1407	18.8750	17.9951	16.8237	19.5645	0.4950	0.5332	0.2349
40	34.1903	34.7214	34.4774	33.9930	0.2816	0.2881	0.0104	20.2561	20.0295	19.3560	20.0458	0.3233	0.4778	0.0251
45	34.3193	35.6612	35.5113	34.0113	0.1167	0.1419	0.0007	20.3387	21.2990	21.5895	20.0939	0.1927	0.4150	0.0023
50	34.8000	36.0445	36.0165	34.0126	0.0469	0.0688	0.0001	20.5000	22.0353	23.5060	20.0984	0.1088	0.3519	0.0002

PL= Panicle length value interpolated cubic spline for 5 days interval, L= Logistic, G= Gompertz, R= Richards

The estimate of *a* and *m* given in the Table 1 explains the difference in growth pattern of the malformed and normal panicles. The major difference in *a* value of both types of panicles is apparent and indicates that the final length which can be attained by the malformed panicle is about 66.67% that of normal panicle.

The present data is one of the classical example where Gompertz function has inadequately fitted normal panicle data during initial growth period (up to 15 days). This underestimated the growth rate during the period. The estimate of final length of the panicle is more in case of Gompertz and logistic function as compared to Richards function (Table 2). Logistic and Gompertz equations both overestimated the growth rate of malformed panicle during 35th to 50th day and resulted higher final growth estimates.

Despite Richards function was introduced in 1959, came in extended use considerably late with the increased use of personal computers in numerical analysis. Causton *et al.* (1978) have shown that it is a reasonable model for the determinate nature of growth. It has been used to elucidate pattern of leaf growth (Dennett *et al.*, 1978) which has growth curve shape similar to mango panicles. Richards model is based on biologically realistic model and has many advantageous uses for growth analysis of mango panicles at different time and treatments.

The better results obtained with the use of Richards model is due to its flexibility for fitting of data of diverse origin. It can be reduced to logistic and

Gompertz models also with the change in *m*. The logistic and Gompertz equations are also the special case of Richards function (Richards, 1969).

The functional analysis of data indicated that Richards function is a suitable model for summarising panicle growth. The model was found to be superior over logistic and Gompertz because of its greater efficiency. Richards function can be successfully used for simulating panicle growth under different conditions and treatments.

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