# Brief Article

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January 17, 2019

This note gives some equations from the paper introducing the HLLD method by Miyoshi and Kusano [1].

Ideal MHD eigenvalues

$$\lambda_{2,6} = u \mp c_A, \;\; \lambda_{1,7} = u \mp c_f, \;\; \lambda_{3,5} = u \mp c_s, \;\; \lambda_4 = u$$

where the Alfvén speed $c_A$, fast wave speed $c_f$ and slow wave speed $c_s$ are given by

$$c_A = \frac{|B_x|}{\sqrt{\rho}}, \;\; c_{f,s} = \left[\frac{\gamma p + |\mathbf{B}|^2 \pm \sqrt{(\gamma p + |\mathbf{B}|^2)^2 - 4\gamma p B_x^2}}{2\rho}\right]^{1/2}$$

HLLD approximation. Entropy wave speed

$$S_M = \frac{(S_\mathrm{R} - u_\mathrm{R})\rho_\mathrm{R} u_\mathrm{R} - (S_\mathrm{L} - u_\mathrm{L})\rho_\mathrm{L} u_\mathrm{L} - p_{T_\mathrm{R}} + p_{T_\mathrm{L}}}{(S_\mathrm{R} - u_\mathrm{R})\rho_\mathrm{R} - (S_\mathrm{L} - u_\mathrm{L})\rho_\mathrm{L}} \tag{1}$$

We have $p^*_{T_\mathrm{L}} = p^{**}_{T_\mathrm{L}} = p^{**}_{T_\mathrm{R}} = p^*_{T_\mathrm{R}} = p^*_T$

$$p^*_T = \frac{(S_\mathrm{R} - u_\mathrm{R})\rho_\mathrm{R} p_{T_\mathrm{L}} - (S_\mathrm{L} - u_\mathrm{L})\rho_\mathrm{L} p_{T_\mathrm{R}} + \rho_\mathrm{L}\rho_\mathrm{R}(S_\mathrm{R} - u_\mathrm{R})(S_\mathrm{L} - u_\mathrm{L})(u_\mathrm{R} - u_\mathrm{L})}{(S_\mathrm{R} - u_\mathrm{R})\rho_\mathrm{R} - (S_\mathrm{L} - u_\mathrm{L})\rho_\mathrm{L}} \tag{2}$$

With $\alpha = L, R$,

$$v^*_\alpha = v_\alpha - B_x B_{y\alpha} \frac{S_M - u_\alpha}{\rho_\alpha(S_\alpha - u_\alpha)(S_\alpha - S_M) - B_x^2} \tag{3}$$

$$B^*_{y\alpha} = B_{y\alpha} \frac{\rho_\alpha(S_\alpha - u_\alpha)^2 - B_x^2}{\rho_\alpha(S_\alpha - u_\alpha)(S_\alpha - S_M) - B_x^2} \tag{4}$$

$$w^*_\alpha = w_\alpha - B_x B_{z\alpha} \frac{S_M - u_\alpha}{\rho_\alpha(S_\alpha - u_\alpha)(S_\alpha - S_M) - B_x^2} \tag{5}$$

$$B^*_{z\alpha} = B_{z\alpha} \frac{\rho_\alpha(S_\alpha - u_\alpha)^2 - B_x^2}{\rho_\alpha(S_\alpha - u_\alpha)(S_\alpha - S_M) - B_x^2} \tag{6}$$

$$e_\alpha^* = \frac{(S_\alpha - u_\alpha)e_\alpha - p_{T\alpha}u_\alpha + p_T^* S_M + B_x\left(\mathbf{v}_\alpha \cdot \mathbf{B}_\alpha - \mathbf{v}_\alpha^* \cdot \mathbf{B}_\alpha^*\right)}{S_\alpha - S_M} \tag{7}$$

We have $\rho_\alpha^{**} = \rho_\alpha^*$ and $p_{T_\alpha}^{**} = p_{T_\alpha}^*$.

Propagation speed of Alfvén waves in the intemediate states:

$$S_{\mathrm{L}}^* = S_M - \frac{|B_x|}{\sqrt{\rho_{\mathrm{L}}^*}} \tag{8}$$

$$S_{\mathrm{R}}^* = S_M + \frac{|B_x|}{\sqrt{\rho_{\mathrm{R}}^*}} \tag{9}$$

Equalities:

$$v_{\mathrm{L}}^{**} = v_{\mathrm{R}}^{**} \equiv v^{**} \tag{10}$$

$$w_{\mathrm{L}}^{**} = w_{\mathrm{R}}^{**} \equiv w^{**} \tag{11}$$

$$B_{yL}^{**} = B_{yR}^{**} \equiv B_y^{**} \tag{12}$$

$$B_{zL}^{**} = B_{zR}^{**} \equiv B_z^{**} \tag{13}$$

$$v^{**} = \frac{\sqrt{\rho_{\mathrm{L}}^*} v_{\mathrm{L}}^* + \sqrt{\rho_{\mathrm{R}}^*} v_{\mathrm{R}}^* + \left(B_{y_{\mathrm{R}}}^* - B_{y_{\mathrm{L}}}^*\right) \operatorname{sgn}(B_x)}{\sqrt{\rho_{\mathrm{L}}^*} + \sqrt{\rho_{\mathrm{R}}^*}} \tag{14}$$

$$w^{**} = \frac{\sqrt{\rho_{\mathrm{L}}^*} w_{\mathrm{L}}^* + \sqrt{\rho_{\mathrm{R}}^*} w_{\mathrm{R}}^* + \left(B_{y_{\mathrm{R}}}^* - B_{y_{\mathrm{L}}}^*\right) \operatorname{sgn}(B_x)}{\sqrt{\rho_{\mathrm{L}}^*} + \sqrt{\rho_{\mathrm{R}}^*}} \tag{15}$$

$$B_y^{**} = \frac{\sqrt{\rho_{\mathrm{L}}^*} B_{y_{\mathrm{R}}}^* + \sqrt{\rho_{\mathrm{R}}^*} B_{y_{\mathrm{L}}}^* + \sqrt{\rho_{\mathrm{L}}^* \rho_{\mathrm{R}}^*}\left(v_{\mathrm{R}}^* - v_{\mathrm{L}}^*\right) \operatorname{sgn}(B_x)}{\sqrt{\rho_{\mathrm{L}}^*} + \sqrt{\rho_{\mathrm{R}}^*}} \tag{16}$$

$$B_z^{**} = \frac{\sqrt{\rho_{\mathrm{L}}^*} B_{y_{\mathrm{R}}}^* + \sqrt{\rho_{\mathrm{R}}^*} B_{y_{\mathrm{L}}}^* + \sqrt{\rho_{\mathrm{L}}^* \rho_{\mathrm{R}}^*}\left(w_{\mathrm{R}}^* - w_{\mathrm{L}}^*\right) \operatorname{sgn}(B_x)}{\sqrt{\rho_{\mathrm{L}}^*} + \sqrt{\rho_{\mathrm{R}}^*}} \tag{17}$$

The jump condition for the energy

$$e_{\mathrm{L}}^{**} = e_{\mathrm{L}}^* - \rho_{\mathrm{L}}^*\left(\mathbf{v}_{\mathrm{L}}^* \cdot \mathbf{B}_{\mathrm{L}}^* - \mathbf{v}^{**} \cdot \mathbf{B}^{**}\right) \tag{18}$$

$$e_{\mathrm{R}}^{**} = e_{\mathrm{R}}^* + \rho_{\mathrm{R}}^*\left(\mathbf{v}_{\mathrm{R}}^* \cdot \mathbf{B}_{\mathrm{R}}^* - \mathbf{v}^{**} \cdot \mathbf{B}^{**}\right) \tag{19}$$

The fluxes found from the jump conditions...

Signal speeds for the left and right sides of the Riemann fan are estimated as

$$S_{\mathrm{L}} = \min(u_{\mathrm{L}}, u_{\mathrm{R}}) - \max(c_{fL}, c_{fR}) \tag{20a}$$

$$S_{\mathrm{R}} = \max(u_{\mathrm{L}}, u_{\mathrm{R}}) + \max(c_{fL}, c_{fR}) \tag{20b}$$

# References


[1] Takahiro Miyoshi and Kanya Kusano. A multi-state HLL approximate Riemann solver for ideal magnetohydrodynamics. *Journal of Computational Physics*, 208(1):315–344, 2005.