SARIMAX Equations
Seasonal Auto-Regressive Integrating Moving Average with Exogenous Features

Autoregression
At any time t the output can be written as a function of past samples, a constant and noise.

$$y_t = \beta + \epsilon + \sum_{i=1}^{p} \theta y_{t-i}$$

where:

$$\beta \equiv Constant$$

$$\epsilon \equiv Noise$$

$$p \equiv Number\ of\ lags\ for\ the\ output$$

If we define the lag operator L we can rewrite this as:

$$L^n y_t = y_{t-n}$$

We can also define the polynomial function $\Theta(L)^p$ of order p we can rewrite the equation as:

$$y_t = \Theta(L)^p y_t + \epsilon_t$$

Note that β is part of θ.

Moving Average

$$y_t = \Phi(L)^q \epsilon_t + \epsilon$$

where q is the number of lags for the error ϵ and Φ is the polynomial function of L with order q. Therefore, the ARMA (Autoregressive Moving Average) model is:

$$y_t = \Theta(L)^p * y_t + \Phi(L)^q \epsilon_t + \epsilon_t$$

Integration Operator
The integration component of the model helps with non-stationarity of data. If we define the Δ^d as follows:

$$y_t^{[d]} = \Delta^d * y_t = y_t^{[d-1]} - y_{t-1}^{[d-1]}$$

where $y^{[0]}$ is y_t and d is the order of the differencing used. Substituting $y_t^{[d]}$ into our ARMA equation.

$$y_t^{[d]} = \Theta(L)^p y_t^{[d]} + \Phi(L)^q \epsilon_t^{[d]} + \epsilon_t^{[d]}$$

We then get the following ARIMA equation:

$$\Delta^d y_t = \Theta(L)^p \Delta^d y_t + \Phi(L)^q \Delta^d \epsilon_t + \Delta^d \epsilon_t$$

Which can be written in the following simplified form:

$$\Theta(L)^p \Delta^d y_t = \Phi(L)^q \Delta^d \epsilon_t$$

Seasonal Components
Seasonal components are handled with a similar equation applied to lags if seasonal period s. We can define a seasonal difference Δ_s^D with D is the differencing for seasonal lags. If we let $L^s \equiv$ seasonal lag operator and P and Q are also seasonal lags then we can write our SARIMA equation as:

$$\Delta_s^D y_t = \theta(L^s)^P \Delta_s^D y_t + \phi(L^s)^Q \Delta_s^D \epsilon_t$$

Which simplifies to:

$$\theta(L^s)^P \Delta_s^D y_t = \phi(L^s)^Q \Delta_s^D \epsilon_t$$

Therefore, the general SARIMAX equation becomes:

$$\Theta(L)^p \theta(L^s)^P \Delta^d \Delta_s^D y_t = \Phi(L)^q \phi(L^s)^Q \Delta^d \Delta_s^D \epsilon_t$$

Exogenous Regressors
Adding exogenous we get ARIMA becomes ARIMAX:

$$\Theta(L)^p \Delta^d y_t = \Phi(L)^q \Delta^d \epsilon_t + \sum_{i=1}^{n} \beta_i x_t^i$$

and SARIMA becomes SARIMAX:

$$\Theta(L)^p \theta(L^s)^P \Delta^d \Delta_s^D y_t = \Phi(L)^q \phi(L^s)^Q \Delta^d \Delta_s^D \epsilon_t + \sum_{i=1}^{n} \beta_i x_t^i$$