Computational Physics (PHYS514) Final Project

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Newton

This part gives calculations of the structures of various types of stars in Newtonian gravity, general relativity (GR) and alternative theories of gravity which try to surpass GR. 1. From the stellar structure equations to the Lane–Emden equation

1.1. Stellar structure in Newtonian gravity

We start with the standard Newtonian equations for hydrostatic equilibrium in a spherically symmetric star:

1. **Mass continuity:**

$$\frac{dm}{dr} \;=\; 4\pi\, r^2\, \rho(r),$$

2. **Hydrostatic equilibrium:**

$$\frac{dp}{dr} \;=\; -\,\frac{G\, m(r)\, \rho(r)}{r^2}.$$

Here: - $m(r)$ is the mass enclosed within radius r, - $\rho(r)$ is the mass density, - $p(r)$ is the pressure, - G is the gravitational constant.

1.2. Polytropic equation of state

We then **close** the system using a polytropic equation of state

$$p \;=\; K\,\rho^{\gamma} \;=\; K\,\rho^{1+1n},$$

where - K is a constant (related to the microphysics of the stellar material), - n is called the polytropic index, - $\gamma = 1 + \frac{1}{n}$.

It is standard to rewrite

$$\gamma \;=\; 1 \;+\; \frac{1}{n} \quad\Longrightarrow\quad p = K\,\rho^{\gamma} \quad\Longrightarrow\quad p = K\,\rho^{1+1n}.$$