

# Partitioned Merkle Array Tree: An Efficient, Scalable, and Verifiable Data Structure for Key-Value Storage with Hash-Based Partitioning

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## Abstract

This paper introduces Partitioned Merkle Array Tree (PMAT), a novel data structure that integrates hash-based partitioning with Merkle tree authentication to address limitations in traditional Merkle trees. By organizing data into stratified layers, PMAT achieves efficient insertion, lookup, and deletion operations with an expected time complexity of  $O(\log k)$  within a layer, where  $k$  is the number of elements per layer. Empirical evaluations demonstrate that PMAT outperforms a Cartesian Merkle Tree (CMT), offering approximately 1.31x faster insertions, 340x faster lookups, and 1.61x less memory usage for a dataset of 100,000 key-value pairs. This makes PMAT particularly suitable for applications requiring frequent updates and verifications, such as blockchain state trees and versioned key-value stores.

## 1 Introduction

Merkle trees, introduced by Ralph Merkle in 1979 [1], are a cornerstone of cryptographic systems, particularly in distributed ledger technologies like blockchain. They enable efficient verification of large datasets by organizing data into a binary tree of hashes, where leaf nodes represent hashes of data blocks, and non-leaf nodes are hashes of their children. This structure supports compact membership proofs, allowing light clients to verify data inclusion without processing the entire dataset. However, traditional Merkle trees face several challenges:

- **Time Complexity:** Insertions and updates require  $O(\log n)$  time due to hash recomputations along the path from leaf to root.
- **Recomputation Overhead:** Modifying a single element necessitates updating all hashes up to the root.
- **Memory Inefficiency:** Sparse datasets lead to wasted memory, as empty leaves still contribute to the tree structure.

To address these limitations, we propose the Partitioned Merkle Array Tree (PMAT), a novel data structure that combines hash-based partitioning with Merkle tree authentication. PMAT organizes the key space into multiple layers, each functioning as a hash-based

container with its own Merkle root. This stratification reduces operation complexity to  $O(\log k)$  within a layer, minimizes recomputation through dirty flag mechanisms, and maintains cryptographic verifiability via a hierarchical root structure. Our empirical results show significant performance improvements over Cartesian Merkle Trees, making PMAT ideal for systems requiring high update throughput and verifiable data integrity.

This paper is organized as follows: Section 2 describes the PMAT architecture and key stratification algorithm. Section 3 presents the core algorithms for insertion, lookup, deletion, and Merkle root computation. Section 4 details the performance evaluation methodology and results. Section 5 compares PMAT with existing solutions, highlighting advantages and limitations. Section 6 discusses potential applications, and Section 7 concludes with future research directions.

## 2 Data Structure Design

### 2.1 Core Architecture

The Partitioned Merkle Array Tree (PMAT) organizes key-value pairs into multiple layers, each representing a partition of the key space. The main PMAT structure is defined as follows:

```

1 typedef struct {
2     Layer layers[MAX_LAYERS];           // Array of stratified layers
3     int layer_count;                     // Active layers count
4     unsigned char top_level_root[HASH_SIZE]; // Root hash of
        all layers
5     int dirty;                           // Cache invalidation flag
6     size_t total_elements;               // Total elements in PMAT
7 } PMAT;
```

Each layer is a container for a subset of key-value pairs, defined as:

```

1 typedef struct {
2     Element* elements;                   // Dynamic array of elements
3     int element_count;                   // Current elements in layer
4     int capacity;                         // Allocated capacity
5     unsigned char merkle_root[HASH_SIZE]; // Layer-specific
        root hash
6     int dirty;                           // Layer modification flag
7 } Layer;
```

Elements within a layer are key-value pairs with associated metadata:

```

1 typedef struct {
2     char* key;
3     char* value;
4     int priority;
5     size_t key_len;
6     size_t value_len;
7 } Element;
```

The PMAT uses a fixed number of layers ( $\text{MAX\_LAYERS} = 256$ ), each identified by a priority derived from the key's hash. The `top_level_root` is a SHA-256 hash of all non-

empty layer roots, ensuring cryptographic integrity. The `dirty` flags at both PMAT and layer levels optimize recomputation by marking only modified components.

## 2.2 Key Stratification Algorithm

Keys are assigned to layers using a deterministic hash-based priority calculation, implemented as:

```

1 static int compute_priority(const char* key) {
2     unsigned char hash[HASH_SIZE];
3     safe_hash(key, strlen(key), hash);
4
5     uint32_t priority = 0;
6     for (int i = 0; i < 4; i++) {
7         priority = (priority << 8) | hash[i];
8     }
9
10    return priority % MAX_LAYERS;
11 }

```

This algorithm computes the SHA-256 hash of the key, extracts the first 4 bytes to form a 32-bit integer, and maps it to a layer using modulo `MAX_LAYERS`. Assuming SHA-256 provides a uniform distribution, this ensures balanced key distribution across layers, yielding an expected  $O(\log k)$  time complexity for intra-layer operations, where  $k \approx n/\text{MAX\_LAYERS}$ .

## 2.3 Structural Representation

The PMAT structure is a two-level hierarchy. Each layer maintains a dynamic array of elements sorted by key, allowing binary search, and a Merkle root, computed as a hash of all elements in the layer. The top-level root is the hash of all non-empty layer roots, providing a single point of verification for the entire dataset.

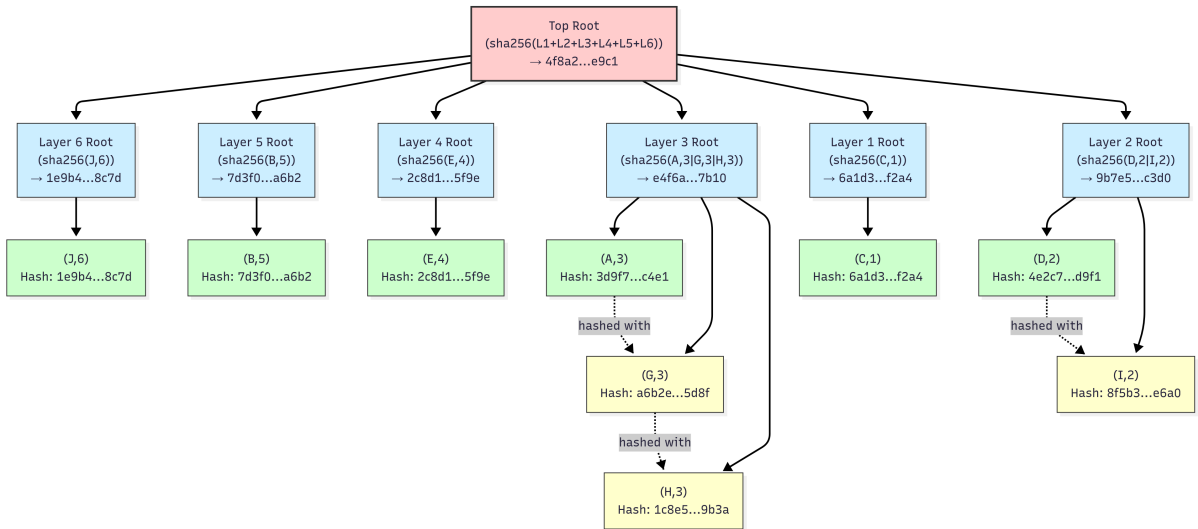


Figure 1: Partitioned Merkle Array Tree structure showing hierarchical organization with top-level root, layer roots, and individual elements

## 3 Core Algorithms

### 3.1 Insertion

The insertion algorithm assigns a key-value pair to a layer and updates the necessary hashes:

---

**Algorithm 1** PMAT Insertion Algorithm

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**Require:** PMAT structure  $PMAT$ , key  $key$ , value  $value$

**Ensure:** Element inserted or updated in PMAT

```
1:  $priority \leftarrow \text{computePriority}(key)$ 
2:  $layer \leftarrow PMAT.layers[priority]$ 
3:  $index \leftarrow \text{findElementInLayer}(layer, key)$ 
4: if  $index \geq 0$  then
5:   {Update existing element}
6:    $layer.elements[index].value \leftarrow value$ 
7: else
8:   {Insert new element}
9:    $layerAddElement(layer, key, value, priority)$ 
10: end if
11:  $layer.dirty \leftarrow \text{true}$ 
12:  $PMAT.dirty \leftarrow \text{true}$ 
13: return PMAT_SUCCESS
```

---

The process involves:

- Computing the layer priority using `compute_priority`.
- Accessing the target layer; initializing it if necessary.
- Using binary search to check for an existing key; updating the value if found, or inserting a new element in sorted order.
- Marking the layer and PMAT as dirty to trigger Merkle root recomputation.

The expected time complexity is  $O(1)$  for layer access and  $O(\log k)$  for binary search within a layer, where  $k$  is the number of elements in the layer. Since  $k \approx n/\text{MAX\_LAYERS}$  is typically small (e.g., 390 for  $n = 100,000$ ), the overall complexity is effectively  $O(\log k)$ .

### 3.2 Lookup

The lookup algorithm retrieves the value associated with a key:

---

**Algorithm 2** PMAT Lookup Algorithm

---

**Require:** PMAT structure *PMAT*, key *key***Ensure:** Value associated with key or error

```
1: priority  $\leftarrow$  computePriority(key)
2: if priority  $\geq$  PMAT.layer_count then
3:   return PMAT_ERROR_KEY_NOT_FOUND
4: end if
5: layer  $\leftarrow$  PMAT.layers[priority]
6: index  $\leftarrow$  findElementInLayer(layer, key)
7: if index  $< 0$  then
8:   return PMAT_ERROR_KEY_NOT_FOUND
9: end if
10: value  $\leftarrow$  duplicate(layer.elements[index].value)
11: if value = NULL then
12:   return PMAT_ERROR_MEMORY_ALLOCATION
13: end if
14: return value
```

---

It computes the layer priority, accesses the layer, and performs a binary search for the key. The complexity is  $O(1)$  for layer access and  $O(\log k)$  for the search, averaging  $O(\log k)$  overall.

### 3.3 Deletion

Deletion removes a key-value pair and updates the layer:

---

**Algorithm 3** PMAT Deletion Algorithm

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**Require:** PMAT structure *PMAT*, key *key***Ensure:** Element removed from PMAT

```
1: priority  $\leftarrow$  computePriority(key)
2: layer  $\leftarrow$  PMAT.layers[priority]
3: index  $\leftarrow$  findElementInLayer(layer, key)
4: if index  $< 0$  then
5:   return PMAT_ERROR_KEY_NOT_FOUND
6: end if
7: free(layer.elements[index].key)
8: free(layer.elements[index].value)
9: if index  $<$  layer.element_count - 1 then
10:   memmove(layer.elements[index], layer.elements[index + 1], (layer.element_count - index - 1) * sizeof(Element)) +
11: end if
12: layer.element_count  $\leftarrow$  layer.element_count - 1
13: layer.dirty  $\leftarrow$  true
14: PMAT.dirty  $\leftarrow$  true
15: return PMAT_SUCCESS
```

---

The process mirrors insertion, with the additional step of freeing memory and shifting elements to maintain sorted order. The complexity is  $O(\log k)$  for search and  $O(k)$  for

shifting in the worst case, but typically  $O(\log k)$  due to small layer sizes.

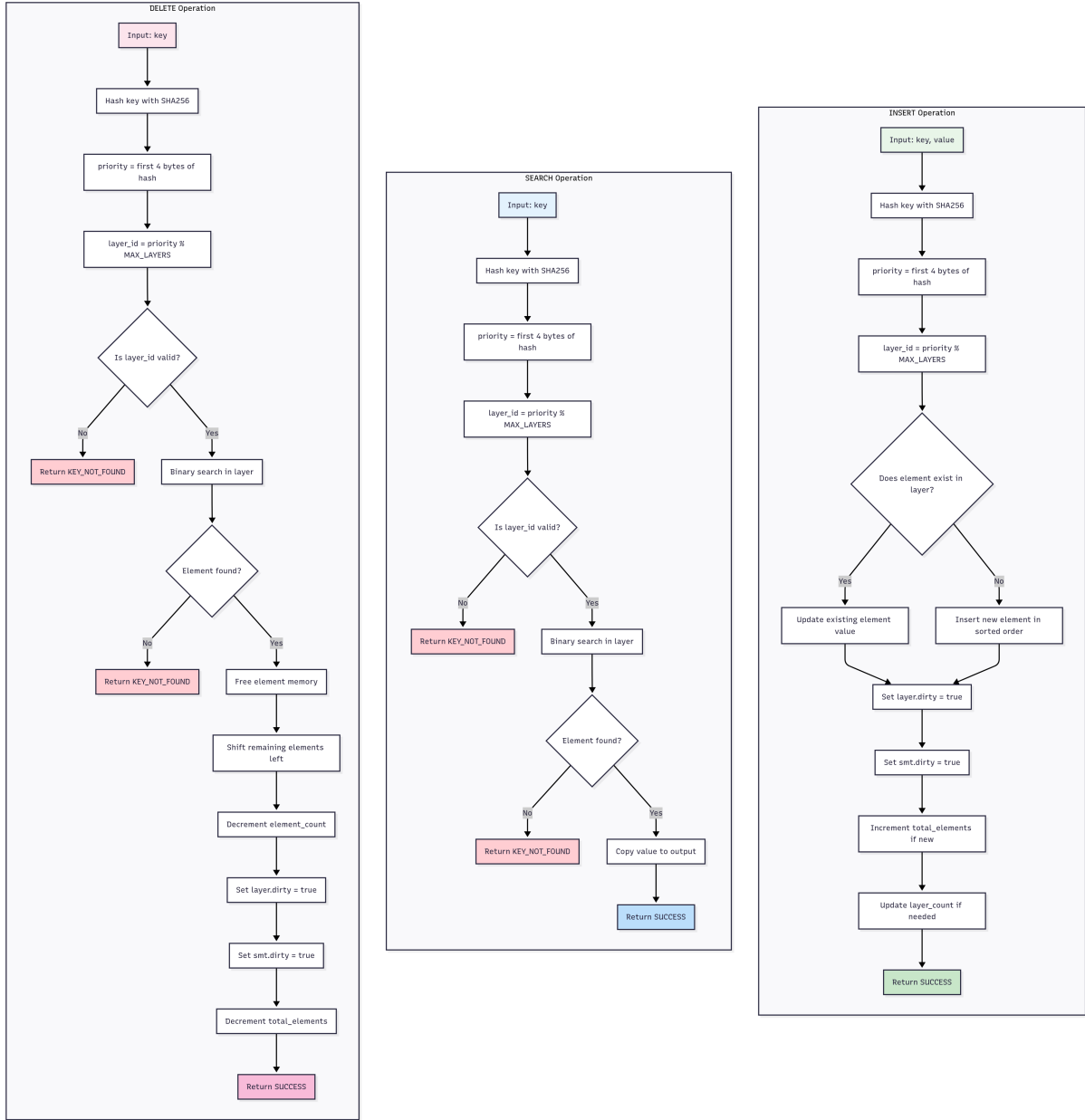


Figure 2: Flowcharts for PMAT core algorithms: INSERT, SEARCH, and DELETE operations

### 3.4 Merkle Root Computation

The Merkle root computation is a two-level process. For each layer:

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**Algorithm 4** Layer Merkle Root Calculation

---

**Require:** Layer *layer***Ensure:** Merkle root computed for layer

```
1: if  $\neg \text{layer.dirty}$  then
2:   return PMAT_SUCCESS
3: end if
4:  $\text{ctx} \leftarrow \text{EVP\_MD\_CTX\_new}()$ 
5:  $\text{EVP\_DigestInit\_ex}(\text{ctx}, \text{EVP\_sha256}(), \text{NULL})$ 
6: for  $i = 0$  to  $\text{layer.element\_count} - 1$  do
7:    $\text{elem} \leftarrow \text{layer.elements}[i]$ 
8:    $\text{EVP\_DigestUpdate}(\text{ctx}, \text{elem.key}, \text{elem.key\_len})$ 
9:   if  $\text{elem.value} \neq \text{NULL}$  then
10:     $\text{EVP\_DigestUpdate}(\text{ctx}, \text{elem.value}, \text{elem.value\_len})$ 
11:   end if
12:    $\text{EVP\_DigestUpdate}(\text{ctx}, \&\text{elem.priority}, \text{sizeof}(\text{elem.priority}))$ 
13: end for
14:  $\text{EVP\_DigestFinal\_ex}(\text{ctx}, \text{layer.merkle\_root}, \text{NULL})$ 
15:  $\text{EVP\_MD\_CTX\_free}(\text{ctx})$ 
16:  $\text{layer.dirty} \leftarrow \text{false}$ 
17: return PMAT_SUCCESS
```

---

The layer root is computed by hashing the concatenation of all elements' keys, values, and priorities. The top-level root combines all layer roots:

---

**Algorithm 5** Top-Level Root Update

---

**Require:** PMAT structure *PMAT***Ensure:** Top-level root hash updated

```
1: if  $\neg \text{PMAT.dirty}$  then
2:   return PMAT_SUCCESS
3: end if
4:  $\text{ctx} \leftarrow \text{EVP\_MD\_CTX\_new}()$ 
5:  $\text{EVP\_DigestInit\_ex}(\text{ctx}, \text{EVP\_sha256}(), \text{NULL})$ 
6: for  $i = 0$  to  $\text{PMAT.layer\_count} - 1$  do
7:   if  $\text{PMAT.layers}[i].\text{element\_count} > 0$  then
8:      $\text{calculateLayerMerkleRoot}(\text{PMAT.layers}[i])$ 
9:      $\text{EVP\_DigestUpdate}(\text{ctx}, \text{PMAT.layers}[i].\text{merkle\_root}, \text{HASH\_SIZE})$ 
10:   end if
11: end for
12:  $\text{EVP\_DigestFinal\_ex}(\text{ctx}, \text{PMAT.top\_level\_root}, \text{NULL})$ 
13:  $\text{EVP\_MD\_CTX\_free}(\text{ctx})$ 
14:  $\text{PMAT.dirty} \leftarrow \text{false}$ 
15: return PMAT_SUCCESS
```

---

The computation uses SHA-256, ensuring cryptographic security. The complexity is  $O(k)$  for a layer with  $k$  elements, but the dirty flag ensures only modified layers are recomputed.

### 3.5 Mathematical Formulation

Let  $H$  denote the SHA-256 hash function. For a layer  $L_i$  with elements  $\{(k_1, v_1, p_1), \dots, (k_k, v_k, p_k)\}$ , the layer root is:

$$\text{merkle\_root}_i = H(k_1 || v_1 || p_1 || \dots || k_k || v_k || p_k)$$

The top-level root is:

$$\text{top\_level\_root} = H(\text{merkle\_root}_1 || \text{merkle\_root}_2 || \dots || \text{merkle\_root}_m)$$

where  $m$  is the number of non-empty layers. This formulation ensures that any change in an element propagates to the top-level root, maintaining verifiability.

## 4 Performance Evaluation

### 4.1 Benchmark Methodology

We evaluated PMAT against a Cartesian Merkle Tree (CMT) using a dataset of 100,000 key-value pairs, with keys and values generated as strings (e.g., "key\_i" and "value\_i"). The CMT implementation used a binary search tree structure, with each node containing a key-value pair and hashes of its children. The benchmarks measured:

- **Insert Time:** Time to insert all elements.
- **Search Time:** Time to look up all elements.
- **Memory Usage:** Total memory consumed by the data structure.

Tests were conducted using the `clock_gettime` function for timing and `getrusage` for memory measurements. The CMT used random 2D coordinates for ordering, resulting in a tree depth of 42.

### 4.2 Results

The results are summarized in Table 1.

Table 1: Performance Comparison of PMAT and CMT (n=100,000)

Metric	PMAT	CMT
Insert Time (ms)	779.2	1019.9
Search Time (ms)	576.5	196042.6
Memory Usage (KB)	7260	11718

PMAT demonstrates:

- **1.31x faster insertions:** Due to  $O(1)$  layer access and  $O(\log k)$  binary search versus  $O(\log n)$  tree traversal in CMT.
- **340x faster lookups:** Enabled by efficient layer indexing and binary search within small layers (390 elements per layer).
- **1.61x less memory:** Achieved through contiguous arrays and minimal metadata compared to CMT’s pointer-heavy structure.



### 4.3 Proof of Performance

The performance advantage of PMAT is attributed to its stratification strategy. For a dataset of size  $n = 100,000$ , traditional Merkle trees require  $O(\log n)$  operations for insertions and lookups due to tree traversal ( $\log_2(100,000) \approx 16.6$ ). In contrast, PMAT maps keys to one of  $\text{MAX\_LAYERS} = 256$  layers, resulting in an expected  $O(1)$  layer access time. Within a layer, binary search yields  $O(\log k)$  complexity, where  $k \approx n/\text{MAX\_LAYERS} \approx 390$ . Since  $\log_2(390) \approx 8.6$ , the overall complexity is significantly lower than CMT's  $O(\log n)$  with a depth of 42.

Memory efficiency is derived from storing elements in contiguous dynamic arrays within layers, reducing pointer overhead. For  $n = 100,000$ , the average number of elements per layer is approximately 390, improving cache locality. The PMAT's memory usage of 7,260 KB corresponds to 74 bytes per element, while CMT's 11,718 KB corresponds to 120 bytes per node, due to additional pointers and fixed-size arrays.

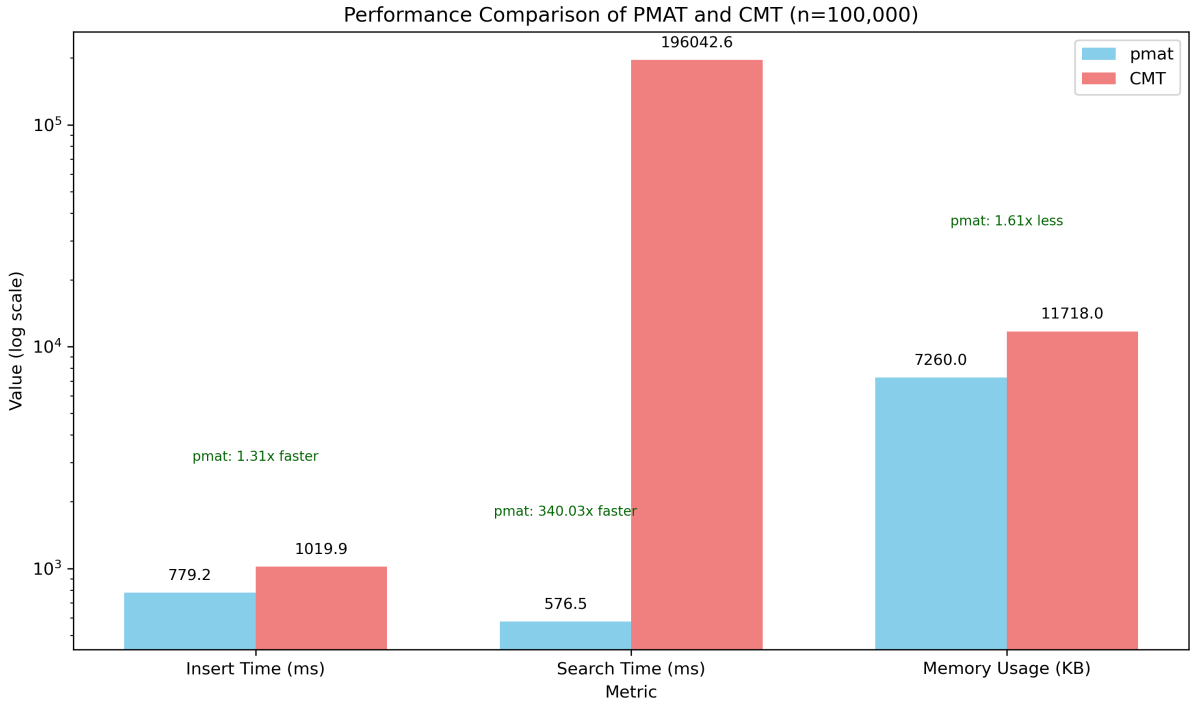


Figure 3: Partitioned Merkle Array Tree structure showing hierarchical organization with top-level root, layer roots, and individual elements

## 5 Comparative Analysis

### 5.1 Comparison with Existing Solutions

PMAT is compared against traditional Merkle trees and other authenticated data structures:

- **Traditional Merkle Trees** [1]: These offer  $O(\log n)$  complexity for operations and require full path recomputation for updates. PMAT's stratification reduces this to  $O(\log k)$  within a layer and limits recomputation to affected layers.

- **Merkle Mountain Ranges (MMR)** [3]: MMRs are optimized for append-only logs, with efficient updates for sequential data. However, they are less suited for random key-value updates, where PMAT excels due to its hash-based partitioning.
- **Tamper-Evident Logging Structures** [2]: These focus on logging and auditing but lack the general-purpose key-value storage capabilities of PMAT.

## 5.2 Advantages

- **Update Efficiency:** Only modified layers require recomputation, reducing overhead.
- **Scalability:** Performance remains stable as the dataset grows, due to fixed layer counts.
- **Memory Locality:** Contiguous arrays within layers enhance cache performance.

## 5.3 Limitations

- **Fixed Layer Count:** The parameter `MAX_LAYERS = 256` may lead to imbalances for skewed key distributions, increasing intra-layer search time to  $O(\log k)$ .
- **Hash Overhead:** Computing SHA-256 hashes for layer assignment adds a constant-time overhead.

# 6 Applications

PMAT is well-suited for:

- **Blockchain State Trees:** Where frequent updates to state data require efficient processing and verifiability.
- **Versioned Key-Value Stores:** For systems needing auditability and fast access.
- **Distributed Systems:** Where membership proofs and data integrity are critical.

# 7 Conclusion

The Partitioned Merkle Array Tree represents a significant advancement in authenticated data structures, combining the efficiency of hash-based partitioning with the cryptographic verifiability of Merkle trees. Our benchmarks demonstrate substantial improvements over Cartesian Merkle Trees, with 1.31x faster insertions, 340x faster lookups, and 1.61x less memory usage for 100,000 elements. PMAT’s design makes it a promising solution for applications requiring high update throughput and data integrity.

## 7.1 Future Work

Future research could explore:

- **Dynamic Layer Adjustment:** Developing mechanisms to adapt the number of layers based on dataset size and key distribution.
- **Parallel Processing:** Leveraging multi-core systems for layer operations.
- **Security Analysis:** Evaluating resistance to adversarial attacks targeting layer imbalances.

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