PHY 329

Josephson Junctions & Artificial Atoms

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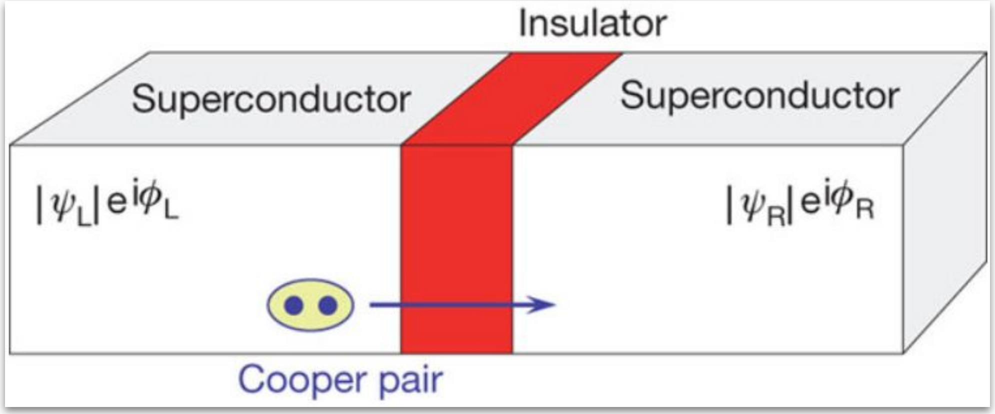
Motivation

- 2025 Physics Nobel
- Understanding Josephson Dynamics in RCSJ context
- Josephson Junctions and artificial atoms

“for the discovery of macroscopic quantum mechanical tunnelling and energy quantization in an electric circuit”

Theory: Josephson Junctions

A Josephson junction is comprised of two superconductors with a thin insulator between them.



If a voltage is applied, then the energy difference between each side is 2eV

The wavefunctions have the following form:

$$\psi_A = \sqrt{n_A}e^{i\phi_A}, \psi_B = \sqrt{n_B}e^{i\phi_B}$$

Theory: Josephson Junctions

Using the Schrodinger equation and the energy difference, we can derive the Josephson equations

$$I(t) = I_c \sin(\varphi(t))$$

$$\frac{\partial \varphi}{\partial t} = \frac{2eV(t)}{\hbar}$$

These equations govern the dynamics of the Josephson junction, and provide a basis for our chosen model.

Theory: The RCSJ Model

Resistively and Capacitively Shunted Junction Model

By treating the junction as a capacitor, we find three terms contributing to the current.

$$V(t) = \frac{\hbar}{2eV}\dot{\varphi}(t)$$

$$I_s = I_c \sin(\varphi)$$

$$I_R = \frac{V}{R}$$

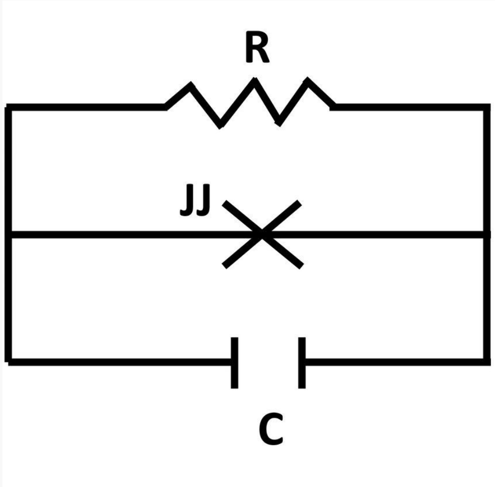
$$I_C = C\frac{dV}{dt}$$

$$C\ddot{\varphi} + \frac{1}{R}\dot{\varphi} + \frac{2eI_c}{\hbar}\sin(\varphi) = \frac{2e}{\hbar}I$$

This lets us view the junction as a circuit.

Theory: The RCSJ Model

By introducing new variables and making a change of variables to dimensionless time, we get the normalized version

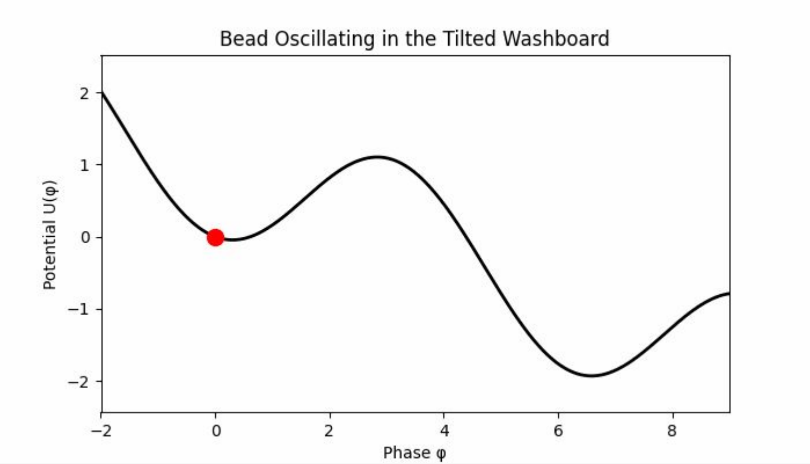


$$\frac{d^2\varphi}{d\tau^2} + \alpha\frac{d\varphi}{d\tau} + \sin(\varphi) = i$$

Setting up and solving this equation forms the basis of our analysis on Josephson junctions

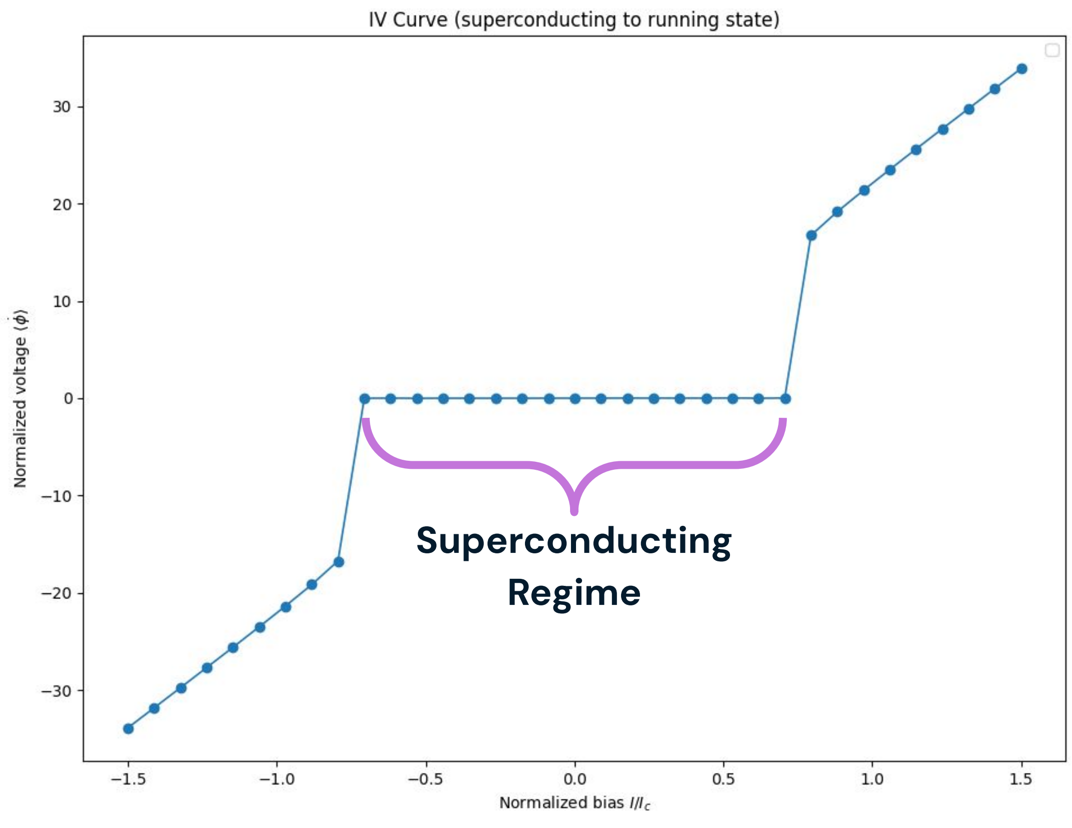
Theory: Washboard Potential

The potential term takes on a washboard potential form.
This shows where the critical current arises from.



$$U(\varphi) = 1 - \cos(\varphi) - i\varphi$$

V-I Curve of Josephson Junction



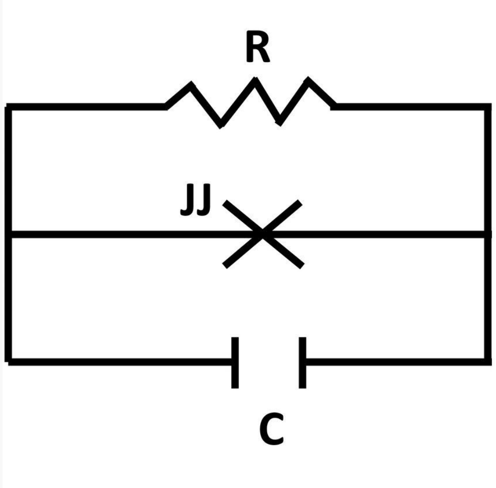
Sweep current

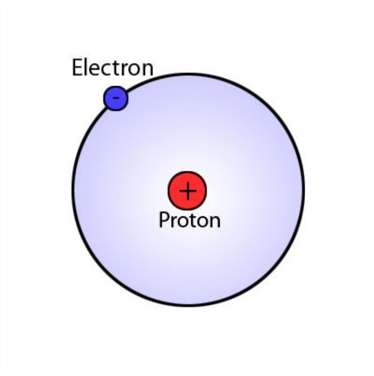
R = V/I = 0

“Artificial” Atomic Models

Parallels:

- For Hydrogen: 1D problems (coordinate and phase)
- Potential well (coulomb and washboard)
- Bound states

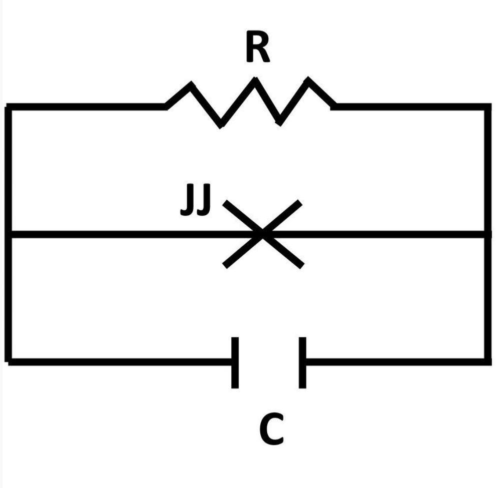


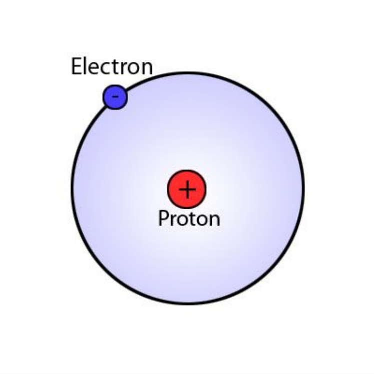


“Artificial” Atomic Models

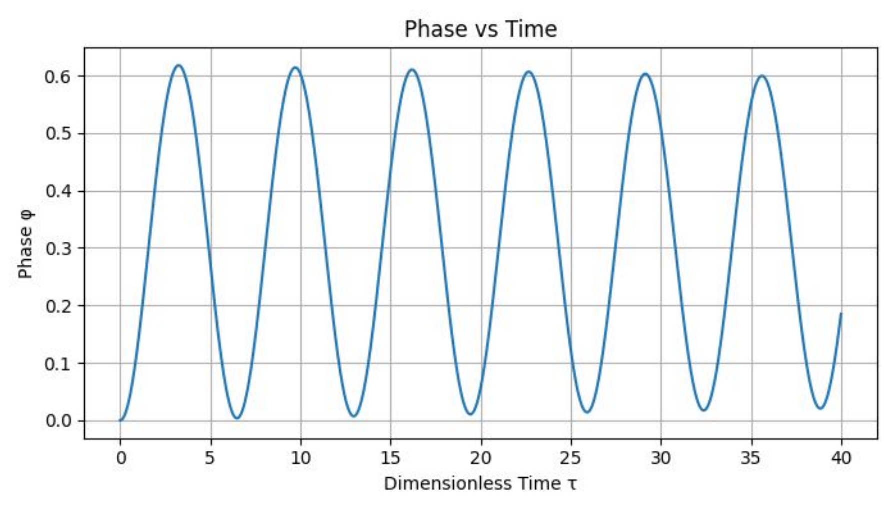
More Parallels:

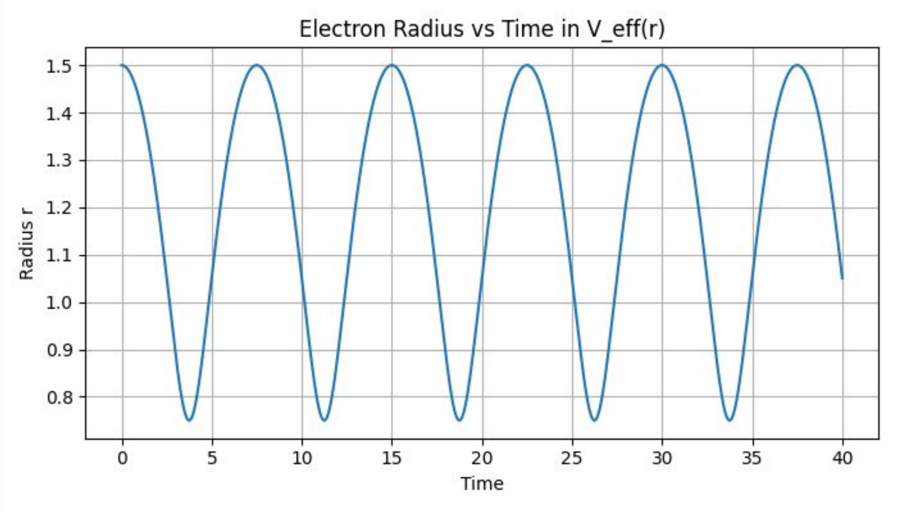
- Discrete nonlinear quantum levels
- Excitation frequencies (Light and Microwave AC)
- Tunneling



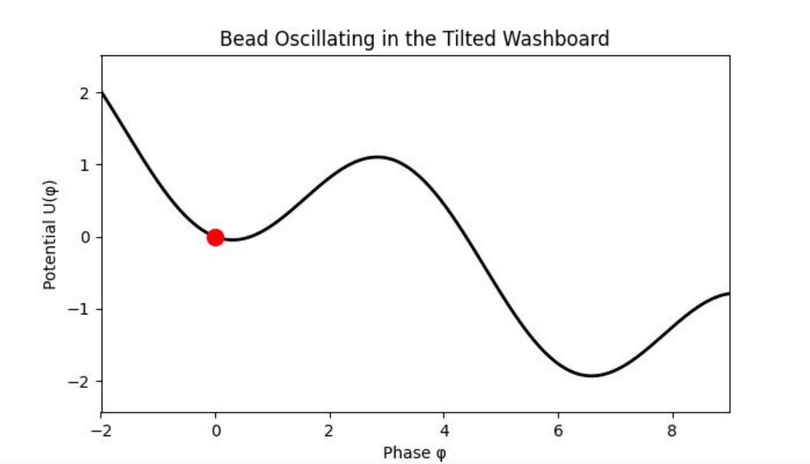


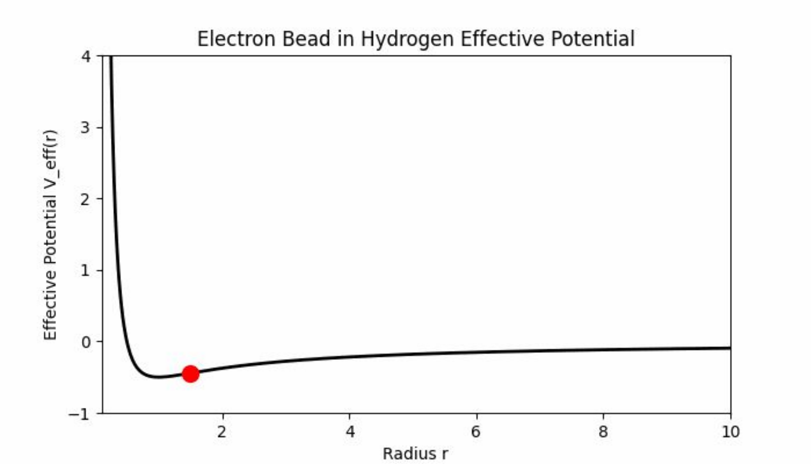
Artificial Hydrogen Model: Phase Difference vs Electron Radius





Artificial Hydrogen Model: Washboard vs Hydrogen Potential



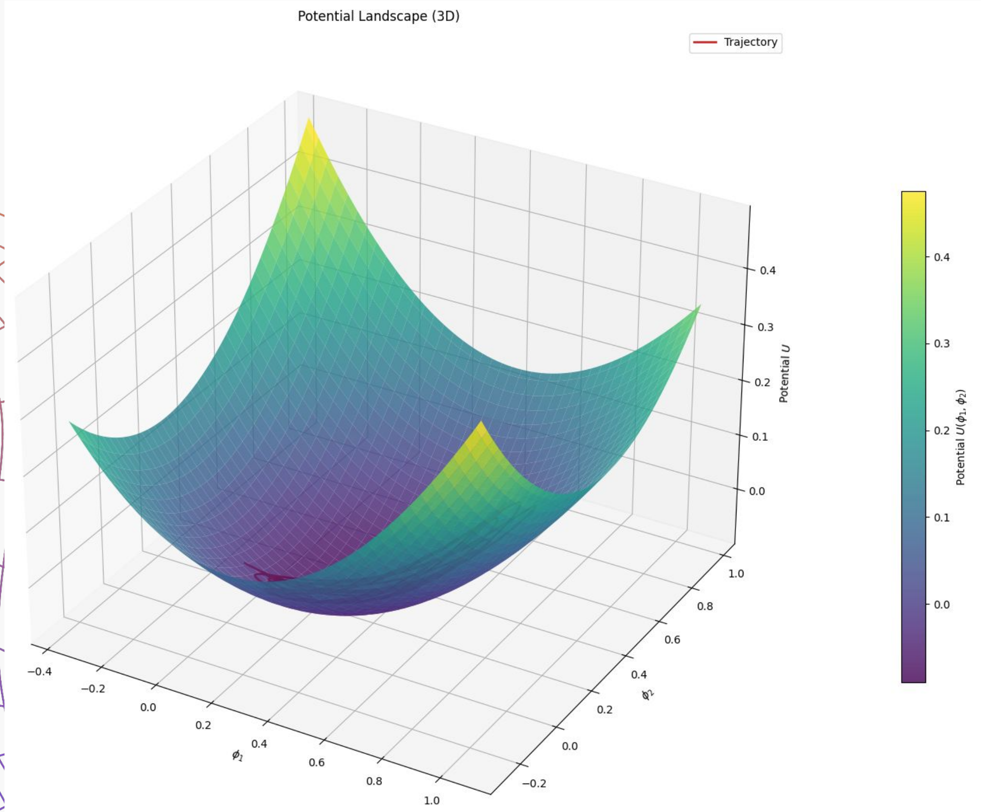


Artificial Helium Model: Mathematics

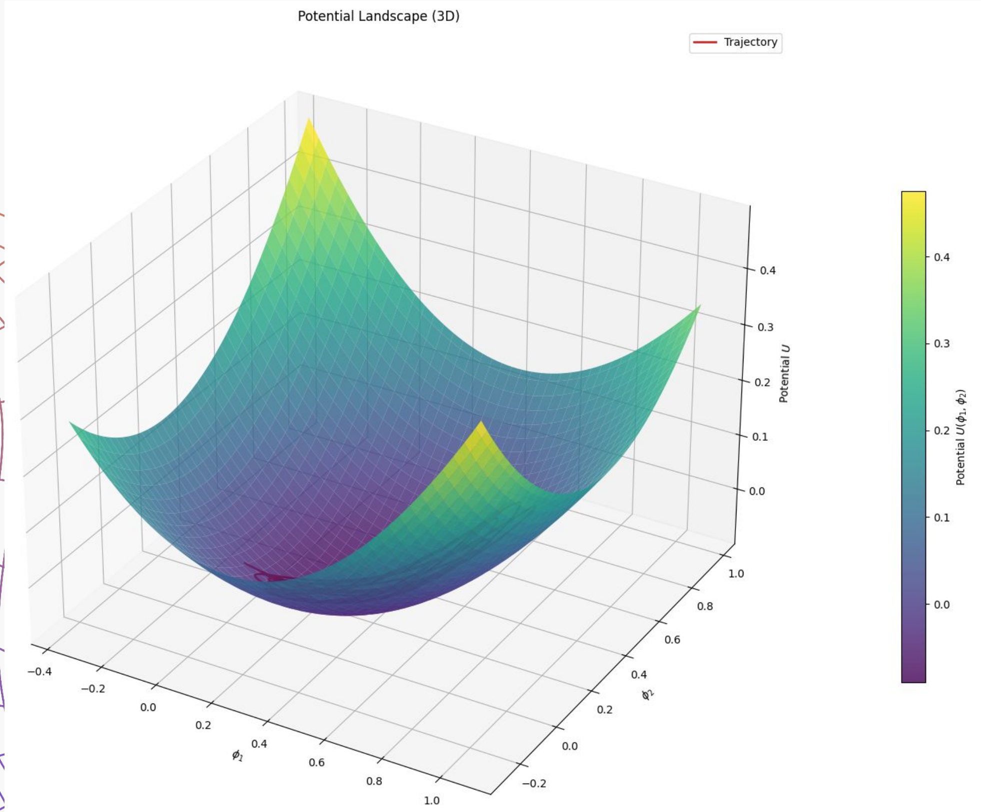
$$E = \frac{1}{2}(p_1^2 + p_2^2) + \left[-\frac{2}{r_1}\right] + \left[-\frac{2}{r_2}\right] + \frac{1}{|r_1 - r_2|}$$

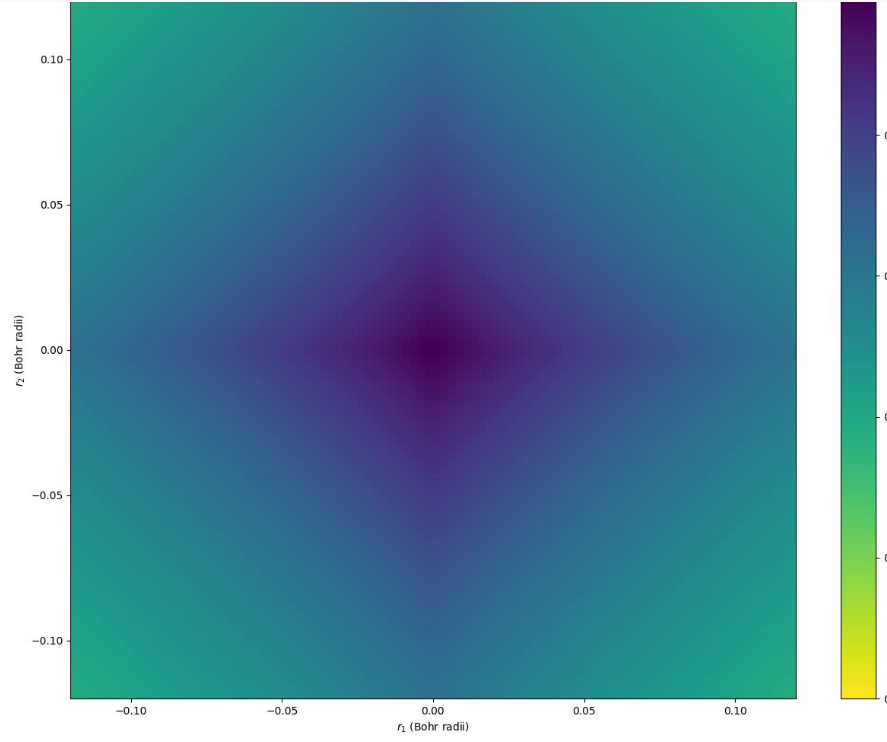
$$E = \frac{1}{2}\left(\dot{\phi}_1^2 + \dot{\phi}_2^2\right) + [1 - \cos\phi_1 - i_{\text{dc}}\phi_1] + [1 - \cos\phi_2 - i_{\text{dc}}\phi_2] + \frac{\kappa}{2}(\phi_1 - \phi_2)^2$$

Artificial Helium Model: Energy



Artificial Helium Model: Energy





Accomplishments, Dead Ends, and Next steps

- Reproduced features of Josephson junctions using a python RCSJ model
- Successfully modeled Artificial Hydrogen, both mathematically and visually
- Successfully modeled Artificial Helium mathematically

Limitations of JJ's in modeling atoms:

- Dimensions
- Interaction mathematics

Next steps:

- Shapiro Steps using Voltage Drive
- Improving models of Hydrogen and Helium
  - Tunnelling
  - damping/lifetime

Questions?