



# Stochastic Gradient Descent

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Optimization methods. MIPT

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$$\min_{x \in \mathbb{R}^p} f(x) = \min_{x \in \mathbb{R}^p} \frac{1}{n} \sum_{i=1}^n f_i(x)$$

The gradient descent acts like follows:

$$x_{k+1} = x_k - \frac{\alpha_k}{n} \sum_{i=1}^n \nabla f_i(x) \quad (\text{GD})$$

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Let's switch from the full gradient calculation to its unbiased estimator, when we randomly choose  $i_k$  index of point at each iteration uniformly:

$$x_{k+1} = x_k - \alpha_k \nabla f_{i_k}(x_k) \quad (\text{SGD})$$

With  $p(i_k = i) = \frac{1}{n}$ , the stochastic gradient is an unbiased estimate of the gradient, given by:

$$\mathbb{E}[\nabla f_{i_k}(x)] = \sum_{i=1}^n p(i_k = i) \nabla f_i(x) = \sum_{i=1}^n \frac{1}{n} \nabla f_i(x) = \frac{1}{n} \sum_{i=1}^n \nabla f_i(x) = \nabla f(x)$$

This indicates that the expected value of the stochastic gradient is equal to the actual gradient of  $f(x)$ .

## Results for Gradient Descent

Stochastic iterations are  $n$  times faster, but how many iterations are needed?

If  $\nabla f$  is Lipschitz continuous then we have:

Assumption	Deterministic Gradient Descent	Stochastic Gradient Descent
PL	$O(\log(1/\varepsilon))$	$O(1/\varepsilon)$
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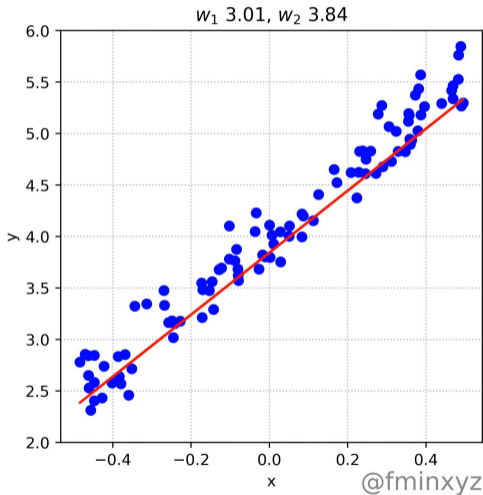
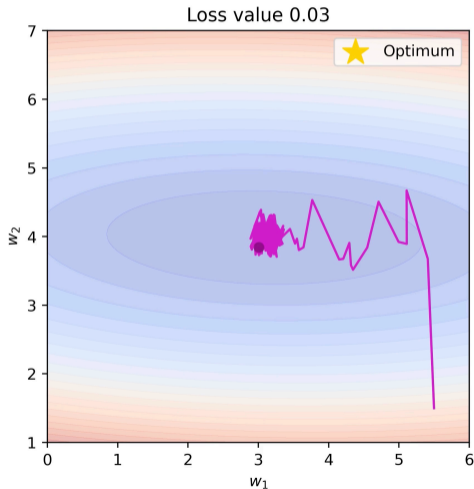
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  - Bounds are unimprovable under standard assumptions.
  - Oracle returns an unbiased gradient approximation with bounded variance.
- Momentum and Quasi-Newton-like methods do not improve rates in stochastic case. Can only improve constant factors (bottleneck is variance, not condition number).

## Stochastic Gradient Descent (SGD)

# Typical behaviour

Stochastic Gradient Descent. Batch = 2



# Convergence

Lipschitz continuity implies:

$$f(x_{k+1}) \leq f(x_k) + \langle \nabla f(x_k), x_{k+1} - x_k \rangle + \frac{L}{2} \|x_{k+1} - x_k\|^2$$

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$$f(x_{k+1}) \leq f(x_k) - \alpha_k \langle \nabla f(x_k), \nabla f_{i_k}(x_k) \rangle + \alpha_k^2 \frac{L}{2} \|\nabla f_{i_k}(x_k)\|^2$$

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Using linearity of expectation:

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Since uniform sampling implies unbiased estimate of gradient:  $\mathbb{E}[\nabla f_{i_k}(x_k)] = \nabla f(x_k)$ :

$$\mathbb{E}[f(x_{k+1})] \leq f(x_k) - \alpha_k \|\nabla f(x_k)\|^2 + \alpha_k^2 \frac{L}{2} \mathbb{E}[\|\nabla f_{i_k}(x_k)\|^2]$$

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This inequality simply requires that the gradient grows faster than a quadratic function as we move away from the optimal function value. Note, that strong convexity implies PL, but not vice versa. Using PL we can write:

$$\mathbb{E}[f(x_{k+1})] - f^* \leq (1 - 2\alpha_k\mu)[f(x_k) - f^*] + \alpha_k^2 \frac{L}{2} \mathbb{E}[\|\nabla f_{i_k}(x_k)\|^2]$$

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Thus, we have

$$\mathbb{E}[f(x_{k+1}) - f^*] \leq (1 - 2\alpha_k\mu)[f(x_k) - f^*] + \frac{L\sigma^2\alpha_k^2}{2}.$$

## Convergence. Smooth PL case.

1. Consider **decreasing stepsize** strategy with  $\alpha_k = \frac{2k+1}{2\mu(k+1)^2}$  we obtain

$$\mathbb{E}[f(x_{k+1}) - f^*] \leq \frac{k^2}{(k+1)^2} [f(x_k) - f^*] + \frac{L\sigma^2(2k+1)^2}{8\mu^2(k+1)^4}$$

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$$\begin{aligned}\delta_f(k+1) &\leq \delta_f(k) + \frac{L\sigma^2(2k+1)^2}{8\mu^2(k+1)^2} \\ &\leq \delta_f(k) + \frac{L\sigma^2}{2\mu^2},\end{aligned}$$

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where the second line follows from  $\frac{2k+1}{k+1} < 2$ . Summing up this inequality from  $k = 0$  to  $k$  and using the fact that  $\delta_f(0) = 0$  we get

$$\delta_f(k+1) \leq \delta_f(0) + \frac{L\sigma^2}{2\mu^2} \sum_{i=0}^k 1 \leq \frac{L\sigma^2(k+1)}{2\mu^2} \Rightarrow (k+1)^2 \mathbb{E}[f(x_{k+1}) - f^*] \leq \frac{L\sigma^2(k+1)}{2\mu^2}$$

which gives the stated rate.

## Convergence. Smooth PL case.

3. **Constant step size:** Choosing  $\alpha_k = \alpha$  for any  $\alpha < 1/2\mu$  yields

$$\begin{aligned}\mathbb{E}[f(x_{k+1}) - f^*] &\leq (1 - 2\alpha\mu)^k [f(x_0) - f^*] + \frac{L\sigma^2\alpha^2}{2} \sum_{i=0}^k (1 - 2\alpha\mu)^i \\ &\leq (1 - 2\alpha\mu)^k [f(x_0) - f^*] + \frac{L\sigma^2\alpha^2}{2} \sum_{i=0}^{\infty} (1 - 2\alpha\mu)^i \\ &= (1 - 2\alpha\mu)^k [f(x_0) - f^*] + \frac{L\sigma^2\alpha}{4\mu},\end{aligned}$$

where the last line uses that  $\alpha < 1/2\mu$  and the limit of the geometric series.

## Convergence. Smooth convex case.

## Convergence. Non-smooth convex case.

## Mini-batch SGD

# Mini-batch SGD

## Approach 1: Control the sample size

The deterministic method uses all  $n$  gradients:

$$\nabla f(x_k) = \frac{1}{n} \sum_{i=1}^n \nabla f_i(x_k).$$

The stochastic method approximates this using just 1 sample:

$$\nabla f_{i_k}(x_k) \approx \frac{1}{n} \sum_{i=1}^n \nabla f_i(x_k).$$

A common variant is to use a larger sample  $B_k$  (“mini-batch”):

$$\frac{1}{|B_k|} \sum_{i \in B_k} \nabla f_i(x_k) \approx \frac{1}{n} \sum_{i=1}^n \nabla f_i(x_k),$$

particularly useful for vectorization and parallelization.

For example, with 16 cores set  $|B_k| = 16$  and compute 16 gradients at once.

## Mini-Batching as Gradient Descent with Error

The SG method with a sample  $B_k$  (“mini-batch”) uses iterations:

$$x_{k+1} = x_k - \alpha_k \left( \frac{1}{|B_k|} \sum_{i \in B_k} \nabla f_i(x_k) \right).$$

Let’s view this as a “gradient method with error”:

$$x_{k+1} = x_k - \alpha_k (\nabla f(x_k) + e_k),$$

where  $e_k$  is the difference between the approximate and true gradient.

If you use  $\alpha_k = \frac{1}{L}$ , then using the descent lemma, this algorithm has:

$$f(x_{k+1}) \leq f(x_k) - \frac{1}{2L} \|\nabla f(x_k)\|^2 + \frac{1}{2L} \|e_k\|^2,$$

for any error  $e_k$ .

# Effect of Error on Convergence Rate

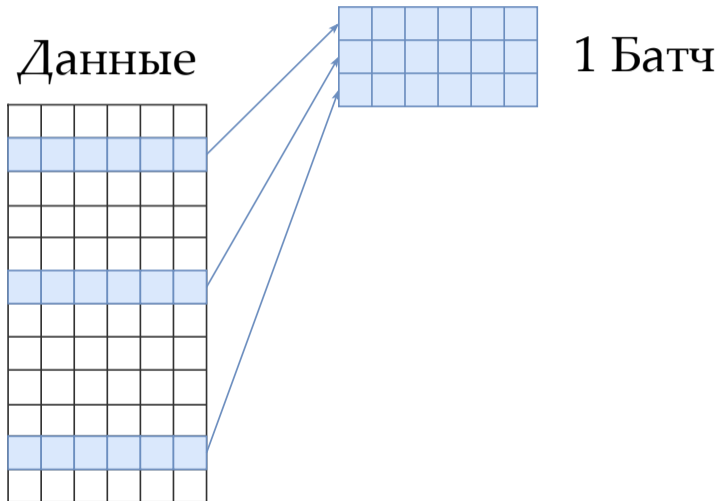
Our progress bound with  $\alpha_k = \frac{1}{L}$  and error in the gradient of  $e_k$  is:

$$f(x_{k+1}) \leq f(x_k) - \frac{1}{2L} \|\nabla f(x_k)\|^2 + \frac{1}{2L} \|e_k\|^2.$$

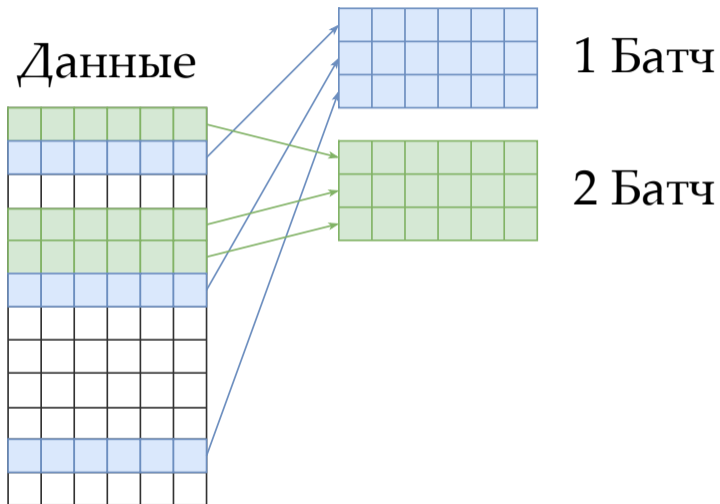
## Идея SGD и батчей

Данные

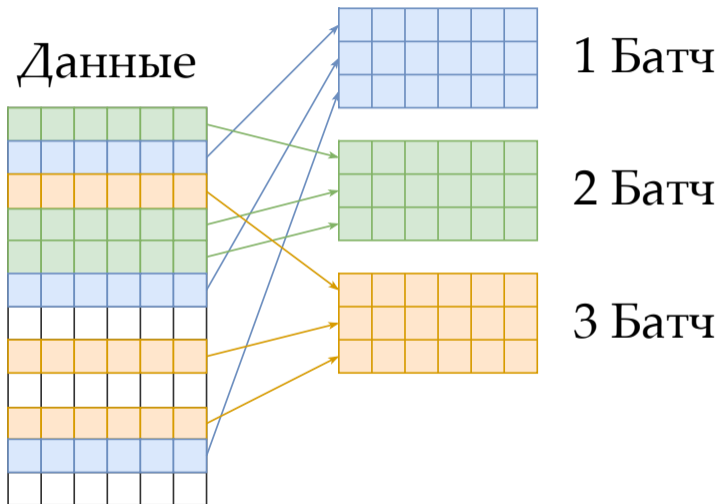

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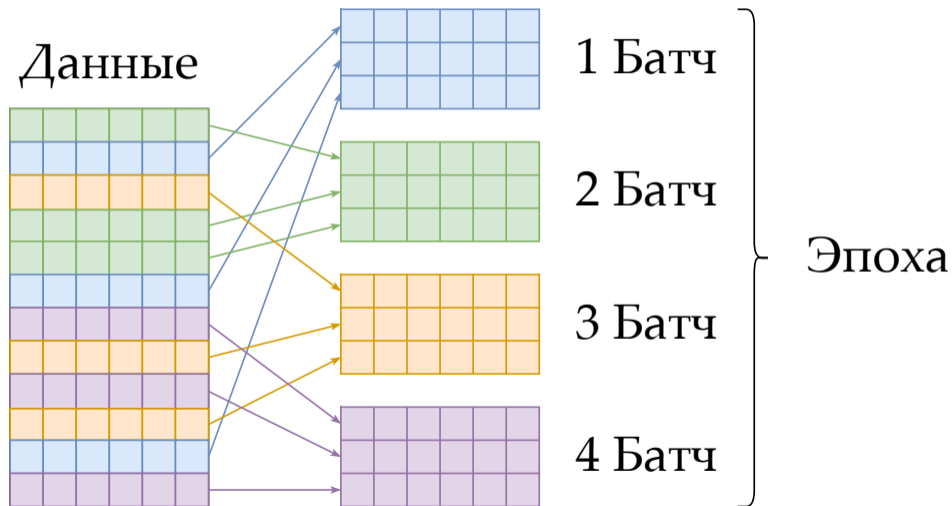
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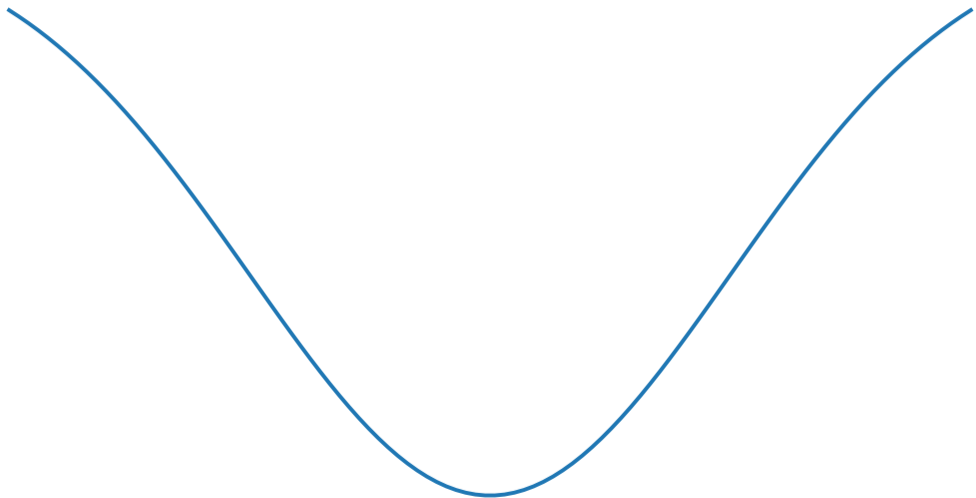
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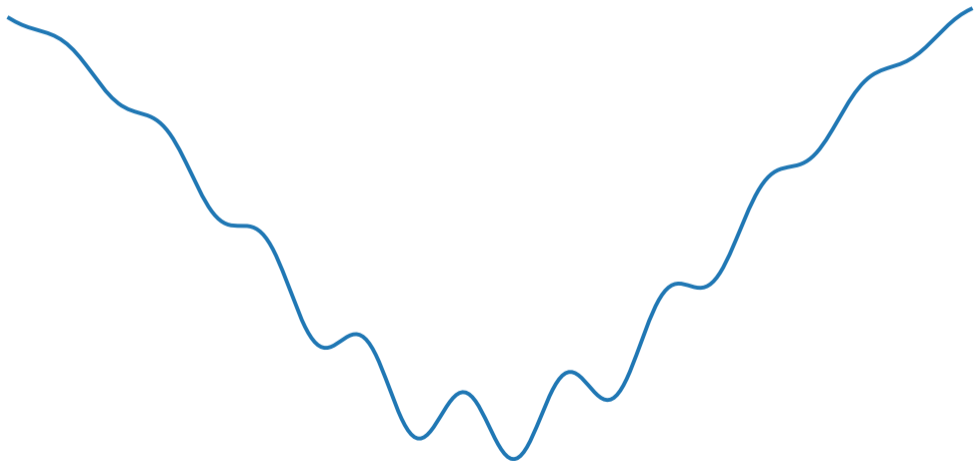
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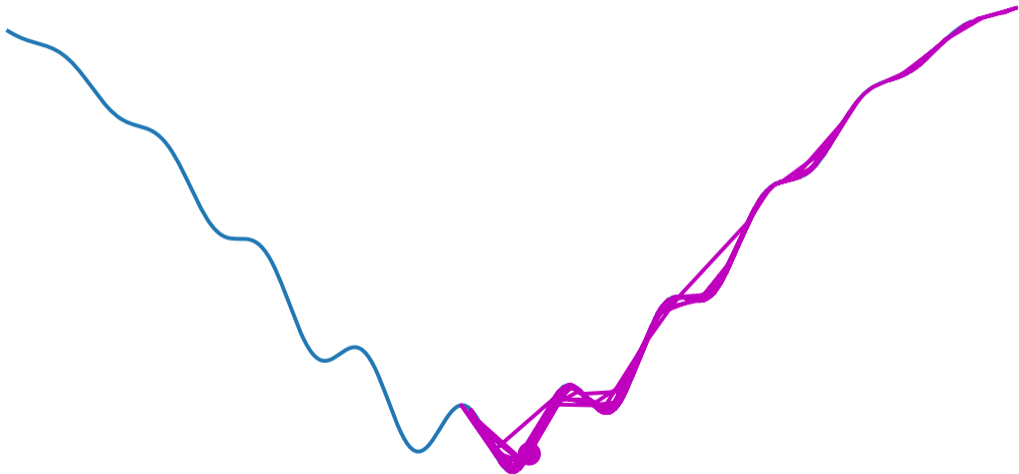
# Градиентный спуск сходится к локальному минимуму



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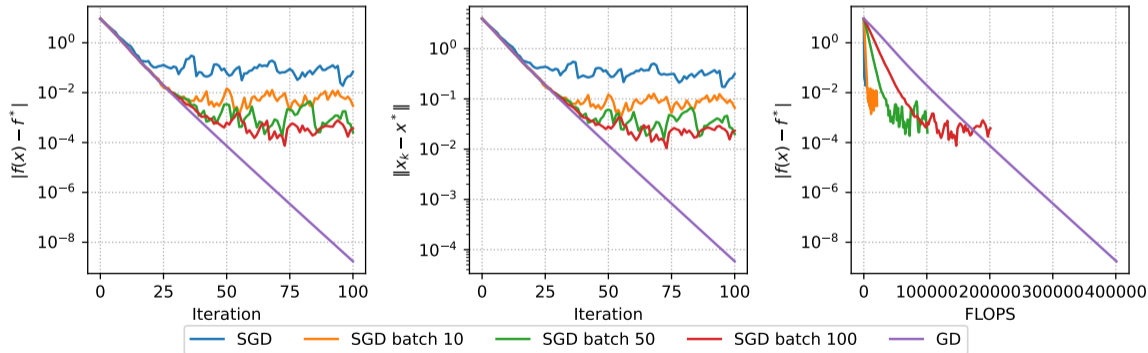
# Стохастический градиентный спуск выпрыгивает из локальных минимумов



# Main problem of SGD

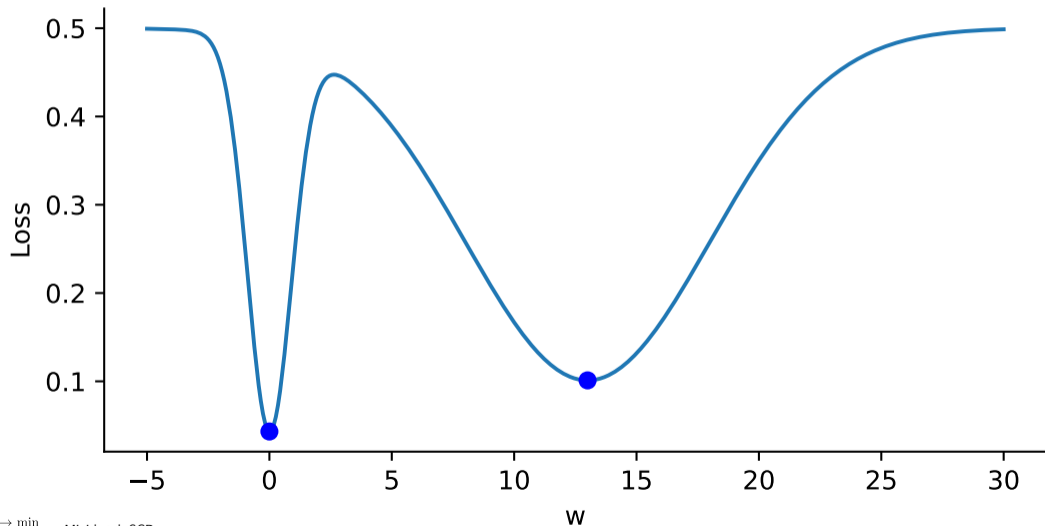
$$f(x) = \frac{\mu}{2} \|x\|_2^2 + \frac{1}{m} \sum_{i=1}^m \log(1 + \exp(-y_i \langle a_i, x \rangle)) \rightarrow \min_{x \in \mathbb{R}^n}$$

Strongly convex binary logistic regression.  $m=200$ ,  $n=10$ ,  $\mu=1$ .



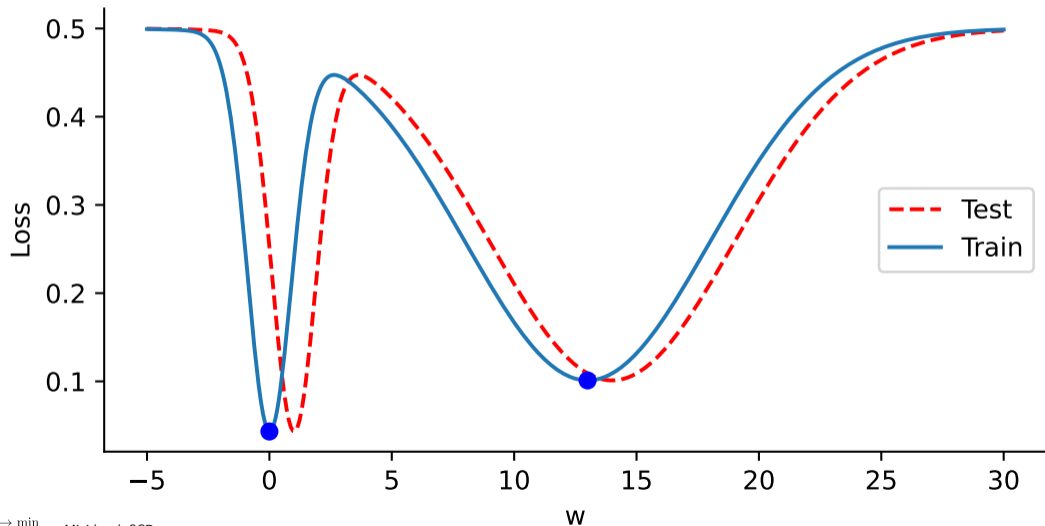
## Ширина локальных минимумов

Узкие и широкие локальные минимумы



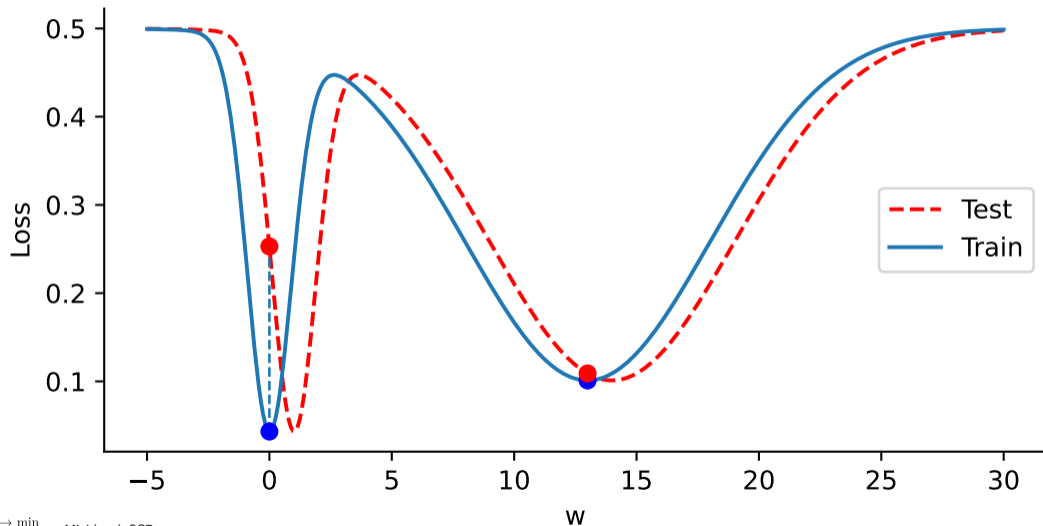
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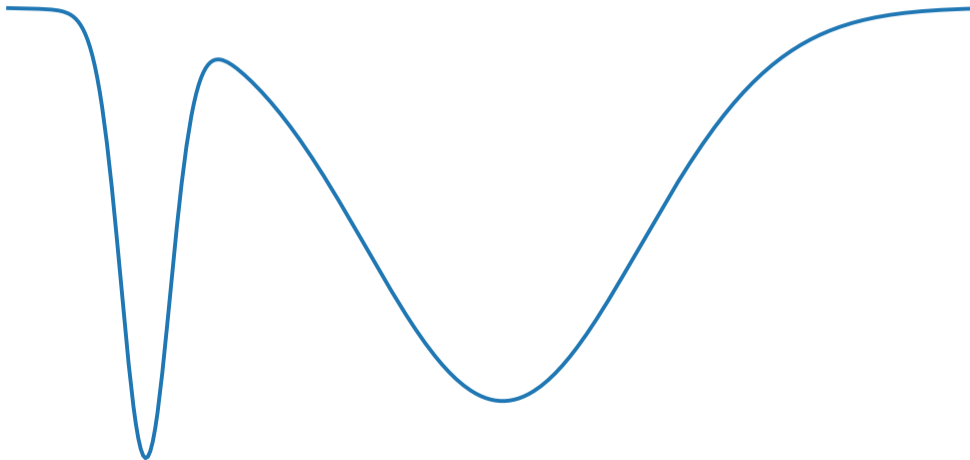


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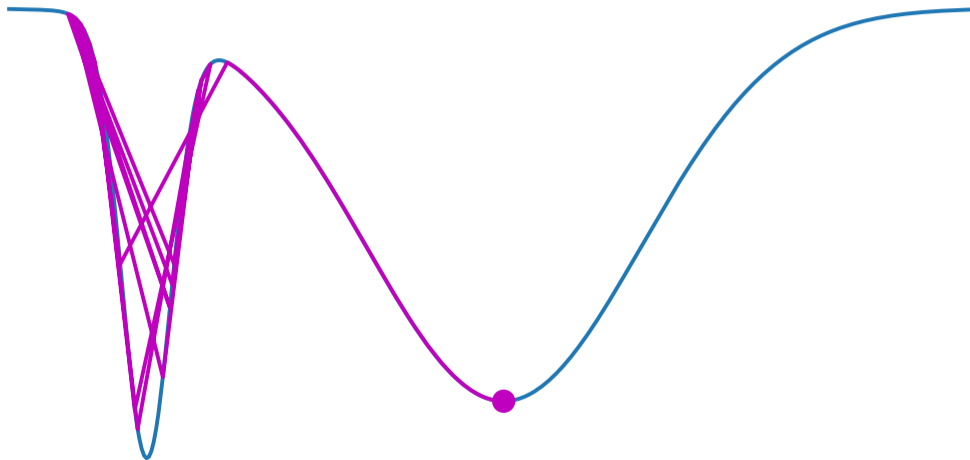
Узкие и широкие локальные минимумы



# Градиентный спуск с маленьким шагом сходится в узкий локальный минимум



# Градиентный спуск с большим шагом избегает узкого локального минимума



# Основные результаты сходимости SGD

**i** Пусть  $f$  -  $L$ -гладкая  $\mu$ -сильно выпуклая функция, а дисперсия стохастического градиента конечна ( $\mathbb{E}[\|\nabla f_i(x_k)\|^2] \leq \sigma^2$ ). Тогда траектория стохастического градиентного спуска с постоянным шагом  $\alpha < \frac{1}{2\mu}$  будет гарантировать:

$$\mathbb{E}[f(x_{k+1}) - f^*] \leq (1 - 2\alpha\mu)^k [f(x_0) - f^*] + \frac{L\sigma^2\alpha}{4\mu}.$$

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$$\mathbb{E}[f(x_{k+1}) - f^*] \leq \frac{L\sigma^2}{2\mu^2(k+1)}$$

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- SGD achieves sublinear convergence with rate  $\mathcal{O}\left(\frac{1}{k}\right)$  for PL-case.
- Nesterov/Polyak accelerations do not improve convergence rate
- Two-phase Newton-like method achieves  $\mathcal{O}\left(\frac{1}{k}\right)$  without strong convexity.