



Zero order methods. Gradient free optimization. Global optimization

- Шпаргалка по результатам в безградиентной оптимизации
- RL и эволюционные алгоритмы

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• Global optimization illustration

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Nevergrad library

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- Optuna quickstart
- Демонстрация медленности методов нулевого порядка

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- Подбор гиперпараметров модели машинного обучения в Keras с помощью Optuna
- A Tutorial on Zero-Order Optimization

Case 1: 2-Point & Multi-Point Estimators

• A naïve approach:

$$\mathsf{G}_{f}^{2n}(x;u) = \sum_{i=1}^{n} \frac{f(x+ue_{i}) - f(x-ue_{i})}{2u} e_{i}$$

When f is L-smooth, we have

In L-smooth, we have
$$\|\mathsf{G}_f^{2n}(x;u)-\nabla f(x)\|\leq \frac{1}{2}uL\sqrt{n}$$

$$f(x)=\mathbb{E}_{y\sim \bar{\lambda}}[f(x+uy)] \text{ is a smooth version of } f$$

where $f^u(x) = \mathbb{E}_{y \sim \tilde{\lambda}}[\overline{f(x+uy)}]$ is a smooth version of f

Case 1: 2-Point & Multi-Point Estimators

• 2-point gradient estimator:

$$\mathsf{G}_{f}^{(2)}(x;u,z)=n\,rac{f(x+uz)-f(x-uz)}{2u}\,z\qquad z\sim\lambda$$

where λ is spherically symmetric with $\mathbb{E}_{\mathbf{z}} \sim \lambda \left[\|z\|^2 \right] = 1$ • Some facts for L-smooth / convex / μ -strongly convex function f:

$$\begin{aligned} & \cdot \quad f^u \text{ is L-smooth / convex / μ-strongly convex} \\ & \cdot \quad |f^u(x) - f(x)| \leq \frac{1}{2} u^2 L \cdot \frac{n}{n+2} \mathbb{E}_{z \sim \lambda} \big[\|z\|^4 \big] \\ & \quad \|\nabla f^u(x) - \nabla f(x)\| \leq uL \cdot \frac{n}{n+1} \mathbb{E}_{z \sim \lambda} \big[\|z\|^3 \big] \end{aligned} \\ & \quad = \frac{1}{4\kappa} \mathbb{E}_{z \sim \lambda} \big[\|z\|^4 \big] \cdot n \|\nabla f(x)\|^2 \\ & \quad + \frac{1+\kappa}{4\kappa} \mathbb{E}_{z \sim \lambda} \big[\|z\|^6 \big] \cdot n^2 u^2 L^2 \end{aligned}$$

$$\mathbb{E}_{z \sim \lambda} \left[\left\| \mathsf{G}_{f}^{(2)}(x; u, z) \right\|^{2} \right]$$

$$\leq (1 + \kappa) \mathbb{E}_{z \sim \lambda} \left[\|z\|^{4} \right] \cdot n \|\nabla f(x)\|^{2}$$

$$+ \frac{1 + \kappa}{4\kappa} \mathbb{E}_{z \sim \lambda} \left[\|z\|^{6} \right] \cdot n^{2} u^{2} L^{2}$$