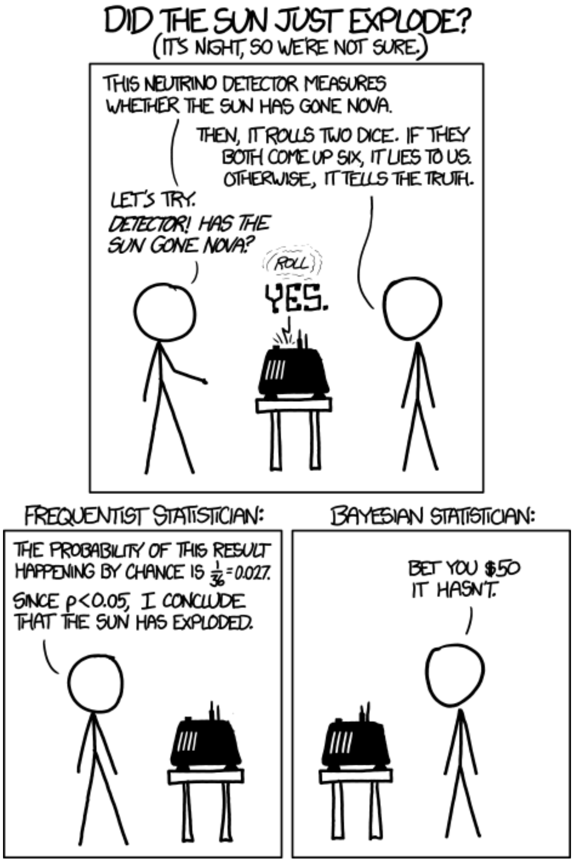
Problem Sheet 1



This is an xkcd cartoon that someone posted to the unit reddit last year, it is `https://xkcd.com/1132/`. It is also worth reading Randall Monroe's comment on the response to this cartoon:

> Hey! I was kinda blindsided by the response to this comic.
>
> Im in the middle of reading a series of books about forecasting errors (including Nate Silvers book, which I really enjoyed), and again and again kept hitting examples of mistakes caused by blind application of the textbook confidence interval approach.
>
> Someone asked me to explain it in simple terms, but I realized that in the common examples used to illustrate this sort of error, like the cancer screening/drug test false positive ones, the correct result is surprising or unintuitive. So I came up with the sun-explosion example, to illustrate a case where nave application of that significance test can give a result thats obviously nonsense.
>
> I seem to have stepped on a hornets nest, though, by adding Frequentist and Bayesian titles to the panels. This came as a surprise to me, in part because I actually added them as an afterthought, along with the final punchline. (I originally had the guy on the right making some other cross-panel comment, but I thought the bet thing was cuter.)
>
> The truth is, I genuinely didnt realize Frequentists and Bayesians were actual camps of peopleall of whom are now emailing me. I thought they were loosely-applied la-

belsperhaps just labels appropriated by the books I had happened to read recently-for the standard textbook approach we learned in science class versus an approach which more carefully incorporates the ideas of prior probabilities.

I meant this as a jab at the kind of shoddy misapplications of statistics I keep running into in things like cancer screening (which is an emotionally wrenching subject full of poorly-applied probability) and political forecasting. I wasnt intending to characterize the merits of the two sides of what turns out to be a much more involved and ongoing academic debate than I realized.

A sincere thank you for the gentle corrections; Ive taken them to heart, and you can be confident I will avoid such mischaracterizations in the future!

At least, 95.45% confident.

Thanks again to the person who posted this.

Useful facts

- **Combinations** The number of ways of choosing r items out of n is

$$\left(\begin{array}{c} n \\ r \end{array}\right) = \frac{n(n-1)(n-2)\dots(n-r+1)}{r(r-1)(r-2)\dots 1} = \frac{n!}{r!(n-r)!} \tag{1}$$

- **Combinations** The number of ways of splitting n items into sets of size r_1, r_2 through to r_k with

$$r_1 + r_2 + \dots + r_k = n \tag{2}$$

  is

$$\left(\begin{array}{c} n \\ r_1, r_2, \dots, r_k \end{array}\right) = \frac{n!}{r_1! r_2! \dots r_k!} \tag{3}$$

- **Bayes's rule**

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \tag{4}$$

- **Set notation**:
  - The bar '|' in sets should be read as 'such that', so $A = \{x|\text{some stuff}\}$ should be read as A is the set of x **such that** 'some stuff' is true and $A = \{x \in \mathbf{Z}|x > 3 \text{ and } x < 10\}$ is the set $A = \{4, 5, 6, 7, 8, 9\}$. $\mathbf{Z}$ by the way is the set of integers.
  - $A \cup B$ is the union so $A \cup B = \{x|x \in A \text{ or } x \in B\}$. If $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$ then $A \cup B = \{1, 2, 3, 4, 5\}$
  - $A \cap B$ is the intersection so $A \cap B = \{x|x \in A \text{ and } x \in B\}$. If $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$ then $A \cup B = \{3\}$
  - $A \setminus B$ is the set minus so $A \setminus B = \{x|x \in A \text{ and } x \notin B\}$. If $A = \{1, 2, 3, 4\}$ and $B = \{1, 3, 5\}$ then $A \setminus B = \{2, 4\}$
  - If C is a subset, the complement of C, that is the set of all the elements not in C, is written $\bar{C}$. If $X = \{1, 2, 3, 4\}$ and $C = \{1, 2\}$ then $\bar{C} = \{3, 4\}$.

For events, $A \cup B$ is the event of A or B happening, $A \cap B$ is the event of A and B happening, $A \setminus B$ is the event of A happening but B not happening and $\bar{C}$ is the event of C not happening.

- **Cards**: 52 cards made up of four suits; in each suit there are 13 values, ace, two through to ten and the jack, queen, king.
- **Poker hands**: the number of poker hands is

$$\binom{52}{5} = 2598960 \tag{5}$$

Questions

Five questions each worth two marks and two marks for attendance but with a maximum of ten marks.

1. In the poker hand *two pair* there are two pairs of cards with each card in the pair matched by value; the fifth card has a different value to either pair. What is the probability of two pairs when five cards are drawn randomly.
2. In a *full house* there is one pair and one triple, what is the probability of getting a full house?
3. A student answers a multiple choice question with four options, one of which is correct. 80% of students know the answer, 20% of students guess and choose randomly. If a student gets the answer correct what is the chance they knew the answer.
4. In the xkcd cartoon above, what is the chance the Bayesian will win his or her bet if the chance the sun has exploded is one in a million? In reality the chance is, of course, much less than one in a million! Show the answer to six decimal places.
5. A three-sided dice is rolled three times. X is the sum of the largest two values. Write down the probability distribution for X.

Extra questions

These are for you to do on your own, not for handing up. Solutions will be included in the solutions section.

1. When it started in 1987 the Irish lottery has 36 numbers; participants paid 50 Irish pence to buy a combination of six different numbers; they would win if these numbers matched the six drawn. In the last week in May in 1992 a syndicate tried to buy all combinations of numbers. If they had succeeded how many numbers would they have bought? In the event the lottery shut down lots of the lottery machines so they only bought most of the numbers, they nonetheless had the winning number but shared the prize three ways. However, because of the roll-over prize and the match-5 and match-4 prizes, they are thought to have made a substantial profit. The lottery was redesigned after this to have more numbers.

2. From a group of three undergraduates and five graduate students, rour students are randomly selected to act as TAs. What is the chance there will be exactly two undergraduate TAs?

3. Prove
$$\binom{n}{r} = \binom{n}{n-r} \tag{6}$$

4. Two events A and B have probabilities $P(A) = 0.2$, $P(B) = 0.3$ and $P(A \cup B) = 0.4$. Find
   a) Find $P(A \cap B)$.
   b) Find $P(\bar{A} \cap \bar{B})$.
   c) Find $P(A|B)$.

5. One night in a bar in Las Vegas you meet a dodgy character who tells you that there are two types of slot machine in the Topicana, one that pays out 10% of the time, the other 20%. One sort of machine is blue, the other red. Unfortunately the dodgy character is too drunk to remember which is which. The next day you randomly select red to try, you find a red machine and put in a coin. You lose. Assuming the dodgy character was telling the truth, what is the chance the red machine is the one that pays out more. If you had won instead of losing, what would the chance be?[1]

[1] I stole this problem from `courses.smp.uq.edu.au/MATH3104/`