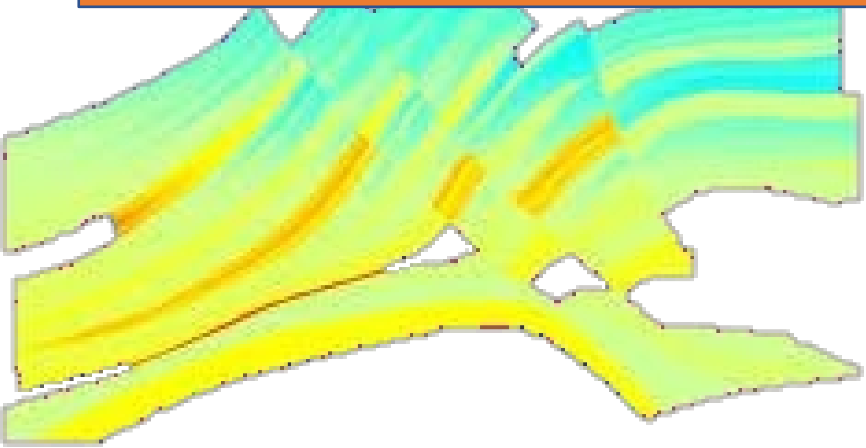
# Project report
# seismic imaging :

Stolt method or
Migration of
Stolt f-k



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# I - Preliminaries

## 1- Fourier Transform

For a quantity f(t,x,z), a function of the three variables t,x and z, the transforms of Direct and inverse Fourier can be written as :

$$F(\omega, k_x, k_z) = \iiint f(t, x, z) e^{+i\omega t - ik_x x - ik_z z} dt\,dx\,dz \qquad (1a)$$

$$f(t, x, z) = \iiint F(\omega, k_x, k_z) e^{-i\omega t + ik_x x + ik_z z} d\omega\,dk_x\,dk_z \qquad (1b)$$

where $\omega$, $k_x$ and $k_z$ are the pulsation, and the wavenumbers at x and z respectively.
For simplicity, the constant ($1/4\pi^3$) has been omitted from the inverse transform.
In the expression of this inverse transform, we find the term ($e^{-i\omega t + ik_x x + ik_z z}$) which represents the sinusoidal travelling plane wave. In addition, the sign convention chosen for writing the direct and inverse Fourier transforms is representative of the idea that the wave propagates from the source towards infinity.

## 2- Wave equation and dispersion relation

D'alembert's equation, also known as the wave equation, is the general equation that describes the propagation of a wave.
In a homogeneous, linear and isotropic space, this equation is written for a field f :

$$\nabla^2 \bar{f} = \frac{1}{v^2}\frac{\partial^2 \bar{f}}{\partial t^2} \qquad (2a)$$

In a Cartesian space (2a) is written :

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 \bar{f}}{\partial t^2} \qquad (2b)$$

In the Fourier domain, relation (2b) gives the dispersion relation for two-dimensional propagation (x,z):

$$k_x^2 + k_z^2 = \frac{\omega^2}{v^2} \qquad (3)$$

For a plane wave propagating in direction $\bar{k}$, we have :

$$k = \frac{\omega}{v} \; ; \begin{cases} k_x = \frac{\omega \sin\theta}{v} \\ k_z = \frac{\omega \cos\theta}{v} \end{cases} \Longrightarrow k_x^2 + k_z^2 = \frac{\omega^2}{v^2} \Longrightarrow k_z = \pm \frac{\omega}{v}\sqrt{1 - \frac{v^2 k_x^2}{\omega^2}} \qquad (4)$$

$k$ being the modulus of the wave vector.

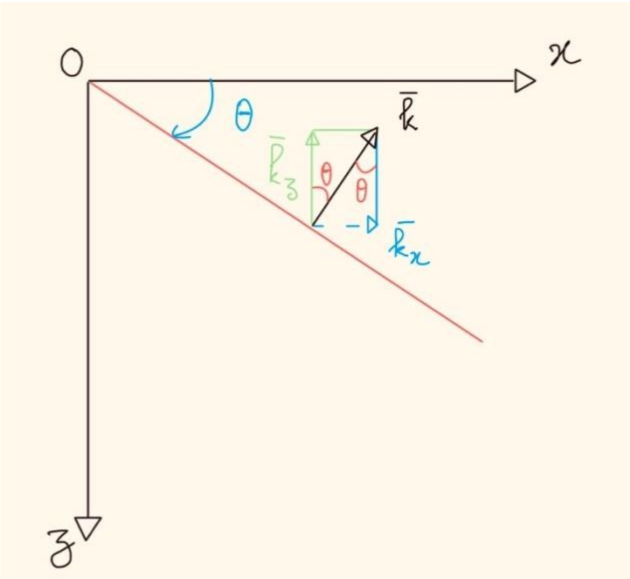



Figure 1: Wave plane and wave vector

## 3- Rising wave/ Falling wave

Consider a plane (x,z) with the Oz axis pointing downwards. The wave propagating downwards from the source (increasing z) is called the downward wave, while the wave propagating upwards (decreasing z) is the upward wave. So, in order to obey the principle of propagation, the signs of the quantities $\omega$, $k_x$ et $k_z$ must agree.

Indeed, consider plane wave $e^{-i\omega t + ik_z z}$, under the assumption that the wave is plane stationary, the phase $\varphi = i(k_z z - \omega t)$ is a constant. So for a descending plane wave ( z increasing ) we have :

1er cases: $\omega > 0$

Then $-\omega t < 0$ as t increases, and so for $\varphi = i(k_z z - \omega t) = cste$, $k_z > 0$ is required because $z \to +\infty$

2ième cases: $\omega < 0$

Then $-\omega t > 0$ as t increases, and so for $\varphi = i(k_z z - \omega t) = cste$, we need $k_z < 0$, with $z \to +\infty$

In both cases we find that for a downward wave, :

$$sign(\omega) = sign(k_z)$$

Applying the same reasoning to a rising wave, we find that we need for an uplink wave that :

$$sign(\omega) = -sign(kz)$$

# II - Stolt f-k migration

## 1- Stolt's idea

After laying down the basic tools for reflection, this part is based on the presentation of Robert H. Stolt's approach to imaging.
Given $f(t)$ and its TF $F(\omega)$, we can shift $f(t)$ by an amount $t_0$ if we multiply $F(\omega)$ by $e^{i\omega t_0}$. This manipulation also works on the z axis if we know $F(k_z)$, we could shift it from the earth's surface $z = 0$ until all $z_0$ by multiplying by $e^{i\omega k z_0}$. But we don't know $F(k_z)$. What we do know is the wave field $p(t, x)$ measured at the surface
$z = 0$ and a 2D TF gives us $P(\omega, k_x)$. If we assume that we have measured a wave field, then we have the dispersion relation (3) and technically we can calculate $P(k_x, k_z)$. We are now ready to extrapolate the waves from the surface ($z = 0$) to any depth $z$.

## 2 - Algorithmic process

First of all, the important point to emphasise here is that the stolt imaging is based on the principle of a medium in which waves propagate at the same speed $V$, which is constant and varies neither laterally nor in depth:

$$v(x, z) = V = cste.$$

Stolt's $f$ - $k$ method can be summarized as follows:

$$p(t,x,z=0) \rightarrow P(\omega, k_x, z=0) \rightarrow P'\left(k_x, k_z = \sqrt{\frac{\omega^2}{V^2} - k_x^2}\right) \rightarrow p'(x,z)$$

Where $p(t,x,z=0)$ is the wavefield recorded at surface $z=0$ and $P(\omega, k_x, z=0)$ is the direct 2D TF of $p(t,x,z=0)$, so by multiplying $P(\omega, k_x, z=0)$ by $e^{i\omega k_z z}$, we can find the field $P(\omega, k_x, z)$:

$$P(\omega, k_x, z) = P(\omega, k_x, z=0).\, e^{i\omega k_z z}$$

Then, using a 2D inverse TF, we have :

$$p(t,x,z) = \iint P(\omega, k_x, z=0)\, e^{-i\omega t + ik_x x + ik_z z}\, d\omega\, dk_x$$

So, applying the idea that the image at (x,z) is the reflector exploding in $t=0$,
We have :

$$Image(x,z) = \iint P(\omega, k_x, z=0)\, e^{ik_x x + ik_z z}\, d\omega\, dk_x \qquad (5)$$

The migrated image is therefore the one obtained by equation (5), which involves a double integration on the variables $\omega$ and $k_x$ for each depth level. However, on the surface, equation (5) looks like an inverse 2D TF with this time $k_x$ *et* $k_z$ which would be the integration variables. In order to change the variable from $\omega$ to $k_z$ Stolt came up with the idea of using the relation dispersion (3) which links the variables $\omega$, $k_x$ $et$ $k_z$ by writing for a rising wave:

$$\omega = sign(k_z) V \sqrt{k_x^2 + k_z^2} \quad \Longrightarrow \frac{\partial \omega}{\partial k_z} = -sign(k_z) V \frac{k_z}{\sqrt{k_x^2 + k_z^2}}$$

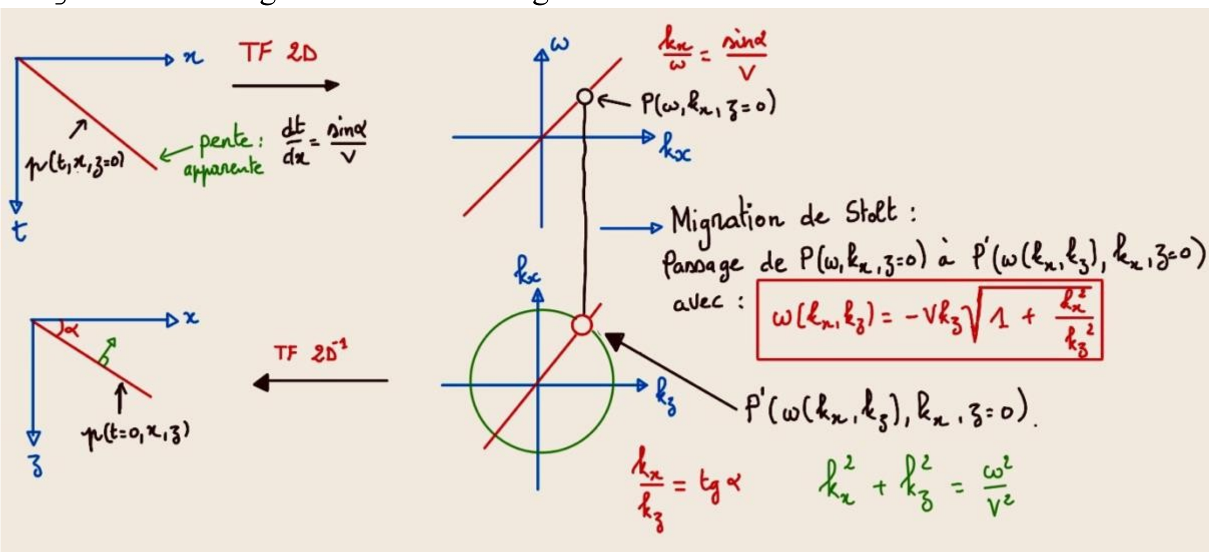
$$\Longrightarrow \; d\omega = - \frac{V\,|k_z|}{\sqrt{k_x^2 + k_z^2}} \qquad (6)$$

Injecting (6) into (5) gives :

$$Image = \iint \frac{V|k_z|}{\sqrt{k_x^2 + k_z^2}} P(\omega(k_x, k_z), k_x, z = 0) e^{ik_x x + ik_z z}\, dk_z\, dk_x \qquad (7)$$

Finally we obtain the inverse 2D TF in ($k_x$ , $k_z$ ) which provides the image in ($x$, $z$) definitive.
Stolt's algorithm can be summarised as follows:

Figure 2: Diagram of the Stolt Migration method




Stolt's algorithm is based numerically on interpolating the value of $P'(k_x , k_z )$ to *partir de* $P(\omega(k_x , k_z ), k_x , z = 0)$ which after an inverse 2D TF will provide the image in ($x$, $z$). This interpolation is performed by the line shown below taken from the "mig_stolt_bis.py" code in the appendix.

given = (1-rw)*data_TF2d[iw][ikx-1] + rw*data_TF2d[iw+1][ikx-1]

However, linear interpolation does not seem to work well with complex values. Figures 3a,b,c allow us to conclude that
Stolt's algorithm only works for part (half) of the seismic section. So to image the whole section, all you have to do is multiply the size of the seismic section by two, adding zeros (0).

Figure 3a: Example of Stolt migration of a pulse placed at 1s in $(x,t)$, migrated into a semicircle whose centre is at surface $z$ = 0.

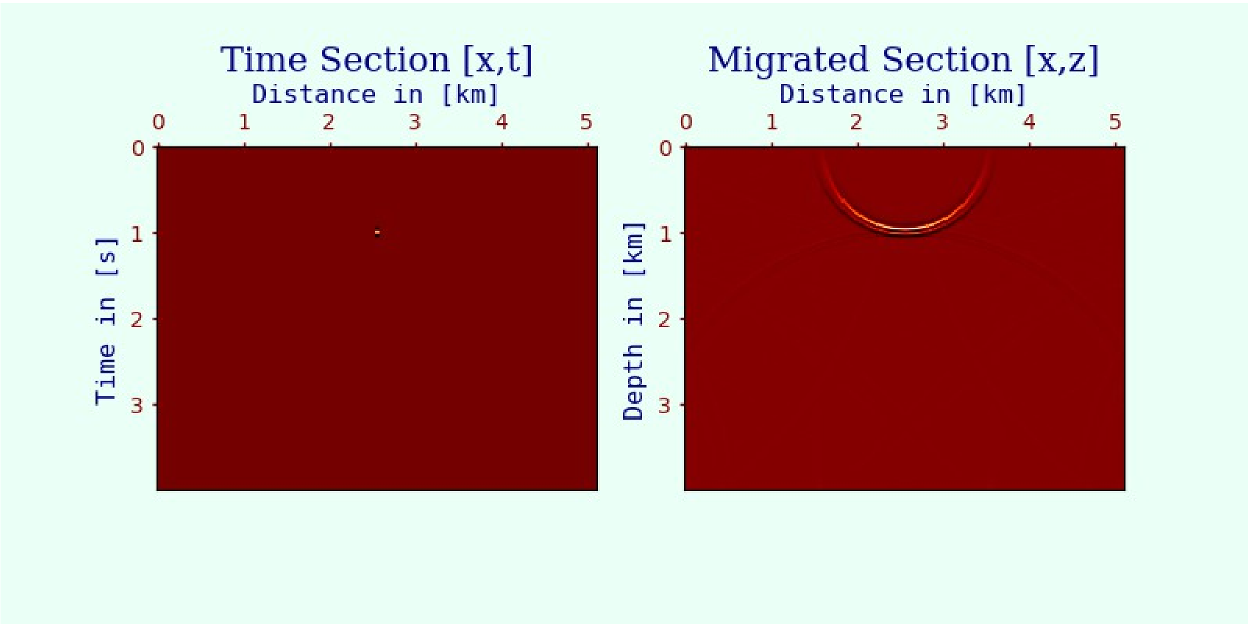
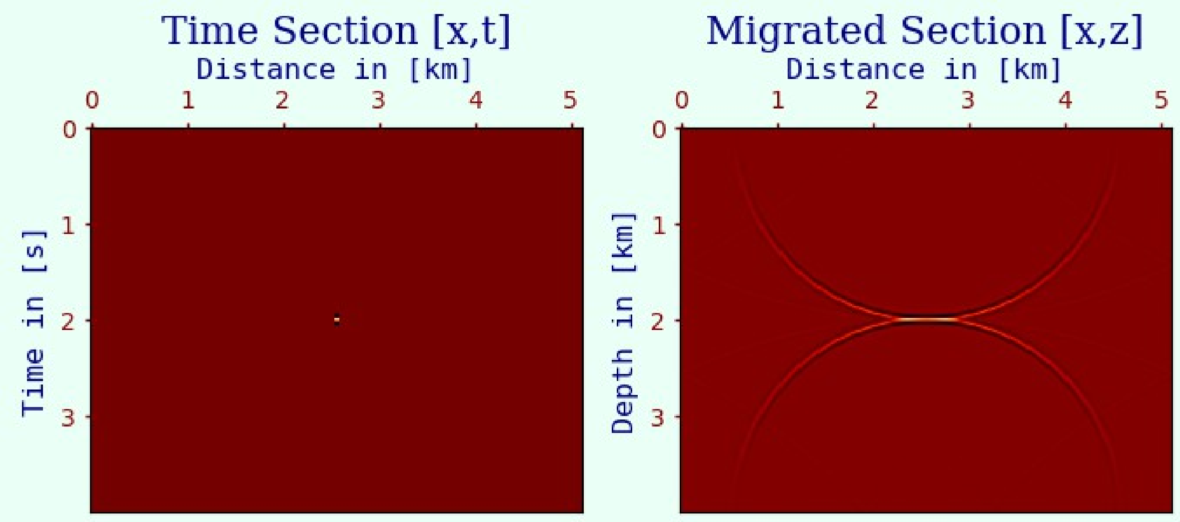


Figure 3b: Impulse placed at 2s in $(x, t)$ leading to two identical semi-circles, the good one facing upwards.



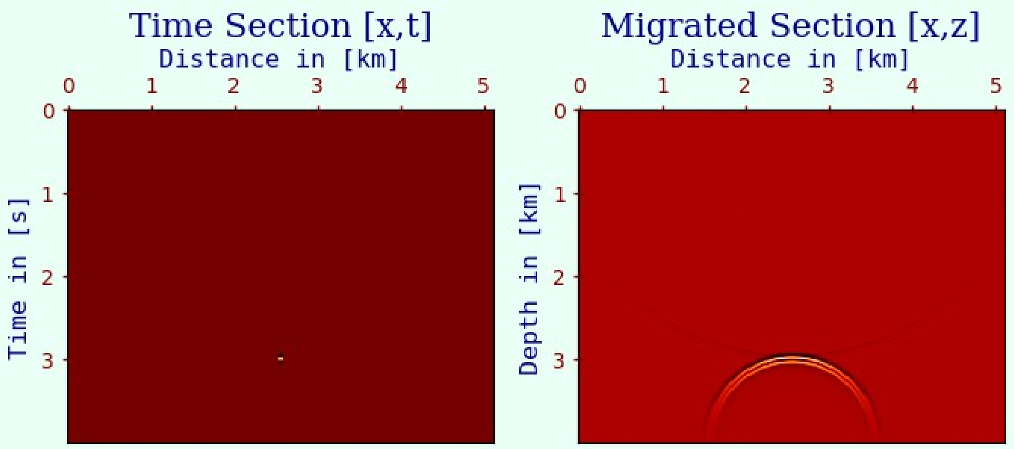
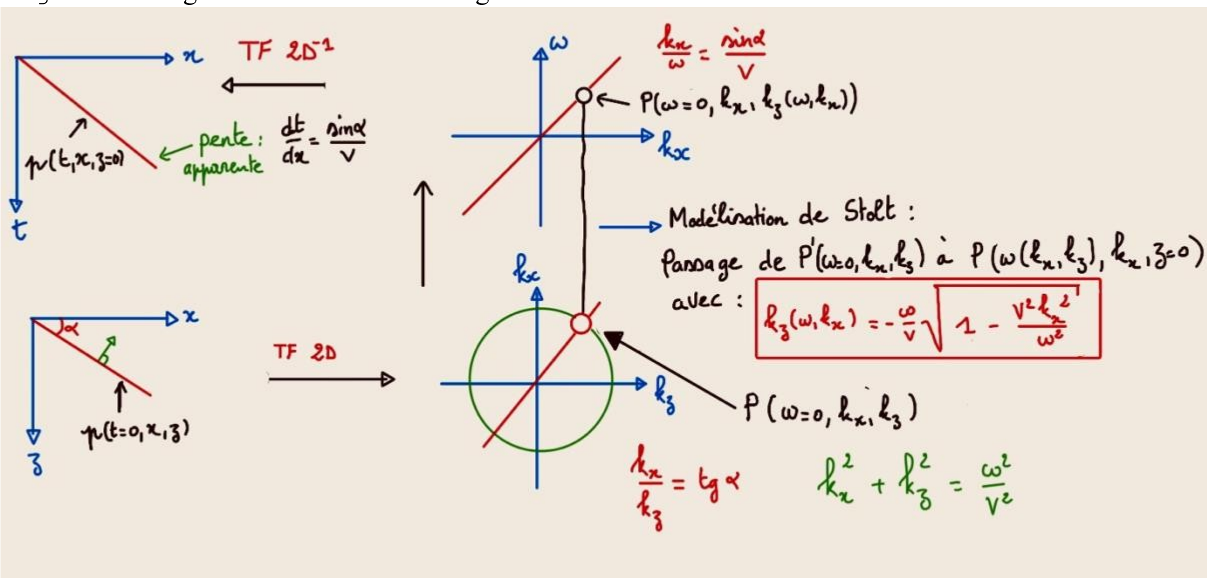




Figure 3c: Impulse placed at 3s in $(x, t)$ migrated into a semicircle whose centre is on the surface $z = z_{Max}$, ***the correct circle is the one pointing upwards.***

# III - Modelling using the Stolt method

Another revolutionary aspect of Stolt's method is that it not only migrates a seismic section, but also models a seismic section from an image ($Image(x, z)$). Indeed, figure () shows the migration pattern that is traversed in the reverse direction of migration $p(t, x) \rightarrow p'(x, z)$ identically mimicking the migration pattern to within one variable change. By traversing the migration scheme in the opposite direction, we can model the seismic section in time.

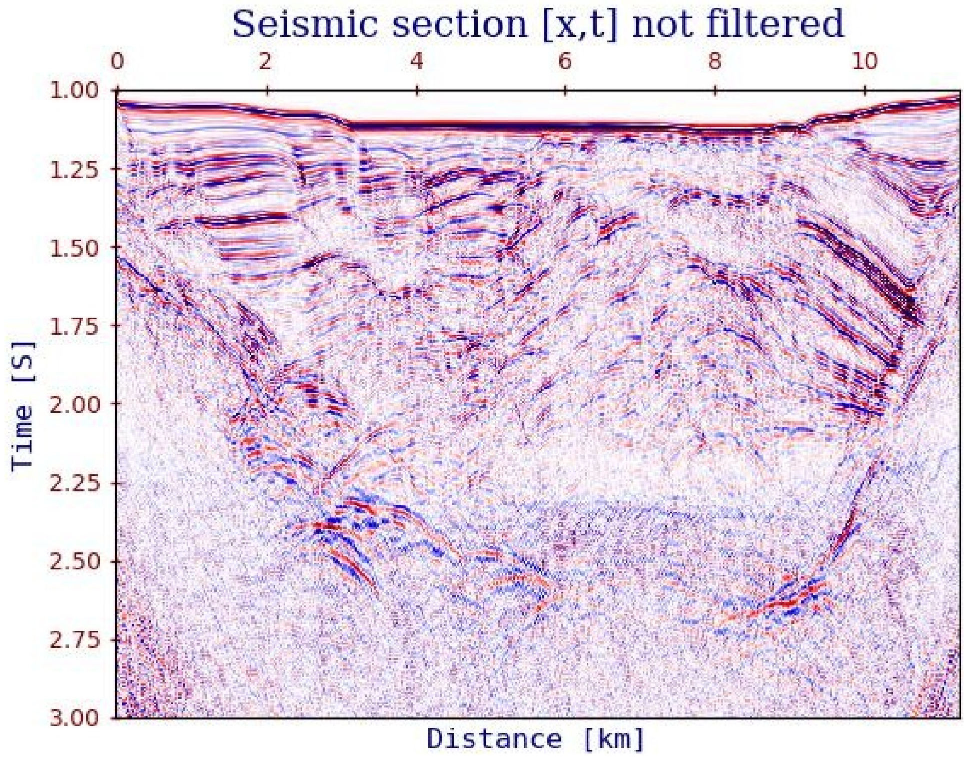
Figure 4: diagram of the Stolt modelling method




# IV - Application to real data

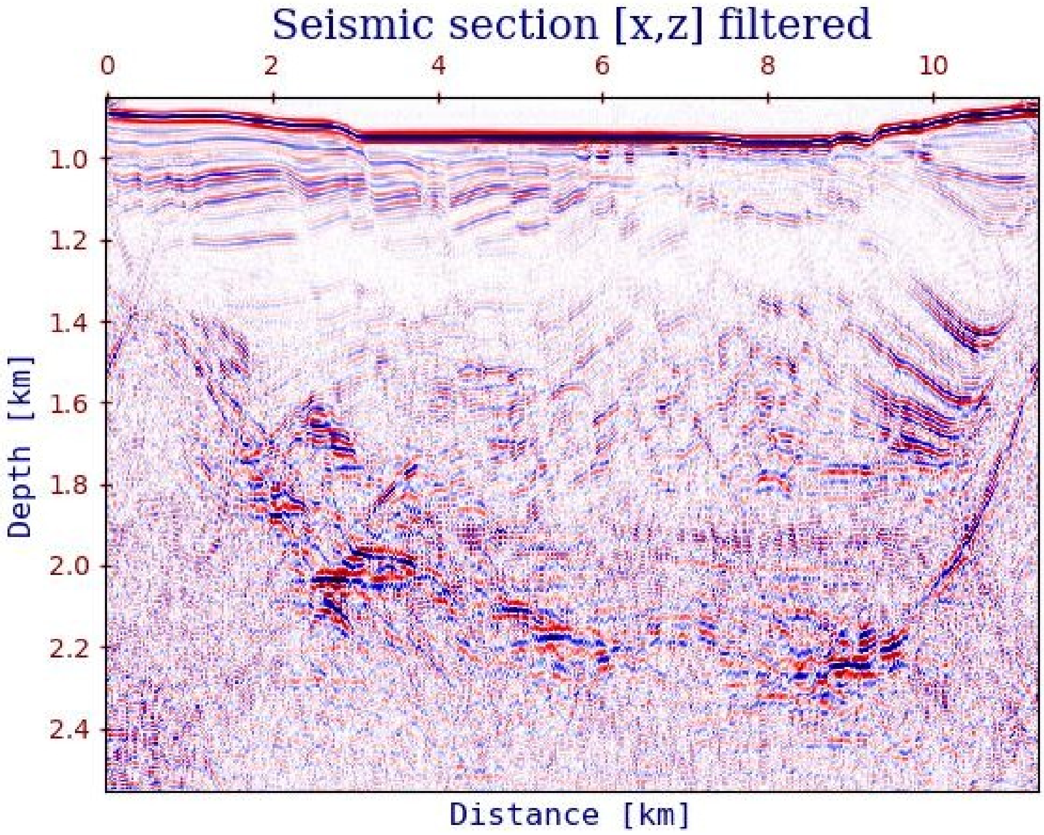
We applied the imaging method invented by Stolt to real data from a seismic acquisition in the Gulf of Corinth, for a constant wave speed equal to 1.7 km.$s^{-1}$ . First of all, we should note the reflection of the water bottom which occurs at 1s on the seismic section (Stack section). The water depth is approximately 800m. see figure(5)

Figure 5: Unfiltered seismic section (x,t) with a reflection from the water bottom around 1s




It should also be noted that the time length of the seismic section was doubled in order to correctly image the second part of the section. Nevertheless, on the migrated section (figure 6), we can observe artefacts linked to the periodicity of the Fourier Transform along the axis (Ox). The artefacts are visible through the slopes on the left and right edges of the
the image (Figure 6).

Figure 6: Migrated seismic section with periodicity artefacts at the edges.




To avoid this aliasing problem, we need to add columns of zero traces at the end of the section. This increases the observation window $[0, _{xMax}]$ , which constitutes the period in $x$. Increasing this $_{xMax}$ leads to a decrease in the spatial sampling step in Fourier domain $dk_x$ . Figure 7 shows the (x,z) image after zero-trace processing, and Figure 8 shows the filtered time seismic slice opposite the same slice after migration.

In particular, the hyperbolic shapes present in the (x,t) section have disappeared in favour of shapes that are much more representative of the geology of the Gulf of Corinth, in the migrated section.

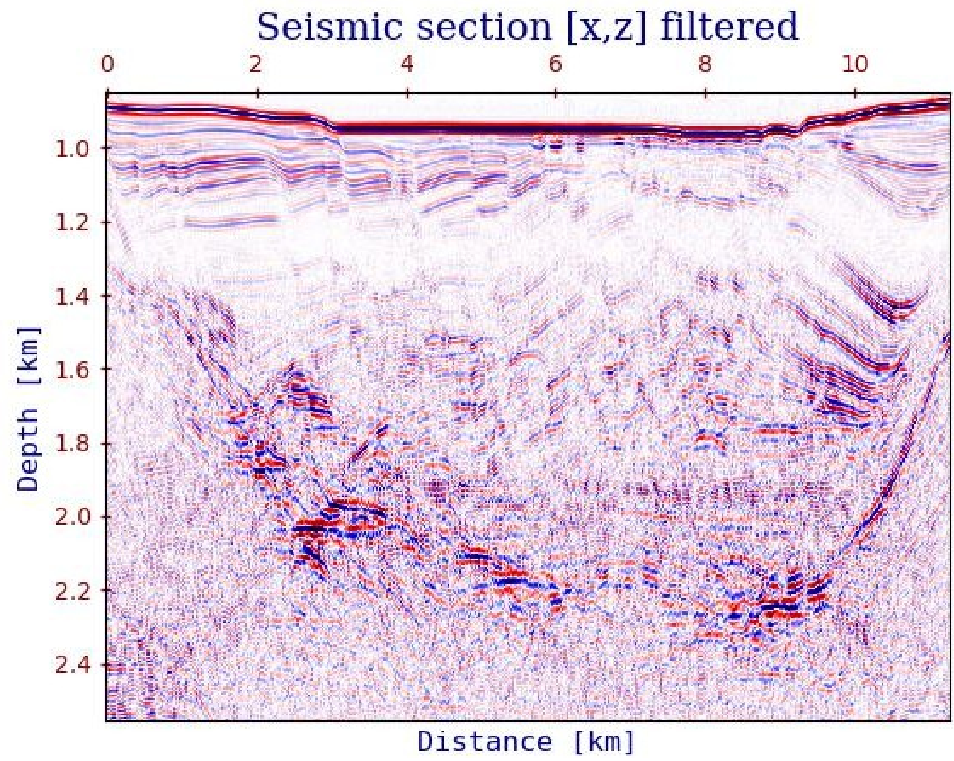
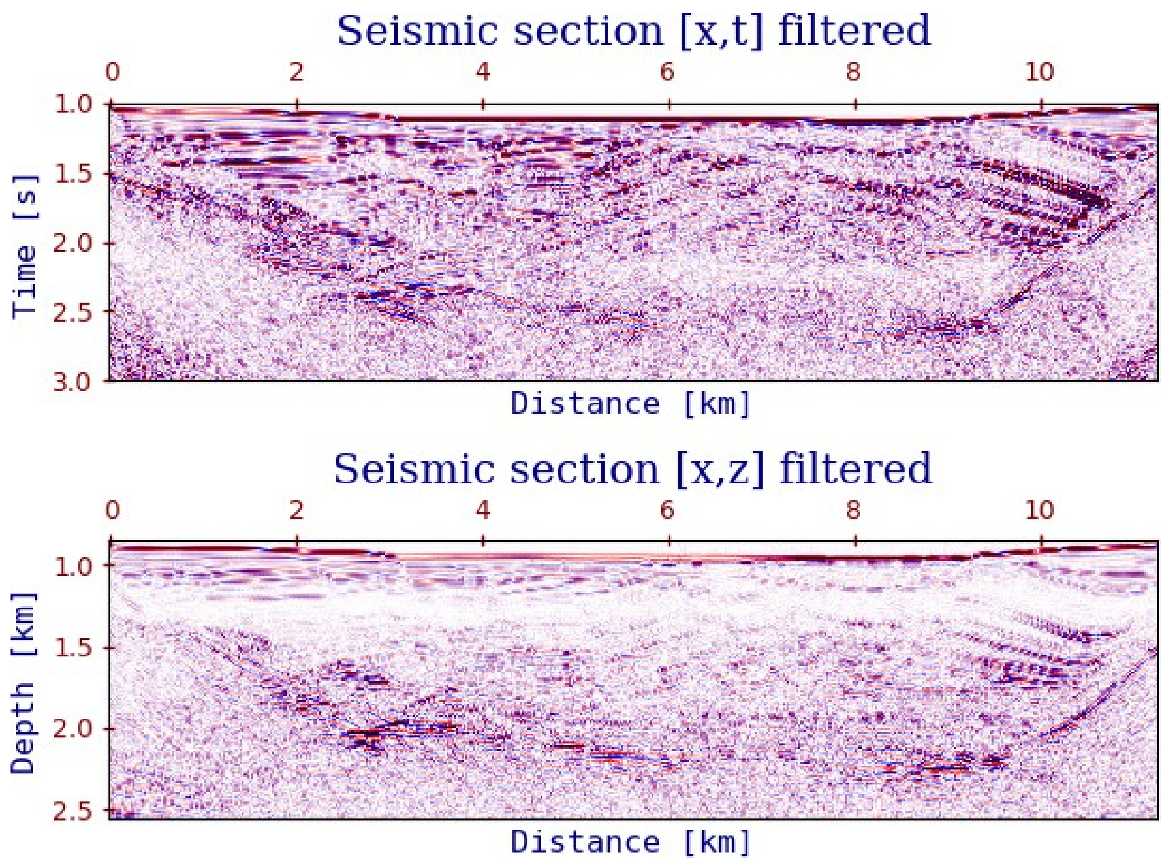



Figure 7: Seismic section migrated by adding null traces. More artefacts are visible

Figure 8: Comparison of seismic sections in time and depth



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