Mapping the Perception-space of Facial Expressions (Online Participants)

Supplementary Materials

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1 General approach

The Geneva Emotion Wheel [GEW; Scherer (2005)] allows having an intuitive and informative way to collect participants' responses in a facial expression perception task. Specifically, in a single measurement is possible to have information about the facial expression *category* (i.e., the response angle around the circle) and *intensity* (i.e., the distance from the center).

1.1 Facial expression category

In order to measure the response angle for each trial we transformed Cartesian coordinates ((x_i, y_i)) into polar coordinates ((r_i, θ_i)) as in Equation (1).

$$\theta_{ij} = tan^{-1}(\frac{y_{ij}}{x_{ij}}) \tag{1}$$

In this way we have the *pressed angle* for each trial. Given that each emotion has an absolute location on the GEW, we calculated a *position-free* index of performance computing the difference between the *pressed angle* and the *ideal angle* (i.e., the GEW location of the presented emotion).

Then we calculated the *ideal* angle for each presented emotion, in the middle of each wheel circle. To obtain a measure comparable between emotion, we calculated the angular difference between the *ideal* and the pressed angle using the Equation (2)

$$Bias = ((ideal - pressed) + 180) \mod 360 - 180 \tag{2}$$

This new measure (*bias*) has several advantages. Despite each emotion have a different location within the wheel, each response is now expressed in a position-free metric. The *bias* is centered on 0 if there is no response tendency away from the *ideal* value. Otherwise, a systematic shift would move the circular mean away from 0, clockwise (positive values) or anticlockwise (negative values). Other than the circular mean, also the spread on the circle (i.e., *uncertainty*) is an important performance measure. The *bias* and the *uncertainty* are can be considered independent measures.

Given the periodicity of circular data, we cannot use standard statistical modeling tools (Cremers, Mulder, and Klugkist 2018; Cremers and Klugkist 2018). There are different ways to model circular data (see Cremers, Mulder, and Klugkist 2018 for an overview). We decided to use a generalized linear mixed-effect model using the *von Mises* likelihood function. The *von Mises* distribution is an alternative to the Gaussian distribution for circular data, bounded in the range $[-\pi, \pi]$. The two parameters of the von Mises distribution, μ and k[1] representing our *bias* and *uncertainty* parameters. To facilitate the interpretation of models' parameters, we transformed k into the circular variance using Equation (3).

$$\sigma^2 = 1 - \frac{I_1(k)}{I_0(k)} \tag{3}$$

The circular variance ranges between 0 (no *uncertainty*) to 1 (maximum *uncertainty*). The transformation is computed using the modified Bessel function $I_i(k)$ of order i (Evans, Hastings, and Peacock 2011).

1.2 Perceived Intensity

The emotion *intensity* is expressed as the difference from the center of the GEW. Values close or far from the center represent respectively neutral and high facial expression intensity. We calculated the *intensity* for

[1] In fact, k is a concentration parameter that can be conceptually considered as the inverse of the standard deviation. When the concentration is 0 the distribution is *uniform*

each trial as the *euclidean distance* between the *center* and the *pressed location*. Given that the GEW has been centered (i.e., the center has coordinates $x = 0, y = 0$), the distance from the center is calculated as Equation (4).

$$I_{ij} = \sqrt{x^2 + y^2} \quad (4)$$

1.3 Statistical models

For the response angle (i.e., *bias* and *uncertainty*) we decided to use a *scale-location* mixed-effect model (Bürkner 2018; Rigby and Stasinopoulos 2005). Under this framework, all parameters of a distribution can be predicted. In particular, we are predicting the *circular mean* (i.e., *bias*) and the *concentration* (i.e., *uncertainty*) Von Mises parameters as a function of Intensity (full and subtle) and Emotion (anger, happiness, disgust, fear, surprise and sadness). For the perceived intensity, we used a regular general linear mixed-effect model.

We estimated both models under a Bayesian framework the R software (R Core Team 2021) using the Brms package (Bürkner 2017) based on the STAN probabilistic programming language (Carpenter et al. 2017). The Bayesian statistics consist in combining information from prior knowledge (i.e. *priors*) and the data (i.e., *likelihood*) to obtain the *posterior* distribution (Kruschke and Liddell 2018).

In terms of contrast coding, for categorical predictors, we used `sum contrasts` using the `contr.sum()` function.

1.3.1 `brms`

We fitted our models using the `brms` package. According to different models the `brm` setup could be different in terms of `backend`, number of `iterations` and `chains` and the parallelization approach. The general approach for `bias/uncertainty` models is the following:

```
# the scale-location specification
form <- bf(theta_cen ~ ... + (1|id),
           kappa ~ ... + (1|id))

brm(formula, # model formula
    data = data,
    prior = priors,
    family = von_mises(link = "tan_half", link_kappa = "log"),
    chains = 15,
    cores = 15,
    iter = 4000,
    sample_prior = "yes",
    save_pars = save_pars(all = TRUE),
    seed = 2022)
```

For the `perceived intensity`

```
brm(int ~ ... + (1|id),
    data = data,
    prior = priors,
    family = gaussian(),
    chains = 15,
    cores = 15,
```

```
    iter = 4000,
    save_pars = save_pars(all = TRUE),
    sample_prior = "yes",
    seed = 2022)
```

When fitting models with uninformative or flat priors, we used a different chains/iteration approach to improve model fitting (especially for the Von Mises model). In particular we used the *within-chains* parallelization (https://cran.r-project.org/web/packages/brms/vignettes/brms_threading.html) for `bias/uncertainty` models:

```
# the scale-location specification
form <- bf(theta_cen ~ ... + (1|id),
           kappa ~ ... + (1|id))

brm(form,
    data = data,
    family = von_mises(link = "tan_half", link_kappa = "log"),
    chains = 4,
    prior = priors, # uninformative or flat
    cores = 4,
    iter = 10000,
    sample_prior = "yes",
    threads = threading(6),  # within-chains parallelization
    save_pars = save_pars(all = TRUE),
    seed = seed)
```

For the `perceived intensity` models we use the same approach as the main models given the simpler fitting process.

1.4 Raw data

The figure S2 represents all participants' responses for each experimental condition, directly plotted on the GEW. The figure S1 represents the GEW legend and the responses to the neutral condition.

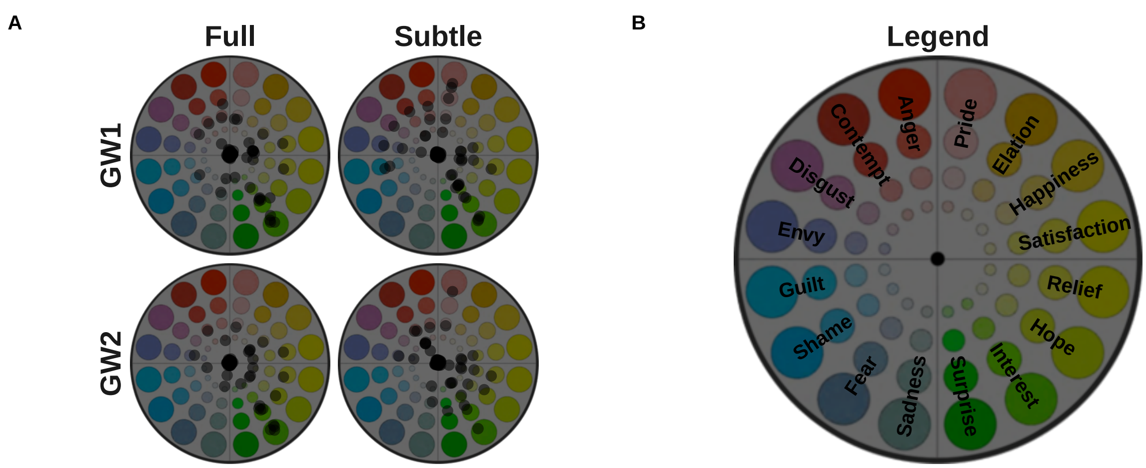

Figure S1: GEW legend (B) and responses to neutral facial expressions as a function of the Intensity and Mapping Wheel 1 and 2 (A)

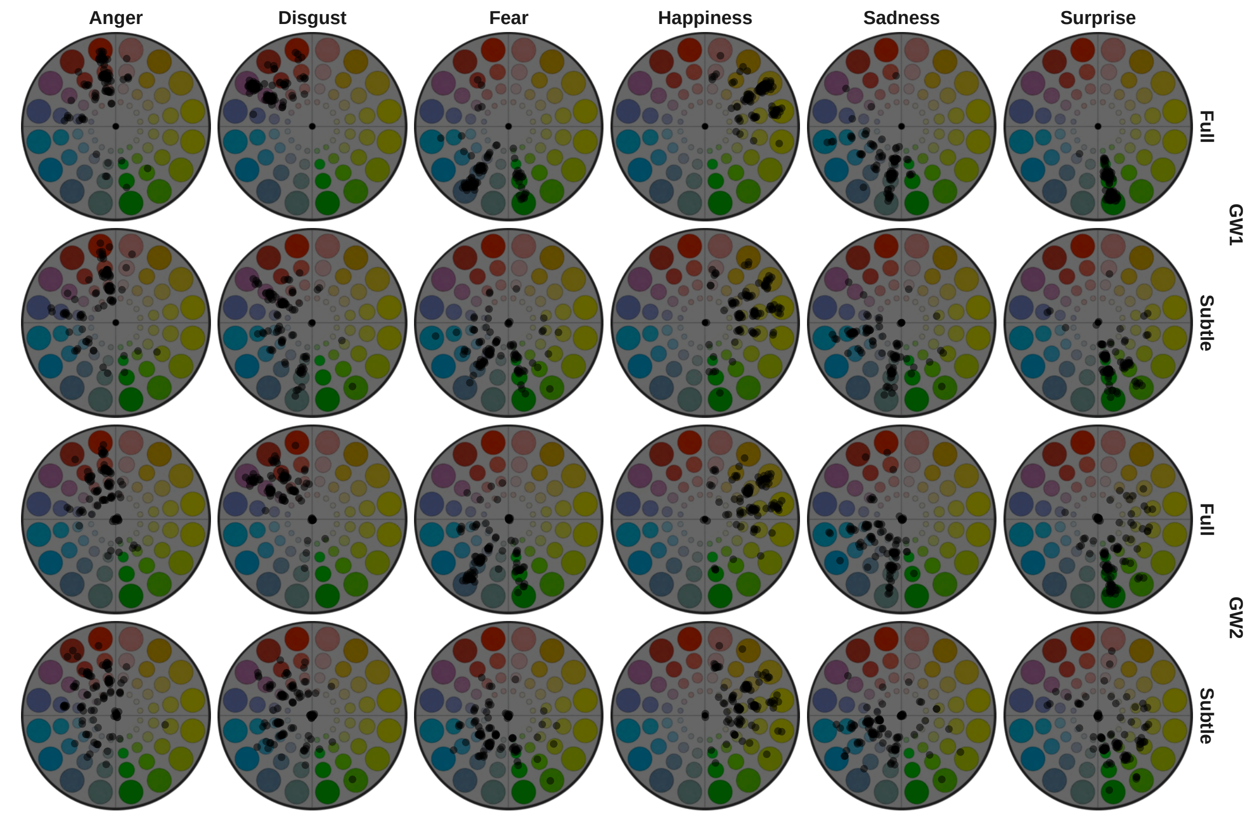

Figure S2: GEW responses as a function of displayed Emotion, Intensity and Mapping Wheel 1 and 2.

2 Fitted Models

Table S1 depicts all fitted models with main parameters. To read the table:

- `fit_ri_int`: the random-intercept two-way interaction model for *bias/uncertainty* and *perceived intensity*
- `fit_ri_no2int`: the random-intercept model without the two-way interaction *bias/uncertainty* and *perceived intensity*
- `fit_*un/flat`: models with completely uninformative or flat priors

Table S1: Table with model formulas, names and fitting parameters

model	name	chains	iter	warmup	samples
diff_theta ~ emotion * intensity + (1 | id) kappa ~ emotion * intensity + (1 | id)	dataset_fit_ri_int	15	4,000	2,000	30,000
theta_cen ~ 0 + Intercept + (1 | id) kappa ~ 0 + Intercept + (1 | id)	dataset_fit_ri_neu	15	4,000	2,000	30,000
diff_theta ~ emotion + intensity + (1 | id) kappa ~ emotion + intensity + (1 | id)	dataset_fit_ri_no2int	15	4,000	2,000	30,000
int ~ 0 + Intercept + emotion * intensity + (1 | id)	dataset_fit_ri_int	15	4,000	2,000	30,000
int ~ 0 + Intercept + (1 | id)	dataset_fit_ri_neu	15	4,000	2,000	30,000
int ~ 0 + Intercept + emotion + intensity + (1 | id)	dataset_fit_ri_no2int	15	4,000	2,000	30,000

In the next section we presented all fitted models using the same approach:

- the model name (the same name as the R object)
- prior distributions for each parameter
- model output

For the prior tables:

- `prior`: is the prior distribution with parameters. All parameters without a proper prior (i.e., different from a flat prior) are not reported in the table.
- `class`: is the type of parameter (`b` is for β and `sd` for a standard deviation parameter e.g., by-subject intercept or residual σ)
- `coef`: is the specific model parameters. If a prior is defined only for a *class*, then all parameters of that class will have the same prior
- `dpar`: is for *distributional parameters*. In the case of the von Mises model refers to k coefficients

For the model tables:

- `param`: is the model parameter name
- `estimate`: is the mean of the posterior distribution
- `Est.Error`: is the standard error of the posterior distribution
- `95% CI`: is the 95% credible interval
- `Rhat`: is the Gelman and Rubin (Gelman and Rubin 1992) convergence index. When is below 1.1 the parameters has converged.
- `Bulk/Tail Effective Sample Size`: can be considered as the amount of information used for estimating a parameter. In general higher is better (see https://mc-stan.org/docs/2_18/reference-manual/effective-sample-size-section.html). Is calculated from the number of iterations and chains of the models.

2.1 Bias/Uncertainty

2.1.1 dataset_fit_ri_int

2.1.1.1 Priors

Table S2:

prior	class	coef	dpar
normal(0, 2)	b		
normal(0, 2)	b		kappa
student_t(3, 0, 2.5)	Intercept		
normal(5.0, 0.8)	Intercept		kappa
student_t(3, 0, 2.5)	sd		
student_t(3, 0, 2.5)	sd		kappa

2.1.1.2 Model

Table S3:

param	median	se	lower	upper	Rhat	Bulk_ESS	Tail_ESS
emotion1	-0.11577	0.02491	-0.16514	-0.06794	1.00039	35,792.29214	24,099.78366
emotion1:intensity1	0.04238	0.02471	-0.00626	0.09098	1.00050	35,971.65325	23,669.07040
emotion2	-0.08574	0.02135	-0.12895	-0.04499	1.00026	24,478.58369	22,595.57802
emotion2:intensity1	0.12300	0.02130	0.08186	0.16490	1.00017	23,330.25693	22,417.88336
emotion3	-0.04853	0.02115	-0.09059	-0.00799	1.00035	37,284.04441	25,158.03886
emotion3:intensity1	-0.01718	0.02107	-0.05831	0.02395	1.00038	36,405.85358	23,868.44862
emotion4	0.08767	0.01562	0.05736	0.11831	1.00013	33,128.29099	25,174.62594
emotion4:intensity1	-0.06964	0.01574	-0.10071	-0.03955	1.00011	33,128.84316	24,451.50993
emotion5	0.20085	0.02226	0.15647	0.24390	1.00002	33,570.25696	24,939.35274
emotion5:intensity1	-0.10603	0.02253	-0.15154	-0.06294	1.00022	34,329.23364	24,255.86124
intensity1	0.01409	0.00921	-0.00412	0.03208	1.00063	36,621.17522	24,694.33769
Intercept	0.00080	0.00935	-0.01814	0.01868	1.00054	34,012.56727	24,039.34585
kappa_emotion1	-0.88162	0.08952	-1.05695	-0.70953	1.00060	50,438.83963	23,648.78254
kappa_emotion1:intensity1	-0.55484	0.08935	-0.72490	-0.37501	1.00089	46,500.62889	22,692.13865
kappa_emotion2	0.01333	0.08964	-0.16293	0.18736	1.00024	52,196.56625	22,378.58262
kappa_emotion2:intensity1	0.31033	0.08921	0.13016	0.48205	1.00060	49,254.02069	23,171.48194
kappa_emotion3	-0.59587	0.08721	-0.76630	-0.42595	1.00044	47,608.80334	23,893.12391
kappa_emotion3:intensity1	-0.42369	0.08925	-0.59632	-0.24918	1.00058	45,474.26047	21,615.14281

param	median	se	lower	upper	Rhat	Bulk_ESS	Tail_ESS
kappa_emotion4	0.13712	0.09094	-0.04336	0.31284	1.00060	47,995.87225	22,586.45969
kappa_emotion4:intensity1	-0.32506	0.08901	-0.49469	-0.14893	1.00013	50,452.23505	24,110.21474
kappa_emotion5	-0.59117	0.08782	-0.77018	-0.42447	1.00020	48,094.52292	22,418.78350
kappa_emotion5:intensity1	-0.41000	0.08736	-0.58434	-0.24143	1.00039	46,650.18766	22,184.54389
kappa_intensity1	0.83063	0.04192	0.74945	0.91253	1.00032	49,805.43378	22,201.56211
kappa_Intercept	1.82676	0.24658	1.39118	2.33861	1.00267	4,935.91798	8,126.98380
sd(Intercept)	0.00404	0.00375	0.00000	0.01184	1.00090	12,027.67666	15,097.94032
sd(kappa_Intercept)	0.69913	0.24117	0.37767	1.21601	1.00176	6,704.51776	10,185.36430

2.1.2 dataset_fit_ri_neu

2.1.2.1 Priors

Table S4:

prior	class	coef	dpar
uniform(-3.141593, 3.141593)	b		
normal(0, 2)	b		kappa
student_t(3, 0, 2.5)	sd		
student_t(3, 0, 2.5)	sd		kappa

2.1.2.2 Model

Table S5:

param	median	se	lower	upper	Rhat	Bulk_ESS	Tail_ESS
Intercept	-0.81268	0.52553	-1.91335	0.18745	1.00126	6,175.18053	10,181.92417
kappa_Intercept	-0.35478	0.50416	-1.40755	0.57519	1.00147	7,479.90394	11,941.26084
sd(Intercept)	1.43909	0.47347	0.78255	2.45705	1.00112	9,680.74060	14,365.14617
sd(kappa_Intercept)	1.47660	0.49706	0.79487	2.53982	1.00088	9,700.76637	13,893.90982

2.1.3 dataset_fit_ri_no2int

2.1.3.1 Priors

Table S6:

prior	class	coef	dpar
normal(0, 2)	b		
normal(0, 2)	b		kappa
student_t(3, 0, 2.5)	Intercept		
normal(5.0, 0.8)	Intercept		kappa
student_t(3, 0, 2.5)	sd		
student_t(3, 0, 2.5)	sd		kappa

2.1.3.2 Model

Table S7:

param	median	se	lower	upper	Rhat	Bulk_ESS	Tail_ESS
emotion1	-0.08834	0.02114	-0.12970	-0.04678	1.00051	36,635.26322	21,738.53023
emotion2	0.00405	0.01434	-0.02440	0.03224	1.00088	39,833.92249	22,610.65459
emotion3	-0.06262	0.01712	-0.09594	-0.02915	1.00024	41,207.45935	22,148.04181
emotion4	0.04070	0.01283	0.01584	0.06600	1.00104	41,389.67755	22,827.17159
emotion5	0.12566	0.01890	0.08886	0.16284	1.00008	39,355.58206	22,063.61307
intensity1	0.02291	0.00790	0.00689	0.03787	1.00023	37,071.06907	23,828.26662
Intercept	-0.00444	0.01010	-0.02392	0.01585	1.00042	28,356.92264	19,444.94103
kappa_emotion1	-0.76276	0.10291	-0.96990	-0.56992	1.00041	36,861.76452	21,775.93078
kappa_emotion2	0.03551	0.09402	-0.14751	0.22283	1.00007	33,006.92194	23,942.25040
kappa_emotion3	-0.35040	0.09255	-0.54026	-0.17645	1.00074	38,920.41118	23,048.05791
kappa_emotion4	0.32536	0.09198	0.14699	0.50737	1.00073	40,690.90270	22,757.88179
kappa_emotion5	-0.49009	0.09429	-0.67548	-0.30644	1.00046	38,326.48302	23,008.47089
kappa_intensity1	0.74659	0.05103	0.64842	0.84746	1.00007	34,588.76328	23,842.82041
kappa_Intercept	1.52577	0.13925	1.27789	1.82382	1.00115	9,846.40811	11,673.97900
sd(Intercept)	0.01127	0.00868	0.00000	0.02843	1.00024	10,751.29409	16,062.78418
sd(kappa_Intercept)	0.41417	0.12721	0.22714	0.68637	1.00108	9,597.94446	14,310.41526

2.2 Perceived intensity

2.2.1 dataset_fit_ri_int

2.2.1.1 Priors

Table S8:

prior	class	coef	dpar
normal(0, 50)	b		
normal(150, 100)	b	Intercept	
student_t(3, 0, 73.3)	sd		
student_t(3, 0, 73.3)	sigma		

2.2.1.2 Model

Table S9:

param	median	se	lower	upper	Rhat	Bulk_ESS	Tail_ESS
emotion1	-9.12751	2.91322	-14.77151	-3.37663	1.00030	31,307.02867	22,688.50716
emotion1:intensity1	-9.68978	2.92192	-15.39165	-3.94756	1.00036	31,007.36247	23,093.63475
emotion2	4.21392	2.88005	-1.32542	9.93550	1.00023	31,725.51330	22,469.26596
emotion2:intensity1	7.49754	2.89363	1.90703	13.29611	1.00040	31,827.03057	23,739.16216
emotion3	-6.44682	2.89581	-12.15740	-0.84802	1.00049	31,076.61242	23,451.09598
emotion3:intensity1	10.44558	2.87395	4.88791	16.15655	1.00023	31,867.48667	23,050.01512
emotion4	23.33171	2.84179	17.68467	28.86713	1.00047	29,818.78978	23,013.74947
emotion4:intensity1	-3.00723	2.87351	-8.57920	2.66032	1.00022	31,805.68826	22,535.03571
emotion5	-17.68282	2.92852	-23.32054	-11.89165	1.00003	31,114.77378	23,072.40002
emotion5:intensity1	-12.06066	2.95506	-17.85851	-6.33906	1.00008	31,001.25866	22,571.22367
intensity1	19.76942	1.29421	17.22271	22.28959	1.00060	35,107.86275	23,140.95614
Intercept	175.95613	5.48875	165.38172	187.19248	1.00192	7,373.36541	11,664.82695
sd(Intercept)	17.92092	4.62737	11.03811	27.96330	1.00079	8,016.76319	14,507.62950

2.2.2 dataset_fit_ri_neu

2.2.2.1 Priors

Table S10:

prior	class	coef	dpar
normal(0, 50)	b		
normal(150, 100)	b	Intercept	
student_t(3, 0, 6.5)	sd		
student_t(3, 0, 6.5)	sigma		

2.2.2.2 Model

Table S11:

param	median	se	lower	upper	Rhat	Bulk_ESS	Tail_ESS
Intercept	44.09534	13.37081	18.15515	71.19100	1.00478	3,440.76832	5,243.24029
sd(Intercept)	45.01356	9.30684	30.31093	64.87365	1.00100	5,871.38332	9,278.86391

2.2.3 dataset_fit_ri_no2int

2.2.3.1 Priors

Table S12:

prior	class	coef	dpar
normal(0, 50)	b		
normal(150, 100)	b	Intercept	
student_t(3, 0, 73.3)	sd		
student_t(3, 0, 73.3)	sigma		

2.2.3.2 Model

Table S13:

param	median	se	lower	upper	Rhat	Bulk_ESS	Tail_ESS
emotion1	-9.14336	2.95525	-14.88024	-3.32928	1.00010	28,921.47204	23,195.76636
emotion2	4.30457	2.92373	-1.41506	10.03496	1.00047	29,688.75934	23,784.49224
emotion3	-6.36217	2.92281	-12.09495	-0.66197	1.00025	27,843.35293	22,913.32066
emotion4	23.41703	2.95518	17.49283	29.08029	1.00033	28,406.51204	22,711.48359
emotion5	-17.96003	2.96206	-23.95241	-12.23642	1.00033	28,746.89089	23,073.95167
intensity1	19.86470	1.30928	17.28930	22.40732	0.99995	35,188.10851	22,042.63382
Intercept	175.96062	5.52007	164.93550	186.90922	1.00102	6,528.09033	10,160.18207
sd(Intercept)	17.91262	4.71946	11.03395	28.30958	1.00180	6,541.37053	11,308.48578

3 Suggestions for meta-analysis

In this section, there are some suggestions for including these results into a meta-analysis. Firstly, if the presented results are not sufficient, the online OSF repository (https://osf.io/e2kcw/) contains raw data to compute all relevant measures. In general, for Bayesian models, each parameter or posterior contrast has a full posterior probability. This makes the computation of new measures (e.g., standardized effect sizes) and standard errors relatively easy. The only difference from standard calculations is that each new measure will have a full posterior distribution. These new distributions can be summarized (e.g., using the median) and used for the meta-analytic model.

3.1 Bias

To our knowledge, for the *bias*, there is no straightforward standardized effect size measure to compute, especially for a meta-analytic model. A possibility is using a general index of overlap between two posterior distributions (e.g., for a specific post-hoc contrast) as proposed by Pastore and Calcagnì (2019). However, the meta-analytic comparison with standard effect sizes index is not straightforward.

3.2 Uncertainty

For the *uncertainty* it is possible to use directly the values from the posterior contrasts. The *uncertainty* (i.e., *circular variance*) is expressed on a scale from 0 to 1 (similar to a probability). All posterior contrasts can be interpreted as probability ratios and odds ratios. Also, the standard error can be calculated as the standard deviation of the posterior distribution. Furthermore, it is also possible to convert from odds ratio (or similar measures) to other effect size indexes (e.g., Cohen's d, see https://easystats.github.io/effectsize/reference/d_to_r.html).

3.3 Perceived Intensity

For the perceived intensity it is possible to use a standard Cohen's d measure. The only general caveat about calculating a Cohen's d with multilevel models concerns which standard deviation(s) to use (Brysbaert and Stevens 2018; Westfall, Kenny, and Judd 2014)

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