Mapping the Perception-space of Facial Expressions (Moebius Participants)

Supplementary Materials

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Contents

1 General approach

The Geneva Emotion Wheel [GEW; Scherer (2005)] allows having an intuitive and informative way to collect participants' responses in a facial expression perception task. Specifically, in a single measurement is possible to have information about the facial expression *category* (i.e., the response angle around the circle) and *intensity* (i.e., the distance from the center).

1.1 Facial expression category

In order to measure the response angle for each trial we transformed Cartesian coordinates ((x_i, y_i)) into polar coordinates ((r_i, θ_i)) as in Equation (1).

$$\theta_{ij} = tan^{-1}(\frac{y_{ij}}{x_{ij}}) \tag{1}$$

In this way we have the *pressed angle* for each trial. Given that each emotion has an absolute location on the GEW, we calculated a *position-free* index of performance computing the difference between the *pressed angle* and the *ideal angle* (i.e., the GEW location of the presented emotion).

Then we calculated the *ideal* angle for each presented emotion, in the middle of each wheel circle. To obtain a measure comparable between emotion, we calculated the angular difference between the *ideal* and the pressed angle using the Equation (2)

$$Bias = ((ideal - pressed) + 180) \mod 360 - 180 \tag{2}$$

This new measure (*bias*) has several advantages. Despite each emotion have a different location within the wheel, each response is now expressed in a position-free metric. The *bias* is centered on 0 if there is no response tendency away from the *ideal* value. Otherwise, a systematic shift would move the circular mean away from 0, clockwise (positive values) or anticlockwise (negative values). Other than the circular mean, also the spread on the circle (i.e., *uncertainty*) is an important performance measure. The *bias* and the *uncertainty* are can be considered independent measures.

Given the periodicity of circular data, we cannot use standard statistical modeling tools (Cremers, Mulder, and Klugkist 2018; Cremers and Klugkist 2018). There are different ways to model circular data (see Cremers, Mulder, and Klugkist 2018 for an overview). We decided to use a generalized linear mixed-effect model using the *von Mises* likelihood function. The *von Mises* distribution is an alternative to the Gaussian distribution for circular data, bounded in the range $[-\pi, \pi]$. The two parameters of the von Mises distribution, μ and k[1] representing our *bias* and *uncertainty* parameters. To facilitate the interpretation of models' parameters, we transformed k into the circular variance using Equation (3).

$$\sigma^2 = 1 - \frac{I_1(k)}{I_0(k)} \tag{3}$$

The circular variance ranges between 0 (no *uncertainty*) to 1 (maximum *uncertainty*). The transformation is computed using the modified Bessel function $I_i(k)$ of order i (Evans, Hastings, and Peacock 2011).

1.2 Perceived Intensity

The emotion *intensity* is expressed as the difference from the center of the GEW. Values close or far from the center represent respectively neutral and high facial expression intensity. We calculated the *intensity* for

[1] In fact, k is a concentration parameter that can be conceptually considered as the inverse of the standard deviation. When the concentration is 0 the distribution is *uniform*

each trial as the *euclidean distance* between the *center* and the *pressed location*. Given that the GEW has been centered (i.e., the center has coordinates $x = 0, y = 0$), the distance from the center is calculated as Equation (4).

$$I_{ij} = \sqrt{x^2 + y^2} \tag{4}$$

1.3 Statistical models

For the response angle (i.e., *bias* and *uncertainty*) we decided to use a *scale-location* mixed-effect model (Bürkner 2018; Rigby and Stasinopoulos 2005). Under this framework, all parameters of a distribution can be predicted. In particular, we are predicting the *circular mean* (i.e., *bias*) and the *concentration* (i.e., *uncertainty*) Von Mises parameters as a function of Intensity (full and subtle) and Emotion (anger, happiness, disgust, fear, surprise and sadness). For the perceived intensity, we used a regular general linear mixed-effect model.

We estimated both models under a Bayesian framework the R software (R Core Team 2021) using the Brms package (Bürkner 2017) based on the STAN probabilistic programming language (Carpenter et al. 2017). The Bayesian statistics consist in combining information from prior knowledge (i.e. *priors*) and the data (i.e., *likelihood*) to obtain the *posterior* distribution (Kruschke and Liddell 2018).

In terms of contrast coding, for categorical predictors, we used `sum contrasts` using the `contr.sum()` function.

1.3.1 `brms`

We fitted our models using the `brms` package. According to different models the `brm` setup could be different in terms of `backend`, number of `iterations` and `chains` and the parallelization approach. The general approach for `bias/uncertainty` models is the following:

```r
# the scale-location specification
form <- bf(theta_cen ~ ... + (1|id),
           kappa ~ ... + (1|id))

brm(formula, # model formula
    data = data,
    prior = priors,
    family = von_mises(link = "tan_half", link_kappa = "log"),
    chains = 15,
    cores = 15,
    iter = 4000,
    sample_prior = "yes",
    save_pars = save_pars(all = TRUE),
    seed = 2022)
```

For the `perceived intensity`

```r
brm(int ~ ... + (1|id),
    data = data,
    prior = priors,
    family = gaussian(),
    chains = 15,
    cores = 15,
```

```
    iter = 4000,
    save_pars = save_pars(all = TRUE),
    sample_prior = "yes",
    seed = 2022)
```

When fitting models with uninformative or flat priors, we used a different chains/iteration approach to improve model fitting (especially for the Von Mises model). In particular we used the *within-chains* parallelization (https://cran.r-project.org/web/packages/brms/vignettes/brms_threading.html) for `bias/uncertainty` models:

```
# the scale-location specification
form <- bf(theta_cen ~ ... + (1|id),
           kappa ~ ... + (1|id))

brm(form,
    data = data,
    family = von_mises(link = "tan_half", link_kappa = "log"),
    chains = 4,
    prior = priors, # uninformative or flat
    cores = 4,
    iter = 10000,
    sample_prior = "yes",
    threads = threading(6),  # within-chains parallellization
    save_pars = save_pars(all = TRUE),
    seed = seed)
```

For the `perceived intensity` models we use the same approach as the main models given the simpler fitting process.

1.4 Raw data

The figure S2 represents all participants' responses for each experimental condition, directly plotted on the GEW. The figure S1 represents the GEW legend and the responses to the neutral condition.

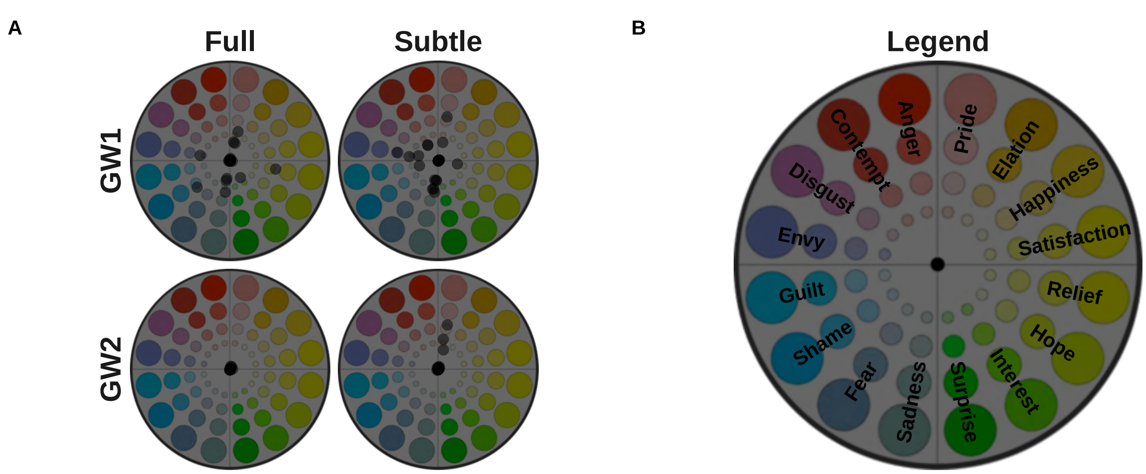

Figure S1: GEW legend (B) and responses to neutral facial expressions as a function of the Intensity and Mapping Wheel 1 and 2 (A)

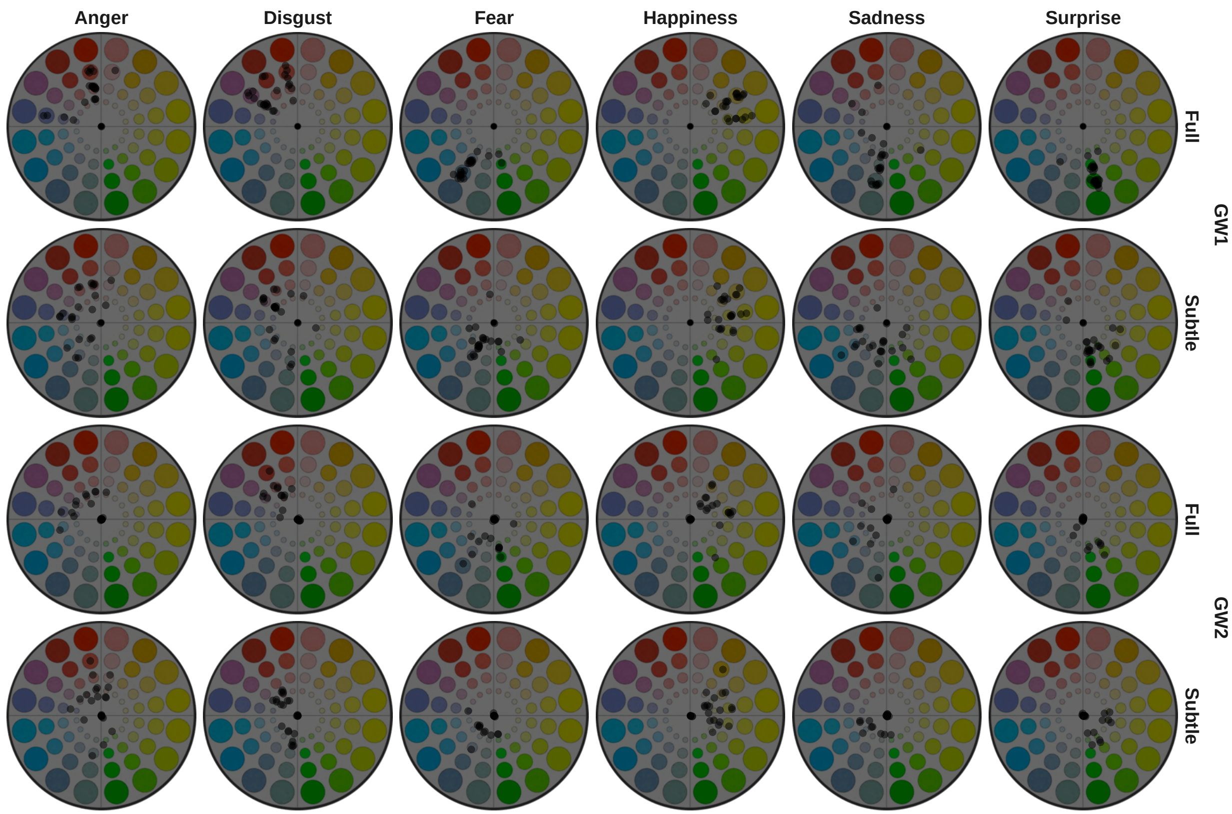

Figure S2: GEW responses as a function of displayed Emotion, Intensity and Mapping Wheel 1 and 2.

2 Fitted Models

Table S1 depicts all fitted models with main parameters. To read the table:

- `fit_ri_int`: the random-intercept two-way interaction model for *bias/uncertainty* and *perceived intensity*
- `fit_ri_no2int`: the random-intercept model without the two-way interaction *bias/uncertainty* and *perceived intensity*
- `fit_*un/flat`: models with completely uninformative or flat priors

Table S1: Table with model formulas, names and fitting parameters

model	name	chains	iter	warmup	samples
diff_theta ~ emotion * intensity + (1 | id) kappa ~ emotion * intensity + (1 | id)	dataset_fit_ri_int	15	4,000	2,000	30,000
theta_cen ~ 0 + Intercept + (1 | id) kappa ~ 0 + Intercept + (1 | id)	dataset_fit_ri_neu	15	4,000	2,000	30,000
diff_theta ~ emotion + intensity + (1 | id) kappa ~ emotion + intensity + (1 | id)	dataset_fit_ri_no2int	15	4,000	2,000	30,000
int ~ 0 + Intercept + emotion * intensity + (1 | id)	dataset_fit_ri_int	15	4,000	2,000	30,000
int ~ 0 + Intercept + (1 | id)	dataset_fit_ri_neu	15	4,000	2,000	30,000
int ~ 0 + Intercept + emotion + intensity + (1 | id)	dataset_fit_ri_no2int	15	4,000	2,000	30,000

In the next section we presented all fitted models using the same approach:

- the model name (the same name as the R object)
- prior distributions for each parameter
- model output

For the prior tables:

- `prior`: is the prior distribution with parameters. All parameters without a proper prior (i.e., different from a flat prior) are not reported in the table.
- `class`: is the type of parameter (`b` is for β and `sd` for a standard deviation parameter e.g., by-subject intercept or residual σ)
- `coef`: is the specific model parameters. If a prior is defined only for a *class*, then all parameters of that class will have the same prior
- `dpar`: is for *distributional parameters*. In the case of the von Mises model refers to k coefficients

For the model tables:

- `param`: is the model parameter name
- `estimate`: is the mean of the posterior distribution
- `Est.Error`: is the standard error of the posterior distribution
- `95% CI`: is the 95% credible interval
- `Rhat`: is the Gelman and Rubin (Gelman and Rubin 1992) convergence index. When is below 1.1 the parameters has converged.
- `Bulk/Tail Effective Sample Size`: can be considered as the amount of information used for estimating a parameter. In general higher is better (see https://mc-stan.org/docs/2_18/reference-manual/effective-sample-size-section.html). Is calculated from the number of iterations and chains of the models.

2.1 Bias/Uncertainty

2.1.1 dataset_fit_ri_int

2.1.1.1 Priors

Table S2:

prior	class	coef	dpar
normal(0, 2)	b		
normal(0, 2)	b		kappa
student_t(3, 0, 2.5)	Intercept		
normal(5.0, 0.8)	Intercept		kappa
student_t(3, 0, 2.5)	sd		
student_t(3, 0, 2.5)	sd		kappa

2.1.1.2 Model

Table S3:

param	median	se	lower	upper	Rhat	Bulk_ESS	Tail_ESS
emotion1	-0.27260	0.07070	-0.40792	-0.14425	1.00290	6,679.28636	7,654.23640
emotion1:intensity1	0.16720	0.07190	0.03918	0.30479	1.00554	2,294.21735	343.85956
emotion2	0.08775	0.04941	-0.01244	0.18454	1.00703	2,379.25480	685.86719
emotion2:intensity1	0.06564	0.05038	-0.03456	0.16387	1.00883	1,706.16363	866.38807
emotion3	-0.00770	0.03723	-0.08054	0.06310	1.00613	2,478.84519	1,545.67905
emotion3:intensity1	-0.05871	0.03706	-0.12937	0.01590	1.00331	9,544.23527	14,941.28329
emotion4	0.11008	0.02958	0.05151	0.16708	1.00280	8,532.33129	10,768.95519
emotion4:intensity1	-0.11354	0.02903	-0.16863	-0.05678	1.00507	4,326.77576	7,755.41387
emotion5	0.11646	0.05862	0.00732	0.23828	1.00512	5,545.24297	1,263.69565
emotion5:intensity1	-0.06403	0.05760	-0.17533	0.05178	1.00220	7,837.08300	18,260.00975
intensity1	0.04764	0.02203	0.00653	0.09095	1.00687	2,920.85589	1,351.82936
Intercept	-0.01748	0.09778	-0.48709	-0.47342	1.01751	643.80786	532.73046
Intercept	-0.01748	0.09778	-0.20001	0.17300	1.01751	643.80786	532.73046
Intercept	-0.01748	0.09778	0.29604	0.30190	1.01751	643.80786	532.73046
Intercept	-0.01748	0.09778	0.30581	0.33510	1.01751	643.80786	532.73046
kappa_emotion1	-0.65091	0.20654	-1.08790	-0.30587	1.00842	1,737.14193	270.42931
kappa_emotion1:intensity1	-0.07832	0.20254	-0.45153	0.32712	1.00234	11,018.38056	12,777.51540
kappa_emotion2	-0.39371	0.19100	-0.77362	-0.03047	1.00357	4,922.45152	17,440.54474

param	median	se	lower	upper	Rhat	Bulk_ESS	Tail_ESS
kappa_emotion2:intensity1	-0.03769	0.19461	-0.41732	0.35361	1.00428	4,376.18670	2,637.87450
kappa_emotion3	0.30201	0.18974	-0.07952	0.67121	1.00876	4,304.30862	1,032.63327
kappa_emotion3:intensity1	0.15219	0.19154	-0.21818	0.51428	1.00800	1,649.57018	9,378.69704
kappa_emotion4	0.91745	0.20041	0.51558	1.28463	1.00641	2,399.90151	4,959.13180
kappa_emotion4:intensity1	-0.16048	0.19294	-0.55051	0.21852	1.00538	6,613.99209	1,965.08156
kappa_emotion5	-0.97216	0.19267	-1.36205	-0.60992	1.00334	7,585.98903	16,510.01028
kappa_emotion5:intensity1	-0.57048	0.20968	-0.96907	-0.13024	1.00831	2,130.18152	773.49845
kappa_intensity1	0.69716	0.08370	0.53616	0.86093	1.00229	9,893.99334	17,178.99092
kappa_Intercept	4.15943	0.92564	2.21061	5.89540	1.00354	6,089.95164	7,107.53443
sd(Intercept)	0.06658	0.17596	0.00007	0.45183	1.01919	640.56877	195.30618
sd(kappa_Intercept)	2.73953	1.51982	0.37060	5.94360	1.00516	5,916.48896	7,411.77576

2.1.2 dataset_fit_ri_neu

2.1.2.1 Priors

Table S4:

prior	class	coef	dpar
uniform(-3.141593, 3.141593)	b		
normal(0, 2)	b		kappa
student_t(3, 0, 2.5)	sd		
student_t(3, 0, 2.5)	sd		kappa

2.1.2.2 Model

Table S5:

param	median	se	lower	upper	Rhat	Bulk_ESS	Tail_ESS
Intercept	-2.12556	0.67813	-3.14144	-0.88335	1.00175	4,775.93418	3,306.94604
kappa_Intercept	0.22805	0.54887	-0.98099	1.39558	1.00155	6,714.29589	6,757.38365
sd(Intercept)	0.76907	1.02724	0.00029	2.99548	1.00243	5,146.25971	3,798.13619
sd(kappa_Intercept)	0.68994	0.72383	0.00004	2.30972	1.00172	5,865.93590	7,713.25734

2.1.3 dataset_fit_ri_no2int

2.1.3.1 Priors

Table S6:

prior	class	coef	dpar
normal(0, 2)	b		
normal(0, 2)	b		kappa
student_t(3, 0, 2.5)	Intercept		
normal(5.0, 0.8)	Intercept		kappa
student_t(3, 0, 2.5)	sd		
student_t(3, 0, 2.5)	sd		kappa

2.1.3.2 Model

Table S7:

param	median	se	lower	upper	Rhat	Bulk_ESS	Tail_ESS
emotion1	-0.14043	0.04292	-0.22322	-0.05462	1.00298	5,377.66061	2,833.88223
emotion2	0.13183	0.03545	0.06398	0.20239	1.00169	17,105.29465	16,760.04327
emotion3	-0.05319	0.02582	-0.10346	-0.00216	1.00183	11,188.25897	9,870.00786
emotion4	0.02702	0.02363	-0.01822	0.07328	1.00236	7,404.23431	20,311.99136
emotion5	0.06733	0.05298	-0.03855	0.16970	1.00242	11,363.80913	17,926.17070
intensity1	0.00424	0.01693	-0.02849	0.03776	1.00146	18,110.18415	22,233.81346
Intercept	0.01593	0.10980	-0.52141	-0.50793	1.00951	1,760.47473	547.76015
Intercept	0.01593	0.10980	-0.21942	-0.21672	1.00951	1,760.47473	547.76015
Intercept	0.01593	0.10980	-0.20863	0.23088	1.00951	1,760.47473	547.76015
kappa_emotion1	-0.67745	0.19638	-1.06734	-0.30590	1.00240	8,398.48119	20,512.47655
kappa_emotion2	-0.31016	0.19226	-0.69578	0.05311	1.00217	9,398.06511	16,543.44520
kappa_emotion3	0.46160	0.18438	0.11373	0.83219	1.00099	16,583.52596	21,341.76100
kappa_emotion4	0.92771	0.21177	0.50485	1.33538	1.00306	9,889.19256	11,536.20341
kappa_emotion5	-1.10552	0.22727	-1.56673	-0.68360	1.00376	19,584.37837	18,152.99474
kappa_intensity1	0.77245	0.10096	0.57715	0.96551	1.00606	2,097.85935	3,350.43772
kappa_Intercept	4.25072	0.92761	2.45207	6.08126	1.01339	811.57850	278.49737
sd(Intercept)	0.07177	0.16476	0.00006	0.47467	1.00888	1,988.43619	640.95033
sd(kappa_Intercept)	2.97848	1.64204	0.70133	6.65408	1.01047	1,276.50864	276.97834

2.2 Perceived intensity

2.2.1 dataset_fit_ri_int

2.2.1.1 Priors

Table S8:

prior	class	coef	dpar
normal(0, 50)	b		
normal(150, 100)	b	Intercept	
student_t(3, 0, 56.2)	sd		
student_t(3, 0, 56.2)	sigma		

2.2.1.2 Model

Table S9:

param	median	se	lower	upper	Rhat	Bulk_ESS	Tail_ESS
emotion1	-4.62551	5.09662	-15.38165	4.46546	1.00638	2,001.19556	301.15597
emotion1:intensity1	-9.78796	4.89538	-19.06161	-0.07117	1.00111	16,534.79511	21,373.47661
emotion2	0.36625	5.02321	-9.12404	10.39372	1.00372	4,796.21221	18,674.02013
emotion2:intensity1	1.78580	4.97719	-7.70286	11.83704	1.00203	13,421.93157	6,835.13750
emotion3	-4.63298	4.98368	-14.53367	4.81354	1.00256	5,859.99153	11,675.16290
emotion3:intensity1	15.34929	4.99024	5.47367	25.25803	1.00188	9,065.93963	7,394.33408
emotion4	20.34052	4.99123	10.90129	30.25717	1.00266	4,849.78841	19,820.12218
emotion4:intensity1	-12.65585	4.96675	-22.21314	-2.67235	1.00062	23,778.36472	19,463.39729
emotion5	-15.84682	5.12152	-25.97403	-5.65654	1.00964	1,140.17019	284.93902
emotion5	-15.84682	5.12152	-3.37184	-3.04546	1.00964	1,140.17019	284.93902
emotion5:intensity1	-5.94565	4.97516	-15.51631	3.87800	1.00226	21,755.26650	13,232.20493
intensity1	23.94087	2.22978	19.47299	28.16967	1.00116	26,435.12086	20,904.27989
Intercept	133.40126	15.15757	102.91991	167.16732	1.00317	5,933.39251	5,567.69596
sd(Intercept)	20.31635	23.72724	4.19097	63.33229	1.01267	975.80500	239.04756

2.2.2 dataset_fit_ri_neu

2.2.2.1 Priors

Table S10:

prior	class	coef	dpar
normal(0, 50)	b		
normal(150, 100)	b	Intercept	
student_t(3, 0, 4.2)	sd		
student_t(3, 0, 4.2)	sigma		

2.2.2.2 Model

Table S11:

param	median	se	lower	upper	Rhat	Bulk_ESS	Tail_ESS
Intercept	26.62102	6.44670	14.12508	39.61553	1.00153	7,811.75641	6,385.40947
sd(Intercept)	5.92078	5.25994	0.00032	16.65752	1.00175	8,085.13004	8,289.16988

2.2.3 dataset__fit__ri__no2int

2.2.3.1 Priors

Table S12:

prior	class	coef	dpar
normal(0, 50)	b		
normal(150, 100)	b	Intercept	
student_t(3, 0, 56.2)	sd		
student_t(3, 0, 56.2)	sigma		

2.2.3.2 Model

Table S13:

param	median	se	lower	upper	Rhat	Bulk__ESS	Tail__ESS
emotion1	-4.43803	5.19304	-14.27614	6.26860	1.00175	6,774.30687	2,192.35195
emotion2	0.43007	5.06317	-9.39145	10.43009	1.00081	21,662.94091	16,576.24297
emotion3	-4.65002	5.12788	-14.61041	5.48770	1.00099	23,248.59826	16,868.97215
emotion4	20.45660	5.07252	10.41935	30.33126	1.00066	26,937.90925	19,365.83702
emotion5	-15.97701	5.15204	-26.16237	-5.83707	1.00026	27,722.51770	20,502.77364
intensity1	23.97988	2.30086	19.46883	28.45770	1.00064	27,632.95699	19,428.56568
Intercept	133.33681	15.66347	98.88737	167.80773	1.00529	3,294.27765	1,172.68077
sd(Intercept)	20.09940	17.27440	4.54514	60.60068	1.00320	2,962.82859	1,151.59775

3 Suggestions for meta-analysis

In this section, there are some suggestions for including these results into a meta-analysis. Firstly, if the presented results are not sufficient, the online OSF repository (https://osf.io/e2kcw/) contains raw data to compute all relevant measures. In general, for Bayesian models, each parameter or posterior contrast has a full posterior probability. This makes the computation of new measures (e.g., standardized effect sizes) and standard errors relatively easy. The only difference from standard calculations is that each new measure will have a full posterior distribution. These new distributions can be summarized (e.g., using the median) and used for the meta-analytic model.

3.1 Bias

To our knowledge, for the *bias*, there is no straightforward standardized effect size measure to compute, especially for a meta-analytic model. A possibility is using a general index of overlap between two posterior distributions (e.g., for a specific post-hoc contrast) as proposed by Pastore and Calcagnì (2019). However, the meta-analytic comparison with standard effect sizes index is not straightforward.

3.2 Uncertainty

For the *uncertainty* it is possible to use directly the values from the posterior contrasts. The *uncertainty* (i.e., *circular variance*) is expressed on a scale from 0 to 1 (similar to a probability). All posterior contrasts can be interpreted as probability ratios and odds ratios. Also, the standard error can be calculated as the standard deviation of the posterior distribution. Furthermore, it is also possible to convert from odds ratio (or similar measures) to other effect size indexes (e.g., Cohen's *d*, see https://easystats.github.io/effectsize/reference/d_to_r.html).

3.3 Perceived Intensity

For the perceived intensity it is possible to use a standard Cohen's *d* measure. The only general caveat about calculating a Cohen's *d* with multilevel models concerns which standard deviation(s) to use (Brysbaert and Stevens 2018; Westfall, Kenny, and Judd 2014)

References

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