Evaluation of Machine Learning in Empirical Asset Pricing

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Contents

1. Introduction

1.1. Topic

This thesis aims to evaluate the application of machine learning algorithms in empirical asset pricing. While there has been significant recent interest in applying machine learning to the problem of predicting asset returns, there is little literature that focuses on how well these algorithms are at capturing true underlying variables in determining stock returns. 12 different simulated datasets ranging from linear to highly non-linear data generating processes incorporating observed phenomena of cross sectional correlation, persistence, and stochastic volatility, in addition to real world data will be used to assess the performance of linear models, elastic net models, random forests and neural networks. Model performance will be assessed according to their out of sample Mean Absolute Error, Root Mean Square Error, and Predictive R^2, in addition to whether or not they were able to identify the correct variables in the data generating process according to a variable importance metric.

1.2. Background Literature and Motivations

This paper is motivated by evaluating the performance of machine learning algorithms in empirical asset pricing, focusing on how well they deal with the many unique problems in financial returns data. Here, we define "performance" to refer to two forms of metrics conventional in the literature: R^2 (and more specifically, out of sample R^2), and forecast errors (see 2.10 for more details).

We first begin by defining "factors" with the more contemporary definition as suggested by Harvey et al. (2016): a collection of regressors to be used in pricing returns that can be used to proxy for unknown underlying risk factors due to their correlation with cross sectional returns. Most notably, their definition rejects the more strict view that risk factors should be variables that have unpredictable variation through time, and that they should be able explain cross sectional return patterns. Harvey et al. (2016) further groups factors into the two broad categories of "common" and individual firm "characteristics." "Common" factors under this definition can be viewed as proxies for sources of risk

constant across all firms, such as macroeconomic variables. They note that individual firm characteristics are unlikely to satisfy the more strict definition because they are often pre-known and display limited time series variation.

Because of this less strict definition, factors introduced and used in the literature often exhibit properties which makes them unsuitable for inclusion in models, such as high persistence, high levels of non-stationarity and cross sectional correlation.

Goetzmann and Jorion (1993) and Ang and Bekaert (2006) note the persistence present in dividend ratio factors. This means that movements in dividend ratios are dominated by movements in price and therefore dividend ratios are correlated with lagged dependent variables on the right hand side of the regression equation. This violates the assumptions of exogeneous regressors (independent from the error term) required for traditional regression models (ordinary least squares) to be unbiased, resulting in t statistics which are biased upwards and increase with time horizon due to autocorrelated errors. Importantly, Goetzmann and Jorion (1993) show that corrections to t statistics using the Generalized Method of Moments and Newey-West standard errors also appear to be biased upwards, making them unreliable.

Goyal and Welch (2003) provide a more comprehensive study on the performance of lagged dividend price ratios, with specific focus on out of sample predictive performance both in terms of R^2 and forecast errors. They conclude that while models incorporating dividend related factors were able to achieve higher in sample performance prior to 1990 than the historical mean, they could not have outperformed the historical mean *out of sample*. Goyal and Welch (2003) attribute this to the increasing persistence and non-stationarity of dividend ratios, noting that they have become like random walks as of 2001. This mirrors the sentiment of (Lettau and Ludvigson (2001), Schwert (2003) and others) who conclude that models incorporating dividend ratios seemed to break down in the 2000s due to a changing economic environment despite having performed well in the 1990s.

Despite the controversy, the prevailing tone within the literature was that various factors such as dividend ratios, earnings price ratio, interest and inflation and other financial indicators were able to predict excess returns, with Lettau and Ludvigson (2001) remarking that this was now "widely accepted." However, Welch and Goyal (2008) extend upon the work of Goyal and Welch (2003) by including a more comprehensive set of variables and time horizons. They conclude that not a single variable had any statistical forecasting power. Crucially, they demonstrate the non-robustness of models incorporating these factors by showing that the significance values of some factors change with the choice of sample periods.

Despite this, the literature has continued to produce more factors: quantitative trading firms were using 81 factor models as the norm by 2014 (Hsu and Kalesnik, 2014), and Harvey and Liu (2019) currently document well over 600 different factors suggested in the literature.

The dramatic increase in the number of factors alone poses challenges to traditional statistical techniques. Harvey et al. (2016) detail the false discovery problem when the number of potential factors is extremely high. The significance of a factor in a traditional regression setting is determined by a single hypothesis test, which by construction carries a level of significance α controlling the type I error rate: the probability of rejecting a "null" hypothesis that a factor is not important, and hence incorrectly concluding that it is significant. When the number of potential factors is large, it is very likely that a factor will be concluded as significant by pure chance. For example, a factor model consisting of 600 factors would find around 30 factors significant by chance at the 5% significance level. Harvey et al. (2016) produce a multiple testing framework to mitigate this, and conclude that many of the historically discovered would have been deemed significant by chance.

Furthermore, Feng et al. (2019) note that the number of potential factors discovered in the literature has increased to the same scale as, if not greater, than the number of stocks considered in a typical portfolio, or the time horizon, producing highly inefficient covariances in a standard cross sectional regression. Moreover, when the number of factors exceeds the sample size, traditional cross sectional regressions become infeasible and do not produce solutions altogether.

It does not help that many factors are cross sectionally correlated, meaning that factors which are discovered to be significant may simply be so because they are correlated with a true, underlying factor and do not provide independent information themselves, a concern which Cochrane (2011) calls the multidimensional challenge. Freyberger et al. (2017) notes that this is especially challenging for traditional regression models, which make strong functional form assumptions and are sensitive to outliers.

RS: More reasoning on why ML is good here

More recently, machine learning algorithms have emerged within the literature and appear to be well suited to the task of predicting asset returns. The definition of machine learning can be vague and is often context specific; Hastie et al. (2009) in *An Introduction to Statistical Learning* describes statistical (machine) learning as a vast set of tools for understanding data, and *supervised* learning specifically as the process of building a statistical model for the prediction or estimation of an output based on input(s). In the context of asset pricing, we use the term to refer to a diverse collection of:

1. high-dimensional models for statistical prediction,
2. the "regularization" methods for model selection and mitigation of over-fitting input data,
3. and the efficient systematic methods for searching potential model specifications.

RS: provide some examples of regularization, such as train vs validation

The high dimensional and hence flexible nature of machine learning brings more hope to approximating unknown and likely complex data generating processes that underlie excess returns. The flexibility however, comes at a cost of potentially over-fitting in sample data (referred to as training data in the machine learning literature), generalizing poorly and producing poor forecasts. The regularization aspect of machine learning explicitly guards against over-fitting problems and emphasizes out of sample performance. The most explicit example of regularization is the splitting of the dataset into an explicit "training" set used for model fitting, and a "test" set withheld and used solely for evaluating out of sample performance. Finally, machine learning offers tools which are designed to produce an optimal model specification from all possible models with manageable computational cost, all in a systematically consistent way.

RS: better linkage needed here

Kozak et al. (2017), Rapach and Zhou (2013) and Freyberger et al. (2017) all apply shrinkage and selection methods from machine learning in factor selection.

Most importantly, portfolios constructed using machine learning have been demonstrated to outperform traditional models in predicting stock returns (Gu et al. (2018), Hsu and Kalesnik (2014) and Feng et al. (2018)) in terms of out of sample predictive R^2 and Sharpe Ratios. Gu et al. (2018) attribute this to machine learning's ability to evaluate and consider non-linear complexities among factors that cannot be feasibly achieved using traditional techniques.

RS: emphasize why the limited simulation design is a problem

However, there is little work done on how machine learning actually recognises and deals with the challenges of returns prediction documented in the literature. Prior work has been done by Gu et al.

(2018), however; only basic simulation designs which were not representative of real financial data were considered. This is problematic,

Furthermore, Feng et al. (2018) in particular use cross validation as part of their model building procedure, destroying the temporal aspect of returns data, in addition to only using a handful of factors. Gu et al. (2018) produce models using a training sample which ends in the 1970s to ultimately produce forecasts for the most recent 30 years. Given the non-robustness of financial data affecting even traditional regressions which are considered to be more inflexible, more research should be done into the robustness of more flexible machine learning methods with regards to sample selection and periods of returns predictability.

For the aspect of factor selection specifically, Gu et al. (2018) concludes that all of the machine methods agree on the same subset of important factors. However, while their factor importance metrics for regression-tree based methods and neural networks (the most complex methods considered) are mostly consistent, they have differences in terms of the relative importance of each factor, in addition to completely different conclusions for the dividend yield factor.

This paper will be the first in focusing on how machine learning algorithms perform in environments with problems exhibited by financial returns data through extending the simulation designs of Gu et al. (2018). In addition, these algorithms will once again be evaluated on real world data, but with only more recent and representative data included in order to test their short term robustness in predicting stock returns. These two aspects of the study together are able to offer a better glimpse as to how "black box" machine learning algorithms deal with the challenges present in asset pricing, if at all.

2. Methodology

2.1. Overall Model Design

Each model will be presented and explained so that a reader without any machine learning background can understand the basic idea behind each model. All asset excess monthly returns denoted as $r_{i,t+1}$ are modelled as an additive prediction error model conditional on the true and unobservable information set available to market participants up to and including time t, $\mathcal{F}_t$):

$$r_{i,t+1} = E(r_{i,t+1}|\mathcal{F}_t) + \epsilon_{i,t+1} \tag{1}$$

where

$$E(r_{i,t+1}|\mathcal{F}_t) = g^*(z_{i,t}) \tag{2}$$

with stocks indexed as $i = 1, \ldots, N$ and time domain by $t = 1, \ldots, T$. $g^*(z_{i,t})$ represents the model approximation using the P dimensional predictor set $z_{i,t}$. We allow $g^*(z_{i,t})$ to be a flexible function of the predictor set $z_{i,t}$, and most notably, not depend on i or t directly. This means that we do not re-estimate a model for each time period, or independently estimate a model for each stock. Note that $g^*(z_{i,t})$ only contains information in time t for individual stock i, meaning that while the model and its parameters will be estimated using $\mathcal{F}_t$ for stock i, predictions for $r_{i,t+1}$ will only use information at time t as an input, analogous to using variables lagged by one period.

All machine learning methods are designed to approximate the empirical model $E_t(r_{i,t+1}) = g*(z_{i,t})$ defined in equation (2). We define the baseline set of stock-level covariates $z_{i,t}$ as:

$$z_{i,t} = x_t \otimes c_{i,t} \tag{3}$$

where $c_{i,t}$ is a $P_c \times 1$ matrix of characteristics for each stock i, and x_t is a $P_x \times 1$ vector of macroeconomic predictors (and are this common to all stocks, including a constant). Thus $z_{i,t}$ is a $P \times 1$ vector of features for predicting individual stock returns ($P = P_c P_x$) and includes interactions between individual characteristics and macroeconomic characteristics.

2.2. Sample Splitting

RS: unclear, re-do slightly

Imperative to any machine learning technique is the establishment of how the dataset is to be split into training, validation and test sets. The training set is used to initially build the model and provide initial estimates of parameters, whereas the validation set is used to tune model parameters to optimise out of sample performance, thus preventing overfitting. The validation set acts as a simulation of out of sample testing, whereas the test set is used only for evaluation, and is thus truly out of sample.

There are three main approaches to splitting temporal data (such as financial data).

The first is to decide arbitrarily on a single training, validation and test set. This method is straightforward and the least computationally intensive, but is limited and inflexible in evaluating how models perform when more recent data is provided for training.

The second method is a "rolling window" method, where a fixed size or "window" for the training and validation set is first chosen. This window then incrementally move forwards in time to include more recent data, with a set of forecasts for the test sets made for all possible windows.

The third is a "recursive" method, which is the same as the rolling window method, but different in that the training set always contains previous data, with only the validation set staying fixed in size and "rolling" forwards. Hence, it is also referred to as a "growing window."

Both the rolling window and recursive schemes are very computationally intensive. Therefore, a hybrid of the rolling and recursive schemes was considered: the training set is increased by one year with each refit, the validation set remains one year in length but moves forward by one year, and forecasts are made using that model for the subsequent year. The "traditional" cross validation method of randomly sampling to determine a train and validation set was not done to maintain the temporal ordering of the data.

2.3. Loss Function

The choice of the loss function used in models is imperative to machine learning. The loss functions considered are Mean Absolute Error (MAE), Mean Squared Error and Root Mean Squared Error (MSE, and RMSE).

2.3.1. Mean Absolute Error

The mean absolute error (MAE) is simply the average magnitude of errors. Because of this, it places equal weighting to all magnitudes of errors and is more robust to outliers.

$$\text{MAE} = \frac{1}{n} \sum_{j=i}^{n} |y_j - \hat{y_j}| \tag{4}$$

It should be noted that minimizing the MAE criterion is equivalent to minimizing 0.5 quantile loss.

2.3.2. Mean Squared Error and Root Mean Squared Error

The mean squared error (MSE) and root mean squared error (RMSE) are quadratic scoring methods. This means that they place higher weight on large errors. Models that minimize this metric are therefore more sensitive to outliers.

$$\text{MSE} = \frac{1}{n}\sum_{j=i}^{n}(y_j - \hat{y_j})^2 \tag{5}$$

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{j=i}^{n}(y_j - \hat{y_j})^2} \tag{6}$$

2.4. Linear Model

The least complex model considered is the simple linear regression model. The simple linear model assumes that the underlying conditional expectation $g^*(z_{i,t})$ can be modelled as a linear function of the predictors and the parameter vector θ:

$$g(z_{i,t};\theta) = z'_{i,t}\theta \tag{7}$$

Computing this model with respect to minimizing the mean squared error yields the pooled ordinary least squares estimator (POLS), while minimizing the mean absolute error corresponds to quantile regression predicting the 0.5th quantile, also known as the Least Absolute Deviation (LAD) estimator.

The OLS estimator is known to be consistent when the regressors are exogeneous, and is optimal in the class of linear unbiased etimators when the errors are homoscedastic and serially uncorrelated. This unbiasedness consequently means that linear models have a large amount of variance due to the bias-variance trade-off, which can lead to poorer predictive ability as the estimator's variance will be very high and therefore inefficient. Additionally, OLS struggles when multicollinearity (also known as cross sectional correlation in empirical finance settings) is present, as the coefficients of different variables can behave erratically to small changes in the data or model. This impreciseness in the coefficient estimates further leads to poor predictive performance. The LAD estimator attempts to improve upon the OLS estimator through the use of loss function that is more robust to outliers. The mean absolute error weights all residuals equally, compared to mean squared error which through considering squared residuals gives more emphasis to large residuals.

Linear models can capture non-linearities only if the predictor set $z^*_{i,t}$ contains specified non-linear transformations or interaction terms. Despite being expected to perform poorly, linear models were implemented as a "control."

2.5. Penalized Linear Model

Penalized linear models attempt to improve upon linear models through regularization, and are particularly well suited to addressing problems of multicollinearity, which can commonly occur in models with a large number of regressors. They achieve this by lowering the variance of their estimates (thus increasing efficiency) in exchange for introducing a tolerable amount of bias. This can be viewed as the model mechanically decreasing its in sample, in hopes that the model will overfit less, particularly to noise in the dataset, and correctly preserving its fit to true underlying regressors.

Penalized linear models have the same underlying statistical model as simple linear models, but differ in their addition of a new penalty term in the loss function:

$$\mathcal{L}(\theta;.) = \underbrace{\mathcal{L}(\theta)}_{\text{Loss Function}} + \underbrace{\phi(\theta;.)}_{\text{Penalty Term}} \tag{8}$$

Several choices exist for the choice of penalty function $\phi(\theta;.)$. We restrict our scope to the popular "elastic net" penalty (Zou and Hastie, 2005):

$$\phi(\theta;\lambda,\rho) = \lambda(1-\rho)\sum_{j=1}^{P}|\theta_j| + \frac{1}{2}\lambda\rho\sum_{j=1}^{P}\theta_j^2 \tag{9}$$

The elastic net has two hyperparameters: λ, which controls the overall magnitude of the loss, and ρ, which controls the shape of the penalization.

The $\rho = 1$ case corresponds to ridge regression proposed by Hoerl and Kennard (1970), which uses l_2 penalty that shrinks all coefficients closer to 0, but not to 0. Ridge regression is therefore a shrinkage method which prevents coefficients from becoming too large and overpowering.

The $\rho = 0$ case corresponds to the popular LASSO and uses absolute (l_1) parameter penalization proposed by Tibshirani (1996), which geometrically allows the coefficients to be shrunk to 0. This allows it to impose sparsity, and can be thought of as a variable selection tool.

For $0 < \rho < 1$, the elastic net aims to produce parsimonious models through both shrinkage and selection by combining the properties of LASSO and ridge regression.

The hyperparameters λ and ρ are both tuned using the validation sample (see 7.2).

2.6. Classification and Regression Trees

Classification and regression trees are fully non-parametric models that can capture complex multi-way interactions. A tree "grows" in a series of iterations. With each iteration, a split ("branch") is made along one predictor such that it is the best split available at that stage with respect to minimizing the loss function. These steps are continued until each observation is its own node, or more commonly until the stopping criterion is met. The eventual model slices the predictor space into rectangular partitions, and predicts the unknown function $g^*(z_{i,t})$ with the average value of the outcome variable in each partition.

The prediction of a tree, $\mathcal{T}$, with K "leaves" (terminal nodes), and depth L is

$$g(z_{i,t};\theta,K,L) = \sum_{k=1}^{K}\theta_k \mathbf{1}_{z_{i,t}\in C_k(L)} \tag{10}$$

where $C_k(L)$ is one of the K partitions in the model.

For this study, only recursive binary trees (the most common and easy to implement) are considered. Though trees were originally proposed and fit with respect to minimizing mean squared error, they can be grown with respect to a variety of loss functions, including mean absolute error, mean squared error:

$$H(\theta,C) = \frac{1}{|C|}\sum_{z_{i,t}\in C} L(r_{i,t+1}-\theta) \tag{11}$$

where $|C|$ denotes the number of observations in set C (partition). Given C, it is clear that the optimal choice for minimising the loss function when it is mean squared error is simply $\theta = \frac{1}{|C|}\sum_{z_{io,t}\in C} r_{i,t+1}$ i.e. the average of the partition, and the median of the partition when the loss function is mean absolute error.

Trees, grown to a deep enough level, are highly unbiased and flexible. The trade-off is their high variance and instability. Thus, an ensemble method called "Random Forest" was proposed by Breiman (2001) to regularize trees by combining many different trees into a single prediction.

2.7. Random Forests

Random Forests are an extension of regression trees that attempt to address some of their problems. A random forest algorithm creates B different bootstrap samples from the training dataset, fits an overfit (and hence low bias) regression tree to each dataset using only a random subset m size from all available predictors (also known as dropout), and then averages their forecasts as the final output. The overfit trees means that the underlying trees has low bias, and the dropout procedure means that they have low correlation. Thus, averaging these low bias, uncorrelated trees results in a low bias, yet stable model. Specific details of the random forest algorithm are detailed in the appendix.

Random forests were the most computationally intensive, and for feasibility purposes an ensemble of only $B = 30$ trees with $m = 20$ at most was considered for the simulation study, and $B = 50$ trees with $m = 40$ at most was considered for the empirical study.

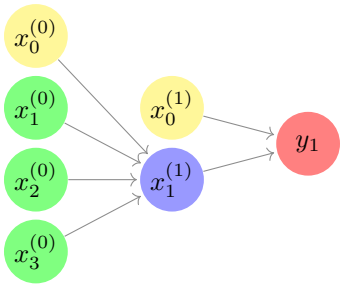


Figure 1. Example Neural Network

2.8. Neural Networks

2.8.1. Introduction

Neural networks have theoretical underpinnings as "universal approximators" for any function, (Hornik et al. (1989)). They are arguably the most complex type of model available, able to capture several non-linear interactions through their many layers, hence its other name "deep learning." On the flipside, their high flexibility often means that they are among the most parameterized and least interpretable models, earning them the reputation as a black box model.

Per this analysis, we focus on traditional "feed-forward" networks. The feed forward network consists of an "input layer" of scaled data inputs, one or more "hidden layers" which interact and non-linearly transform the inputs, and finally an output layer that aggregates the hidden layers and transform them a final time for the final output.

More specifcally, a neural network consists of layers denoted by $l = 0, 1, \ldots, L$, with $l = 0$ denoting the input layer and $l = L$ denoting the output layer, and $K^{(l)}$ denoting the number of neurons in each hidden layer. The input layer is defined using predictors, $x^{(0)} = (1, z_1, \ldots, z_N)'$. The output of neuron k in layer l is then $x_k^{(l)}$. Next, define the vector of outputs for this layer as $x^{(l)} = (1, x_1^{(l)}, \ldots, x_{K^{(l)}}^{(l)})'$. The recursive output formula for the neural network at each neuron in layer $l > 0$ is then:

$$x_k^{(l)} = \alpha(x^{(l-1)'}\theta_k^{l-1}), \tag{12}$$

where $\alpha()$ represents the activation function for that layer (see next section) with the final output

$$g(z;\theta) = x^{(L-1)'}\theta^{L-1} \tag{13}$$

Note that the specification of a constant "1" at the beginning of each layer is the same as specifying a bias term as is popular in other parametrizations.

RS: move this paragraph elsewhere

Neural networks with up to 5 hidden layers were considered, each named NNX where X represents the number of hidden layers. The number of neurons is each layer was chosen according to the geometric pyramid rule (Masters, 1993): NN1 has 32 neurons, NN2 has 32 and 16 neurons in the first and second hidden layers respectively, NN3 has 32, 16, and 8 neurons, NN4 has 32, 16, 8, and 4 neurons, and NN5 has 32, 16, 8, 4, 2 neurons respectively. All units are fully connected; that is, each neurons receives input from all neurons the layer before it (see Figure 2). This mimics the methodology in Gu et al. (2018).

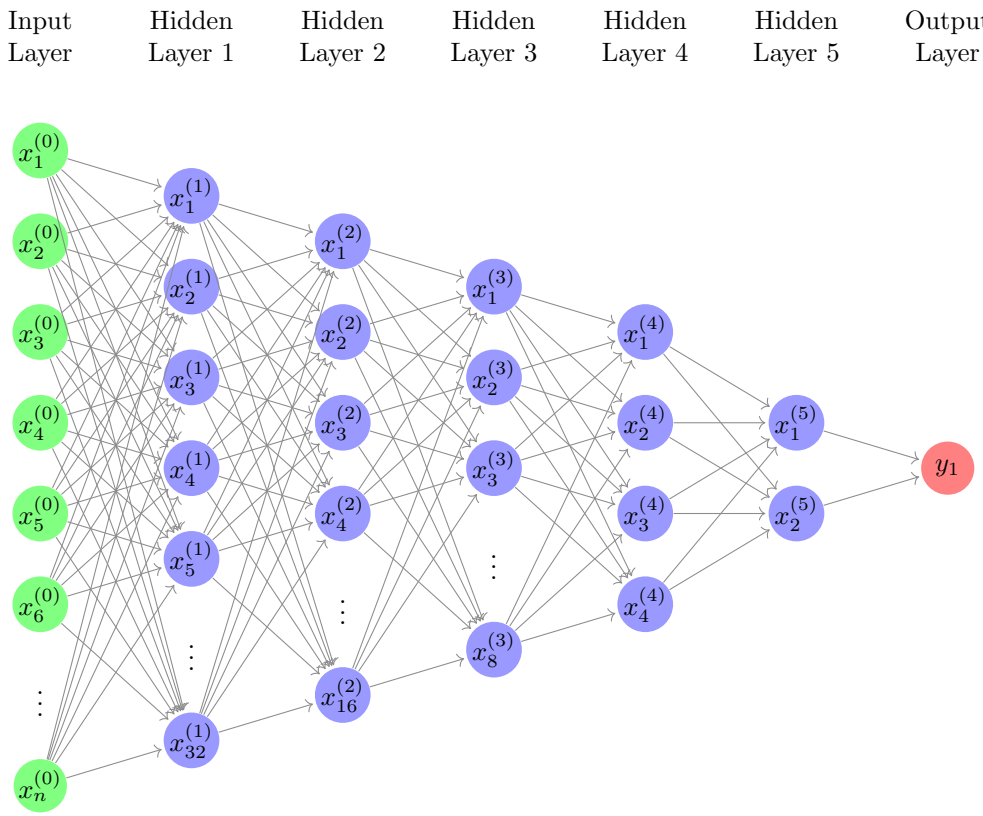


Figure 2. Neural Network 5 (most complex considered), without biases terms drawn

2.8.2. Activation Function

Several choices of activation functions exist in the literature, one of the most popular being the ReLU activation function (see Lecun et al. (2015) and Ramachandran et al. (2017), among others):

$$\text{ReLU}(x) = max(0, x) \tag{14}$$

owing to its high computational speed, ability to introduce sparseness into neural networks, and fewer vanishing gradient problems, as repeatedly applying ReLU tends to saturate less compared to other activation functions such as sigmoid and tanh. However, during the study it was observed that ReLU suffered tremendously from the "dying-ReLU problem," where ReLU neurons receive weight updates that fail to activate (output a 0), hence making it unable to receive further weight updates and learn. This resulted in networks which outputted the same value for the majority of, if not all inputs. To mitigate this, a variant of ReLU known as Leaky ReLU was used instead:

$$\text{lReLU}(x) = \begin{cases} x & \text{if} \quad x > 0; \\ 0.01x & \text{otherwise} \end{cases} \tag{15}$$

This activation function assigns a small non-zero gradient to each weight, hence enabling them to receive further weight updates.

2.8.3. Computation

RS: this, like all neural networks, is a mess. Re write

The neural network's weight and bias parameters for each layer are estimated by minimizing the loss function with respect to the parameters, i.e. by calculating the partial derivative with respect to a specific weight or bias element.

The solution to this is typically found via backpropagation, an iterative alogrithm similar to Gauss-Newton steps and produces a local minimum. These steps can be visualized as a descent towards the local minimum of the loss function; it is thus also known as "gradient descent." Note that the lack of a global minimum is actually desirable, as global minimums tend to be overfit solutions to the problem, (Choromanska et al., 2014). A common solution is to use "stochastic gradient descent" (SGD) where instead of optimising the loss function with respect to the entire training sample, only a small, random subset of the data (batch) is used at each optimisation step.

RS: Define learning rate more properly

Due the noisiness (randomness) introduced by SGD, the path towards the local minimum is more of a quick "zig zag" and has the potential to "overshoot" the local minimum and not converge to a solution. This is typically controlled via a hyperparameter known as the learning rate, which controls the step size of each gradient descent iteration. Note that it is also common to apply a learning rate hyperparameter in non SGD environments as a way to control the magnitude of each step towards the solution to assist with convergence. The learning rate is to be tuned so that the solution path descends quickly enough to be computationally feasible, but slow enough so that it does not overshoot the local minimum and not converge. In this paper, the learning rate shrinkage algorithm which adaptively shrinks the learning rate of each individual weight separately as convergence occurs known as ADAM was employed (see Kingma and Ba (2014)). It should be noted that ADAM still requires a baseline learning rate which acts as a upper bound on the learning rate to be supplied, and this was tuned manually.

2.8.4. Batch Normalization

"Batch normalization" is a technique for addressing a phenomenon known as internal covariate shift, a particularly prevalent problem in training deep, complex neural networks, (Ioffe and Szegedy, 2015). Internal covariate shift occurs when the distributions of each layers' inputs change as the parameters of the previous layer change, resulting in the need for much slower learning rates and more careful initialization of parameters. More crucially, batch normalization has been observed to help normalizing the outputs of activation layers, alleviating issues related to vanishing or exploding gradients. By normalizing (demeaning and variance standardizing) each training step (batch) input, the representative power of each neuron is restored. Additionally, significant gains in computational speed may also be achieved.

2.9. Simulation Design

2.9.1. Overall Design

Though Gu et al. (2018) explore the performance of machine learning on simulated returns series, their design used factors are uncorrelated across i, and, in particular, that the factors which enter the return equation are uncorrelated with the factors that do not enter the return equation. As note by Harvey et al. (2016) and many others, this is not what is observed in practice.

Therefore, we simulate a latent factor model with a stochastic volatility process for excess returns r_{t+1}, for $t = 1, \ldots, T$:

$$r_{i,t+1} = g\left(z_{i,t}\right) + \beta_{i,t+1} v_{t+1} + e_{i,t+1}; \quad z_{i,t} = (1, x_t)' \otimes c_{i,t}, \quad \beta_{i,t} = (c_{i1,t}, c_{i2,t}, c_{i3,t}) \tag{16}$$

$$e_{i,t+1} = \sigma_{i,t+1} \varepsilon_{i,t+1}; \tag{17}$$

$$\log(\sigma^2_{i,t+1}) = \omega + \gamma \log(\sigma^2_t) + \sigma_u u; \quad u \sim N(0,1) \tag{18}$$

Let v_{t+1} be a 3×1 vector of errors, and $w_{t+1} \sim N(0,1)$ and $\varepsilon_{i,t+1} \sim N(0,1)$ scalar error terms. The parameters of these are tuned such that the R squared for each individual return series was around 5%, and the annualized volatility of each return series approximately 22%, as is often observed empirically.

The matrix C_t is an $N \times P_c$ vector of latent factors, where the first three columns correspond to $\beta_{i,t}$, across the $1 \leq i \leq N$ dimensions, while the remaining $P_c - 3$ factors do not enter the return equation. The $P_x \times 1$ vector x_t is a 3×1 multivariate time series, and ε_{t+1} is a $N \times 1$ vector of idiosyncratic errors.

2.9.2. Simulating Characteristics

A simulation mechanism for C_t that gives some correlation across the factors and across time was used. First consider drawing normal random numbers for each $1 \leq i \leq N$ and $1 \leq j \leq P_c$, according to

$$\overline{c}_{ij,t} = \rho_j \overline{c}_{ij,t-1} + \epsilon_{ij,t}; \quad \rho_j \sim \mathcal{U}\left(\frac{1}{2}, 1\right) \tag{19}$$

Then, define the matrix

$$B := \Lambda\Lambda' + \frac{1}{10}\mathbb{I}_n, \quad \Lambda_i = (\lambda_{i1}, \ldots, \lambda_{i4}), \quad \lambda_{ik} \sim N(0, \lambda_{sd}),\ k = 1, \ldots, 4 \tag{20}$$

where different λ_{sd} values were considered to explore different degrees of cross sectional correlation. This is decomposed via the Cholesky Decomposition to yielf the lower triangle matrix L:

$$B = LL' \tag{21}$$

To build in cross-sectional correlation, from the $N \times P_c$ matrix $\bar{C}_t$, we simulate characteristics according to

$$\widehat{C}_t = L\overline{C}_t \tag{22}$$

Finally, the "observed" characteristics for each $1 \leq i \leq N$ and for $j = 1, \ldots, P_c$ are constructed according to:

$$c_{ij,t} = \frac{2}{n+1} \operatorname{rank}\left(\hat{c}_{ij,t}\right) - 1. \tag{23}$$

with the rank transformation normalizing all predictors to be within $[-1, 1]$.

2.9.3. Simulating Macroeconomic Series

For simulation of x_t, a 3×1 multivariate time series, we consider a Vector Autoregression (VAR) model, a generalization of the univariate autoregressive model to multiple time series:

$g(z_{i,t})$	λ_{sd}	error term	θ_0
g_1	0	$\beta_{i,t+1} v_{t+1} + e_{i,t+1}; \quad e_{i,t+1} \sim t_5(0, 0.05^2)$	$(0.02, 0.02, 0.02)'$

$$x_t = Ax_{t-1} + u_t; \quad A = \begin{pmatrix} .95 & 0 & 0 \\ 0 & .95 & 0 \\ 0 & 0 & .95 \end{pmatrix} \quad u_t \sim N\left(\mu = (0,0,0)', \Sigma = I_3\right)$$

Other more complex and interactive matrix specifications of A were briefly explored, but these did not appear to have a significant impact on results. More complex designs were observed to only affect the variable importance metrics, but only to a small degree.

2.9.4. Simulating Return Series

We consider three different function for $g(z_{i,t})$:

(1) $g_1\left(z_{i,t}\right) = \left(c_{i1,t}, c_{i2,t}, c_{i3,t} \times x_t'\right)\theta_0; \quad \theta_0 = (0.02, 0.02, 0.02)'$

(2) $g_2\left(z_{i,t}\right) = \left(c_{i1,t}^2, c_{i1,t} \times c_{i2,t}, \text{sgn}\left(c_{i3,t} \times x_t'\right)\right)\theta_0; \quad \theta_0 = (0.04, 0.035, 0.01)'$

(3) $g_3\left(z_{i,t}\right) = \left(1[c_{i3,t} > 0], c_{i2,t}^3, c_{i1,t} \times c_{i2,t} \times 1[c_{i3,t} > 0], \text{logit}\left(c_{i3,t}\right)\right)\theta_0; \quad \theta_0 = (0.04, 0.035, 0.01, 0.01)'$

$g_1\left(z_{i,t}\right)$ allows the characteristics to enter the return equation linearly, and $g_2\left(z_{i,t}\right)$ allows the characteristics to enter the return equation interactively and non-linearly. These two specifications correspond to the simulation design proposed by Gu et al. (2018).

$g_3\left(z_{i,t}\right)$ allows the characteristics to enter in a highly complex and non-linear fashion.

θ^0 was tuned such that the cross sectional R^2 and predictive R^2 were approximately 5%.

RS: change this

The simulation design results in $3 \times 3 = 12$ different simulated datasets, each with $N = 200$ stocks, $T = 180$ periods and $P_c = 100$ characteristics. Each design was simulated 30 times to assess the robustness of machine learning algorithms. The number of simulations was kept low for computational feasibility reasons.

Our final simulation specifications are as follows:

2.9.5. Sample Splitting

If viewed as monthly periods, $T = 180$ corresponds to 15 years. A data splitting scheme similar to to the scheme to be used in the empirical data study was used: a training:validation length ratio of approximately 1.5 to begin, and a test set that is 1 year in length. We employ the hybrid growing window approach as described earlier in section 2.2 (see Figure 3 for a graphical representation). The sample splitting scheme was chosen in a way which assisted with the stability of the advanced machine learning models, particularly neural networks.

Other popular schemes in the forecasting literature such as using an "inner" rolling window validation loop to find the best hyperparameters on average, finally aggregating them in an "outer" loop for a robust error were considered but not implemented for a variety of reasons. Firstly, many of the models are computationally too intensive for this to be feasible for this to be feasible. More importantly, during the

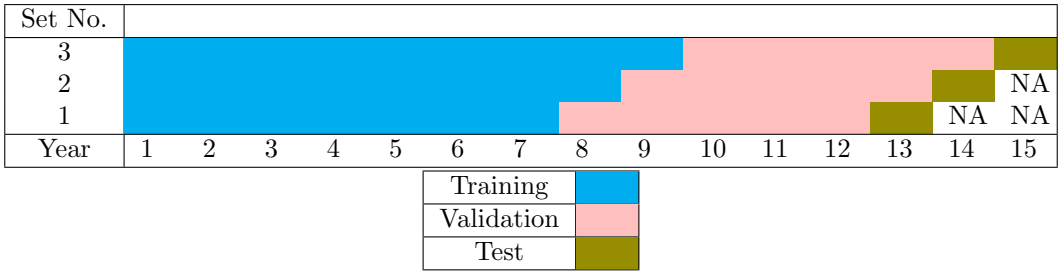


Figure 3. Sample Splitting Procedure

model fitting process it was observed that the optimal hyperparameters for the different rolling windows were highly unstable. Thus, this would have made the selection of the best hyperparameters on average across all windows significantly less meaningful.

2.10. Model Evaluation

2.10.1. R Squared

Overall predictive performance for individual excess stock returns were assessed using the out of sample R^2:

$$R^2_{OOS} = 1 - \frac{\sum_{(i,t)\in\mathcal{T}_3}(r_{i,t+1} - \widehat{r}_{i,t+1})^2}{\sum_{(i,t)\in\mathcal{T}_3}(r_{i,t+1} - \bar{r}_{i,t+1})^2} \tag{24}$$

where $\mathcal{T}_3$ indicates that the fits are only assessed on the test subsample, which is never used for training or tuning.

2.10.2. Diebold-Mariano Test

The Diebold-Mariano test (Diebold and Mariano (2002), and Harvey et al. (1997)) is a procedure which compares the forecast accuracy of two forecast methods. It is different to the overall R squared metric because it tests whether the forecasts from two different models are different to a statistically significant degree.

Under the null hypothesis of the Diebold-Mariano test:

$$S_1^* = \left[\frac{n+1-2h+n^{-1}h(h-1)}{n}\right]^{1/2} S_1; \quad S_1^* \sim N(0,1) \tag{25}$$

$$S_1 = \left[\hat{V}(\bar{d})\right]^{-1/2} \bar{d} \tag{26}$$

$$\hat{\gamma}_k = n^{-1} \sum_{t=k+1}^{n} (d_t - \bar{d})(d_{t-k} - \bar{d}) \tag{27}$$

$$V(\bar{d}) \approx n^{-1}\left[\gamma_0 + 2\sum_{k=1}^{h-1}\gamma_k\right] \tag{28}$$

where d_t represents the difference series between the forecast errors of the two models $e_1 t - e_2 t$, $\hat{\gamma}_k$ represents the sample kth autocovariance for d_t, and S_1 represents the original unmodified Diebold Mariano test statistic.

As all models in this paper will be producing forecasts for an entire cross section of stocks, e_1t and e_2t will instead represent the cross sectional average forecast errors for each model.

It must be noted that the DM test statistic assumes that the loss differential series d_t is covariance stationary and short memory. The precise behaviour of the DM test statistic with regards to more complex non-linear models such as Random Forests and Neural Networks is not fully understood, which may impact on the validity of the null hypothesis distribution.

2.11. Variable Importance

The importance of each predictor j is denoted as VI_j, and is defined as the reduction in predictive R-Squared from setting all values of predictor j to 0, while holding the remaining model estimates fixed. These were then normalized to sum to 1 within each model as a way to assess the relative importance of each predictor for each model. As VI_j can sometimes be negative, this was achieved by shifting all VI_j positively by the magnitude of the smallest VI_j, then dividing all VI_j by the total:

$$VI_{j,norm} = \frac{VI_j + \min(VI_j)}{\Sigma VI_j + \min(VI_j)} \tag{29}$$

This mechanism was chosen because the other popular normalization mechanism "softmax" was observed to be unable to preserve the distances between each original VI_j well enough, thus squishing together all importance metrics.

3. Simulation Study

In our simulation study, we find that the penalized linear models consistently perform the best, followed extremely closely by random forests.

We also find that models fitted with respect to minimizing mean absolute error (quantile loss) generally perform better, even in terms of the mean squared error metrics. This is not a surprising result, especially considering the stochastic error design which introduces significant shocks to the returns process, leading to large outliers which the mean squared error metric is more sensitive to.

In terms of finding the correct underlying causal regressors, we find that the penalized linear models perform the best at identifying the true data generating regressors, and that this appears to be mostly robust regardless of the amount of cross sectional correlation. The penalized linear models are not perfect, particularly in the g_2 specification, where the true covariates enter in the returns process via a *sgn* function, which is more difficult to estimate that other non-linear specifications, such as an indicator function. On these more difficult specifications, the penalized linear models are generally very conservative, sometimes only identifying a single covariate as important.

Of particular note the instability of the penalized linear models' hyperparameters across different training samples. Though the optimal value for α is generally 1 (corresponding to LASSO and thus a sparse representation), it was not uncommon to see α values swinging between values close to 0 (corresponding to ridge regression, and thus a dense representation) to 1 as the training sample moved forwards in time. As the penalized linear models consistently performed the best and still remained able to correctly identify the true covariates this is not a large issue, but it should be noted that this can lead to interpretation issues.

The random forest models were typically able to correctly identify the true data generating covariates, but struggled with discerning them adequately from other covariates. This is to be expected, likely due to how the random forest algorithms work. Recall that the random forest algorithm is an ensemble of

tree models, with each tree model only having access to a subset of all available predictors. If this subset does not include the true data generating predictor, that particular tree will likely select the predictors which have the highest correlation with the true data generating predictor instead. Thus, the resulting ensemble model is likely to believe that cross sectionally correlated predictors are important.

3.0.1. Linear Models

The linear models performed very well in simple linear settings, with notably deteriorating performance as the specifications became more complex. They struggled greatly with cross sectional correlation even in specifications with no cross sectional correlation, as noted by the fact that they identify correlated interaction terms. When cross sectional correlation was increased

The introduction of non-linear interactions in the data generating process affected its performance, though it was still able to pick out transformations such as sgn and logit functions. This is likely due to these transformations not changing the direction of their terms.

3.0.2. Penalized Linear Models

The penalized linear models had markedly better performance than the linear models, though this was only apparent for more complex model specifications.

Of particular note is the instability of the optimal hyperparameter as the validation set moved forward in time. It was quite common to see optimal α values that changed from 1 (equivalent to the sparse case of LASSO regression), to 0.01 (almost equivalent to dense case of ridge regression). Increasing the size of the validation set seemed to help a little, but not by a lot.

Similarly to the linear models, penalized linear models which minimized mean absolute error (corresponding to quantile regression) offered consistent improvements over their mean squared error versions.

3.0.3. Neural Networks

4. Empirical Data

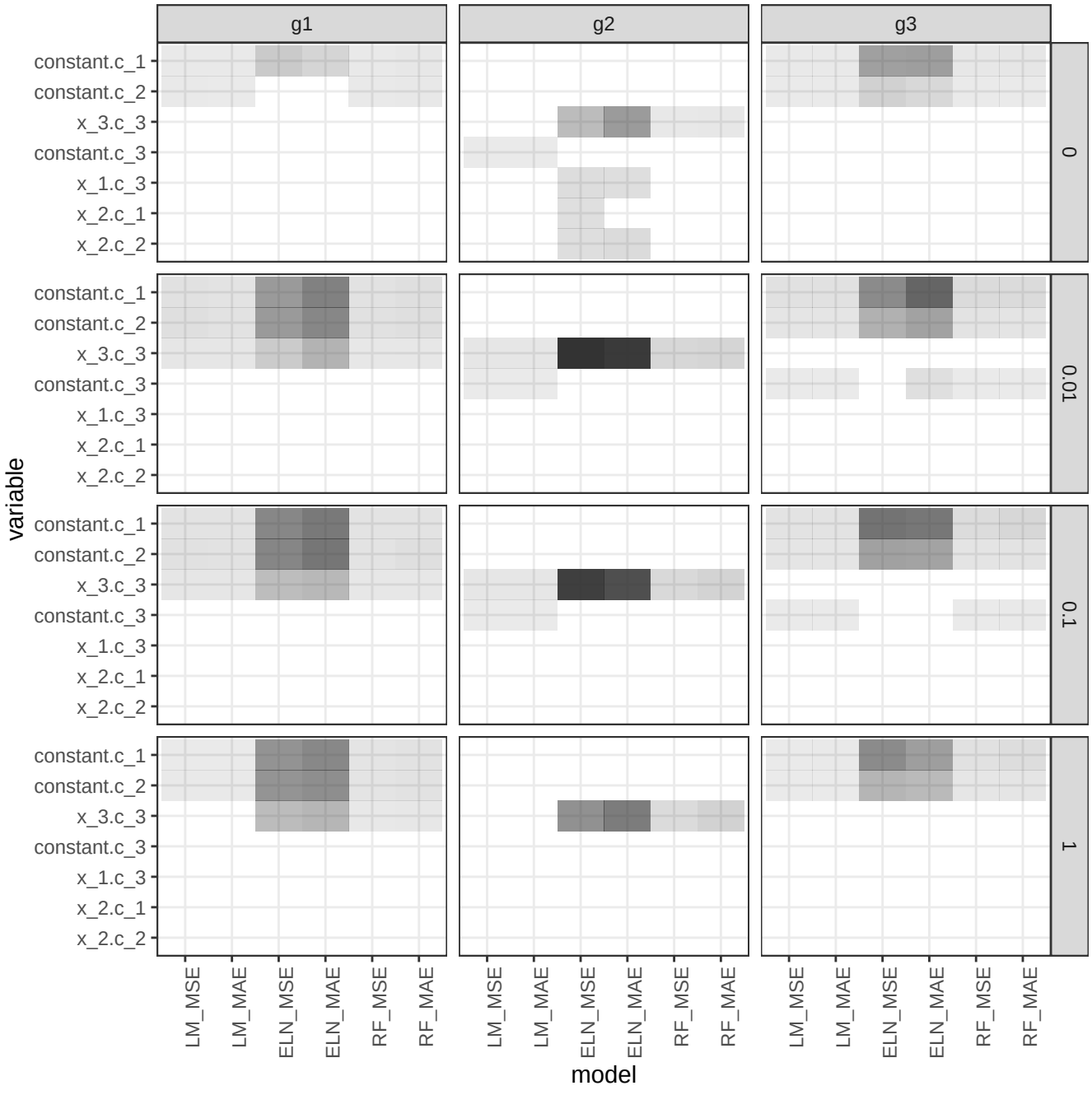
4.1. Data

We mimic the data procedure of Gu et al. (2018). This means that we obtain the dataset provided by Gu on his website. This dataset sample begins in March 1957 (the start date of the S&P 500) and ends in December 2016, totalling 60 years. It contains 94 stock level characteristics: 61 updated annually, 13 updated quarterly and 20 updated monthly, in addition to 74 industry dummies corresponding the the first two digits of the Standard Industrial Classification (SIC) codes. It is noted that this dataset contains all securities traded, including those with a CRSP share code other than 10 or 11 and thus includes instruments such as REITs and mutual funds, and those with a share price of less than $5.

We detail our cleaning procedure of this dataset. To reduce the size of the dataset and increase feasibility, the dataset was filtered so that only stocks traded primarily on NASDAQ were included (using the PRIMEXCH variable from WRDS). Then, penny stocks (also referred to as microcaps in the literature) with a stock price of less than $5 were filtered out, as is commonly done in the literature to reduce variability. Stocks without a share code of 10 or 11 (referring to equities) were filtered out, so that securities that are not equities were not included (such as REITs and trust funds). The dataset is provided in a monthly format, which means that many of the factors which are updated only quarterly or annually have very low levels of variability, which can lead to misleading results in the model fitting process. The

Table 1: Simulation Results

Figure 4. Simulation Variable Importance Plots



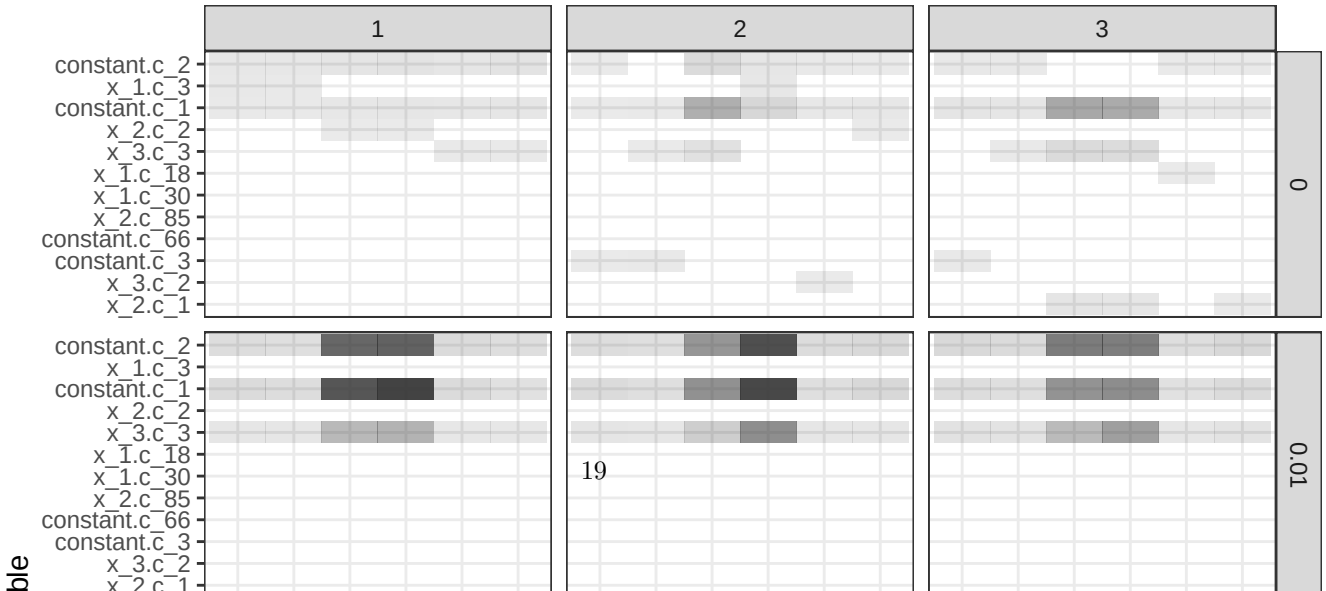


Table 2: Diebold Mariano Tests for Simulation Study

achieve of a balance between having a dataset with enough data points and variability among factors, the dataset was converted to a quarterly format. Quarterly returns were then constructed using the PRC variable according to actual returns (ie not logged differences):

$$RET_t = \frac{PRC_t PRC_{t-1}}{PRC_{t-1}} \tag{30}$$

We allow all stocks which have a quarterly return to enter the dataset, even if they disappear from the dataset for certain periods, as opposed to only keeping stocks which appear continuously throughout the entire period. This was primarily done to reduce survivorship bias in the dataset, which can be very prevalent in financial data, and also allows for stocks which were unlisted and relisted again to feature in the dataset. This has the obvious drawback of introducing some bias in the dataset, as attrition in the dataset is likely to be non-random and correlated with the stocks' returns.

RS: proper example of sic codes being not very useful

The sic2 variable, corresponding to the stocks' Standard Industrial Classification (SIC) codes was also dropped. This is because the SIC code system was established . Indeed, WRDS explicitly cautions the use of SIC codes beyond the use of rough grouping of industries, as SIC codes are not strictly enforced by government agencies for accuracy, as well most large companies belonging to multiple SIC codes over time. Additionally, dropping the sic2 variable also reduced the dimensionality of the dataset by 74 columns, contributing to computational feasibility.

There exists a significant amount of missing data in the dataset. The dataset's columns were first examined, and any characteristics that had over 20% of their data were removed. However, as the amount of missing data increases dramatically going further back in time, a balance between using more periods at the cost of removing more characteristics versus using less periods but keeping more characteristics was needed. 1993 Q3 was determined to be a reasonable time frame to begin the dataset, as was a noticeable increase in data availability after this time. Missing characteristics were then imputed using their cross sectional medians across all years.

This final dataset of individual factors consists of X rows and N columns, with the average number of stocks in each time period being .

We then follow Gu et al. (2018) and construct eight macroeconomic factors following the variable definitions in Welch and Goyal (2008): dividend-price ratio (dp), earnings-price ratio (ep), book-to-market ratio (bm), net equity expansion (ntis), Treasury-bill rate (tbl), term spread (tms), default spread (dfy) and stock variance (svar). The treasury bill rate was also used from this source to construct excess quarterly returns.

The two sets of factors were then combined to form a baseline set of covariates, which we define throughout all methods and analysis as:

$$z_{i,t} = (1, x_t)' \otimes c_{i,t} \tag{31}$$

where $c_{i,t}$ is a P_c matrix of characteristics for each stock i, and $(1, x_t)'$ is a $P_x \times 1$ vector of macroeconomic predictors. $z_{i,t}$ is therefore a $P_x P_c$ vector of features for predicting individual stock returns and includes interactions between stock level characteristics and macroeconomic variables. The total number of covariates in this baseline set is $94 \times (8 + 1) + 74 = 920$.

As the individual and macroeconomic factors can have similar names, individual and macroeconomic

factors were prefixed with ind_ and macro_ respectively.

The dataset was not normalized for all methods, as only penalized regression and neural networks are affected by normalization. For these two methods, the dataset was normalized such that each predictor column had 0 mean and 1 variance.

The final dataset spanned from 1993 Q3 to 2016 Q6 with 20,000

4.2. Empirical Data Model Fitting and Methodology Differences

We detail our model fitting procedure used specifically for the empirical dataset.

We try to mimic the procedure used in the simulation study. This means that the dataset was split such that the training and validation sets were split such that the training set was 1.5 times the length of the validation set, in order to predict a test set that is one year in length.

4.3. Empirical Data Results

In general, results from the simulation study were reproduced in the empirical study.

We similarly see that the penalized linear models generally performing the best, with the random forest models offering slightly worse performance, occasionally outperforming penalized linear models. The neural networks offered lukewarm performance with the earlier training samples, but began to outperform random forests as the amount of training data increased.

Interestingly,

The linear models typically had the worst performance among all the models.

Similar to the simulation study, the random forests overfitted the training sample by a large margin, offering the greatest performance by far for in sample forecasts.

The neural networks offered lukewarm performance in the first two training samples, trailing behind the random forest and penalized linear models.

5. Limitations

There were many limitations in this study.

Dataset limitations The dataset As observed from both the simulation and empirical results, the neural networks' performance increases as the amount of training data does.

Data splitting limitations The data splitting scheme was decided in a pseudo-independent fashion: the training:validation length ratio of 1.5 was chosen to mimic the procedure of Gu et al. (2018), in addition providing more stability for the neural network models. Crucially, this meant that no regards were given to large events affecting stock returns, including but not limited to the dot-com bubble, the 2008 Global Financial Crisis, and the 2015-2016 stock market sell-off.

Model specification limitations For each class of models considered, not all possible or reasonable specifications were explored. The linear and penalized linear models were not supplied with all possible higher order transformations, such as all possible pairwise interaction terms between all regressors as is the norm. This was mainly due to computational issues, as a model frame containing all possible pairwise interactions has exponentially higher memory requirements, in addition to computational feasibility.

6. Conclusion

ind_

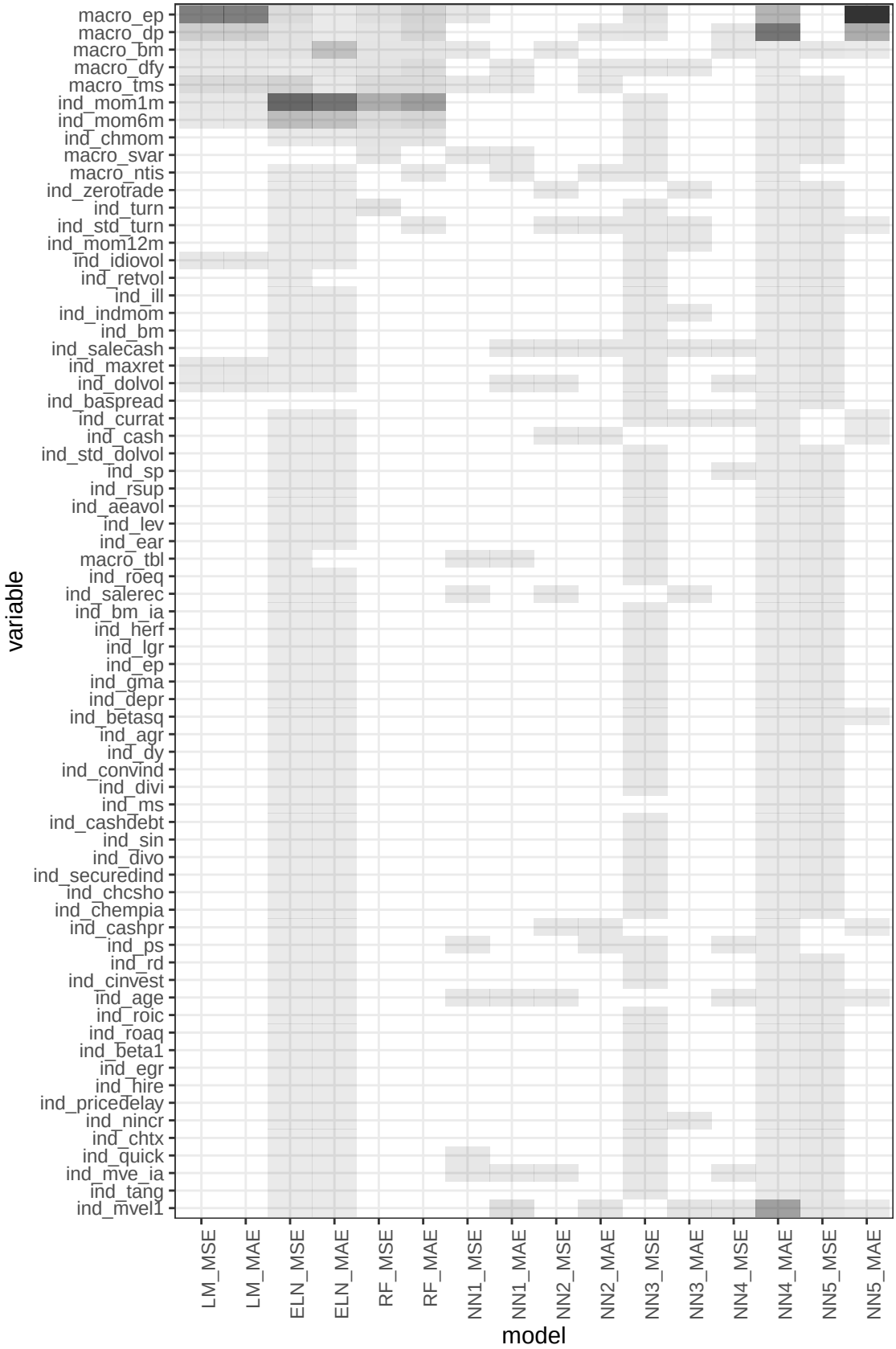


Figure 5. Empirical Data Variable Importance for Sample 1

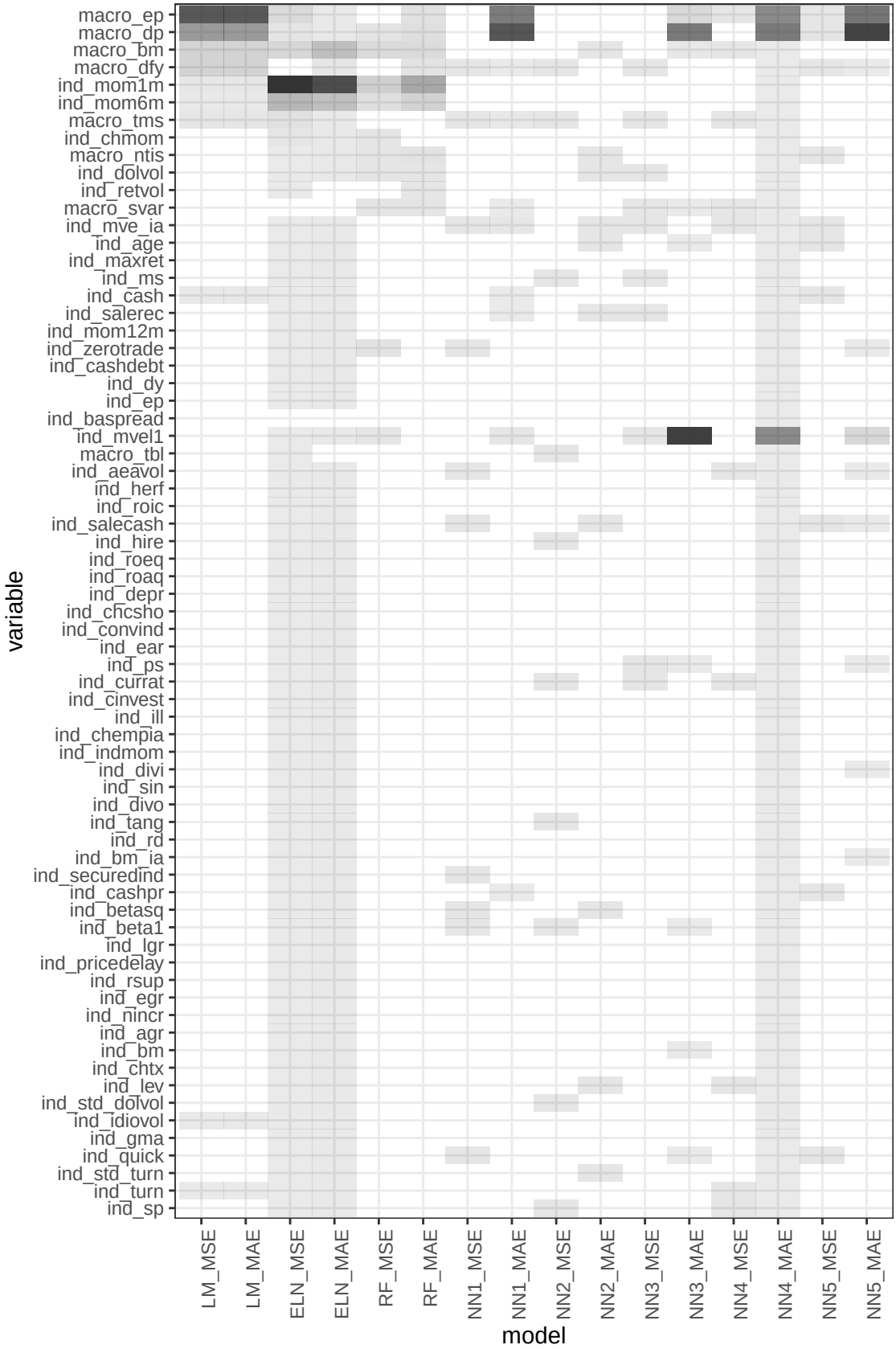


Figure 6. Empirical Data Variable Importance for Sample 2

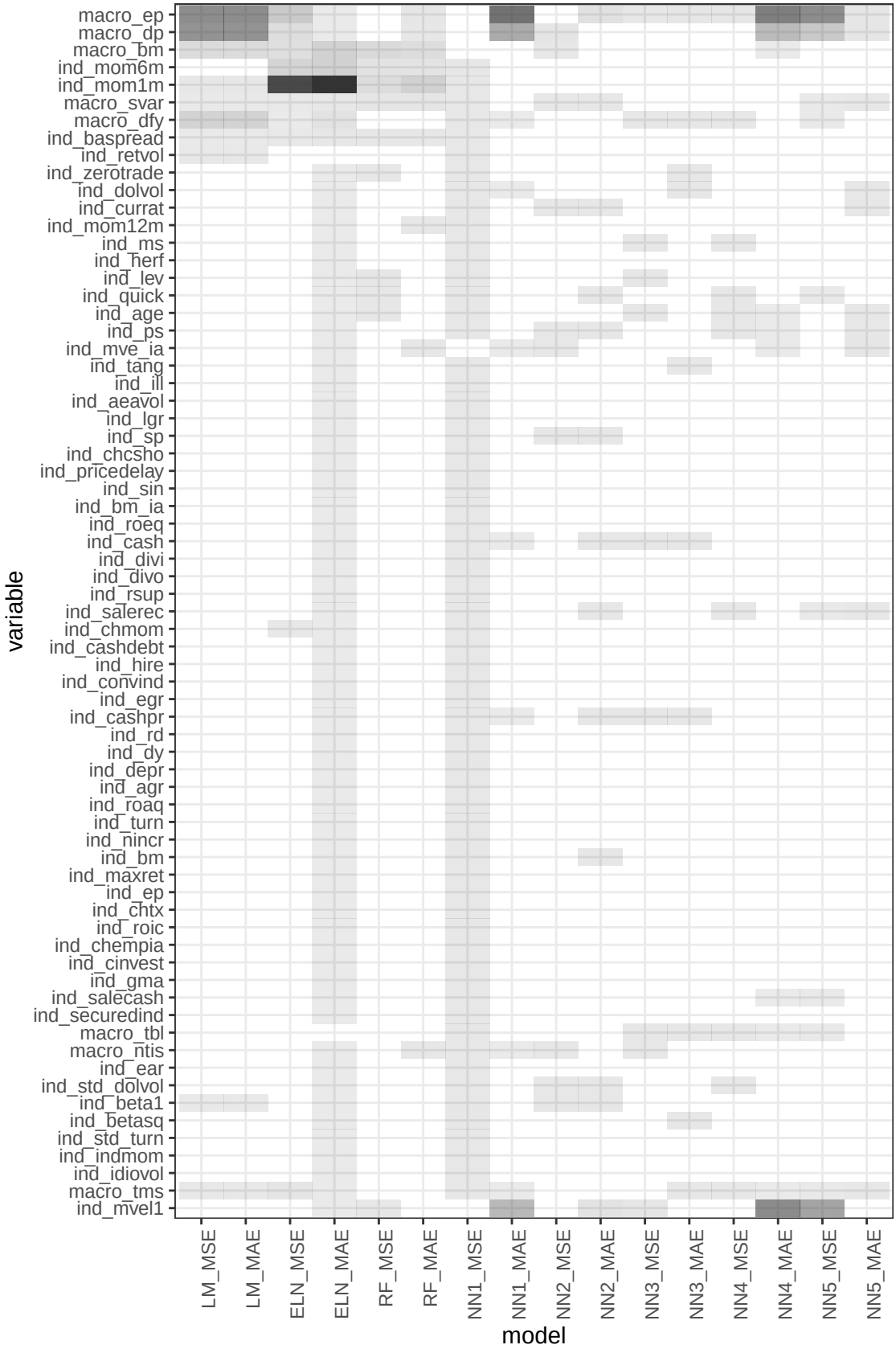


Figure 7. Empirical Data Variable Importance for Sample 3

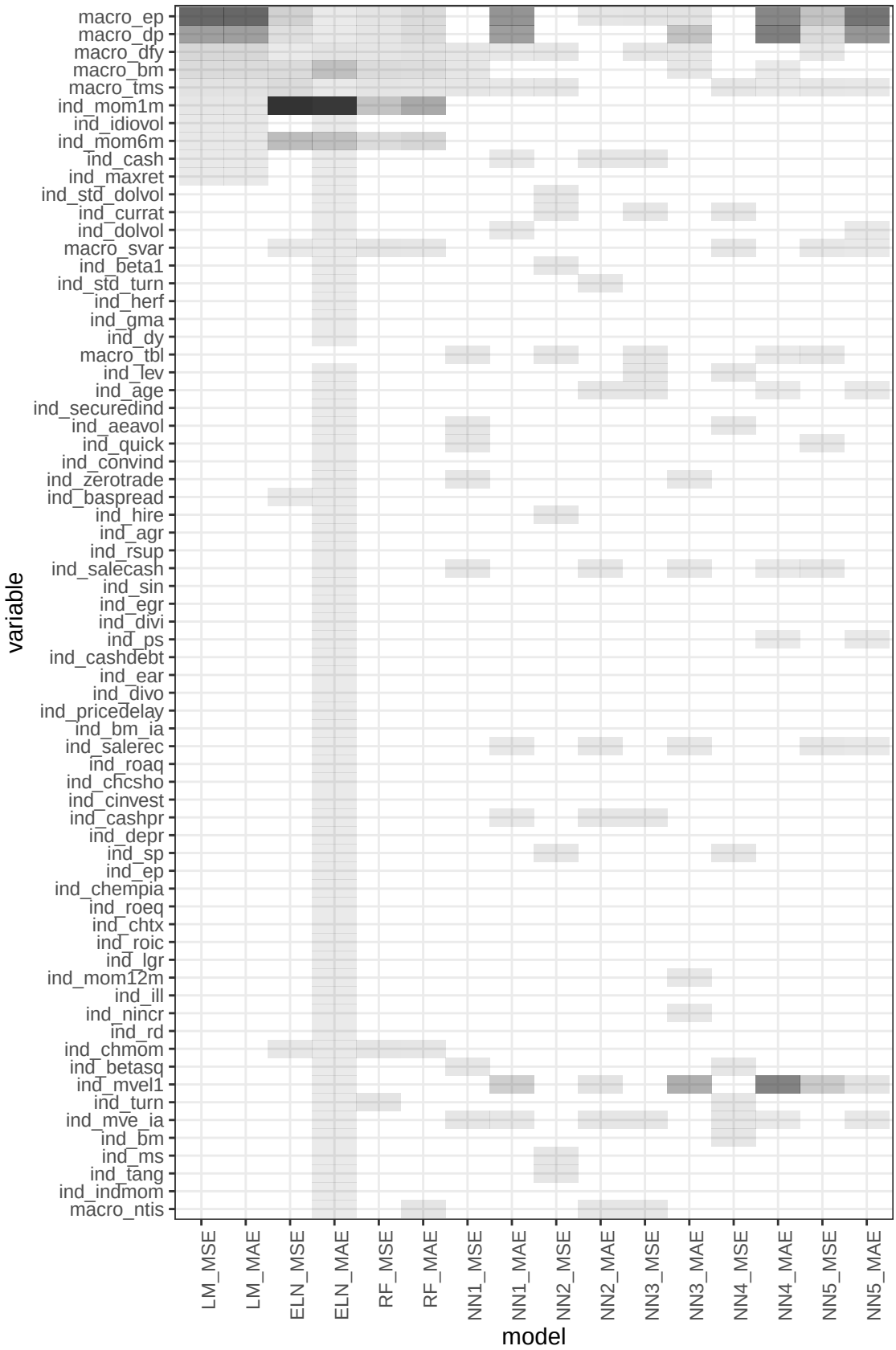


Figure 8. Empirical Data Variable Importance Averaged Across All Samples

7. Appendix

7.1. Data

	Sample	model	train_MAE	train_MSE	train_RMSE	train_RSquare	validation_MAE	validation_M
1	1	LM_MSE	0.16	0.06	0.24	0.37	0.19	0
2	1	LM_MAE	0.16	0.06	0.24	0.37	0.19	0
3	1	ELN_MSE	0.16	0.06	0.25	0.32	0.12	0
4	1	ELN_MAE	0.16	0.06	0.25	0.30	0.12	0
5	1	RF_MSE	0.07	0.01	0.11	0.87	0.13	0
6	1	RF_MAE	0.08	0.02	0.15	0.77	0.13	0
7	1	NN1_MSE	0.16	0.06	0.25	0.34	0.16	0
8	1	NN1_MAE	0.16	0.06	0.25	0.31	0.14	0
9	1	NN2_MSE	0.16	0.06	0.25	0.33	0.16	0
10	1	NN2_MAE	0.16	0.06	0.25	0.32	0.15	0
11	1	NN3_MSE	0.16	0.06	0.25	0.32	0.16	0
12	1	NN3_MAE	0.16	0.07	0.26	0.26	0.14	0
13	1	NN4_MSE	0.17	0.06	0.25	0.31	0.15	0
14	1	NN4_MAE	0.16	0.06	0.25	0.31	0.15	0
15	1	NN5_MSE	0.17	0.06	0.25	0.30	0.16	0
16	1	NN5_MAE	0.16	0.07	0.26	0.29	0.14	0
17	2	LM_MSE	0.15	0.06	0.24	0.37	0.18	0
18	2	LM_MAE	0.15	0.06	0.24	0.37	0.18	0
19	2	ELN_MSE	0.16	0.06	0.25	0.32	0.12	0
20	2	ELN_MAE	0.16	0.06	0.25	0.30	0.12	0
21	2	RF_MSE	0.07	0.01	0.11	0.87	0.13	0
22	2	RF_MAE	0.08	0.02	0.14	0.78	0.12	0
23	2	NN1_MSE	0.16	0.06	0.25	0.33	0.14	0
24	2	NN1_MAE	0.16	0.06	0.25	0.30	0.13	0
25	2	NN2_MSE	0.16	0.06	0.25	0.32	0.14	0
26	2	NN2_MAE	0.16	0.06	0.25	0.31	0.14	0
27	2	NN3_MSE	0.16	0.06	0.25	0.32	0.14	0
28	2	NN3_MAE	0.16	0.06	0.25	0.30	0.13	0
29	2	NN4_MSE	0.16	0.06	0.25	0.31	0.14	0
30	2	NN4_MAE	0.16	0.06	0.25	0.30	0.14	0
31	2	NN5_MSE	0.16	0.06	0.25	0.33	0.15	0
32	2	NN5_MAE	0.16	0.06	0.25	0.30	0.14	0
33	3	LM_MSE	0.15	0.05	0.23	0.37	0.18	0
34	3	LM_MAE	0.15	0.05	0.23	0.37	0.18	0
35	3	ELN_MSE	0.15	0.06	0.24	0.32	0.12	0
36	3	ELN_MAE	0.16	0.06	0.25	0.30	0.13	0
37	3	RF_MSE	0.06	0.01	0.11	0.87	0.13	0
38	3	RF_MAE	0.08	0.02	0.14	0.77	0.13	0
39	3	NN1_MSE	0.16	0.06	0.24	0.33	0.14	0
40	3	NN1_MAE	0.15	0.06	0.25	0.30	0.13	0
41	3	NN2_MSE	0.16	0.06	0.24	0.31	0.15	0
42	3	NN2_MAE	0.16	0.06	0.25	0.30	0.13	0
43	3	NN3_MSE	0.16	0.06	0.24	0.31	0.15	0
44	3	NN3_MAE	0.15	0.06	0.25	0.29	0.13	0
45	3	NN4_MSE	0.16	0.06	0.24	0.31	0.15	0
46	3	NN4_MAE	0.15	0.06	0.25	0.30	0.13	0
47	3	NN5_MSE	0.16	0.06	0.24	0.31	0.14	0
48	3	NN5_MAE	0.16	0.06	0.25	0.28	0.13	0

Table 3: Empirical Data Results

Table 4: Diebold Mariano Tests for Empirical Data

Table 5: Individual Factors

No.	Acronym	Firm Characteristic	Author(s)	Data Source	Frequency
1	ind_absacc[0]	Absolute Accruals	Bandyopadhyay et al. (2010)	Compustat	Annual
2	ind_acc[0]	Working capital accruals	Sloan (1996)	Compustat	Annual
3	ind_aeavol	Abnormal earnings announcement volume	Lerman et al. (2008)	Compustat	Quarterly
4	ind_age	# years since first Compustat coverage	Jiang et al. (2005)	Compustat	Annual
5	ind_agr	Asset growth	Cooper et al. (2008)	Compustat	Annual
6	ind_baspread	Bid-ask spread	Amihud and Mendelson (1989)	Compustat	Monthly
7	ind_beta	Beta	Fama and MacBeth (1973)	Compustat	Monthly
8	ind_betasq	Beta squared	Fama and MacBeth (1973)	Compustat	Monthly
9	ind_bm	Book-to-market	Rosenberg et al. (1985)	Compustat	Annual
10	ind_bm_ia	Industry-adjusted book to market	Asness et al. (2000)	Compustat	Quarterly
11	ind_cash	Cash holdings	Palazzo (2012)	Compustat	Annual
12	ind_cashdebt	Cashflow to debt	Ou and Penman (1989)	Compustat	Annual
13	ind_cashpr	Cash productivity	Chandrashekar and Rao (2009)	Compustat	Annual
14	ind_cfp[0]	Cashflow to price ratio	Desai et al. (2004)	Compustat	Annual
15	ind_cfp_ia[0]	Industry-adjusted cashfow to price ratio	Asness et al. (2000)	Compustat	Annual
16	ind_chatoia[0]	Industry-adjusted change in asset turnover	Soliman (2008)	Compustat	Annual
17	ind_chcsho	Change in shares outstanding	Pontiff and Woodgate (2008)	Compustat	Annual
18	ind_chempia	Industry-adjusted change in employee	Asness et al. (2000)	Compustat	Annual
19	ind_chinv[0]	Change in inventory	Thomas and Zhang (2002)	Compustat	Annual
20	ind_chmom	Change in 6-month momentum	Gettleman and Marks (2006)	Compustat	Monthly
21	ind_chpmia[0]	Industry-adjusted change in profit margin	Soliman (2008)	Compustat	Annual
22	ind_chtx	Change in tax expense	Thomas and Zhang (2011)	Compustat	Quarterly
23	ind_cinvest	Corporate investment	Titman et al. (2004)	Compustat	Quarterly
24	ind_convind	Convertible debt indicator	Valta (2016)	Compustat	Annual
25	ind_currat	Current ratio	Ou and Penman (1989)	Compustat	Annual

[0]The factor was included in the original dataset provided by Gu et al. (2018), but was not used due to missing data issues

26	ind_depr	Depreciation / PP&E	Holthausen and Larcker (1992)	Compustat	Annual
27	ind_divi	Dividend initiation	Michaely et al. (1995)	Compustat	Annual
28	ind_divo	Dividend omission	Michaely et al. (1995)	Compustat	Annual
29	ind_dolvol	Dollar trading volume	Chordia et al. (2001)	Compustat	Monthly
30	ind_dy	Dividend to price	Litzenberger and Ramaswamy (1982)	Compustat	Annual
31	ind_ear	Earnings announcement return	Brandt et al. (2008)	Compustat	Quarterly
32	ind_egr	Growth in common shareholder eq	Richardson et al. (2005)	Compustat	Annual
33	ind_ep	Earnings to price	Basu (1977)	Compustat	Annual
34	ind_gma	Gross profitability	Novy-Marx (2013)	Compustat	Annual
35	ind_grCAPX^0	Growth in capital expenditures	Anderson and Garcia-Feijo (2006)	Compustat	Annual
36	ind_grltnoa^0	Growth in long term net operating assets	Fairfield et al. (2003)	Compustat	Annual
37	ind_herf	Industry sales concentration	Hou and Robinson (2006)	Compustat	Annual
38	ind_hire	Employee growth rate	Belo et al. (2014)	Compustat	Annual
39	ind_idiovol	Idiosyncratic return volatility	Ali et al. (2003)	Compustat	Monthly
40	ind_ill	Illiquidity	Amihud (2002)	Compustat	Monthly
41	ind_indmom	Industry momentum	Moskowitz and Grinblatt (1999)	Compustat	Monthly
42	ind_invest^0	Capital expenditures and inventory	Chen and Zhang (2010)	Compustat	Annual
43	ind_lev	Leverage	Bhandari (1988)	Compustat	Annual
44	ind_lgr	Growth in long-term debt	Richardson et al. (2005)	Compustat	Annual
45	ind_maxret	Maximum daily return	Bali et al. (2011)	Compustat	Monthly
46	ind_mom12m	12-month momentum	Jegadeesh (1990)	Compustat	Monthly
47	ind_mom1	1-month momentum	Jegadeesh and Titman (1993)	Compustat	Monthly
48	ind_mom36m^0	36-month momentum	Jegadeesh and Titman (1993)	Compustat	Monthly
49	ind_mom6m	6-month momentum	Jegadeesh and Titman (1993)	Compustat	Monthly
50	ind_ms	Financial statement score	Mohanram (2005)	Compustat	Quarterly
51	ind_mvel1	Size	Banz (1981)	Compustat	Monthly
52	ind_mve_ia	Industry-adjusted size	Asness et al. (2000)	Compustat	Annual
53	ind_nincr	Number of earnings increases	Barth et al. (1999)	Compustat	Quarterly
54	ind_operprof^0	Operating profitability	Fama and French (2015)	Compustat	Annual

55	ind_orgcap[0]	Organizational capital	Eisfeldt and Papanikolaou (2013)	Compustat	Annual
56	ind_pchcapx_ia[0]	Industry adjusted % change in capital expenditures	Abarbanell and Bushee (1998)	Compustat	Annual
57	ind_pchcurrat[0]	% change in current ratio	Ou and Penman (1989)	Compustat	Annual
58	ind_pchdepr[0]	% change in depreciation	Holthausen and Larcker (1992)	Compustat	Annual
59	ind_pchgm_pchsale[0]	% change in gross margin - % change in sales	Abarbanell and Bushee (1998)	Compustat	Annual
60	ind_pchquick[0]	% change in quick ratio	Ou and Penman (1989)	Compustat	Annual
61	ind_pchsale_pchinvt[0]	% change in sales - % change in inventory	Abarbanell and Bushee (1998)	Compustat	Annual
62	ind_pchsale_pchrect[0]	% change in sales - % change in A/R	Abarbanell and Bushee (1998)	Compustat	Annual
63	ind_pchsale_pchxsga[0]	% change in sales - % change in SG	Abarbanell and Bushee (1998)	Compustat	Annual
64	ind_pchsaleinv[0]	% change sales-to-inventory	Ou and Penman (1989)	Compustat	Annual
65	ind_pctacc[0]	Percent accruals	Hafzalla et al. (2011)	Compustat	Annual
66	ind_pricedelay	Price delay	Hou and Moskowitz (2005)	Compustat	Monthly
67	ind_ps	Financial statements score	Piotroski (2000)	Compustat	Annual
68	ind_quick	Quick ratio	Ou and Penman (1989)	Compustat	Annual
69	ind_rd	R&D increase	Eberhart et al. (2004)	Compustat	Annual
70	ind_rd_mve[0]	R&D to market capitalization	Guo et al. (2006)	Compustat	Annual
71	ind_rd_sale[0]	R&D to sales	Guo et al. (2006)	Compustat	Annual
72	ind_realestate[0]	Real estate holdings	Tuzel (2010)	Compustat	Annual
73	ind_retvol	Return volatility	Ang et al. (2006)	Compustat	Monthly
74	ind_roaq	Return on assets	Balakrishnan et al. (2010)	Compustat	Quarterly
75	ind_roavol[0]	Earnings volatility	Francis et al. (2004)	Compustat	Quarterly
76	ind_roeq	Return on equity	Hou et al. (2015)	Compustat	Quarterly
77	ind_roic	Return on invested capital	Brown and Rowe (2007)	Compustat	Annual
78	ind_rsup	Revenue surprise	Kama (2009)	Compustat	Quarterly
79	ind_salecash	Sales to cash	Ou and Penman (1989)	Compustat	Annual
80	ind_saleinv[0]	Sales to inventory	Ou and Penman (1989)	Compustat	Annual
81	ind_salerec	Sales to receivables	Ou and Penman (1989)	Compustat	Annual
82	ind_secured[0]	Secured debt	Valta (2016)	Compustat	Annual
83	ind_securedind	Secured debt indicator	Valta (2016)	Compustat	Annual

84	ind_sgr^0	Sales growth	Barbee Jr et al. (1996)	Compustat	Annual
85	ind_sin	Sin stocks	Hong and Kacperczyk (2009)	Compustat	Annual
86	ind_sp	Sales to price	Barbee Jr et al. (1996)	Compustat	Annual
87	ind_std_dolvol	Volatility of liquidity (dollar trading volume)	Chordia et al. (2001)	Compustat	Annual
88	ind_std_turn	Volatility of liquidity (share turnover)	Chordia et al. (2001)	Compustat	Monthly
89	ind_stdacc^0	Accrual volatility	Bandyopadhyay et al. (2010)	Compustat	Monthly
90	ind_stdcf^0	Cashflow volatility	Huang (2009)	Compustat	Quarterly
91	ind_tang	Debt capacity/rm tangibility	Almeida and Campello (2007)	Compustat	Quarterly
92	ind_tb^0	Tax income to book income	Lev and Ohlson (1982)	Compustat	Annual
93	ind_turn	Share turnover	Datar et al. (1998)	Compustat	Monthly
94	ind_zerotrade	Zero trading days	Liu (2006)	Compustat	Monthly

Table 6: Macroeconomic Factors

No.	Acronym	Macroeconomic Factor
1	macro_dp	Dividend Price Ratio
2	macro_ep	Earnings Price Ratio
3	macro_bm	Book to Market Ratio
4	macro_ntis	Net Equity Expansion
5	macro_tbl	Treasury Bill Rate
6	macro_tms	Term Spread
7	macro_dfy	Default Spread
8	macro_svar	Stock Variance

7.2. Computational Details

7.2.1. Linear Models

OLS used for fitting wrt MSE.

quantreg package used for fitting wrt MAE. (minimizing 0.5 quantile loss is equivalent to minimizing mae).

7.2.2. Penalized Linear

The package was used to fit penalized regression models with respect to MSE and MAE.

This package efficiently calculates a regularization path of penalization values given a value for α. This means that it is much more efficient to instead only supply a grid for α, let the algorithm decide its own path of penalization values. The combination of these two parameters which produces the best results on the validation set were then chosen.

The more traditional approach of supplying a grid of α and λ was also implemented at first, and this was observed to give almost identical results to the above implementation, but at a much higher computational cost.

7.2.3. Classification and Regression Trees

For full details of the Classification and Regression Tree algorithm see Breiman (1984).

Algorithm 1: Classification and Regression Tree

Initialize ;

for *d from 1 to L* **do**

  for *i in* $C_l(d-1), l = 1, \ldots, 2^{d-1}$ **do**

    For each feature $j = 1, 2, \ldots, P$, and each threshold level α, define a split as $s = (j, \alpha)$ which divides $C_l(d-1)$ into C_{left} and C_{right}:

$$C_{left}s = \{z_j \leq \alpha\} \cap C_l(d-1); C_{right}s = \{z_j > \alpha\} \cap C_l(d-1)$$

    Define the impurity function:

$$\mathcal{L}(C, C_{left}, C_{right}) = \frac{|C_{left}|}{|C|} H(C_{left}) + \frac{|C_{right}|}{|C|} H(C_{right})$$

    where

$$H(C) = \frac{1}{|C|} \sum^{z_{i,t} \in C} (r_{i,t+1} - \theta)^2, \theta = \frac{1}{|C|} \sum_{z_{i,t} \in C} r_{i,t+1}$$

    and $|C|$ denotes the number of observations in set C

    Find the optimal split

$$s^* \leftarrow \underset{s}{argmin} \mathcal{L}(C(s), C_{left}(s), C_{right}(s))$$

    Update nodes (partition the data):

$$C_{2l-1}(d) \leftarrow C_{left}(s^*), C_{2l}(d) \leftarrow C_{right}(s^*)$$

  end

end

Result: The prediction of a regression tree is:

$$g(z_{i,t}; \theta, L) = \sum_{k=1}^{2^L} \theta_k \mathbf{1}_{z_{i,t} \in C_k(L)}; \theta_k = \frac{1}{|C_k(L)|} \sum_{z_{i,t} \in C_k(L)} r_{i,t+1}$$

7.2.4. Random Forest

The randomforestSRC package was used to fit both random forests for mean regression and quantile regression.

To maintain computational feasibility, all random forest models were grown using 50 trees, and only had their *mtry* hyperparameter tuned. This is because it has been empirically observed that the *mtry* hyperparameter is often the most "tunable," i.e. offering the largest amount of performance gains when tuned, when compared to other hyperparameters such as *nodesize*, **?**.

Rather than use the randomly determined out of bag (OOB) observations for hyperparameter tuning, we explicitly use a separate validation set that is closer to the test set in time, in order to maintain temporal ordering of the data.

For full details of the Random Forest algorithm see Breiman (2001).

Algorithm 2: Random Forest

for b *from* 1 *to* B **do**

Draw bootstrap samples $(z_{i,t}, r_{i,t+1}), (i,t) \in Bootstrap(b)$ from the dataset Grow a tree T_b using Algorithm, using only a random subsample, say $\sqrt{P}$ of all features Denote the resulting bth tree as

$$\hat{g}_b(z_{i,t}, \hat{\theta}_b, L) = \sum_{k=1}^{2^L} \theta_b^k \mathbf{1}_{z_{i,t} \in C_k(L)}$$

end

Result: The final random forest prediction is given by the output of all trees:

$$\hat{g}_b(z_{i,t}; L, B) = \frac{1}{B} \sum_{b=1}^{B} \hat{g}_b(z_{i,t}, \hat{\theta}_b, L)$$

7.2.5. Neural Networks

Fit using keras package (cite)

ADAM algorithm for stochastic gradient descent and learning rate shrinkage as detailed by Kingma and Ba (2014).

Algorithm 3: Early stopping via validation

Initialize $j = 0$, $\epsilon = \infty$ and select the patience parameter p (max iterations)
while j ¡ p **do**
  Update θ using the training algorithm Calculate the prediction error from the validation sample, denoted as ϵ'
  if $\epsilon' < \epsilon$ **then**
    $j \leftarrow 0$
    $\epsilon \leftarrow \epsilon'$
    $\theta' \leftarrow \theta$
  else
    $j \leftarrow j + 1$
  end
end
Result: θ' is the final parameter estimate

Batch Normalization Algorithm as detailed by Ioffe and Szegedy (2015).

Algorithm 4: Batch Normalization for one activation over one batch

Input: Values of x for each activation over a batch $\mathcal{B} = x_1, x_2, \ldots, x_N$
$\mu_{\mathcal{B}} \leftarrow \frac{1}{N} \sum_{i=1}^{N} x_i$
$\sigma_{\mathcal{B}}^2 \leftarrow \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu_{\mathcal{B}})^2$
$\hat{x_i} \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}}$
$y_i \leftarrow \gamma \hat{x_i} + \beta := BN_{\gamma,\beta}(x_i)$
Result: $y_i = BN_{\gamma,\beta}(x_i) : i = 1, 2, ..., N$

7.2.6. Tuning of Simulated Datasets

The simulated datasets were tuned according to the following statistics: average individual time series R squared, average annualized volatility, and cross sectional R squared, and true/predictive R squared.

The methodology for evaluating average time series R squared and cross sectional R squared is consistent with that detailed by Cochrane (2005). The steps are reproduced here for reference; for complete details refer to Cochrane (2005).

First evaluate the following OLS model:

$$R_{it} = a_i + \beta_i' f_{it} + \epsilon_{it} \tag{32}$$

where f_{it} represents the *true* factors in the returns process. The corresponding R-squared value for this time series regression is calculated across all stocks and averaged to yield the average time series R-Squared.

A cross sectional regression for the risk premia is then run across assets of average returns on the factor coefficients:

$$\bar{R_{it}} = \alpha_i = \beta_i' \lambda \tag{33}$$

where the β_i' are the estimated coefficients from each time series regressions run previously. The corresponding R squared for this regression is the cross sectional R squared.

The true R squared is a measure of signal to noise ratio i.e. how much of the returns data is due to This is simply calculated by running a pooled ordinary least squares regression on the entire panel, using the underlying g() as the "predictions.' The resulting R squared value is therefore a measure of how much of the panel can be explained by the g() term exclusive of any noise or error terms.

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