Evaluation of Machine Learning in Empirical Asset Pricing

Anonymous Author(s)
Affiliation
Address
`email`

Abstract

Several recent studies have claimed that machine learning methods provide superior predictive accuracy of asset returns, relative to simpler modeling approaches, and can correctly identify factors needed to price portfolio risk. Herein, we demonstrate that this performance is critically dependent on several features of the data being analyzed; including, the training/test sample split, the frequency at which the data is observed, and the chosen loss-function. In contrast to existing studies, which claim that neural nets provide superior predictive accuracy, through a series of realistic examples that mimics the stylized facts of asset returns, we demonstrate that neural methods are easily outperformed by simpler methods, such as random forests and elastic nets.

1 Introduction

The dominance of machine learning (hereafter, ML) methods in terms of predictive accuracy has begun to filter into the empirical asset pricing literature. Arguably, the most common applications of ML methods in empirical finance are for portfolio construction, asset price prediction, and factor selection.

Several studies have now used ML techniques to analyze the cross-section of asset returns and produce portfolios that can capture nonlinear information in the cross-section of asset returns. [14] use tree-based methods to understand which firm-level characteristics best predict the cross-section of stock returns, and use this information to help mitigate portfolio risk. Similarly, [12] uses deep feedforward neural nets (DFNs) to construct portfolios and predict the returns across a cross-sections of US asset returns. However, while [12] demonstrates that DFNs can better capture nonlinear information, no claim is made that deep learning methods are the best approach to exploit this information.

Several studies have now suggested that ML methods can produce better predictions of asset returns ([6], [9] and [4]). The results of [6] suggest that, in terms of predictive performance, as measured by an out-of-sample R^2, tree-based methods and shallow neural nets can provide superior predictive accuracy over other ML methods and simpler model-based approaches.

Similarly, [11], [5], [3] and [15] demonstrate that ML methods can "systematically evaluate the contribution to asset pricing of any new factor" used within an existing linear asset pricing structure. As such, these authors argue that ML can be used, *en masse*, to consistently evaluate the ability of various factors to help price portfolio risk. Such work is particularly pertinent given the literature's obsession with constructing such factors: as of 2014, quantitative trading firms were using 81 factor models ([9]), while [7] currently document that well over 600 different factors have been suggested in the literature.

Submitted to 34th Conference on Neural Information Processing Systems (NeurIPS 2020). Do not distribute.

The above studies all demonstrate the potential benefits of ML methods within empirical finance. However, it is unclear if the above findings generalize to different training and validation periods; different sampling frequencies; and different loss-measures of predictive accuracy. The answer to such questions in the realm of empirical finance are particularly pertinent given that certain ML methods, have known difficulties in dealing with data that display the stylized facts of asset returns, e.g., weak and nonlinear dependence, low signal-to-noise and a lack of conditional independence/sparsity. Moreover, training even standard types of neural networks, such as DFNs, becomes particularly difficult when data displays strong, or nonlinear, dependence ([1]).

In many ways, existing applications of ML to empirical finance have either over-looked, downplayed, or simply ignored the importance of the above issues. [12] and [4] use cross validation as part of their model building procedures, destroying the temporal ordering of data. [6] and [12] produce models using training samples that end much earlier than the data sets which they ultimately produce forecasts. This is particularly worrying as the factors driving returns can be starkly different across different time periods.

The goal of this paper is to provide a systematic, and reproducible study on the ability of ML methods to 1) accurately detect significant factors; and 2) accurately predict returns according to a range of loss measures. It is our belief that any such study is necessary in order for practitioners to reliably apply these methods in their problems of interest.

After giving the general setup in Section two, in Section three we conduct a rigorous study that gives an in-depth comparison of several ML methods used in the empirical finance literature. The analysis demonstrates that persistence in features, and different complexities of the return generating process affect ML method's ability to: 1) accurately predict future returns across a range of loss measures; and 2) correctly identify the significant factors driving returns. In contrast to existing findings, in this realistic simulation design, we find that neural network procedures, such as feedforward nets, LSTM, and DeepAR models ([16]), are among the worst performing methods, while simpler tree-based methods and elastic net are among the best performing methods.

In Section four, the above findings are validated in an empirical exercise that considers individual returns data from CRSP for all firms listed in the NYSE, AMEX and NASDAQ over a 60 year period, where a set of 549 possible factors are used to explain the cross-section of returns. Careful attention is given to the training and test split, with only use the last fourteen years of returns data used to evaluate the different ML methods. Across all ML methods considered, neural net based procedure perform the worst, while tree-based methods and elastic net perform the best.

Our results suggest that the efficacy of ML methods in empirical finance depends on several features of the underlying problem, such as sampling frequency, the particular training test split, and the data period under analysis. As such, while potentially useful, ML methods are not a panacea for predicting, or understanding the factors that drive financial returns.

2 Model and Methods

2.1 Statistical Model

We briefly discuss the statistical model considered for asset returns. Excess monthly returns on asset i, $i \leq n$, at time t, $t \leq T$, are assumed to evolve in an additive fashion:

$$r_{i,t+1} = E(r_{i,t+1}|\mathcal{F}_t) + \epsilon_{i,t+1}, \quad E(\epsilon_{i,t+1}|\mathcal{F}_t) = 0, \tag{1}$$

where $\mathcal{F}_t$ denotes the observable information at time t, and $\epsilon_{i,t+1}$ is a martingale difference sequence. The conditional mean of returns is an unknown function of a P-dimensional vector of features, measurable at time t:

$$E(r_{i,t+1}|\mathcal{F}_t) = g(z_{i,t}) \tag{2}$$

The features, or predictors, $z_{i,t}$ are composed of time-t information, and only depends on the characteristics of stock i. The assumption that the information set can be characterized by the variables $z_{i,t}$, without dependence on the $j \neq i$ return units, is reasonable if the collection of $z_{i,t}$ is rich enough.

In what follows, we represent the space of possible features as the Kronecker product of two pieces

$$z_{i,t} = x_t \otimes c_{i,t} \tag{3}$$

where the variables $c_{i,t}$ represent a $P_c \times 1$ vector of individual-level characteristics for return i, and x_t represents a $P_x \times 1$ vector of macroeconomic predictors, and $\otimes$ represents the Kronecker product. Thus, for $P = P_c \cdot P_x$, $z_{i,t}$ represents a $P \times 1$ feature space that can be used to approximate the unknown function $g(\cdot)$.

2.2 Methods to be compared

Given features $z_{i,t}$, the goal of any ML method is to approximate the unknown function $g(\cdot)$ in 1. Broadly speaking, how different ML methods choose to approximate this function depends on three components:

1. the model used to make predictions;[1]
2. the regularization mechanism employed to mitigate over-fitting;
3. a loss function that penalized poor predictions.

To ensure the results of ML different methods will be comparable, we fix both the regularization mechanisms and loss functions used within each method, and allow only the models used for prediction to vary. This approach seeks to ensure that performances in one method, relative to another, are based on the model structure and not to some feature of how the models were fit. To this end, we first discuss points 2. and 3. above, and then briefly present the models used for our comparison.

Loss functions: All ML methods are implemented using two possible loss functions: Mean Absolute Error (MAE) and Mean Squared Error (MSE): for $\widehat{r}_{i,j}$ denoting the predicted return on asset i at time j,

$$\text{MAE} = \frac{1}{n}\sum_{j=i}^{n} |r_{i,j} - \widehat{r}_{i,j}| \text{ and MSE} = \frac{1}{n}\sum_{j=i}^{n} (r_{i,j} - \widehat{r}_{i,j})^2 ,$$

We consider both loss functions since MAE is less sensitive to outliers in the data which financial returns are known to exhibit, and which are caused by extreme market movements. Given this, we expect MAE to produce predictive results that are more robust to such outlier events.

Sample Splitting: Since returns data is intrinsically dependent, observed data is split into “training”, “validation” and “test” sets according to a schema that respects this dependence structure. To balance computation and accuracy, we use a hybrid “rolling window" and “recursive” approach to training/validation/test splits: for each model refit, the training set is increased by one year observations, i.e., 12 monthly observations; the validation set is fixed at one year and moves forward (by one year) with each model refit; predictions are generated using that model for the subsequent year.

Models In what follows we compare a host of different ML models including elastic net ([18], random forest ([2]), feed-forward neural nets, LSTM, FFORMA ([13]) and DeepAR models ([16]). Details on each model and certain features of its implementation used in this work are given in Appendix A. For each of the different methods, we consider two variants, one based on the MAE loss and one based on the MSE loss.

2.3 Model evaluation measures

Predictive accuracy Predictive performance is assessed using Mean Absolute Error (MAE), Mean Squared Error (MSE) (evaluated over the test set) and an out-of-sample R^2 measure. While out-of-sample R^2 is a common measure, there is no universally agreed-upon definition. As such, we explicitly state the version employed herein as

$$R^2_{OOS} = 1 - \frac{\sum_{(i,t)\in\mathcal{T}_3} (r_{i,t+1} - \widehat{r}_{i,t+1})^2}{\sum_{(i,t)\in\mathcal{T}_3} (r_{i,t+1} - \bar{r}_{i,t+1})^2}, \tag{4}$$

[1] The model used by the ML method need not correspond to the statistical model assumed to describe the data. Herein, our goal will not be to asses the “accuracy” of the statistical model, but to determine how different ML methods accurately determine the salient features of this model.

where $\mathcal{T}_3$ indicates that the fits are only assessed on the test sub-sample, which is never used for training or tuning.

Since R^2 is based on in-sample-fit of a linear model, this measure is less meaningful for most of the ML methods considered in in this paper. However, we report this measure since this measure has also been considered in other applications of ML to empirical finance (see, e.g., [6]).

Factor Selection An important aspect of empirical finance is the knowledge of which features drive risk, i.e., which features are explicitly represented within $z_{i,t}$. To this end, we follow [6] and construct a variable importance (VI) measure to compare the different ML methods. The importance of variable j, VI_j, is defined as the reduction in predictive R^2 from setting all values of predictor j to 0, while holding the remaining model estimates fixed. Each VI_j is then normalized to sum to 1.

However, as VI_j can sometimes be negative, we shift VI_j by the smallest VI_j plus a small constant, then dividing by this sum to alleviate numerical issues[2]. The resulting VI measure is then:

$$VI_{j,norm} = \frac{VI_j + \min(VI_j) + o}{\Sigma VI_j + \min(VI_j) + o} \quad ; \quad o = 10^{-100} \tag{5}$$

3 Preliminary Results

We first explore how ML methods perform in terms of prediction and factor selection for data that exhibit the stylized facts of empirical returns. We simulate according to a design which incorporates a low signal-to-noise ratio, stochastic volatility, persistence and cross-sectional correlated features. Data is generated from a latent factor volatility model for excess returns r_{t+1}, for $t = 1, \ldots, T$:

$$\begin{aligned} r_{i,t+1} &= g\left(z_{i,t}\right) + \beta_{i,t+1} v_{t+1} + e_{i,t+1}; \;\; z_{i,t} = (1, x_t)' \otimes c_{i,t}, \;\; \beta_{i,t} = (c_{i1,t}, c_{i2,t}, c_{i3,t}) \\ e_{i,t+1} &= \sigma_{i,t+1} \varepsilon_{i,t+1}; \\ \log(\sigma^2_{i,t+1}) &= \omega + \gamma \log(\sigma^2_t) + \sigma_u u; \quad u \sim N(0,1) \end{aligned}$$

where v_{t+1} is a 3×1 vector of errors, $w_{t+1} \sim N(0,1)$, $\varepsilon_{i,t+1} \sim N(0,1)$ scalar error terms, matrix C_t is an $N \times P_c$ matrix of latent factors, where the first three columns correspond to $\beta_{i,t}$, across the $1 \leq i \leq N$ dimensions, while the remaining $P_c - 3$ factors do not enter the return equation. The $P_x \times 1$ vector x_t is a 3×1 multivariate time series that captures for macroeconomic factors, and ε_{t+1} is a $N \times 1$ vector of idiosyncratic errors. The parameters of these were tuned such that the annualized volatility of each return series was approximately 22% when viewed as monthly returns, as is often observed empirically.

We consider three different functions for $g(z_{i,t})$:

$$\begin{aligned} &(1)\; g_1\left(z_{i,t}\right) = \left(c_{i1,t}, c_{i2,t}, c_{i3,t} \times x'_t[3,]\right) \theta_0 \\ &(2)\; g_2\left(z_{i,t}\right) = \left(c^2_{i1,t}, c_{i1,t} \times c_{i2,t}, \operatorname{sgn}\left(c_{i3,t} \times x'_t[3,]\right)\right) \theta_0 \\ &(3)\; g_3\left(z_{i,t}\right) = \left(1[c_{i3,t} > 0], c^3_{i2,t}, c_{i1,t} \times c_{i2,t} \times 1[c_{i3,t} > 0], \operatorname{logit}\left(c_{i3,t}\right)\right) \theta_0 \end{aligned}$$

where $x'_t[3,]$ denotes the third element of the x'_t vector. $g_1\left(z_{i,t}\right)$ allows the characteristics to enter the return equation linearly, and $g_2\left(z_{i,t}\right)$ and $g_3\left(z_{i,t}\right)$ allow the characteristics to enter the return equation interactively and non-linearly. [3] θ^0 was tuned such that the predictive R^2 was approximately 5%.

We consider two different levels of cross-sectional correlation for the N factors, $c_{i,t}$, which correspond to a small amount of 0.01 and a large amount, 1.0, or cross-sectional correlation. The specific details regarding the level of cross-sectional correlation, and how it is introduced, is given in Appendix B.1. The macroeconomic factors, x_t, a 3×1 vector, is generated according to a stationary Vector Autoregression (VAR) model with a high-degree of persistence (0.95 for each series) and a diagonal coefficient matrix. See Appendix B.1 for more details.

The simulation design results in 9 different data generating process (DGP). For each DGP we fix with $N = 200$ stocks, $T = 180$ time periods and $P_c = 100$ characteristics. Each DGP was simulated

[2] This mechanism was chosen because the other popular normalization mechanism "softmax" was observed to be unable to preserve the distances between each original VI_j, making discernment between each VI_j difficult.

[3] (g_1, g_2 correspond to the simulation design used by [6].)

10 times to assess the robustness of ML algorithms, with the number of simulations kept low for computational feasibility. We employ the hybrid data splitting approach with a training:validation length ratio of approximately 1.5 and a test set that is 1 year in length.

3.1 Simulation Study Results

Prediction Performance: The complete set of simulation results are detailed in Appendix B.2, however, for brevity we only remark on the most interesting findings in the main paper within the below Table. In contrast to existing studies, we find that elastic nets are the best performing model, followed closely by random forests, then neural networks. Interestingly, all ML models were unaffected by the level of cross-sectional correlation in terms of prediction performance, and typically had better performance when fitted with respect to quantile loss.

Generally, ML models fitted with respect to minimizing MAE (quantile loss) generally perform better, even when evaluated against MSE loss metrics. Although the actual level difference between the loss metrics across the different methods is small, the results are remarkably consistent across the various Monte Carlo designs.

Table 1: Top Models in Simulation Study

		Test MAE			Test MSE		
Corr	model	g1	g2	g3	g1	g2	g3
0.01	ELN.MAE	0.0345786	0.0361950	0.0353345	0.0025652	0.0026882	0.0026210
	RF.MAE	0.0354594	0.0354204	0.0355399	0.0026434	0.0026305	0.0026446
	NN2.MAE	0.0359604	0.0369206	0.0363047	0.0026786	0.0027474	0.0026996
	NN1.MAE	0.0358939	0.0368335	0.0363352	0.0026718	0.0027396	0.0027028
	NN3.MAE	0.0358164	0.0369345	0.0364712	0.0026697	0.0027491	0.0027181
1	ELN.MSE	0.0346142	0.0362761	0.0354437	0.0025676	0.0026980	0.0026300
	RF.MAE	0.0359158	0.0356434	0.0360529	0.0026747	0.0026445	0.0026786
	NN5.MAE	0.0370087	0.0372705	0.0374132	0.0027744	0.0027832	0.0027916
	NN4.MSE	0.0373820	0.0368966	0.0373542	0.0028051	0.0027505	0.0027970
	NN3.MAE	0.0372849	0.0370382	0.0371925	0.0027940	0.0027652	0.0027753

Factor Importance The factor importance results are presented graphically in Figure 1, and demonstrates that overall elastic net outperforms all other models consistently in terms of assigning the correct relative importance to the true underlying features.[4]. However, the performance of elastic net does degrade as the data generating process becomes more non-linear.

Random forests, and to a lesser extent the neural networks, also correctly identified the correct underlying regressors, but struggled with adequately discerning relative importance among correlated regressors. This behavior becomes more pronounced as the degree of cross-sectional correlation increases (see decreasing relative importance of true underlying regressors in Figures **??** and **??** in Appendix ??).

What figures are you referring to here???

4 Empirical analysis

We now investigate the performance of ML methods across a large sample of returns. As we shall see later, the results obtained in Section 3.1 are largely borne out in this empirical exercise.

4.1 Data

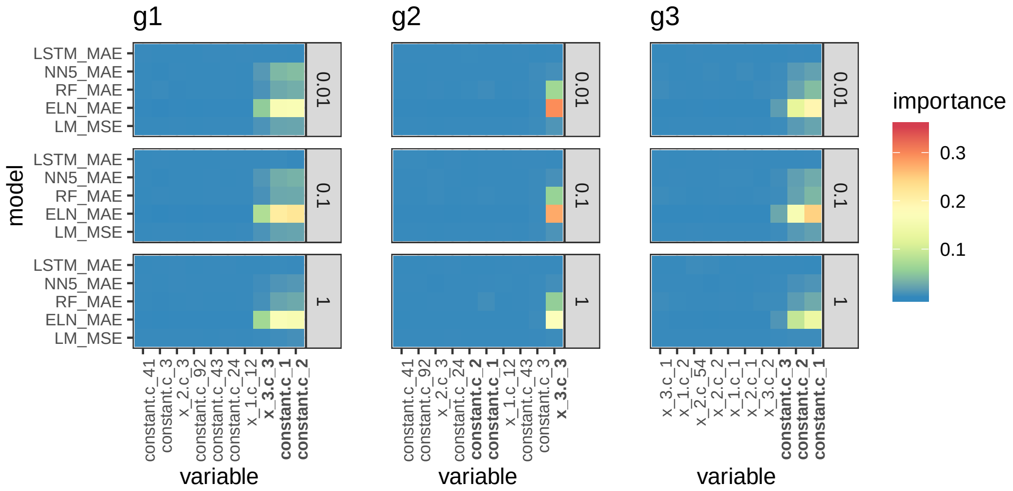
We use the universe of firms listed in the NYSE, AMEX and NASDAQ, starting from 1957 (starting date of the S&P 500) and ending in December 2016, totaling 60 years, that have a quarterly return over this period. This approach allows firms to enter and exit the dataset and helps alleviate the problem of survivorship bias in the dataset. Individual cross-sectional factors, $c_{i,t}$, are constructed following the approach of [6]. **We restrict our dataset to begin from 1993 Q3 and end on 2016 Q4 to alleviate data quality issues.** Our individual factor set contains 94 characteristics: 61 updated

It previously said monthly returns but below you talked about quarterly returns. So, I've taken the lower frequency. Please make sure that is correct.

What do you mean by this

[4] (c_1.constant, c_2.constant and $c_3.x_3$ for $g1$ and g_2 specifications, and c_1.constant, c_2.constant and c_3.constant for g_3)

Figure 1: Simulation variable importance, faceted by simulation specification



annually, 13 updated quarterly and 20 updated monthly.[56] Complete details of the data and the cleaning procedures employed are detailed in Appendix C.1.

Following [17] (see Table 6) we consider eight macroeconomic factors. These factors were lagged by one period so as to be used to predict one period ahead quarterly returns. The The 3-month Treasury Bill rate was also used from this source to proxy for the risk-free rate in order to construct excess quarterly returns. The two sets of factors, $c_{i,t}$ and x_t, are then used to build the baseline set of factors, which we defined as in equation (3); i.e., $z_{i,t} = (1, x_t')' \otimes c_{i,t}$. The total number of features in this baseline set is $61 \times (8 + 1) = 549$.

The final dataset contains 202, 066 individual observations. We note that due to data quality issues, LSTMs, FFORMA and DeepAR are not feasible on empirical data, though the results of the simulation study suggest that even if were to be used, their performance would be underwhelming. [7]

We mimic the sample splitting procedure used in the simulation study: the dataset was split such that the training and validation sets were split such that the training set was approximately 1.5 times the length of the validation set, in order to predict a test set that is one year in length.

4.2 Results

Prediction Accuracy The predictive results for the five best methods, according to the various loss measures, are displayed below. In general, the same patter of results in Section 3.1 is again in evidence: elastic net performs best, followed by the random forests, then the DFNs. We note that the differences between each model using the MSE and MAE loss metrics are much more pronounced on empirical data. In addition, the ML models perform better when fitted with respect to quantile loss instead of MSE. Most notably, the lack of robustness for the DFNs observed in Section 3.1 is amplified on the empirical dataset, which directly contradicts existing results already reported in the literature.

[5] The dataset also included 74 Standard Industrial Classification (SIC) codes, but these were omitted due to their inconsistency, and inadequateness at classifying companies, as noted by WRDS

[6] To deal with missing data, any characteristics that had over 20% of their data missing were omitted. Remaining missing data were then imputed using their cross sectional medians for each year. See Appendix for more details.

[7] The dataset was not normalized for all methods, as only penalized regression and neural networks are sensitive to normalization. For these two methods, the dataset was normalized such that each predictor column had mean zero and unit variance.

Table 2: Top 5 models in empirical study

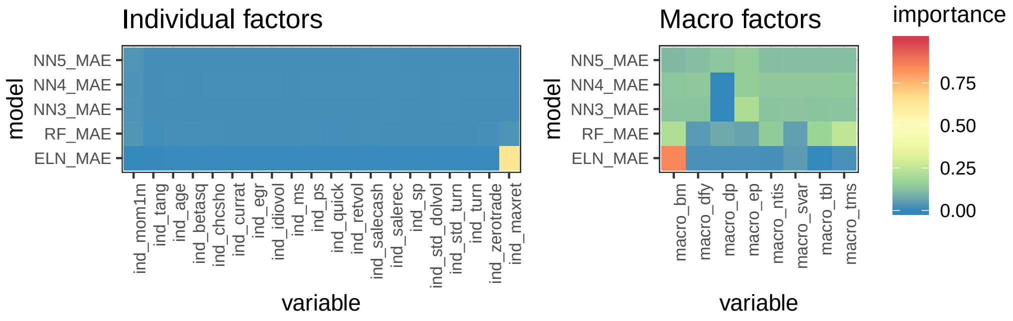
	Sample 1			Sample 2			Sample 3		
model	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2
ELN.MAE	0.131369	0.040718	0.014306	**0.137092**	**0.041892**	**0.017875**	**0.146251**	**0.045207**	**0.000835**
RF.MAE	**0.126703**	**0.036785**	**0.109505**	0.173721	0.057546	-0.349132	0.14692	0.046037	-0.01752
NN5.MAE	0.146411	0.044901	-0.086967	0.18499	0.06461	-0.514744	0.184986	0.063861	-0.411475
NN4.MAE	0.157301	0.050286	-0.217308	0.168815	0.055711	-0.306102	0.167998	0.055129	-0.218463
NN3.MAE	0.140781	0.042832	-0.036882	0.181096	0.06216	-0.4573	0.164896	0.053458	-0.181528

That being said, we do observe some evidence that deeper neural networks perform better, though this result is less apparent due to the lack of robustness of these methods on empirical data (see **??** in Appendix XX for results).

This refernce is broken, and you need to point to where this is in the appendix...

Factor Importance As the data generating process for empirical returns is unknown, the variable importance results cannot be directly compared with those of the simulation study. Even so, we see similar results: the elastic net and random forest models tend to agree on the same subset of predictors, but the random forest struggles to discern between highly correlated regressors. Similar to the prediction performance results, neural networks perform poorly.

Figure 2: Empirical individual and macroeconomic factor importance, averaged over all samples



Individual factors shown on x axis (see Table **??** in Appendix for definitions)

The elastic net, random forest and to a lesser extent DFNs tend to pick out the max return and 1 month momentum factors out of the individual characteristics as important, and the book-to-market factor out of the macroeconomic factors are important. In general, the variable importance metrics are less consistent for the random forests, and it should be noted in particular that the random forest tends to determine factors highly correlated with momentum as important, such as change in momentum, dollar trading volume and return volatility. Within the macroeconomic factors, penalized linear models tend to identify the average book to market ratio and the default spread as the most important. The random forests were inconsistent with the elastic nets, and tended to assign very similar variable importance metrics to most macroeconomic factors.

The overall results of this analysis again question existing results already reported in the literature, which conclude that all ML methods tend to agree on the same subset of important factors (see, e.g., [6]). In our context, we see, at best, only mild agreement between the various ML methods in regards to individual factor selection.

Interestingly, the linear models assign the controversial dividend price ratio macroeconomic factor as highly important, a result mirrored only with the neural networks. Their variable importance for individual factors across different training samples is non-robust, with the important variables almost completely changing year to year. The linear models consistently identified the controversial dividend-price ratio as important, a result that was somewhat consistent with the neural networks.

5 Conclusion

Our findings demonstrate that the field of ML may offer certain tools to improve stock prediction and identification of underlying factors. This study suggest that penalized linear models and to a lesser extent, random forests are the most robust methods for data displaying the stylized facts of asset returns. In contrast to existing results, we find that DFNs fail in the context of return prediction, and variable importance analysis. This result is consistent across a variety of simulated data sets, as well as empirical data.

Therefore, the overall findings of this research differs from the sparse literature on ML methods in empirical finance. However, the performance of the penalized linear models with respect to both out of sample prediction performance and variable importance analysis is promising, and our findings show that ML provides some tools which may aid in the problems of stock return prediction and risk factor selection in the financial world.

Broader Impact

This research calls into question the broad applicability of machine learning methods within empirical finance, at least in the context of return prediction and factor selection. In contrast to existing studies, we find that more complex machine learning methods, such as deep feedforward neural nets, LSTM, and DeepAR, do not perform as well as simpler penalized linear methods and random forest. As such, this research suggests that ML methods are not a panacea for empirical finance, and that great care and diligence is needed in the application of these methods within any financial decision making process.

References

[1] Bengio, Y., Simard, P., and Frasconi, P. (1994). Learning long-term dependencies with gradient descent is difficult. *IEEE transactions on neural networks*, 5(2):157–166.

[2] Breiman, L. (2001). Random forests. *Machine learning*, 45(1):5–32.

[3] Feng, G., Giglio, S., and Xiu, D. (2019). Taming the Factor Zoo: A Test of New Factors. Technical Report w25481, National Bureau of Economic Research, Cambridge, MA.

[4] Feng, G., He, J., and Polson, N. G. (2018). Deep Learning for Predicting Asset Returns. *arXiv:1804.09314 [cs, econ, stat]*. arXiv: 1804.09314.

[5] Freyberger, J., Neuhierl, A., and Weber, M. (2017). Dissecting characteristics nonparametrically. Technical report, National Bureau of Economic Research.

[6] Gu, S., Kelly, B. T., and Xiu, D. (2019). Empirical Asset Pricing via Machine Learning. SSRN Scholarly Paper ID 3159577, Social Science Research Network, Rochester, NY.

[7] Harvey, C. R. and Liu, Y. (2019). A Census of the Factor Zoo. *Social Science Research Network*, page 7.

[8] Hochreiter, S. and Schmidhuber, J. (1997). Long Short-Term Memory. *Neural Computation*, 9(8):1735–1780.

[9] Hsu, J. and Kalesnik, V. (2014). Finding smart beta in the factor zoo. *Research Affiliates (July)*.

[10] Ishwaran, H., Kogalur, U. B., Blackstone, E. H., and Lauer, M. S. (2008). Random survival forests. *Annals of Applied Statistics*, 2(3):841–860.

[11] Kozak, S., Nagel, S., and Santosh, S. (2017). Shrinking the cross section. Technical report, National Bureau of Economic Research.

[12] Messmer, M. (2017). Deep Learning and the Cross-Section of Expected Returns. SSRN Scholarly Paper ID 3081555, Social Science Research Network, Rochester, NY.

[13] Montero-Manso, P., Athanasopoulos, G., Hyndman, R. J., and Talagala, T. S. (2020). FFORMA: Feature-based forecast model averaging. *International Journal of Forecasting*, 36(1):86–92.

[14] Moritz, B. and Zimmermann, T. (2016). Tree-Based Conditional Portfolio Sorts: The Relation between Past and Future Stock Returns. SSRN Scholarly Paper ID 2740751, Social Science Research Network, Rochester, NY.

[15] Rapach, D. and Zhou, G. (2013). Forecasting Stock Returns. In *Handbook of Economic Forecasting*, volume 2, pages 328–383. Elsevier.

[16] Salinas, D., Flunkert, V., Gasthaus, J., and Januschowski, T. (2019). DeepAR: Probabilistic forecasting with autoregressive recurrent networks. *International Journal of Forecasting*.

[17] Welch, I. and Goyal, A. (2008). A Comprehensive Look at The Empirical Performance of Equity Premium Prediction. *Review of Financial Studies*, 21(4):1455–1508.

[18] Zou, H. and Hastie, T. (2005). Regularization and variable selection via the Elastic Net. *Journal of the Royal Statistical Society, Series B*, 67:301–320.

A Additional details: models

In this section, we give a brief overview of all the models considered in the simulation and empirical study.

A.1 Linear models

Linear models model the conditional expectation $g^*(z_{i,t})$ as a linear function of the predictors and the parameter vector θ:

$$g(z_{i,t};\theta) = z_{i,t}'\theta \tag{6}$$

This yields the OLS estimator when optimized with respect to MSE, and the LAD estimator when optimized with respect to MAE.

A.2 Elastic nets

Elastic Nets are similar to linear models but differ via the addition of a penalty term in the loss function:

$$\mathcal{L}(\theta;.) = \underbrace{\mathcal{L}(\theta)}_{\text{Loss Function}} + \underbrace{\phi(\theta;.)}_{\text{Penalty Term}} \tag{7}$$

where the elastic net penalty [18] is:

$$\phi(\theta;\lambda,\rho) = \lambda(1-\rho)\sum_{j=1}^{P}|\theta_j| + \frac{1}{2}\lambda\rho\sum_{j=1}^{P}\theta_j^2 \tag{8}$$

Further details are given in [18].

A.3 Random forests

Random Forests are an extension of Classification and Regression Trees (CART) proposed by [2] (see for more comprehensive details). CART are fully non-parametric models that can capture complex multi-way interactions. A tree "grows" in a series of iterations. With each iteration, a split ("branch") is made along one predictor such that it is the best split available at that stage with respect to minimizing the loss function. These steps are continued until each observation is its own node, or more commonly until the stopping criterion is met. The eventual model slices the predictor space into rectangular partitions, and predicts the unknown function $g^*(z_{i,t})$ with the average value of the outcome variable in each partition. The prediction of a tree, $\mathcal{T}$, with K "leaves" (terminal nodes), and depth L is

$$g(z_{i,t};\theta,K,L) = \sum_{k=1}^{K}\theta_k \mathbf{1}_{z_{i,t}\in C_k(L)} \tag{9}$$

where $C_k(L)$ is one of the K partitions in the model. For this study, only recursive binary trees were considered. Though trees were originally proposed and fit with respect to minimizing MSE, they can be grown with respect to a variety of loss functions, including MAE, where the loss within each C partition is denoted by $H(\theta, C)$:

$$H(\theta, C) = \frac{1}{|C|} \sum_{z_{i,t} \in C} L(r_{i,t+1} - \theta) \tag{10}$$

where $|C|$ denotes the number of observations in set C (partition). Given C, it is clear that the optimal choice for minimising MSE is simply $\theta = \frac{1}{|C|} \sum_{z_{io,t} \in C} r_{i,t+1}$ i.e. the average of the partition, and the median of the partition when minimising MAE.

Trees, grown to a deep enough level, are highly unbiased and flexible, as each partition can potentially predict a single, or low number of observations. The trade-off is their high variance and instability.

A.4 Feed forward neural networks

More specifically, a neural network consists of layers denoted by $l = 0, 1, \ldots, L$, with $l = 0$ denoting the input layer and $l = L$ denoting the output layer. The input layer is defined by the scaled predictor set, $x^{(0)} = (1, z_1, \ldots, z_N)'$. The model adds complexity through the use of one or more hidden layer, each containing $K^{(l)}$ "neurons". Each neuron linearly aggregates the values of the previous layer, and applies some non-linear "activation function" which we denote as α to its aggregated signal before sending its output to the next layer. The output of neuron k in layer l is then $x_k^{(l)}$. Next, define the vector of outputs for this layer as $x^{(l)} = (1, x_1^{(l)}, \ldots, x_{K^{(l)}}^{(l)})'$. The recursive output formula for the neural network at each neuron in layer $l > 0$ is then:

$$x_k^{(l)} = \alpha(x^{(l-1)'} \theta_k^{l-1}), \tag{11}$$

where $\alpha()$ represents the activation function for that layer with the final output [8]

$$g(z; \theta) = x^{(L-1)'} \theta^{L-1} \tag{12}$$

The neural network's weight and bias parameters for each layer are estimated by minimizing the loss function with respect to the parameters, i.e. by calculating the partial derivative with respect to a specific weight or bias element.

Due to the complexity and hence non-existent analytical form for this solution, this is typically found via backpropagation, an algorithm which exploits the chain rule of the partial derivative and iteratively finds a local optimum using a first order gradient based algorithm, also known as "gradient descent." The gradient descent algorithm minimizes some function (such as the loss function in the context of machine learning) by iteratively moving in the direction of steepest descent, defined as the negative gradient. Formally, for a loss function $L(x)$ that is defined and has a gradient defined in the neighbourhood of the parameter set a, the updating algorithm is:

$$a_{n+1} = a_n - \gamma \Delta F(a_n) \tag{13}$$

where γ controls the size of each update. This γ parameter is known as the learning rate in neural network training, and controlling this is critical for good performance. As the loss functions of neural networks can be very complex with many local minima, the learning rate should be high enough such that the parameter updates are large enough to skip or jump over them. Too large of a learning rate however, and the neural may fail to converge to a solution at all. Due to computational limitations, we tune the learning rate manually, and consider a variety of different "optimizers", or algorithms which adapt the learning rate in different ways (see Appendix for computational details).

For our application, we considered the following grid of hyperparameters:

Further details are given in cite().

A.5 Long short term memory networks

Long short term memory (LSTM) networks are , initially proposed by [8] (see for more comprehensive details). The general model setup for an LSTM is as follows:

For our application, we considered the following grid of hyperparameters:

[8] Note that the specification of a constant "1" at the beginning of each layer is the same as specifying a bias term as is popular in other parametrizations.

Table 3: Hyperparameters considered for LSTMs

Hyperparameter	Grid
Activation Function	ReLU, Leaky ReLU, tanh
Optimizer	ADAM, NADAM, SGD, RMSPROP
Learning Rate	(0.1, 0.01, 0.001, 0.0001)
L1 Penalty	(1, 0.1, 0.01, 0.001)
Batch Size	(32, 64, 128, 256, 512, 1024, 2048)
Early Stopping Patience	(10, 20, 30, 40, 50)

Table 4: Hyperparameters considered for LSTMs

Hyperparameter	Grid
Activation Function	ReLU, Leaky ReLU, tanh
Optimizer	ADAM, NADAM, SGD, RMSPROP
Learning Rate	(0.1, 0.01, 0.001, 0.0001)
L1 Penalty	(1, 0.1, 0.01, 0.001)
Batch Size	(32, 64, 128, 256, 512, 1024, 2048)
Early Stopping Patience	(10, 20, 30, 40, 50)

A.6 FFORMA

Feature-based Forecast Model Averaging, ([13]) is an automated method for obtaining weighted forecast combinations for time series. We provide a brief overview of the two phases in this methodology.

We follow cite()'s selection of time series features as inputs to the meta-learner.

To incorporate all regressors in each individual time series model, we applied dimensional reduction techniques of PCA and UMAP to generate new feature mappings for use in GARCH (1, 1) models (generally the best performing of the constituent models). It was noted that none of the new external regressors as generated by these feature mappings improved fit, however.

The constituent models we considered are:

- Naive
- Random walk with drift
- Theta method
- ARIMA
- ETS
- TBATS
- Neural network auto-regressive model
- ARMA (1, 1) with Generalized Error Distribution GARCH(1, 1) errors
- ARMA (1, 1) with Generalized Error Distribution GARCH(1, 1) errors and UMAP external regressors

The time series features used to train the meta-model are detailed in cite(), with the addition of realized volatility.

Note that because financial returns data does not typically exhibit seasonality, features and constituent models related which utilized seasonality were omitted.

A.7 DeepAR

DeepAR is a generalization of traditional Auto Regressive (AR) models to include additional layers into order to introduce non-linearities into the model. We provide the general formulation here - for full details see [16].

DeepAR aims to model the conditional distribution of the

$$P(\mathbf{z}_{i,t_0:T}|\mathbf{z}_{i,1:t_0-1},\mathbf{x}_{i,1:T})$$

of the future of each time series $[z_{i,t_0}, z_{i,t_0+1}, \dots, z_{i,T}] := \mathbf{z}_{i,t_0:T}$ given its past $[z_{i,1}, \dots, z_{i,t_0-2}, z_{i,t_0-1}] := \mathbf{z}_{i,1:t_0-1}$, where t_0 denotes the time point from which we assume $z_{i,t}$ to be unknown at prediction time, and $\mathbf{x}_{i,1:T}$ are covariates that are assumed to be known for all time points. To prevent confusion we avoid the ambiguous terms "past" and "future" and will refer to time ranges $[1, t_0 - 1]$ and $[t_0, T]$ as the conditioning range and prediction range, respectively. During training, both ranges have to lie in the past so that the $z_{i,t}$ are observed, but during prediction $z_{i,t}$ is only available in the conditioning range. Note that the time index t is relative, i.e. $t = 1$ can correspond to a different actual time period for each i.

We assume that our model distribution $Q_\Theta(\mathbf{z}_{i,t_0:T}|\mathbf{z}_{i,1:t_0-1},\mathbf{x}_{i,1:T})$ consists of a product of likelihood factors

$$Q_\Theta(\mathbf{z}_{i,t_0:T}|\mathbf{z}_{i,1:t_0-1},\mathbf{x}_{i,1:T}) = \prod\nolimits_{t=t_0}^{T} Q_\Theta(z_{i,t}|\mathbf{z}_{i,1:t-1},\mathbf{x}_{i,1:T}) = \prod\nolimits_{t=t_0}^{T} \ell(z_{i,t}|\theta(\mathbf{h}_{i,t},\Theta))$$

parametrized by the output $\mathbf{h}_{i,t}$ of an autoregressive recurrent network

$$\mathbf{h}_{i,t} = h\left(\mathbf{h}_{i,t-1}, z_{i,t-1}, \mathbf{x}_{i,t}, \Theta\right) \,, \tag{14}$$

where h is a function implemented by a multi-layer recurrent neural network with LSTM cells.[9] The model is autoregressive, in the sense that it consumes the observation at the last time step $z_{i,t-1}$ as an input, as well as recurrent, i.e. the previous output of the network $\mathbf{h}_{i,t-1}$ is fed back as an input at the next time step. The likelihood $\ell(z_{i,t}|\theta(\mathbf{h}_{i,t}))$ is a fixed distribution whose parameters are given by a function $\theta(\mathbf{h}_{i,t},\Theta)$ of the network output $\mathbf{h}_{i,t}$ (see below).

Information about the observations in the conditioning range $\mathbf{z}_{i,1:t_0-1}$ is transferred to the prediction range through the initial state $\mathbf{h}_{i,t_0-1}$. In the sequence-to-sequence setup, this initial state is the output of an *encoder network*. While in general this encoder network can have a different architecture, in our experiments we opt for using the same architecture for the model in the conditioning range and the prediction range (corresponding to the *encoder* and *decoder* in a sequence-to-sequence model). Further, we share weights between them, so that the initial state for the decoder $\mathbf{h}_{i,t_0-1}$ is obtained by computing (14) for $t = 1, \dots, t_0 - 1$, where all required quantities are observed. The initial state of the encoder $\mathbf{h}_{i,0}$ as well as $z_{i,0}$ are initialized to zero.

Given the model parameters Θ, we can directly obtain joint samples $\tilde{\mathbf{z}}_{i,t_0:T} \sim Q_\Theta(\mathbf{z}_{i,t_0:T}|\mathbf{z}_{i,1:t_0-1},\mathbf{x}_{i,1:T})$ through ancestral sampling: First, we obtain $\mathbf{h}_{i,t_0-1}$ by computing (14) for $t = 1, \dots, t_0$. For $t = t_0, t_0 + 1, \dots, T$ we sample $\tilde{z}_{i,t} \sim \ell(\cdot|\theta(\tilde{\mathbf{h}}_{i,t},\Theta))$ where $\tilde{\mathbf{h}}_{i,t} = h\left(\mathbf{h}_{i,t-1}, \tilde{z}_{i,t-1}, \mathbf{x}_{i,t}, \Theta\right)$ initialized with $\tilde{\mathbf{h}}_{i,t_0-1} = \mathbf{h}_{i,t_0-1}$ and $\tilde{z}_{i,t_0-1} = z_{i,t_0-1}$. Samples from the model obtained in this way can then be used to compute quantities of interest, e.g. quantiles of the distribution of the sum of values for some time range in the future.

Further details are given in cite().

[9] Details of the architecture and hyper-parameters are given in the supplementary material.

B Additional details: simulation design

In this section, we give additional features of the simulation design required to implenent our results. All code and data can be found at:

https://github.com/Meron35/Evaluation-of-Machine-Learning-in-Asset-Pricing.

B.1 Simulation Design

We begin with the simulation study as a way to explore how ML performs with regards to the stylized facts of empirical returns in a controlled environment. We simulate according to a design which incorporates low signal to noise ratio, stochastic volatility in errors, persistence and cross sectional correlation in regressors. Our specification is a latent factor model for excess returns r_{t+1}, for $t = 1, \ldots, T$:

$$r_{i,t+1} = g\left(z_{i,t}\right) + \beta_{i,t+1} v_{t+1} + e_{i,t+1}; \quad z_{i,t} = \left(1, x_t\right)' \otimes c_{i,t}, \quad \beta_{i,t} = \left(c_{i1,t}, c_{i2,t}, c_{i3,t}\right) \tag{15}$$

$$e_{i,t+1} = \sigma_{i,t+1} \varepsilon_{i,t+1}; \tag{16}$$

$$\log(\sigma^2_{i,t+1}) = \omega + \gamma \log(\sigma^2_t) + \sigma_u u; \quad u \sim N(0,1) \tag{17}$$

where v_{t+1} is a 3×1 vector of errors, $w_{t+1} \sim N(0,1)$, $\varepsilon_{i,t+1} \sim N(0,1)$ scalar error terms, matrix C_t is an $N \times P_c$ matrix of latent factors, where the first three columns correspond to $\beta_{i,t}$, across the $1 \leq i \leq N$ dimensions, while the remaining $P_c - 3$ factors do not enter the return equation. The $P_x \times 1$ vector x_t is a 3×1 multivariate time series, and ε_{t+1} is a $N \times 1$ vector of idiosyncratic errors. The parameters of these were tuned such that the annualized volatility of each return series was approximately 22%, as is often observed empirically.

Simulating characteristics We build in correlation across time among factors by drawing normal random numbers for each $1 \leq i \leq N$ and $1 \leq j \leq P_c$, according to :

$$\bar{c}_{ij,t} = \rho_j \bar{c}_{ij,t-1} + \epsilon_{ij,t}; \quad \rho_j \sim \mathcal{U}\left(0.5, 1\right) \tag{18}$$

We then build in cross sectional correlation:

$$\widehat{C}_t = L\overline{C}_t; \quad B = LL' \tag{19}$$

$$B := \Lambda\Lambda' + 0.1\mathbb{I}_n, \quad \Lambda_i = (\lambda_{i1}, \ldots, \lambda_{i4}), \quad \lambda_{ik} \sim N(0, \lambda_{sd}), \; k = 1, \ldots, 4 \tag{20}$$

where B serves as a variance covariance matrix with λ_{sd} its density, and L represents the lower triangle matrix of B via the Cholesky decomposition. λ_{sd} values of 0.01, 0.1 and 1 were used to explore increasing degrees of cross sectional correlation. Characteristics are then normalized to be within $[-1, 1]$ for each $1 \leq i \leq N$ and for $j = 1, \ldots, P_c$ via:

$$c_{ij,t} = \frac{2}{n+1} \operatorname{rank}\left(\hat{c}_{ij,t}\right) - 1. \tag{21}$$

Simulating macroeconomic series We consider a Vector Autoregression (VAR) model for x_t, a 3×1 multivariate time series [10]:

$$x_t = Ax_{t-1} + u_t; \quad A = 0.95I_3; \quad u_t \sim N\left(\mu = (0,0,0)', \Sigma = I_3\right)$$

Simulating return series We consider three different functions for $g(z_{i,t})$:

$$(1)\; g_1\left(z_{i,t}\right) = \left(c_{i1,t}, c_{i2,t}, c_{i3,t} \times x_t'[3,]\right)\theta_0 \tag{22}$$

$$(2)\; g_2\left(z_{i,t}\right) = \left(c^2_{i1,t}, c_{i1,t} \times c_{i2,t}, \operatorname{sgn}\left(c_{i3,t} \times x_t'[3,]\right)\right)\theta_0 \tag{23}$$

$$(3)\; g_3\left(z_{i,t}\right) = \left(1[c_{i3,t} > 0], c^3_{i2,t}, c_{i1,t} \times c_{i2,t} \times 1[c_{i3,t} > 0], \operatorname{logit}\left(c_{i3,t}\right)\right)\theta_0 \tag{24}$$

where $x_t'[3,]$ denotes the third element of the x_t' vector. $g_1\left(z_{i,t}\right)$ allows the characteristics to enter the return equation linearly, and $g_2\left(z_{i,t}\right)$ and $g_3\left(z_{i,t}\right)$ allow the characteristics to enter the return equation interactively and non-linearly. [11] θ^0 was tuned such that the predictive R^2 was approximately 5%.

[10] More complex specifications for A were briefly explored, but these did not have a significant impact on results.

[11] (g_1, g_2 correspond to the simulation design used by [6].)

The simulation design results in $3 \times 3 = 9$ different simulated datasets, each with $N = 200$ stocks, $T = 180$ periods and $P_c = 100$ characteristics. Each design was simulated 10 times to assess the robustness of machine learning algorithms, with the number of simulations kept low for computational feasibility. We employ the hybrid data splitting approach with a training:validation length ratio of approximately 1.5 and a test set that is 1 year in length. Other schemes in the forecasting literature such as using an “inner" rolling window validation loop to find the best hyperparameters on average, finally aggregating them in an “outer" loop for a more robust error were considered but not implemented due to a) computational feasibility and b) the relative instability of optimal hyperparameters across different different windows.

B.2 Simulation Study Results

B.2.1 Prediction Performance

Table 5: Simulation Study Loss Statistics

		g1			g2			g3		
model	Corr	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2
LM.MSE	0.01	0.0366775	0.0027400	0.0082732	0.0382548	0.0028801	-0.1117880	0.0373098	0.0027954	-0.0320680
	0.10	0.0369652	0.0027653	-0.0110198	0.0385796	0.0029144	-0.1429443	0.0375694	0.0028168	-0.0549404
	1.00	0.0429486	0.0034141	-0.4387965	0.0453765	0.0037172	-0.7809535	0.0434339	0.0034688	-0.4887785
LM.MAE	0.01	0.0366417	0.0027373	0.0090496	0.0383478	0.0028862	-0.1163694	0.0373235	0.0027967	-0.0351619
	0.10	0.0368113	0.0027555	0.0029188	0.0387449	0.0029275	-0.1525797	0.0374894	0.0028098	-0.0476746
	1.00	0.0423399	0.0033445	-0.3930442	0.0453420	0.0036847	-0.7699555	0.0435349	0.0034682	-0.5445237
ELN.MSE	0.01	0.0345878	0.0025663	0.1403351	0.0362229	0.0026898	0.0368766	0.0353534	0.0026227	0.0991416
	0.10	0.0345630	0.0025643	0.1442376	0.0361830	0.0026860	0.0372585	0.0352923	0.0026167	0.1002410
	1.00	0.0346142	0.0025676	0.1671841	0.0362761	0.0026980	0.0378391	0.0354437	0.0026300	0.1198755
ELN.MAE	0.01	0.0345786	0.0025652	0.1409821	0.0361950	0.0026882	0.0391694	0.0353345	0.0026210	0.1004424
	0.10	0.0345582	0.0025637	0.1446272	0.0361730	0.0026877	0.0388747	0.0352851	0.0026167	0.1009186
	1.00	0.0345989	0.0025667	0.1677712	0.0363047	0.0027028	0.0365834	0.0354652	0.0026310	0.1180225
RF.MSE	0.01	0.0357752	0.0026710	0.0634257	0.0357179	0.0026571	0.0676147	0.0358032	0.0026613	0.0702977
	0.10	0.0357695	0.0026649	0.0667382	0.0356845	0.0026525	0.0691389	0.0358666	0.0026704	0.0628386
	1.00	0.0362325	0.0026977	0.0687741	0.0359893	0.0026833	0.0571035	0.0362129	0.0026952	0.0698868
RF.MAE	0.01	0.0354594	0.0026434	0.0833385	0.0354204	0.0026305	0.0876529	0.0355399	0.0026446	0.0865291
	0.10	0.0355153	0.0026489	0.0814253	0.0354894	0.0026345	0.0834048	0.0355688	0.0026438	0.0816426
	1.00	0.0359158	0.0026747	0.0870806	0.0356434	0.0026445	0.0809651	0.0360529	0.0026786	0.0753573
NN1.MSE	0.01	0.0364516	0.0027219	0.0163443	0.0367677	0.0027319	-0.0039174	0.0366874	0.0027384	0.0093355
	0.10	0.0364624	0.0027191	0.0204223	0.0367762	0.0027345	-0.0072588	0.0367326	0.0027372	0.0029550
	1.00	0.0375452	0.0028206	-0.0144520	0.0370492	0.0027638	-0.0146973	0.0374589	0.0027975	-0.0124689
NN1.MAE	0.01	0.0359604	0.0026786	0.0558139	0.0369206	0.0027474	-0.0151053	0.0363047	0.0026996	0.0393707
	0.10	0.0360823	0.0026866	0.0506976	0.0370100	0.0027503	-0.0205616	0.0363220	0.0027022	0.0323034
	1.00	0.0378894	0.0028338	-0.0431818	0.0379790	0.0028445	-0.0840747	0.0373056	0.0027926	0.0021783
NN2.MSE	0.01	0.0370187	0.0027850	-0.0217869	0.0373197	0.0027752	-0.0433537	0.0370890	0.0027745	-0.0173037
	0.10	0.0369775	0.0027651	-0.0212763	0.0370088	0.0027478	-0.0275384	0.0369898	0.0027584	-0.0206446
	1.00	0.0375360	0.0028138	-0.0139783	0.0369035	0.0027518	-0.0058664	0.0375157	0.0028087	-0.0169336
NN2.MAE	0.01	0.0358939	0.0026718	0.0577427	0.0368335	0.0027396	-0.0071579	0.0363352	0.0027028	0.0363052
	0.10	0.0358898	0.0026681	0.0603096	0.0369367	0.0027503	-0.0170774	0.0362701	0.0026960	0.0371567
	1.00	0.0374795	0.0028142	-0.0095290	0.0377146	0.0028226	-0.0653904	0.0374711	0.0028038	-0.0101183
NN3.MSE	0.01	0.0367827	0.0027568	-0.0067616	0.0368397	0.0027379	-0.0075249	0.0370360	0.0027644	-0.0200783
	0.10	0.0369384	0.0027613	-0.0153994	0.0368517	0.0027384	-0.0151060	0.0368743	0.0027573	-0.0044063
	1.00	0.0374242	0.0028081	-0.0129638	0.0369376	0.0027543	-0.0063529	0.0374202	0.0027991	-0.0103479
NN3.MAE	0.01	0.0358164	0.0026697	0.0654321	0.0369345	0.0027491	-0.0163983	0.0364712	0.0027181	0.0299484
	0.10	0.0358935	0.0026771	0.0620017	0.0368590	0.0027406	-0.0118497	0.0362000	0.0026932	0.0406114
	1.00	0.0370087	0.0027744	0.0213288	0.0372705	0.0027832	-0.0296437	0.0374132	0.0027916	-0.0083067
NN4.MSE	0.01	0.0368808	0.0027586	-0.0206197	0.0368555	0.0027423	-0.0077152	0.0371255	0.0027752	-0.0265634
	0.10	0.0368772	0.0027610	-0.0145791	0.0372207	0.0027615	-0.0487112	0.0368718	0.0027480	-0.0088940
	1.00	0.0373820	0.0028051	-0.0064811	0.0368966	0.0027505	-0.0053689	0.0373542	0.0027970	-0.0077389
NN4.MAE	0.01	0.0359348	0.0026782	0.0577196	0.0368974	0.0027487	-0.0109166	0.0367079	0.0027376	0.0070464
	0.10	0.0358281	0.0026651	0.0650415	0.0369333	0.0027494	-0.0191117	0.0362730	0.0026954	0.0377039
	1.00	0.0370948	0.0027786	0.0198663	0.0373230	0.0027947	-0.0293767	0.0373013	0.0027871	-0.0018876
	0.01	0.0372306	0.0027846	-0.0499701	0.0369309	0.0027474	-0.0170017	0.0371140	0.0027720	-0.0218954
	0.10	0.0370264	0.0027669	-0.0321897	0.0371758	0.0027623	-0.0394362	0.0369093	0.0027565	-0.0113522

Table 5: Simulation Study Loss Statistics

		g1			g2			g3		
model	Corr	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2
NN5.MSE	1.00	0.0373642	0.0027949	-0.0104952	0.0369277	0.0027552	-0.0053762	0.0374751	0.0028071	-0.0149737
NN5.MAE	0.01	0.0358880	0.0026693	0.0585792	0.0368354	0.0027380	-0.0086455	0.0366851	0.0027371	0.0046430
	0.10	0.0360381	0.0026803	0.0509764	0.0367451	0.0027273	-0.0049349	0.0364843	0.0027103	0.0181920
	1.00	0.0372849	0.0027940	0.0025412	0.0370382	0.0027652	-0.0127290	0.0371925	0.0027753	0.0025723
LSTM.MSE	0.01	0.0372963	0.0027982	-0.0432886	0.0372268	0.0027764	-0.0447640	0.0375909	0.0028180	-0.0625164
	0.10	0.0372369	0.0027946	-0.0319550	0.0371342	0.0027674	-0.0382547	0.0371984	0.0027845	-0.0303936
	1.00	0.0381282	0.0028506	-0.0820266	0.0373821	0.0027921	-0.0442426	0.0377803	0.0028300	-0.0443304
LSTM.MAE	0.01	0.0374310	0.0028046	-0.0564056	0.0373372	0.0027801	-0.0518537	0.0376270	0.0028169	-0.0674327
	0.10	0.0374461	0.0028036	-0.0629523	0.0371178	0.0027679	-0.0325442	0.0372409	0.0027931	-0.0333196
	1.00	0.0380266	0.0028456	-0.0614833	0.0374152	0.0027902	-0.0455057	0.0377435	0.0028252	-0.0458837
FFORMA.MSE	0.01	0.0382767	0.0028820	-0.1326717	0.0384600	0.0028893	-0.1473902	0.0424656	0.0033108	-0.4861451
	0.10	0.0383581	0.0028947	-0.1407652	0.0384795	0.0028912	-0.1600616	0.0423231	0.0032914	-0.4739906
	1.00	0.0388747	0.0029647	-0.1312392	0.0388080	0.0029331	-0.1659900	0.0430130	0.0033713	-0.4709541
FFORMA.MAE	0.01	0.0387548	0.0029387	-0.1797483	0.0387472	0.0029178	-0.1740938	0.0429893	0.0033651	-0.5279094
	0.10	0.0389359	0.0029511	-0.1927930	0.0387959	0.0029457	-0.1759939	0.0430966	0.0034057	-0.5863752
	1.00	0.0392468	0.0029721	-0.1636559	0.0393873	0.0029960	-0.2116186	0.0437090	0.0034483	-0.5260813
DeepAR	0.01	0.0382993	0.0029000	-0.1289295	0.0384895	0.0029121	-0.1325183	0.0393898	0.0030161	-0.2049803
	0.10	0.0388318	0.0029353	-0.1816633	0.0384345	0.0029045	-0.1318744	0.0391770	0.0029932	-0.1905583
	1.00	0.0405348	0.0031590	-0.2391417	0.0387870	0.0029524	-0.1440285	0.0396918	0.0030422	-0.1823646

B.3 Random Forest VIMPs

We note that random forest methods typically have their own methodologies to calculate variable importance (VIMP) which are different to the VIMP metric presented in the main body of the paper. We provide two popular schemes of calculating random forest VIMP - Breiman-cutler VIMP, ([2]) and Ishwaran-Kogalur VIMP , ([10]), and show that importantly, the overall conclusion regarding factor selection does not change with respect to which methodology employed.

Figure 3: Simulation Breiman-Cutler VIMP

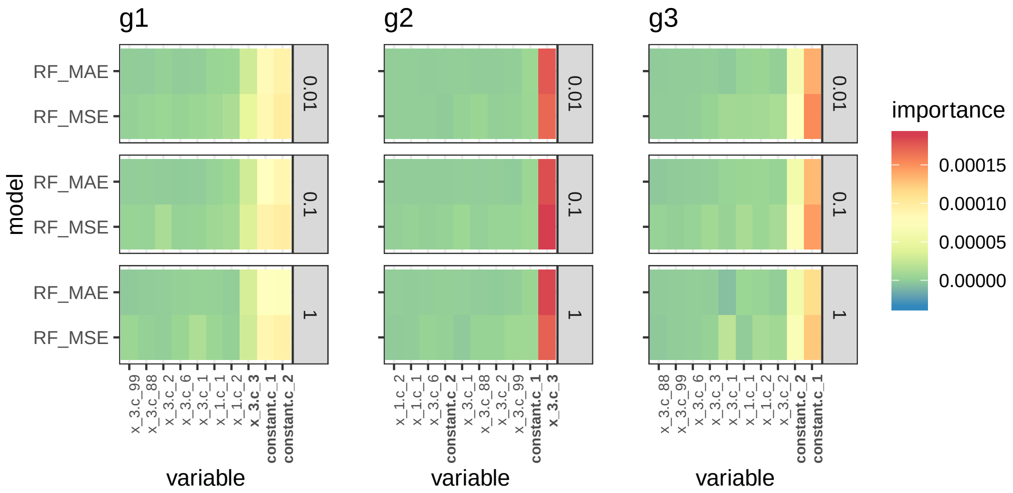
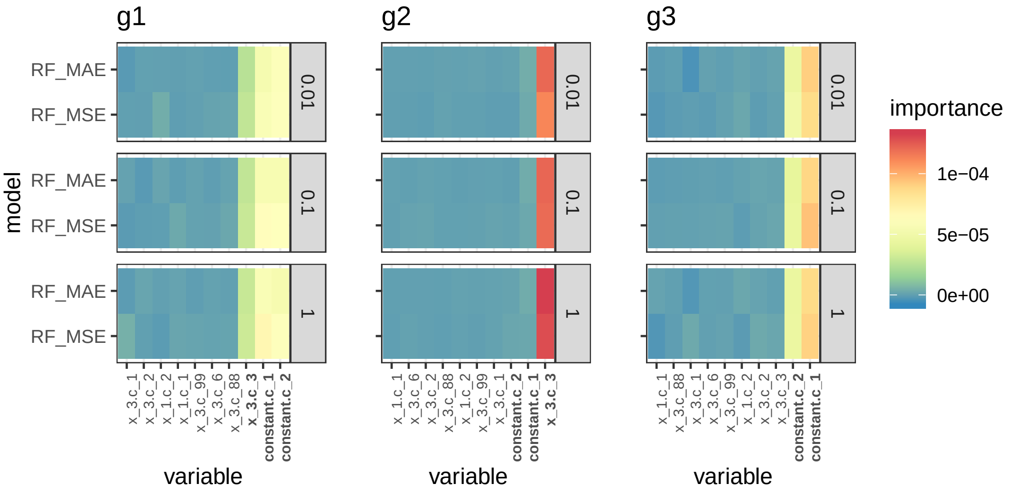


Figure 4: Simulation Ishwaran-Kogalur VIMP



C Additional details: Empirical analysis

C.1 Data & cleaning

We begin by obtaining monthly individual price data from CRSP for all firms listed in the NYSE, AMEX and NASDAQ, starting from 1957 (starting date of the S&P 500) and ending in December 2016, totalling 60 years. To build individual factors, we construct a factor set based on the cross section of returns literature. This is the same data used in [6]. We restrict our dataset to begin from 1993 Q3 and end on 2016 Q4 to alleviate data quality issues. Our individual factor set contains 94 characteristics: 61 updated annually, 13 updated quarterly and 20 updated monthly [12].

We detail our cleaning procedure of this dataset. To reduce the size of the dataset and increase feasibility, we only consider equities with a share price larger than $5 traded primarily on the NASDAQ. To achieve a balance between having a dataset with enough data points and variability among factors, the dataset was converted to a quarterly format. Quarterly returns were then constructed using the PRC variable according to:

$$RET_t = (PRC_t - PRC_{t-1})/PRC_{t-1} \tag{25}$$

We allow all stocks which have a quarterly return to enter the dataset, even if they disappear from the dataset for certain periods. To deal with missing data, any characteristics that had over 20% of their data missing were omitted. Remaining missing data were then imputed using their cross sectional medians for each year.

We then follow [6] and construct eight macroeconomic factors following the variable definitions in [17] (see Table 6). These factors were lagged by one period so as to be used to predict one period ahead quarterly returns. The 3-month Treasury Bill rate was also used from this source to proxy for the risk free rate in order to construct excess quarterly returns.

The two sets of factors were then combined to form a baseline set of covariates, which we define throughout all methods and analysis as:

$$z_{i,t} = (1, x_t)' \otimes c_{i,t} \tag{26}$$

where $c_{i,t}$ is a P_c matrix of characteristics for each stock i, and $(1, x_t)'$ is a $P_x \times 1$ vector of macroeconomic predictors, , and $\otimes$ represents the Kronecker product. $z_{i,t}$ is therefore a $P_x P_c$ vector of features for predicting individual stock returns and includes interactions between stock level

[12] The dataset also included 74 Standard Industrial Classification (SIC) codes, but these were omitted due to their inconsistency, and inadequateness at classifying companies, as noted by WRDS

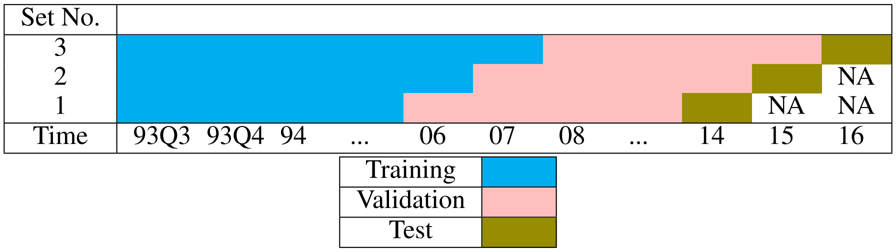
Table 6: Macroeconomic Factors, ([17])

No.	Acronym	Macroeconomic Factor
1	macro_dp	Dividend Price Ratio
2	macro_ep	Earnings Price Ratio
3	macro_bm	Book to Market Ratio
4	macro_ntis	Net Equity Expansion
5	macro_tbl	Treasury Bill Rate
6	macro_tms	Term Spread
7	macro_dfy	Default Spread
8	macro_svar	Stock Variance

characteristics and macroeconomic variables. The total number of covariates in this baseline set is $61 \times (8+1) = 549$[13]. The final dataset contains 202, 066 individual observations.

We mimic the sample splitting procedure used in the simulation study: the dataset was split such that the training and validation sets were split such that the training set was approximately 1.5 times the length of the validation set, in order to predict a test set that is one year in length (see Figure 5).

Figure 5: Empirical Data Sample Splitting Procedure

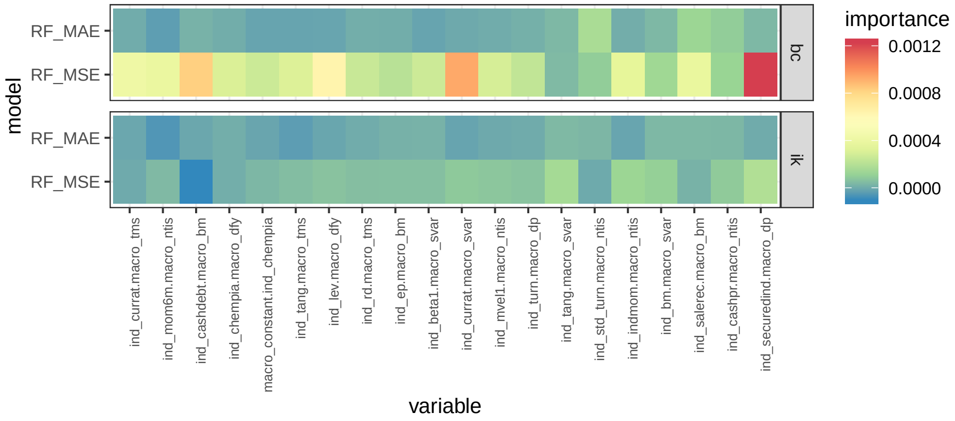


[13] As the individual and macroeconomic factors can have similar names, individual and macroeconomic factors were prefixed with ind_ and macro_ respectively.

Table 7: Empirical Study Loss Statistics

	Sample 1			Sample 2			Sample 3		
model	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2
LM.MSE	0.229034	0.116015	-1.808481	0.397573	0.312653	-6.329935	0.566307	0.83804	-17.522476
LM.MAE	0.273452	0.15894	-2.8476	0.555673	0.742223	-16.400898	0.651614	1.225121	-26.077774
ELN.MSE	0.133887	0.039947	0.032956	0.140402	0.04277	-0.002712	**0.14433**	**0.043761**	**0.032789**
ELN.MAE	0.131369	0.040718	0.014306	**0.137092**	**0.041892**	**0.017875**	0.146251	0.045207	0.000835
RF.MSE	0.130366	**0.036629**	**0.113289**	0.195817	0.070642	-0.656158	0.157934	0.05122	-0.132066
RF.MAE	**0.126703**	0.036785	0.109505	0.173721	0.057546	-0.349132	0.14692	0.046037	-0.01752
NN1.MSE	0.169127	0.057044	-0.380909	0.207662	0.074751	-0.752494	0.192125	0.069738	-0.541369
NN1.MAE	0.157324	0.050418	-0.22052	0.191762	0.066746	-0.564818	0.18547	0.063053	-0.393606
NN2.MSE	0.168773	0.059436	-0.43883	0.181808	0.063232	-0.482433	0.180584	0.062745	-0.386797
NN2.MAE	0.162667	0.055447	-0.342256	0.194277	0.069386	-0.626702	0.185173	0.065186	-0.440746
NN3.MSE	0.154784	0.050152	-0.21408	0.180103	0.060193	-0.411175	0.177604	0.060404	-0.335065
NN3.MAE	0.146411	0.044901	-0.086967	0.18499	0.06461	-0.514744	0.184986	0.063861	-0.411475
NN4.MSE	0.153802	0.048641	-0.177503	0.193066	0.067515	-0.582833	0.172707	0.057774	-0.276929
NN4.MAE	0.157301	0.050286	-0.217308	0.168815	0.055711	-0.306102	0.167998	0.055129	-0.218463
NN5.MSE	0.149436	0.047279	-0.14452	0.183584	0.064137	-0.503653	0.170238	0.056992	-0.259652
NN5.MAE	0.140781	0.042832	-0.036882	0.181096	0.06216	-0.4573	0.164896	0.053458	-0.181528

Figure 6: Empirical study random forest vimps



C.2 Empirical study robustness checks & results

Better description of these is necessary. You can't just present the results here...

In addition to the main study, we provide four additional robustness checks for our empirical study, with regards to different training/validation splitting schemes, missing data imputation and additional regressors. Importantly, our overall results are consistent across all checks.

We consider training:validation length ratios of 1:1 and 1:2 in addition to 1:1.5 in the main study.

We consider changing the missing data threshold to be 10% - that is, any regressors with over 10% missing data were omitted before being imputed.

We finally consider supplementing our macroeconomic regressor set with the five Fama-French factors.

C.3 Empirical Data Results

C.3.1 Prediction Accuracy

Table 8: Missing Data Threshold Robustness Check Loss Statistics

	Sample 1			Sample 2			Sample 3		
model	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2
LM.MSE	0.247457	0.130166	-2.151058	0.541089	0.700574	-15.424468	0.615714	1.188991	-25.279238
LM.MAE	0.214055	0.102848	-1.489727	0.372683	0.259976	-5.094954	0.507397	0.766373	-15.93847
ELN.MSE	0.133887	0.039947	0.032956	0.140402	0.04277	-0.002712	**0.14433**	**0.043761**	**0.032789**
ELN.MAE	0.131338	0.040465	0.020421	**0.137083**	**0.041804**	**0.019938**	0.146589	0.045362	-0.002596
RF.MSE	0.129226	0.035869	0.131692	0.198914	0.072749	-0.705542	0.168068	0.05777	-0.276838
RF.MAE	**0.124319**	**0.035103**	**0.150229**	0.167845	0.053578	-0.256106	0.15463	0.051594	-0.140342
NN1.MSE	0.153785	0.048726	-0.179553	0.221019	0.084867	-0.98964	0.172557	0.058354	-0.289742
NN1.MAE	0.154534	0.048854	-0.18266	0.199647	0.073699	-0.727823	0.176348	0.061359	-0.356155
NN2.MSE	0.158513	0.057061	-0.381324	0.233631	0.095004	-1.227299	0.154083	0.048353	-0.068708
NN2.MAE	0.138489	0.043364	-0.049759	0.215253	0.078792	-0.847234	0.164459	0.055049	-0.216706
NN3.MSE	0.167392	0.058508	-0.416345	0.19754	0.071293	-0.671422	0.156873	0.049602	-0.096299
NN3.MAE	0.144457	0.045293	-0.096445	0.210372	0.077747	-0.822723	0.159841	0.05152	-0.138704
NN4.MSE	0.147989	0.047211	-0.142888	0.184277	0.064247	-0.506225	0.152214	0.048185	-0.064987
NN4.MAE	0.15851	0.052021	-0.259326	0.18643	0.063032	-0.477746	0.177651	0.064046	-0.415562
NN5.MSE	0.153187	0.050053	-0.211683	0.181622	0.060313	-0.413989	0.161028	0.051221	-0.132095
NN5.MAE	0.149496	0.050779	-0.229251	0.165726	0.053988	-0.265712	0.156151	0.049772	-0.100061

Figure 7: Missing Data Threshold Robustness Check Individual Factor Importance

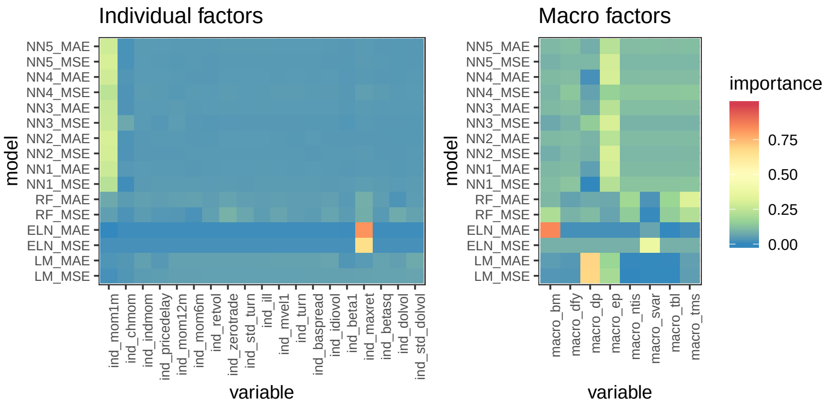


Figure 8: Missing Data Threshold Robustness Check RF VIMP

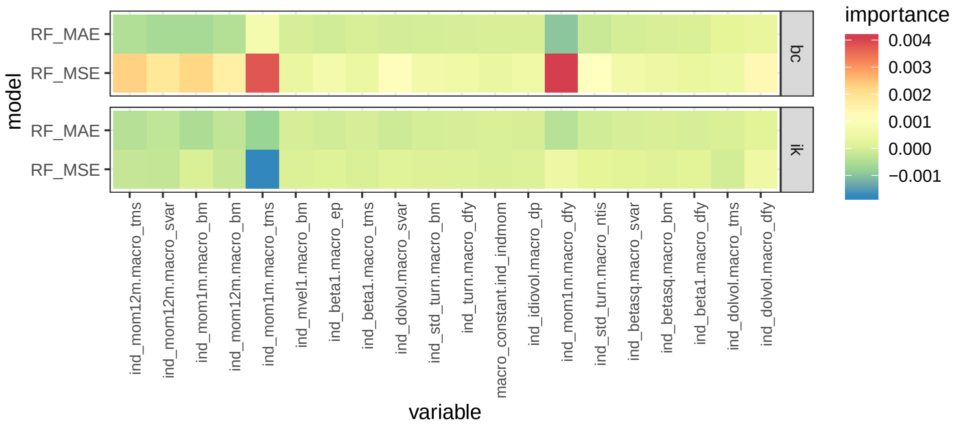


Table 9: Train:Validation 1:1 Robustness Check Loss Statistics

	Sample 1			Sample 2			Sample 3		
model	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2
LM.MSE	0.915703	2.495094	-59.401029	0.717	1.553454	-35.419641	0.451206	0.375505	-7.299459
LM.MAE	0.751551	1.583265	-37.32754	0.469831	0.524686	-11.300895	0.675112	1.105759	-23.43964
ELN.MSE	0.134609	**0.040072**	**0.029933**	0.141434	0.043169	-0.012055	**0.144375**	**0.043705**	**0.034019**
ELN.MAE	**0.131668**	0.040748	0.013583	**0.137494**	**0.042135**	**0.012178**	0.146776	0.045753	-0.01123
RF.MSE	0.155282	0.046655	-0.129427	0.210936	0.078006	-0.828784	0.229147	0.092622	-1.047155
RF.MAE	0.13882	0.04016	0.027805	0.185338	0.063217	-0.482087	0.182753	0.063873	-0.411736
NN1.MSE	0.218129	0.087699	-1.123002	0.238606	0.110201	-1.583582	0.260721	0.120908	-1.672321
NN1.MAE	0.202259	0.072844	-0.763409	0.205092	0.073567	-0.724721	0.239051	0.096477	-1.132346
NN2.MSE	0.239446	0.101312	-1.452556	0.206109	0.078412	-0.838305	0.228591	0.095126	-1.102488
NN2.MAE	0.19141	0.068261	-0.652455	0.184095	0.062366	-0.462125	0.220087	0.086888	-0.920403
NN3.MSE	0.193117	0.069206	-0.675336	0.193859	0.070747	-0.658609	0.205093	0.076497	-0.690745
NN3.MAE	0.191596	0.066926	-0.620138	0.176555	0.060022	-0.407183	0.234768	0.091003	-1.011359
NN4.MSE	0.191361	0.07068	-0.71101	0.175311	0.059253	-0.389136	0.18148	0.061718	-0.364096
NN4.MAE	0.139659	0.041096	0.005158	0.179318	0.05976	-0.401027	0.188921	0.066144	-0.461932
NN5.MSE	0.17209	0.056982	-0.379418	0.164756	0.054398	-0.275325	0.202012	0.074051	-0.636691
NN5.MAE	0.170945	0.056029	-0.356356	0.180669	0.059697	-0.399552	0.189149	0.065921	-0.456988

Figure 9: Train:Validation = 1:1 Robustness Check Individual Factor Importance

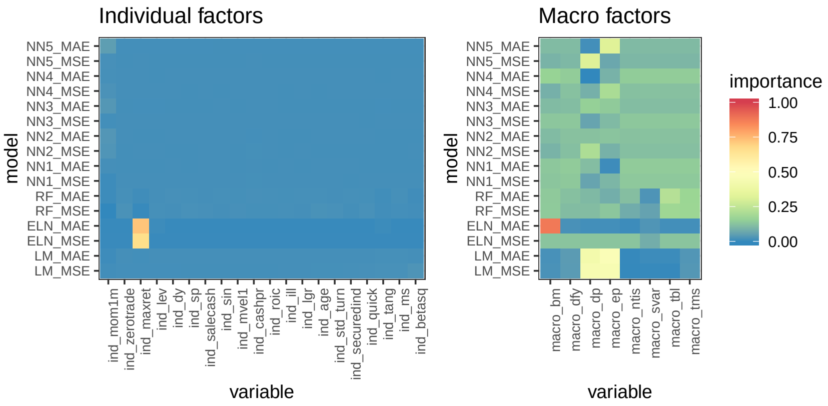


Figure 10: Train:Validation = 1:1 Robustness Check RF VIMP

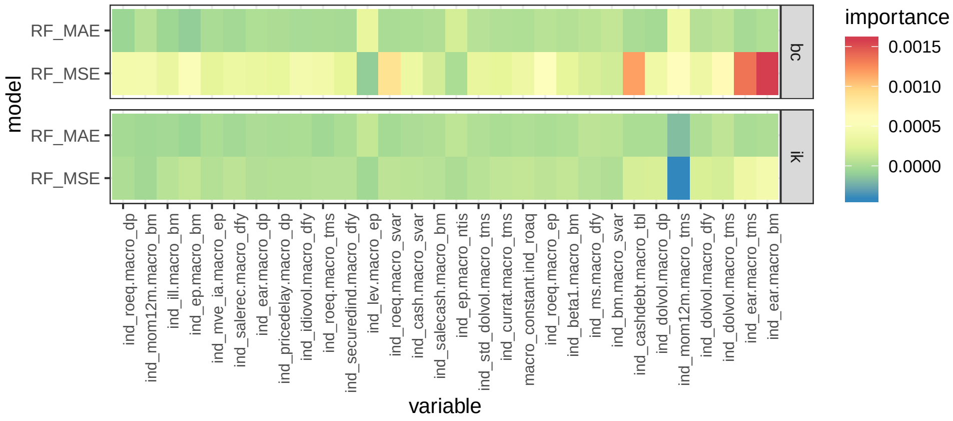


Table 10: Train:Validation 2:1 Robustness Check Loss Statistics

	Sample 1			Sample 2			Sample 3		
model	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2
LM.MSE	0.277087	0.164599	-2.98459	0.383421	0.31299	-6.337839	0.523418	0.740288	-15.361936
LM.MAE	0.246936	0.147979	-2.582262	0.277044	0.161215	-2.779579	0.487285	0.631575	-12.95915
ELN.MSE	0.133715	0.039919	0.033647	0.139723	0.042525	0.003028	**0.145034**	**0.044306**	**0.020752**
ELN.MAE	0.131237	0.040361	0.022952	**0.137205**	**0.041858**	**0.018674**	0.174408	0.064513	-0.425873
RF.MSE	0.130808	0.036982	0.104754	0.162762	0.051118	-0.198417	0.155264	0.048661	-0.075516
RF.MAE	**0.127013**	**0.036722**	**0.111033**	0.146758	0.043961	-0.030633	0.168905	0.055983	-0.237348
NN1.MSE	0.155088	0.050284	-0.217281	0.165871	0.053459	-0.253309	0.181984	0.064621	-0.428262
NN1.MAE	0.159797	0.050566	-0.224107	0.163397	0.052329	-0.226828	0.181636	0.062407	-0.379326
NN2.MSE	0.155815	0.050954	-0.233492	0.168576	0.055738	-0.306745	0.170991	0.057453	-0.269824
NN2.MAE	0.148149	0.047617	-0.152709	0.166334	0.054058	-0.26734	0.163141	0.052639	-0.163436
NN3.MSE	0.154141	0.04976	-0.204586	0.166218	0.053402	-0.251967	0.169539	0.05661	-0.251204
NN3.MAE	0.142464	0.043771	-0.059594	0.154233	0.048682	-0.141321	0.184217	0.064175	-0.418401
NN4.MSE	0.166547	0.056184	-0.360092	0.150748	0.047566	-0.115162	0.168447	0.056575	-0.250437
NN4.MAE	0.150167	0.046919	-0.135802	0.16197	0.05226	-0.225199	0.171676	0.057352	-0.267598
NN5.MSE	0.155784	0.052258	-0.265047	0.139699	0.043082	-0.010018	0.166166	0.055027	-0.216219
NN5.MAE	0.161161	0.053216	-0.28825	0.149207	0.046344	-0.086511	0.149424	0.047544	-0.050824

Figure 11: Train:Validation = 2:1 Robustness Check Individual Factor Importance

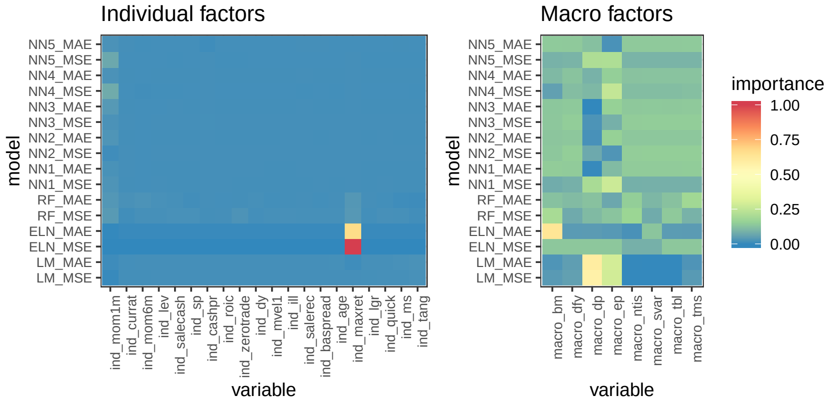


Figure 12: Train:Validation = 2:1 Robustness Check RF VIMP

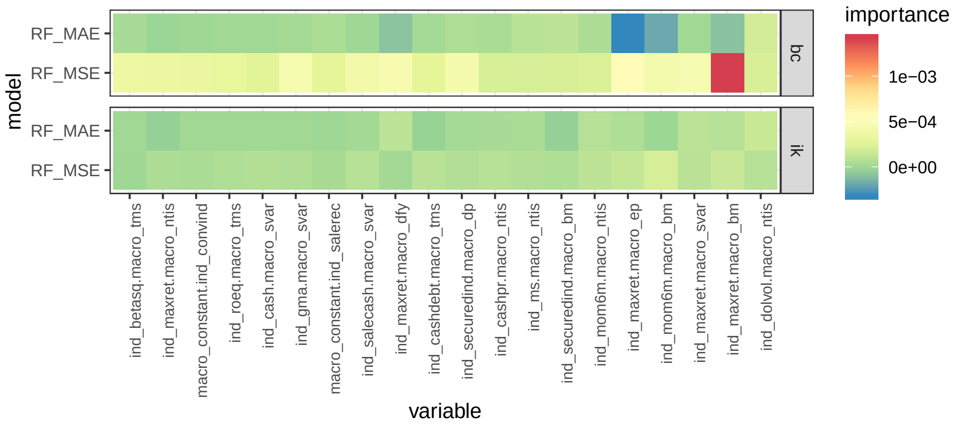


Table 11: Fama French Factor Robustness Check Loss Statistics

	Sample 1			Sample 2			Sample 3		
model	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2
LM.MSE	0.288636	0.182966	-3.42923	0.367636	0.264918	-5.210825	1.101604	5.012469	-109.78624
LM.MAE	0.280535	0.179777	-3.352038	0.376163	0.279476	-5.552114	1.25341	7.06036	-155.048996
ELN.MSE	0.13383	0.039956	0.032746	0.14022	0.0427	-0.00107	**0.144472**	**0.043852**	**0.030769**
ELN.MAE	**0.128936**	**0.039665**	**0.039798**	**0.13716**	**0.042144**	**0.011965**	0.172148	0.063154	-0.395841
RF.MSE	0.146318	0.042607	-0.031434	0.151137	0.047091	-0.104011	0.177125	0.064664	-0.429221
RF.MAE	0.138266	0.04005	0.030475	0.138714	0.042246	0.009583	0.152068	0.048488	-0.071698
NN1.MSE	0.168063	0.055354	-0.340017	0.192143	0.068904	-0.61541	0.275195	0.138165	-2.053731
NN1.MAE	0.161596	0.051507	-0.246873	0.199416	0.068181	-0.598444	0.23054	0.093434	-1.065082
NN2.MSE	0.169842	0.056899	-0.377415	0.179733	0.058966	-0.382416	0.252929	0.117102	-1.588199
NN2.MAE	0.155816	0.046809	-0.133147	0.185008	0.060854	-0.426679	0.219342	0.085115	-0.881213
NN3.MSE	0.1621	0.053165	-0.287008	0.182996	0.059643	-0.398278	0.232226	0.099353	-1.195903
NN3.MAE	0.161255	0.050737	-0.228237	0.191625	0.064676	-0.516291	0.218355	0.085297	-0.885238
NN4.MSE	0.166036	0.055575	-0.345349	0.191589	0.066207	-0.552182	0.23417	0.097348	-1.151607
NN4.MAE	0.148375	0.045227	-0.094843	0.168623	0.054176	-0.270114	0.20837	0.077667	-0.7166
NN5.MSE	0.147379	0.044503	-0.077315	0.166006	0.054935	-0.287914	0.20667	0.077866	-0.721013
NN5.MAE	0.150541	0.045723	-0.106868	0.172466	0.055402	-0.298865	0.218796	0.084938	-0.877301

Figure 13: Fama French Factors Robustness Check Individual Factor Importance

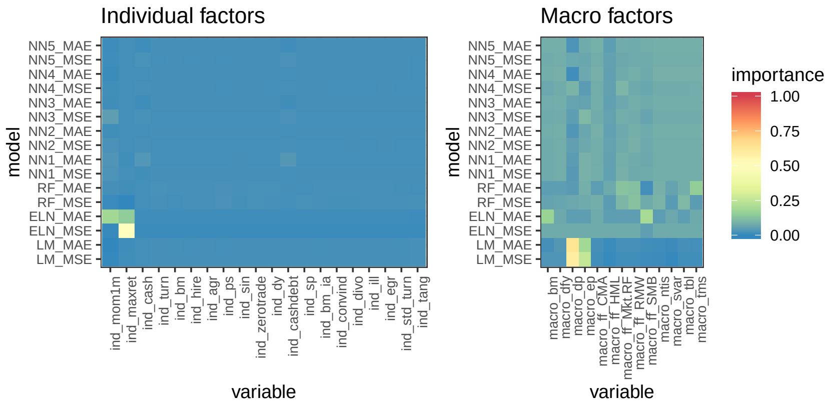
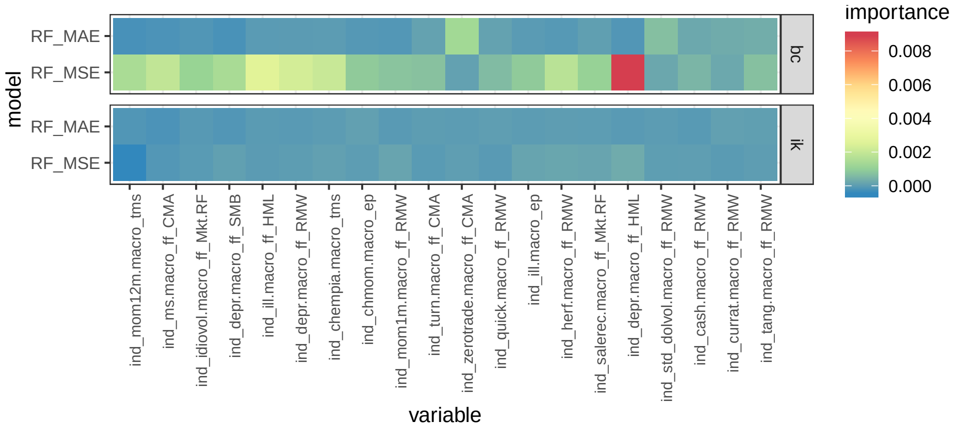


Figure 14: Fama French Factors Robustness Check RF VIMP


model
RF_MAE
RF_MSE
RF_MAE
RF_MSE
bc
ik
ind_mom12m.macro_tms
ind_ms.macro_ff_CMA
ind_idiovol.macro_ff_Mkt.RF
ind_depr.macro_ff_SMB
ind_ill.macro_ff_HML
ind_depr.macro_ff_RMW
ind_chempia.macro_tms
ind_chmom.macro_ep
ind_mom1m.macro_ff_RMW
ind_turn.macro_ff_CMA
ind_zerotrade.macro_ff_CMA
ind_quick.macro_ff_RMW
ind_ill.macro_ep
ind_herf.macro_ff_RMW
ind_salerec.macro_ff_Mkt.RF
ind_depr.macro_ff_HML
ind_std_dolvol.macro_ff_RMW
ind_cash.macro_ff_RMW
ind_currat.macro_ff_RMW
ind_tang.macro_ff_RMW
variable
importance