Evaluation of Machine Learning in Empirical Asset Pricing

Anonymous Author(s)
Affiliation
Address
email

Abstract

Several recent studies have claimed that machine learning methods provide superior predictive accuracy of asset returns, relative to simpler modelling approaches, and can correctly identify factors needed to price portfolio risk. Herein, we demonstrate that this performance is critically dependent on several features of the data being analysed; including, the training/test sample split, the frequency at which the data is observed, and the chosen loss-function. In contrast to existing studies, which claim that neural nets provide superior predictive accuracy, through a series of realistic examples that mimics the stylized facts of asset returns, we demonstrate that neural methods are easily outperformed by simpler methods, such as random forest and elastic nets.

1 Introduction

The dominance of machine learning methods in terms of predictive accuracy has now begun to filter into the application and assessment of asset pricing. The most common application of machine learning methods within finance are for portfolio construction, asset price prediction, and factor selection.

Several studies have now used machine learning techniques to analyze the cross-section of asset returns and produce portfolios that can capture nonlinear information in the cross-section of asset returns. Mortiz and Zimmermann (2016) use tree-based methods in an attempt to understand which firm-level characteristics best predict the cross-section of stock returns, where this information can then be used within portfolio sorting to help mitigate risk. Similarly, Messemer (2017) uses deep feedforward neural nets (DFNs) to construct portfolios and predict the returns across a cross-sections of US asset returns. While Messemer (2017) demonstrates that such DFNs can better capture nonlinear information, and outperform portfolios generated from linear benchmarks, the author does not claim that deep learning methods are the best methods to exploit these nonlinear interactions.

In addition, several studies have now suggested that machine learning methods can produce better predictions of asset returns ([**?**], [**?**] and [**?**]). In particular, the results of Gu et al. (2019) suggest that, in terms of predictive performance, as measured by an out-of-sample R^2, tree-based methods and shallow neural nets can provide superior predictive accuracy over other machine learning methods and simpler model-based approaches. This finding is born out both in terms of simulated data, and an empirical example with monthly returns data from 1957 to 2016. [**?**] attribute this to machine learning's ability to evaluate and consider non-linear complexities among factors that cannot be feasibly achieved using traditional techniques.

Similarly, work by Kozak et al, (2018), Freyberger et al. (2018), Feng et al., (2019) and Rapach and Zhou (2013), demonstrate that machine learning methods can "systematically evaluate the contribution to asset pricing of any new factor" used within an existing linear asset pricing structure.

Submitted to 34th Conference on Neural Information Processing Systems (NeurIPS 2020). Do not distribute.

In addition, Gu et al. (2019) use variable importance metrics to quantify the differential impact of factors across a large set of possible factors available for asset pricing. As such, machine learning methods can be used, *en masse*, to consistently evaluate the ability of various factors to help price portfolio risk. Such work is particularly useful given the literature's seeming obsession with the XXX and constructing such factors: as of 2014, quantitative trading firms were using 81 factor models (Hsu and Kalesnik, 2014), while Harvey and Liu (2019) currently document that well over 600 different factors have been suggested in the literature.

While the above studies all demonstrate the potential benefits of machine learning methods within empirical finance, it is unclear whether the findings in these papers are easily generalizable to: one, different training and validation periods; two, different sampling frequencies, which result in stock returns with significant different characteristics (e.g., daily volatility is significantly higher than monthly volatility); and three, different loss-measures of predictive accuracy. The answer to such questions are particularly pertinent given that the machine learning literature has already documented the difficulties of certain methods, including those references above, in dealing with data that displays the stylized facts of asset returns. For instance, methods such as penalized regression and tree-based models assume a form of conditional independence between observations, which is violated by the state dependence that exists within, and across, asset returns. In addition, it has already been noted that training more standard types of neural networks, such as the feed forward kind considered in Gu et al, becomes particularly difficult when data displays strong dependence, ([**?**]). In addition, more complex machine learning approaches require extremely large amounts of data, as well as specialized sample splitting and cross-validation schemes, to deal with possible model over-fitting.

In some ways, existing applications of machine learning to empirical asset pricing have either overlooked, downplayed, or simply ignored the importance of the above issues. For example, Messemer (2017) and [**?**] use cross validation as part of their model building procedures, thereby destroying the temporal ordering of data. In addition, [**?**] and Messemer (2017) produce models using training samples that end much earlier than the data sets which they ultimately produce forecasts for: in the case of Messemer (1970), the training period ends in 1981, while the which ends in the 1970s to ultimately produce forecasts for the most recent 30 years; in the case of [**?**], the training ends in the 1970s, with predictions ultimately produced only for the period of returns from 1987-2016. This is particularly worrying as the factors driving daily or monthly returns in the 1980s, are starkly different than those driving returns in, say, 2001 onwards. However, both of these papers suggest that the training and validation sets used for the various methods does not impact the test set results.

While some combination of machine learning methods can undoubtedly lead to better performance than simpler model-based solutions, a more systematic treatment on the ability of these methods to 1) accurately detect significant factors; and 2) accurately predict returns according to a range of loss measures, must be formulated before researchers can rely on such methods in practice. The goal of this paper is to bridge this gap and thereby provide a systematic, rigorous, realistic, and reproducible study on the performance of several machine learning methods that have been used in empirical asset pricing.

First, through a rigorous simulation study, which captures the stylized facts of asset returns, we give an in-depth comparison of several machine learning methods used in the literature. The simulation study explicitly explores how different aspects of financial data such as persistence in regressors, cross sectional correlation and different complexities of data generating process can affect a method's ability to: 1) accurately predict future returns across a range of loss measures; and 2) correctly identify the significant factors driving returns. In contrast to existing findings, in this realistic simulation design, we find that neural network procedures, such as feedforward nets, LSTM (CITE), and DeepAR models (CITE), are among the worst performing methods, while tree-based methods and elastic net are among the best performing methods. We also demonstrate that this result is consistent across various levels of volatility, cross-sectional correlation, return signal, and different loss functions. In addition, we demonstrate that elastic net and tree-based methods also outperform neural net based approach in terms of correctly identifying significant factors.

Next, we validate these findings using a empirical data set of asset returns that considers quarterly individual price data from CRSP for all firms listed in the NYSE, AMEX and NASDAQ. The starting period of the data is January first 1957 (starting date of the S&P 500) and the ending date is December 2016, totalling 60 years. A set of 549 possible factors are used to explain the cross-section of returns. We pay careful attention to the training and test split, and only use the last fourteen years of quarterly returns to evaluate the different machine learning methods. The results found in the empirical study agree completely with those in the aforementioned simulation study: across all machine learning methods, neural net based procedure perform the worst across various loss functions, while tree-based methods and elastic net perform the best.

The results of this study suggest that great care and diligence is required if one wishes to implement machine learning methods within empirical finance. Indeed, our results suggest that the efficacy of machine learning methods within empirical finance depends are highly-dependent on the samples used for training and testing, the loss functions used for evaluation, and the specific nature of the data series one wishes to predict. As such, while potentially quite useful in empirical finance, machine leaning methods are not necessarily a panacea to correctly predict future asset prices or to correctly disentangle which factors are relevant.

The remainder of the paper is organized as follows....

2 Model and Methods

2.1 Statistical Model

In this section we briefly discuss the statistical model considered for asset returns. Excess monthly returns on asset i, $i = 1, \dots, n$, at time t, $t = 1, \dots, T$, are assumed to evolve in an additive fashion:

$$r_{i,t+1} = E(r_{i,t+1}|\mathcal{F}_t) + \epsilon_{i,t+1}, \quad E(\epsilon_{i,t+1}|\mathcal{F}_t) = 0 \tag{1}$$

where $\mathcal{F}_t$ denotes the observable information at time t, and $\epsilon_{i,t+1}$ is a martingale difference sequence (hereafter, mds). We further consider that the conditional mean of returns is an unknown function of a P-dimensional vector of features, assumed measurable at time t, such that

$$E(r_{i,t+1}|\mathcal{F}_t) = g(z_{i,t}) \tag{2}$$

The features, or predictors, $z_{i,t}$ are assumed to be composed of time-t information, and depends only the characteristics of stock i. It is not assumed that all $z_{i,t}$ are present within the function $g(\cdot)$ across all i units. That is, the function $g(\cdot)$ need not depend on the same $z_{i,t}$ as i varies. The assumption that the information set can be characterized by the variables $z_{i,t}$ without dependence on the $j \neq i$ return units, is reasonable given that the collection of $z_{i,t}$ is rich enough.

In what follows, we represent the space of possible features as the Kronecker product of two pieces

$$z_{i,t} = x_t \otimes c_{i,t} \tag{3}$$

where the variables $c_{i,t}$ represent a $P_c \times 1$ vector of individual-level characteristics for return i, and x_t represents a $P_x \times 1$ vector of macroeconomic predictors, and $\otimes$ represents the Kronecker product. Thus, for $P = P_c \cdot P_x$, $z_{i,t}$ represents a $P \times 1$ feature space that can be used to approximate the unknown function $g(\cdot)$.

2.2 Methods

Given features $z_{i,t}$, the goal of any machine learning method is to approximate the unknown function $g(\cdot)$ in 1. Broadly speaking, how different ML methods choose to approximate this function depends on three components:

1. the model used to make predictions,[1]
2. the regularization mechanism employed to mitigate over-fitting;
3. a loss function that penalized poor predictions.

To ensure the results of ML different methods will be comparable, we fix both the regularization mechanisms and loss functions used within each method, and allow only the models used for prediction to vary. This approach seeks to ensure that performances in one method, relative to another, are based on the model structure and not to some feature of how the models were fit. To this end, we first discuss points 2. and 3. above, and then briefly present the models used for our comparison.

[1]The model used by the ML method need not correspond to the statical models assumed to describe the data. Herein, our goal will not be to asses the "accuracy" of the statistical model, but to determine how different ML methods accurately determine the salient features of this model.

Loss functions: The choice of loss function used to fit the ML methods is instrumental in the methods' ultimate performance. Herein, we consider two separate loss functions: Mean Absolute Error (MAE) and Mean Squared Error (MSE):

$$\text{MAE} = \frac{1}{n}\sum_{j=i}^{n} |y_j - \hat{y_j}| \text{ and MSE} = \frac{1}{n}\sum_{j=i}^{n} (y_j - \hat{y_j})^2 \, ,$$

We consider both loss functions since MAE is less sensitive to outliers in the data which financial returns are known to exhibit, and which are caused by extreme market movements. Given this, we expect MAE to produce predictive results that are more robust to such outlier events.

Mitigating over-fitting: ML methods guard against over-fitting by emphasizing out-of-sample performance. To this end, observed data is split into "training", "validation" and "test" sets. Since returns data is intrinsically dependent, when constructing such a split we must consider a schema that respects this dependence structure.

Throughout our experiments/applications, to balance computation and accuracy, we use a hybrid "rolling window" and "recursive" approach to training/validation/test splits: for each model refit, the training set is increased by one year observations, i.e., 12 monthly observations; the validation set is fixed at one year and moves forward (by one year) with each model refit; predictions are generated using that model for the subsequent year.

Models The remaining specification for the ML methods is the chosen model used to generate predictions. Herein, we consider a host of different models: including elastic net (Hastie et al., XXX), Random forest (XXX), feed-forward neural nets (XXX), LSTM (XXX), DeepAR models (XXX). To keep the details as brief as possible, we give full details on each model and certain features of its implementation used in this work in the appendix. For each of the different methods, we consider two variants, one based on the MAE loss and one based on the MSE loss.

2.3 Model evaluation measures

Predictive accuracy Predictive performance for individual excess returns are assessed using Mean Absolute Error (MAE), Mean Squared Error (MSE) (evaluated over the test set) and an out-of-sample R^2 measure. While out-of-sample R^2 is a common measure, there is no universally agreed-upon definition. As such, we explicitly state the version employed herein as

$$R^2_{OOS} = 1 - \frac{\sum_{(i,t)\in\mathcal{T}_3} (r_{i,t+1} - \widehat{r}_{i,t+1})^2}{\sum_{(i,t)\in\mathcal{T}_3} (r_{i,t+1} - \bar{r}_{i,t+1})^2} \tag{4}$$

where $\mathcal{T}_3$ indicates that the fits are only assessed on the test sub-sample, which is never used for training or tuning.

Since R^2 is based on in-sample-fit of a linear model, this measure is less meaningful for most of the ML methods considered in in this paper. However, we report this measure since this measure has also been considered in other applications of ML to empirical finance (see, e.g., Gu et al., 2019).

Factor Selection An important aspect of empirical finance is the understanding of which features drive risk. That is, which features are explicitly represented within $z_{i,t}$ and can thus be used to help price risk using equation 1. To this end, we define a simple variable importance (VI) measure to be applied across all ML methods in this research. To this end, we mirror the measure produced in [**?**] and define VI_j as the reduction in predictive R^2 from setting all values of predictor j to 0, while holding the remaining model estimates fixed. Each VI_j is then normalized to sum to 1.

However, as VI_j can sometimes be negative, we shift VI_j by the smallest VI_j plus a small constant, then dividing by this sum to alleviate numerical issues[2]. The resulting VI measure is then.

$$VI_{j,norm} = \frac{VI_j + \min(VI_j) + o}{\Sigma VI_j + \min(VI_j) + o} \quad ; \quad o = 10^{-100} \tag{5}$$

[2]This mechanism was chosen because the other popular normalization mechanism "softmax" was observed to be unable to preserve the distances between each original VI_j, making discernment between each VI_j difficult.

3 Simulation Study

We begin with the simulation study as a way to explore how machine learning performs with regards to the stylized facts of empirical returns in a controlled environment. We simulate according to a design which incorporates low signal to noise ratio, stochastic volatility in errors, persistence and cross sectional correlation in regressors. Our specification is a latent factor model for excess returns r_{t+1}, for $t = 1, \ldots, T$:

$$r_{i,t+1} = g\left(z_{i,t}\right) + \beta_{i,t+1} v_{t+1} + e_{i,t+1}; \quad z_{i,t} = \left(1, x_t\right)' \otimes c_{i,t}, \quad \beta_{i,t} = \left(c_{i1,t}, c_{i2,t}, c_{i3,t}\right) \tag{6}$$

$$e_{i,t+1} = \sigma_{i,t+1} \varepsilon_{i,t+1}; \tag{7}$$

$$\log(\sigma^2_{i,t+1}) = \omega + \gamma \log(\sigma^2_t) + \sigma_u u; \quad u \sim N(0,1) \tag{8}$$

where v_{t+1} is a 3×1 vector of errors, $w_{t+1} \sim N(0,1)$, $\varepsilon_{i,t+1} \sim N(0,1)$ scalar error terms, matrix C_t is an $N \times P_c$ matrix of latent factors, where the first three columns correspond to $\beta_{i,t}$, across the $1 \leq i \leq N$ dimensions, while the remaining $P_c - 3$ factors do not enter the return equation. The $P_x \times 1$ vector x_t is a 3×1 multivariate time series, and ε_{t+1} is a $N \times 1$ vector of idiosyncratic errors. The parameters of these were tuned such that the annualized volatility of each return series was approximately 22%, as is often observed empirically.

Simulating Characteristics We build in correlation across time among factors by drawing normal random numbers for each $1 \leq i \leq N$ and $1 \leq j \leq P_c$, according to :

$$\bar{c}_{ij,t} = \rho_j \bar{c}_{ij,t-1} + \epsilon_{ij,t}; \quad \rho_j \sim \mathcal{U}\left(\frac{1}{2}, 1\right) \tag{9}$$

To build in cross sectional correlation, we define the positive-semidefinite matrix B:

$$B := \Lambda\Lambda' + \frac{1}{10}\mathbb{I}_n, \quad \Lambda_i = (\lambda_{i1}, \ldots, \lambda_{i4}), \quad \lambda_{ik} \sim N(0, \lambda_{sd}),\ k = 1, \ldots, 4 \tag{10}$$

to serve as a variance covariance matrix with λ_{sd} controlling the density of the matrix, and hence degree of cross sectional correlation. λ_{sd} values of 0.01, 0.1 and 1 were used to explore increasing degrees of cross sectional correlation.

To build this into our $N \times P_c$ characteristics matrix $\bar{C}_t$, we simulate characteristics according to

$$\widehat{C}_t = L\overline{C}_t; \quad B = LL' \tag{11}$$

where L represents the lower triangle matrix of B using the Cholesky decomposition.

Finally, the "observed" characteristics for each $1 \leq i \leq N$ and for $j = 1, \ldots, P_c$ are constructed according to:

$$c_{ij,t} = \frac{2}{n+1} \operatorname{rank}\left(\hat{c}_{ij,t}\right) - 1. \tag{12}$$

with the rank transformation normalizing all predictors to be within $[-1, 1]$.

3.0.1 Simulating Macroeconomic Series

For simulation of x_t, a 3×1 multivariate time series, we consider a Vector Autoregression (VAR) model [3]:

$$x_t = Ax_{t-1} + u_t; \quad A = 0.95 I_3; \quad u_t \sim N\left(\mu = (0,0,0)', \Sigma = I_3\right)$$

3.0.2 Simulating Return Series

We consider three different functions for $g(z_{i,t})$:

$$(1)\ g_1\left(z_{i,t}\right) = \left(c_{i1,t}, c_{i2,t}, c_{i3,t} \times x'_t[3,]\right)\theta_0 \tag{13}$$

$$(2)\ g_2\left(z_{i,t}\right) = \left(c^2_{i1,t}, c_{i1,t} \times c_{i2,t}, \operatorname{sgn}\left(c_{i3,t} \times x'_t[3,]\right)\right)\theta_0 \tag{14}$$

$$(3)\ g_3\left(z_{i,t}\right) = \left(1[c_{i3,t} > 0], c^3_{i2,t}, c_{i1,t} \times c_{i2,t} \times 1[c_{i3,t} > 0], \operatorname{logit}\left(c_{i3,t}\right)\right)\theta_0 \tag{15}$$

[3] Other more complex and interactive matrix specifications of A were briefly explored, but these did not appear to have a significant impact on results. More complex designs were observed to only affect the variable importance metrics, but to an insignificant degree

Table 1: Top Models by MAE in Simulation Study

Corr	model	Test MAE g1	Test MAE g2	Test MAE g3
0.01	ELN.MAE	**0.034579**	0.036195	**0.035334**
	RF.MAE	0.035459	**0.03542**	0.03554
	NN2.MAE	0.03596	0.036921	0.036305
	NN1.MAE	0.035894	0.036834	0.036335
	NN3.MAE	0.035816	0.036934	0.036471
1	ELN.MSE	**0.034614**	0.036276	**0.035444**
	RF.MAE	0.035916	**0.035643**	0.036053
	NN5.MAE	0.037009	0.03727	0.037413
	NN4.MSE	0.037382	0.036897	0.037354
	NN3.MAE	0.037285	0.037038	0.037193

where $x_t'[3,]$ denotes the third element of the x_t' vector.

$g_1(z_{i,t})$ allows the characteristics to enter the return equation linearly, and $g_2(z_{i,t})$ and $g_3(z_{i,t})$ allow the characteristics to enter the return equation interactively and non-linearly. [4]g_1, g_2 correspond to the simulation design used by [**?**].) θ^0 was tuned such that the predictive R^2 was approximately 5%.

The simulation design results in $3 \times 3 = 9$ different simulated datasets, each with $N = 200$ stocks, $T = 180$ periods and $P_c = 100$ characteristics. Each design was simulated 10 times to assess the robustness of machine learning algorithms. The number of simulations was kept low for computational feasibility.

3.0.3 Sample Splitting

If viewed as monthly periods, $T = 180$ corresponds to 15 years. A data splitting scheme similar to the scheme to be used in the empirical data study was used: a training:validation length ratio of approximately 1.5 to begin, and a test set that is 1 year in length. We employ the hybrid growing window approach as described earlier.

3.1 Simulation Study Results

Prediction Performance We observe that in general elastic nets are the best performing model, followed closely by random forests, then neural networks. All machine learning models were unaffected by cross sectional correlation in terms of prediction performance, and had better performance when fitted with respect to quantile loss. Random forest models only outperformed the elastic nets on highly non-linear specifications. The neural network models were not observed to outperform any of the machine learning models.

This is in stark contrast to the linear models, whose prediction performance is severely affected by both non-linearities, and increasing cross sectional correlation. This result is consistent across all loss metrics, and is most obvious when looking at the out-of-sample R-squared metrics.

Machine learning models fitted with respect to minimizing MAE (quantile loss) generally perform better, even when evaluated against MSE loss metrics. This is not a surprising result, especially considering the stochastic error design which introduces significant shocks to the returns process. Though the actual difference between the loss metrics between the penalized linear models, random forests and neural networks are very small, when considering the consistency of the results across numerous Monte Carlo simulations, the differences in prediction performance, though small, is robust and significant.

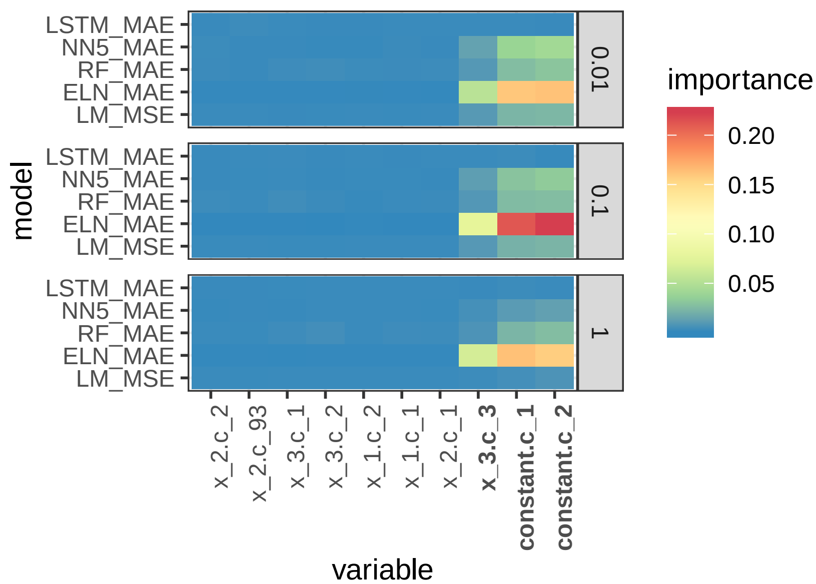
We note that most of these results contradict the sparse literature.

[4] (

Table 2: Top Models by MSE in Simulation Study

Corr	model	Test MSE g1	Test MSE g2	Test MSE g3
0.01	ELN.MAE	**0.002565**	0.002688	**0.002621**
	RF.MAE	0.002643	**0.00263**	0.002645
	NN2.MAE	0.002679	0.002747	0.0027
	NN1.MAE	0.002672	0.00274	0.002703
	NN3.MAE	0.00267	0.002749	0.002718
1	ELN.MSE	**0.002568**	0.002698	**0.00263**
	RF.MAE	0.002675	**0.002644**	0.002679
	NN5.MAE	0.002774	0.002783	0.002792
	NN3.MAE	0.002805	0.002751	0.002797
	NN4.MSE	0.002794	0.002765	0.002775

Figure 1: Simulation g1 Variable Importance



Factor Importance We observe that the elastic net outperforms all other models consistently in terms of assigning the correct relative importance to the true underlying regressors [5], even in settings with high cross sectional correlation.

Elastic net models perform the best at identifying the true data generating regressors, and this appears to be mostly robust regardless of the amount of cross sectional correlation, though their performance worsens as the data generating process becomes more non-linear. On more difficult specifications, the elastic net models are conservative and typically identify a single regressor as importance - most apparent on the g_2 specification. Occasionally, the elastic nets identified the incorrect covariates, though the relative importance assigned to them was small.

The random forests and to a lesser extent the neural networks also correctly identified the correct underlying regressors, but struggled with adequately discerning relative importance among correlated regressors. This was became more apparent as the degree of cross sectional correlation increased (see decreasing relative importance of true underlying regressors in Figures **??** and **??** in Appendix).

The linear models unsurprisingly struggled with factor significance analysis with respect to both increasing cross sectional correlation and increasing non-linearities. This highlights the non-robustness

[5] (c_1.constant, c_2.constant and $c_3.x_3$ for $g1$ and g_2 specifications, and c_1.constant, c_2.constant and c_3.constant for g_3)

Figure 2: Simulation g2 Variable Importance

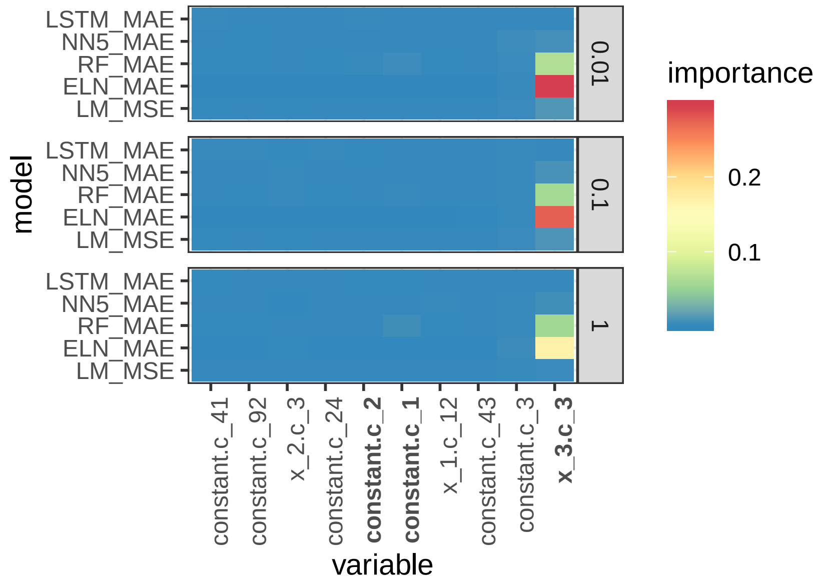
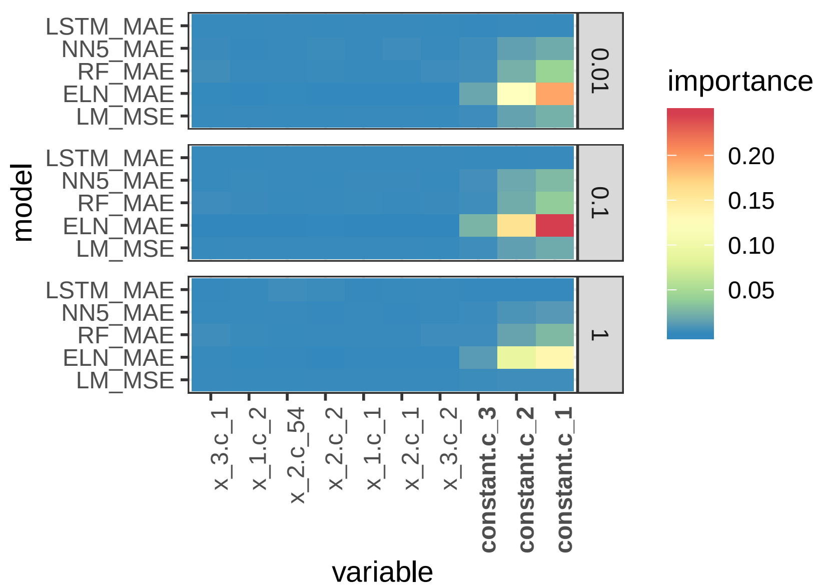


Figure 3: Simulation g3 Variable Importance



and ineffectiveness of using traditional linear regression as documented by the literature; linear models were consistently observed to identify irrelevant regressors as important, especially as the degree of cross sectional correlation increased. Considering that the graphs represent the averaged variable importance metrics over different simulation realisations, this means that on a single simulation realization, the performance of linear models is significantly worse.

4 Empirical analysis

We conduct an empirical study as a final way to corroborate the findings of the properties of machine learning models which we observed in the simulation study. Though our simulation study was aimed at capturing the main features of observed data, the underlying data generating process for empirical returns is unknown. This study thus acts as a robustness check as to how machine learning performs on real world data, which can be significantly more complex and noisy than simulated contexts. Our two studies together can be thought of a repeated sampling exercise in exploring how machine learning methods perform on datasets which feature the "stylized facts" of empirical returns. This empirical study also acts as a final validation against what has been reported in the literature.

Importantly, we find that our findings from the simulation study are largely corroborated for empirical returns data.

4.1 Data

We begin by obtaining monthly individual price data from CRSP for all firms listed in the NYSE, AMEX and NASDAQ, starting from 1957 (starting date of the S&P 500) and ending in December 2016, totalling 60 years. To build individual factors, we construct a factor set based on the cross section of returns literature. This data was sourced from and is the same data used in [**?**]. Like our initial returns sample, it begins in March 1957 and ends in December 2016, totalling 60 years. It contains 94 stock level characteristics: 61 updated annually, 13 updated quarterly and 20 updated monthly [6].

We detail our cleaning procedure of this dataset. To reduce the size of the dataset and increase feasibility, we only consider non-penny equities traded primarily on the NASDAQ. To achieve a balance between having a dataset with enough data points and variability among factors, the dataset was converted to a quarterly format. Quarterly returns were then constructed using the PRC variable according to actual returns (ie not logged differences):

$$RET_t = \frac{PRC_t - PRC_{t-1}}{PRC_{t-1}} \tag{16}$$

We allow all stocks which have a quarterly return to enter the dataset, even if they disappear from the dataset for certain periods, as opposed to only keeping stocks which appear continuously throughout the entire period. This was primarily done to reduce survivorship bias in the dataset, and also allows for stocks which were unlisted and relisted again to feature in the dataset. [7]

We then follow [**?**] and construct eight macroeconomic factors following the variable definitions in [**?**] (see Table 6). These factors were lagged by one period so as to be used to predict one period ahead quarterly returns. The treasury bill rate was also used from this source to proxy for the risk free rate in order to construct excess quarterly returns.

The two sets of factors were then combined to form a baseline set of covariates, which we define throughout all methods and analysis as:

$$z_{i,t} = (1, x_t)' \otimes c_{i,t} \tag{17}$$

[6] The dataset also included 74 Standard Industrial Classification (SIC) codes, but these were omitted due to their inconsistency, and inadequateness at classifying companies, as noted by WRDS

[7] To deal with missing data, any characteristics that had over 20% of their data missing were omitted. Remaining missing data were then imputed using their cross sectional medians for each year. We start our sample from 1993 Q3 as there was a noticeable increase in data availability and quality after this time, thus striking a balance between using more periods at the cost of removing more characteristics versus using less periods but keeping more characteristics.

Table 3: Macroeconomic Factors, ([?])

No.	Acronym	Macroeconomic Factor
1	macro_dp	Dividend Price Ratio
2	macro_ep	Earnings Price Ratio
3	macro_bm	Book to Market Ratio
4	macro_ntis	Net Equity Expansion
5	macro_tbl	Treasury Bill Rate
6	macro_tms	Term Spread
7	macro_dfy	Default Spread
8	macro_svar	Stock Variance

where $c_{i,t}$ is a P_c matrix of characteristics for each stock i, and $(1, x_t)'$ is a $P_x \times 1$ vector of macroeconomic predictors, , and $\otimes$ represents the Kronecker product. $z_{i,t}$ is therefore a $P_x P_c$ vector of features for predicting individual stock returns and includes interactions between stock level characteristics and macroeconomic variables. The total number of covariates in this baseline set is $61 \times (8 + 1) = 549$[8]. The final dataset spanned from 1993 Q3 to 2016 Q4 with 202, 066 individual observations. [9]

We mimic the procedure used in the simulation study. For the sample splitting procedure, the dataset was split such that the training and validation sets were split such that the training set was approximately 1.5 times the length of the validation set, in order to predict a test set that is one year in length.

4.2 Empirical Data Results

In general, the empirical results are in remarkable agreement with the those obtained in the simulation study: the penalized linear models general perform the best, with the random forest models offering slightly worse performance. Machine learning models fitted with respect to median quantile loss were similarly observed to typically offer improvements across all machine learning models across all loss metrics.

Prediction Accuracy In terms of prediction accuracy, we can see that in general the results of the simulation study were repeated: the elastic net models perform the best, followed by the random forests, then the neural networks, and finally the linear models. We note that the differences between each model using the MSE and MAE loss metrics are much more pronounced on empirical data. Even so, the predictive performance between the elastic net models and the quantile random forests is not particularly large, and we observe the quantile random forests outperforming the elastic nets in the first data sample. We similarly see that machine learning models perform better when fitted with respect to quantile loss instead of MSE. Most notably, we start to see the neural network models performing poorly on the empirical data, a direct contradiction to what has been reported in the literature.

Figure 4: Empirical Test MSE

Figure 5: Empirical Test MAE

Figure 6: Empirical Test Predictive R-Squared

Focusing on the neural networks specifically, their non-robustness is amplified on the empirical dataset, with some neural networks in some samples even performing worse than linear models. This was observed to be somewhat more common on neural networks fitted with respect to MSE,

[8] As the individual and macroeconomic factors can have similar names, individual and macroeconomic factors were prefixed with ind_ and macro_ respectively.

[9] The dataset was not normalized for all methods, as only penalized regression and neural networks are sensitive to normalization. For these two methods, the dataset was normalized such that each predictor column had 0 mean and 1 variance.

suggesting that they are indeed very sensitive to outliers in training data. We similarly observe some evidence that deeper neural networks perform better, though this result is less apparent due to the lower robustness on empirical data (see **??** in Appendix for results).

Factor Importance As the data generating process for empirical returns is unknown, the variable importance results cannot be directly compared with those of the simulation study. Even so, we see similar results: the elastic net and random forest models tend to agree on the same subset of predictors, but the random forest struggles to discern between highly correlated regressors. Similar to the prediction performance results, neural networks perform poorly.

Figure 7: Empirical Individual Factor Importance, averaged across all training samples

Figure 8: Empirical Macroeconomic Factor Importance, faceted by training sample

If we focus on the two top performing models of elastic net and random forest, we see that they consistently pick out the 1 month and 6 month momentum factors out of the individual characteristics as important, and the book-to-market and default yield spread factors out of the macroeconomic factors are important. In general though, the variable importance metrics are less consistent for the random forests, and it should be noted in particular that the random forest tends to determine factors highly correlated with momentum as important, such as change in moment, dollar trading volume and return volatility. Looking at the macroeconomic factors, penalized linear models tend to identify the average book to market ratio and the default spread as the most important macroeconomic factors. On the macroeconomic factor set, the random forests were inconsistent with the elastic nets, and tended to assign very similar variable importance metrics to most macroeconomic factors.

The neural networks tended to believe that the market value factor was the most important among the individual factors, a result not repeated by any of the other models considered. Within the macroeconomic factors, the neural networks identified and the dividend-price ratio and earnings-price ratio as the most important among the macroeconomic factors, though these results were highly non-robust across different architectures, loss functions and training samples.

Interestingly, we find that the linear models assign the controversial dividend price ratio macroeconomic factor as highly important, a result mirrored only with the neural networks Their variable importance for individual factors across different training samples is highly non-robust, with the important variables almost completely changing year to year. The linear models consistently identified the controversial dividend-price ratio as important, a result that was somewhat consistent with the neural networks.

The overall results again contradict the results of [**?**], who conclude that all of the machine methods agree on the same subset of important factors. Indeed, we only see consistency in variable importance between the elastic nets and random forests on the individual factors only - all other variable importance metrics were either inconsistent between different models, or non-robust.

All models considered typically preferred sparse parameterizations. That is, most if not all of the individual factors had little to no importance across all models.[10]

5 Conclusion

Our findings demonstrate that the field of machine learning may offer certain tools to improve stock prediction and identification of true underlying factors. Penalized linear models and to a lesser extent, random forests are the best performing methods in the analysis undertaken.

Importantly, we find that DFNs fail in the context of stock return prediction, at both prediction performance and variable importance analysis. This result is consistent across a variety of simulated datasets, as well as empirical data.

Lastly, we find that the top performing models - the elastic nets and random forests, tend to agree and correctly identify the correct underlying regressors in simulated contexts, and agree on the

[10]Note that because the variable importance here was not evaluated explicitly for each pairwise interaction term, some of the individual factors appear as slightly important. This is because setting an individual factor to zero also sets some of the macroeconomic pairwise terms to zero, increasing its apparent importance.

same subset of factors which are important in empirical contexts. We find that of all the models considered, the elastic nets are the most consistent at identifying true underlying regressors through the simulation study. We find that in the empirical setting, among the individual factors the 1 and 6 month momentum factors are the most powerful predictors of stock returns, according to the penalized linear models and random forests.

Across all models except for linear models, we find that minimizing quantile loss yields better prediction performance.

The overall findings of this paper differ from the sparse literature on machine learning methods in empirical finance. However, the performance of the penalized linear models with respect to both out of sample prediction performance and variable importance analysis is promising, and our findings show that machine learning provides some tools which may aid in the problems of stock return prediction and risk factor selection in the financial world.

5.1 Retrieval of style files

The style files for NeurIPS and other conference information are available on the World Wide Web at

http://www.neurips.cc/

The file `neurips_2020.pdf` contains these instructions and illustrates the various formatting requirements your NeurIPS paper must satisfy.

The only supported style file for NeurIPS 2020 is `neurips_2020.sty`, rewritten for $\LaTeX\,2_{\varepsilon}$. **Previous style files for LaTeX 2.09, Microsoft Word, and RTF are no longer supported!**

The LaTeX style file contains three optional arguments: `final`, which creates a camera-ready copy, `preprint`, which creates a preprint for submission to, e.g., arXiv, and `nonatbib`, which will not load the `natbib` package for you in case of package clash.

Preprint option If you wish to post a preprint of your work online, e.g., on arXiv, using the NeurIPS style, please use the `preprint` option. This will create a nonanonymized version of your work with the text "Preprint. Work in progress." in the footer. This version may be distributed as you see fit. Please **do not** use the `final` option, which should **only** be used for papers accepted to NeurIPS.

At submission time, please omit the `final` and `preprint` options. This will anonymize your submission and add line numbers to aid review. Please do *not* refer to these line numbers in your paper as they will be removed during generation of camera-ready copies.

The file `neurips_2020.tex` may be used as a "shell" for writing your paper. All you have to do is replace the author, title, abstract, and text of the paper with your own.

The formatting instructions contained in these style files are summarized in Sections 6, 7, and 8 below.

6 General formatting instructions

The text must be confined within a rectangle 5.5 inches (33 picas) wide and 9 inches (54 picas) long. The left margin is 1.5 inch (9 picas). Use 10 point type with a vertical spacing (leading) of 11 points. Times New Roman is the preferred typeface throughout, and will be selected for you by default. Paragraphs are separated by ½ line space (5.5 points), with no indentation.

The paper title should be 17 point, initial caps/lower case, bold, centered between two horizontal rules. The top rule should be 4 points thick and the bottom rule should be 1 point thick. Allow ¼ inch space above and below the title to rules. All pages should start at 1 inch (6 picas) from the top of the page.

For the final version, authors' names are set in boldface, and each name is centered above the corresponding address. The lead author's name is to be listed first (left-most), and the co-authors' names (if different address) are set to follow. If there is only one co-author, list both author and co-author side by side.

Please pay special attention to the instructions in Section 8 regarding figures, tables, acknowledgments, and references.

7 Headings: first level

All headings should be lower case (except for first word and proper nouns), flush left, and bold.

First-level headings should be in 12-point type.

7.1 Headings: second level

Second-level headings should be in 10-point type.

7.1.1 Headings: third level

Third-level headings should be in 10-point type.

Paragraphs There is also a `\paragraph` command available, which sets the heading in bold, flush left, and inline with the text, with the heading followed by 1 em of space.

8 Citations, figures, tables, references

These instructions apply to everyone.

8.1 Citations within the text

The `natbib` package will be loaded for you by default. Citations may be author/year or numeric, as long as you maintain internal consistency. As to the format of the references themselves, any style is acceptable as long as it is used consistently.

The documentation for `natbib` may be found at

http://mirrors.ctan.org/macros/latex/contrib/natbib/natnotes.pdf

Of note is the command `\citet`, which produces citations appropriate for use in inline text. For example,

```
\citet{hasselmo} investigated\dots
```

produces

Hasselmo, et al. (1995) investigated…

If you wish to load the `natbib` package with options, you may add the following before loading the `neurips_2020` package:

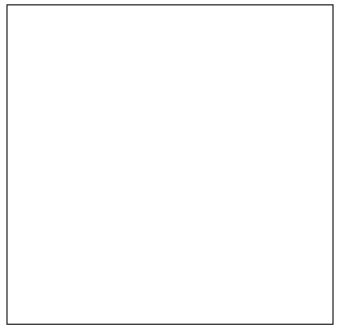
```
\PassOptionsToPackage{options}{natbib}
```

If `natbib` clashes with another package you load, you can add the optional argument `nonatbib` when loading the style file:

```
\usepackage[nonatbib]{neurips_2020}
```

As submission is double blind, refer to your own published work in the third person. That is, use "In the previous work of Jones et al. [4]," not "In our previous work [4]." If you cite your other papers that are not widely available (e.g., a journal paper under review), use anonymous author names in the citation, e.g., an author of the form "A. Anonymous."

Figure 9: Sample figure caption.



Macroeconomic Factors shown on x axis (see Table 6 for definitions)

Table 4: Sample table title

Part		
Name	Description	Size (μm)
Dendrite	Input terminal	$\sim$100
Axon	Output terminal	$\sim$10
Soma	Cell body	up to 10^6

8.2 Footnotes

Footnotes should be used sparingly. If you do require a footnote, indicate footnotes with a number[11] in the text. Place the footnotes at the bottom of the page on which they appear. Precede the footnote with a horizontal rule of 2 inches (12 picas).

Note that footnotes are properly typeset *after* punctuation marks.[12]

8.3 Figures

All artwork must be neat, clean, and legible. Lines should be dark enough for purposes of reproduction. The figure number and caption always appear after the figure. Place one line space before the figure caption and one line space after the figure. The figure caption should be lower case (except for first word and proper nouns); figures are numbered consecutively.

You may use color figures. However, it is best for the figure captions and the paper body to be legible if the paper is printed in either black/white or in color.

8.4 Tables

All tables must be centered, neat, clean and legible. The table number and title always appear before the table. See Table 4.

Place one line space before the table title, one line space after the table title, and one line space after the table. The table title must be lower case (except for first word and proper nouns); tables are numbered consecutively.

Note that publication-quality tables *do not contain vertical rules.* We strongly suggest the use of the `booktabs` package, which allows for typesetting high-quality, professional tables:

https://www.ctan.org/pkg/booktabs

This package was used to typeset Table 4.

[11] Sample of the first footnote.
[12] As in this example.

9 Final instructions

Do not change any aspects of the formatting parameters in the style files. In particular, do not modify the width or length of the rectangle the text should fit into, and do not change font sizes (except perhaps in the **References** section; see below). Please note that pages should be numbered.

10 Preparing PDF files

Please prepare submission files with paper size "US Letter," and not, for example, "A4."

Fonts were the main cause of problems in the past years. Your PDF file must only contain Type 1 or Embedded TrueType fonts. Here are a few instructions to achieve this.

- You should directly generate PDF files using `pdflatex`.
- You can check which fonts a PDF files uses. In Acrobat Reader, select the menu Files>Document Properties>Fonts and select Show All Fonts. You can also use the program `pdffonts` which comes with `xpdf` and is available out-of-the-box on most Linux machines.
- The IEEE has recommendations for generating PDF files whose fonts are also acceptable for NeurIPS. Please see http://www.emfield.org/icuwb2010/downloads/IEEE-PDF-SpecV32.pdf
- `xfig` "patterned" shapes are implemented with bitmap fonts. Use "solid" shapes instead.
- The `\bbold` package almost always uses bitmap fonts. You should use the equivalent AMS Fonts:

  ```
  \usepackage{amsfonts}
  ```

  followed by, e.g., `\mathbb{R}`, `\mathbb{N}`, or `\mathbb{C}` for $\mathbb{R}$, $\mathbb{N}$ or $\mathbb{C}$. You can also use the following workaround for reals, natural and complex:

  ```
  \newcommand{\RR}{I\!\!R} %real numbers
  \newcommand{\Nat}{I\!\!N} %natural numbers
  \newcommand{\CC}{I\!\!\!\!C} %complex numbers
  ```

  Note that `amsfonts` is automatically loaded by the `amssymb` package.

If your file contains type 3 fonts or non embedded TrueType fonts, we will ask you to fix it.

10.1 Margins in LaTeX

Most of the margin problems come from figures positioned by hand using `\special` or other commands. We suggest using the command `\includegraphics` from the `graphicx` package. Always specify the figure width as a multiple of the line width as in the example below:

```
\usepackage[pdftex]{graphicx} ...
\includegraphics[width=0.8\linewidth]{myfile.pdf}
```

See Section 4.4 in the graphics bundle documentation (http://mirrors.ctan.org/macros/latex/required/graphics/grfguide.pdf)

A number of width problems arise when LaTeX cannot properly hyphenate a line. Please give LaTeX hyphenation hints using the `\-` command when necessary.

Broader Impact

Authors are required to include a statement of the broader impact of their work, including its ethical aspects and future societal consequences. Authors should discuss both positive and negative outcomes, if any. For instance, authors should discuss a) who may benefit from this research, b) who may be put at disadvantage from this research, c) what are the consequences of failure of the system, and d) whether the task/method leverages biases in the data. If authors believe this is not applicable to them, authors can simply state this.

Use unnumbered first level headings for this section, which should go at the end of the paper. **Note that this section does not count towards the eight pages of content that are allowed.**

References

References follow the acknowledgments. Use unnumbered first-level heading for the references. Any choice of citation style is acceptable as long as you are consistent. It is permissible to reduce the font size to `small` (9 point) when listing the references. **Note that the Reference section does not count towards the eight pages of content that are allowed.**

[1] Alexander, J.A. & Mozer, M.C. (1995) Template-based algorithms for connectionist rule extraction. In G. Tesauro, D.S. Touretzky and T.K. Leen (eds.), *Advances in Neural Information Processing Systems 7*, pp. 609–616. Cambridge, MA: MIT Press.

[2] Bower, J.M. & Beeman, D. (1995) *The Book of GENESIS: Exploring Realistic Neural Models with the GEneral NEural SImulation System.* New York: TELOS/Springer–Verlag.

[3] Hasselmo, M.E., Schnell, E. & Barkai, E. (1995) Dynamics of learning and recall at excitatory recurrent synapses and cholinergic modulation in rat hippocampal region CA3. *Journal of Neuroscience* **15**(7):5249-5262.

A Additional details: simulation design

In this section, we give additional features of the simulation design required to implenent our results. All code and data can be found at XXXX.

A.1 Simulation Design

We simulate a latent factor model with a stochastic volatility process for excess returns r_{t+1}, for $t = 1, \ldots, T$:

$$r_{i,t+1} = g\left(z_{i,t}\right) + \beta_{i,t+1} v_{t+1} + e_{i,t+1}; \quad z_{i,t} = (1, x_t)' \otimes c_{i,t}, \quad \beta_{i,t} = (c_{i1,t}, c_{i2,t}, c_{i3,t}) \tag{18}$$
$$e_{i,t+1} = \sigma_{i,t+1} \varepsilon_{i,t+1}; \tag{19}$$
$$\log(\sigma^2_{i,t+1}) = \omega + \gamma \log(\sigma^2_t) + \sigma_u u; \quad u \sim N(0,1) \tag{20}$$

Let v_{t+1} be a 3×1 vector of errors, and $w_{t+1} \sim N(0,1)$ and $\varepsilon_{i,t+1} \sim N(0,1)$ scalar error terms.

The matrix C_t is an $N \times P_c$ matrix of latent factors, where the first three columns correspond to $\beta_{i,t}$, across the $1 \leq i \leq N$ dimensions, while the remaining $P_c - 3$ factors do not enter the return equation. The $P_x \times 1$ vector x_t is a 3×1 multivariate time series, and ε_{t+1} is a $N \times 1$ vector of idiosyncratic errors.

The parameters of these were tuned such that the annualized volatility of each return series was approximately 22%, as is often observed empirically.

Note that we also reproduce [**?**]'s error specification as a case where there is no stochastic volatility:

$$v_{t+1} \sim N(0, 0.05^2 \times I_3) \tag{21}$$
$$e_{i,t+1} \sim t_5(0, 0.05^2) \tag{22}$$

A.1.1 Simulating Characteristics

We build in correlation across time among factors by drawing normal random numbers for each $1 \leq i \leq N$ and $1 \leq j \leq P_c$, according to

$$\bar{c}_{ij,t} = \rho_j \bar{c}_{ij,t-1} + \epsilon_{ij,t}; \quad \rho_j \sim \mathcal{U}\left(\frac{1}{2}, 1\right) \tag{23}$$

To build in cross sectional correlation, we define the positive-semidefinite matrix B:

$$B := \Lambda\Lambda' + \frac{1}{10}\mathbb{I}_n, \quad \Lambda_i = (\lambda_{i1}, \ldots, \lambda_{i4}), \quad \lambda_{ik} \sim N(0, \lambda_{sd}),\ k = 1, \ldots, 4 \tag{24}$$

to serve as a variance covariance matrix with λ_{sd} controlling the density of the matrix, and hence degree of cross sectional correlation. λ_{sd} values of 0.01, 0.1 and 1 were used to explore increasing degrees of cross sectional correlation.

To build this into our $N \times P_c$ characteristics matrix $\bar{C}_t$, we simulate characteristics according to

$$\widehat{C}_t = L\overline{C}_t; \quad B = LL' \tag{25}$$

where L represents the lower triangle matrix of B using the Cholesky decomposition.

Finally, the "observed" characteristics for each $1 \le i \le N$ and for $j = 1, \ldots, P_c$ are constructed according to:

$$c_{ij,t} = \frac{2}{n+1} \operatorname{rank}\left(\hat{c}_{ij,t}\right) - 1. \tag{26}$$

with the rank transformation normalizing all predictors to be within $[-1, 1]$.

A.1.2 Simulating Macroeconomic Series

For simulation of x_t, a 3×1 multivariate time series, we consider a Vector Autoregression (VAR) model [13]:

$$x_t = Ax_{t-1} + u_t; \quad A = \begin{pmatrix} .95 & 0 & 0 \\ 0 & .95 & 0 \\ 0 & 0 & .95 \end{pmatrix} \quad u_t \sim N\left(\mu = (0,0,0)', \Sigma = I_3\right)$$

A.1.3 Simulating Return Series

We consider three different functions for $g(z_{i,t})$:

$$(1)\ g_1\left(z_{i,t}\right) = \left(c_{i1,t}, c_{i2,t}, c_{i3,t} \times x'_t[3,]\right)\theta_0 \tag{27}$$

$$(2)\ g_2\left(z_{i,t}\right) = \left(c_{i1,t}^2, c_{i1,t} \times c_{i2,t}, \operatorname{sgn}\left(c_{i3,t} \times x'_t[3,]\right)\right)\theta_0 \tag{28}$$

$$(3)\ g_3\left(z_{i,t}\right) = \left(1[c_{i3,t} > 0], c_{i2,t}^3, c_{i1,t} \times c_{i2,t} \times 1[c_{i3,t} > 0], \operatorname{logit}\left(c_{i3,t}\right)\right)\theta_0 \tag{29}$$

where $x'_t[3,]$ denotes the third element of the x'_t vector.

$g_1\left(z_{i,t}\right)$ allows the characteristics to enter the return equation linearly, and $g_2\left(z_{i,t}\right)$ allows the characteristics to enter the return equation interactively and non-linearly. The true underlying regressors for these specifications are $(c_{i1,t}, c_{i2,t}, c_{i3,t} \times x'_t[3,])$.These two specifications correspond to the simulation design used by [**?**].

$g_3\left(z_{i,t}\right)$ allows the characteristics to enter in a complex and non-linear fashion. The true underlying regressors for this specification are $(c_{i1,t}, c_{i2,t}, c_{i3,t})$.

It should be noted however, that because $g_2\left(z_{i,t}\right)$ has a large part of its signal entering through a sgn function, this should make it the most difficult to estimate given the regressors and resulting returns process.

θ^0 was tuned such that the predictive R^2 was approximately 5%.

The simulation design results in $3 \times 3 = 12$ different simulated datasets, each with $N = 200$ stocks, $T = 180$ periods and $P_c = 100$ characteristics. Each design was simulated 10 times to assess the robustness of machine learning algorithms. The number of simulations was kept low for computational feasibility.

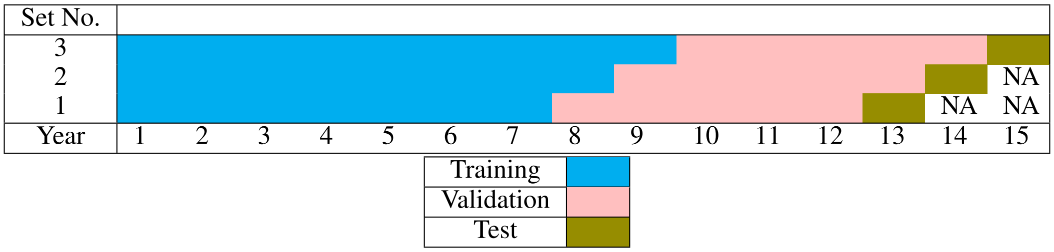
A.1.4 Sample Splitting

If viewed as monthly periods, $T = 180$ corresponds to 15 years. A data splitting scheme similar to the scheme to be used in the empirical data study was used: a training:validation length ratio of approximately 1.5 to begin, and a test set that is 1 year in length. We employ the hybrid growing window approach as described earlier in section **??** (see Figure 10 for a graphical representation).

Other schemes in the forecasting literature such as using an “inner" rolling window validation loop to find the best hyperparameters on average, finally aggregating them in an “outer" loop for a more robust error were

[13] Other more complex and interactive matrix specifications of A were briefly explored, but these did not appear to have a significant impact on results. More complex designs were observed to only affect the variable importance metrics, but to an insignificant degree

Figure 10: Sample Splitting Procedure



considered but not implemented for a variety of reasons. Firstly, many of the models were computationally too intensive for this to be feasible. More importantly, during the model fitting process it was observed that the optimal hyperparameters for the different rolling windows were highly unstable. Thus, this would have made the selection of the best hyperparameters on average across all windows significantly less meaningful.

A.2 Simulation Study Results

A.2.1 Prediction Performance

Table 5: Simulation Study Loss Statistics

		g1			g2			g3		
model	Corr	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2
LM.MSE	0.01	0.0366775	0.0027400	0.0082732	0.0382548	0.0028801	-0.1117880	0.0373098	0.0027954	-0.0320680
	0.10	0.0369652	0.0027653	-0.0110198	0.0385796	0.0029144	-0.1429443	0.0375694	0.0028168	-0.0549404
	1.00	0.0429486	0.0034141	-0.4387965	0.0453765	0.0037172	-0.7809535	0.0434339	0.0034688	-0.4887785
LM.MAE	0.01	0.0366417	0.0027373	0.0090496	0.0383478	0.0028862	-0.1163694	0.0373235	0.0027967	-0.0351619
	0.10	0.0368113	0.0027555	0.0029188	0.0387449	0.0029275	-0.1525797	0.0374894	0.0028098	-0.0476746
	1.00	0.0423399	0.0033445	-0.3930442	0.0453420	0.0036847	-0.7699555	0.0435349	0.0034682	-0.5445237
ELN.MSE	0.01	0.0345878	0.0025663	0.1403351	0.0362229	0.0026898	0.0368766	0.0353534	0.0026227	0.0991416
	0.10	0.0345630	0.0025643	0.1442376	0.0361830	0.0026860	0.0372585	0.0352923	0.0026167	0.1002410
	1.00	0.0346142	0.0025676	0.1671841	0.0362761	0.0026980	0.0378391	0.0354437	0.0026300	0.1198755
ELN.MAE	0.01	0.0345786	0.0025652	0.1409821	0.0361950	0.0026882	0.0391694	0.0353345	0.0026210	0.1004424
	0.10	0.0345582	0.0025637	0.1446272	0.0361730	0.0026877	0.0388747	0.0352851	0.0026167	0.1009186
	1.00	0.0345989	0.0025667	0.1677712	0.0363047	0.0027028	0.0365834	0.0354652	0.0026310	0.1180225
RF.MSE	0.01	0.0357752	0.0026710	0.0634257	0.0357179	0.0026571	0.0676147	0.0358032	0.0026613	0.0702977
	0.10	0.0357695	0.0026649	0.0667382	0.0356845	0.0026525	0.0691389	0.0358666	0.0026704	0.0628386
	1.00	0.0362325	0.0026977	0.0687741	0.0359893	0.0026833	0.0571035	0.0362129	0.0026952	0.0698868
RF.MAE	0.01	0.0354594	0.0026434	0.0833385	0.0354204	0.0026305	0.0876529	0.0355399	0.0026446	0.0865291
	0.10	0.0355153	0.0026489	0.0814253	0.0354894	0.0026345	0.0834048	0.0355688	0.0026438	0.0816426
	1.00	0.0359158	0.0026747	0.0870806	0.0356434	0.0026445	0.0809651	0.0360529	0.0026786	0.0753573
NN1.MSE	0.01	0.0364516	0.0027219	0.0163443	0.0367677	0.0027319	-0.0039174	0.0366874	0.0027384	0.0093355
	0.10	0.0364624	0.0027191	0.0204223	0.0367762	0.0027345	-0.0072588	0.0367326	0.0027372	0.0029550
	1.00	0.0375452	0.0028206	-0.0144520	0.0370492	0.0027638	-0.0146973	0.0374589	0.0027975	-0.0124689
NN1.MAE	0.01	0.0359604	0.0026786	0.0558139	0.0369206	0.0027474	-0.0151053	0.0363047	0.0026996	0.0393707
	0.10	0.0360823	0.0026866	0.0506976	0.0370100	0.0027503	-0.0205616	0.0363220	0.0027022	0.0323034
	1.00	0.0378894	0.0028338	-0.0431818	0.0379790	0.0028445	-0.0840747	0.0373056	0.0027926	0.0021783
NN2.MSE	0.01	0.0370187	0.0027850	-0.0217869	0.0373197	0.0027752	-0.0433537	0.0370890	0.0027745	-0.0173037
	0.10	0.0369775	0.0027651	-0.0212763	0.0370088	0.0027478	-0.0275384	0.0369898	0.0027584	-0.0206446
	1.00	0.0375360	0.0028138	-0.0139783	0.0369035	0.0027518	-0.0058664	0.0375157	0.0028087	-0.0169336
	0.01	0.0358939	0.0026718	0.0577427	0.0368335	0.0027396	-0.0071579	0.0363352	0.0027028	0.0363052

Table 5: Simulation Study Loss Statistics

		g1			g2			g3		
model	Corr	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2
NN2.MAE	0.10	0.0358898	0.0026681	0.0603096	0.0369367	0.0027503	-0.0170774	0.0362701	0.0026960	0.0371567
	1.00	0.0374795	0.0028142	-0.0095290	0.0377146	0.0028226	-0.0653904	0.0374711	0.0028038	-0.0101183
NN3.MSE	0.01	0.0367827	0.0027568	-0.0067616	0.0368397	0.0027379	-0.0075249	0.0370360	0.0027644	-0.0200783
	0.10	0.0369384	0.0027613	-0.0153994	0.0368517	0.0027384	-0.0151060	0.0368743	0.0027573	-0.0044063
	1.00	0.0374242	0.0028081	-0.0129638	0.0369376	0.0027543	-0.0063529	0.0374202	0.0027991	-0.0103479
NN3.MAE	0.01	0.0358164	0.0026697	0.0654321	0.0369345	0.0027491	-0.0163983	0.0364712	0.0027181	0.0299484
	0.10	0.0358935	0.0026771	0.0620017	0.0368590	0.0027406	-0.0118497	0.0362000	0.0026932	0.0406114
	1.00	0.0370087	0.0027744	0.0213288	0.0372705	0.0027832	-0.0296437	0.0374132	0.0027916	-0.0083067
NN4.MSE	0.01	0.0368808	0.0027586	-0.0206197	0.0368555	0.0027423	-0.0077152	0.0371255	0.0027752	-0.0265634
	0.10	0.0368772	0.0027610	-0.0145791	0.0372207	0.0027615	-0.0487112	0.0368718	0.0027480	-0.0088940
	1.00	0.0373820	0.0028051	-0.0064811	0.0368966	0.0027505	-0.0053689	0.0373542	0.0027970	-0.0077389
NN4.MAE	0.01	0.0359348	0.0026782	0.0577196	0.0368974	0.0027487	-0.0109166	0.0367079	0.0027376	0.0070464
	0.10	0.0358281	0.0026651	0.0650415	0.0369333	0.0027494	-0.0191117	0.0362730	0.0026954	0.0377039
	1.00	0.0370948	0.0027786	0.0198663	0.0373230	0.0027947	-0.0293767	0.0373013	0.0027871	-0.0018876
NN5.MSE	0.01	0.0372306	0.0027846	-0.0499701	0.0369309	0.0027474	-0.0170017	0.0371140	0.0027720	-0.0218954
	0.10	0.0370264	0.0027669	-0.0321897	0.0371758	0.0027623	-0.0394362	0.0369093	0.0027565	-0.0113522
	1.00	0.0373642	0.0027949	-0.0104952	0.0369277	0.0027552	-0.0053762	0.0374751	0.0028071	-0.0149737
NN5.MAE	0.01	0.0358880	0.0026693	0.0585792	0.0368354	0.0027380	-0.0086455	0.0366851	0.0027371	0.0046430
	0.10	0.0360381	0.0026803	0.0509764	0.0367451	0.0027273	-0.0049349	0.0364843	0.0027103	0.0181920
	1.00	0.0372849	0.0027940	0.0025412	0.0370382	0.0027652	-0.0127290	0.0371925	0.0027753	0.0025723
LSTM.MSE	0.01	0.0372963	0.0027982	-0.0432886	0.0372268	0.0027764	-0.0447640	0.0375909	0.0028180	-0.0625164
	0.10	0.0372369	0.0027946	-0.0319550	0.0371342	0.0027674	-0.0382547	0.0371984	0.0027845	-0.0303936
	1.00	0.0381282	0.0028506	-0.0820266	0.0373821	0.0027921	-0.0442426	0.0377803	0.0028300	-0.0443304
LSTM.MAE	0.01	0.0374310	0.0028046	-0.0564056	0.0373372	0.0027801	-0.0518537	0.0376270	0.0028169	-0.0674327
	0.10	0.0374461	0.0028036	-0.0629523	0.0371178	0.0027679	-0.0325442	0.0372409	0.0027931	-0.0333196
	1.00	0.0380266	0.0028456	-0.0614833	0.0374152	0.0027902	-0.0455057	0.0377435	0.0028252	-0.0458837
FFORMA.MSE	0.01	0.0382767	0.0028820	-0.1326717	0.0384600	0.0028893	-0.1473902	0.0424656	0.0033108	-0.4861451
	0.10	0.0383581	0.0028947	-0.1407652	0.0384795	0.0028912	-0.1600616	0.0423231	0.0032914	-0.4739906
	1.00	0.0388747	0.0029647	-0.1312392	0.0388080	0.0029331	-0.1659900	0.0430130	0.0033713	-0.4709541
FFORMA.MAE	0.01	0.0387548	0.0029387	-0.1797483	0.0387472	0.0029178	-0.1740938	0.0429893	0.0033651	-0.5279094
	0.10	0.0389359	0.0029511	-0.1927930	0.0387959	0.0029457	-0.1759939	0.0430966	0.0034057	-0.5863752
	1.00	0.0392468	0.0029721	-0.1636559	0.0393873	0.0029960	-0.2116186	0.0437090	0.0034483	-0.5260813
DeepAR	0.01	0.0382993	0.0029000	-0.1289295	0.0384895	0.0029121	-0.1325183	0.0393898	0.0030161	-0.2049803
	0.10	0.0388318	0.0029353	-0.1816633	0.0384345	0.0029045	-0.1318744	0.0391770	0.0029932	-0.1905583
	1.00	0.0405348	0.0031590	-0.2391417	0.0387870	0.0029524	-0.1440285	0.0396918	0.0030422	-0.1823646

A.3 Random Forest VIMPs

We note that random forest methods typically have their own built-in ways to calculate variable importance which are different to the variable importance metric presented in the main body of the paper. Here we provide two popular schemes of calculating random forest vairable importance metrics - . Importantly, the overall conclusion regarding factor selection does not change with respect to which vimp methodology employed.

Figure 11: g1 BC VIMP

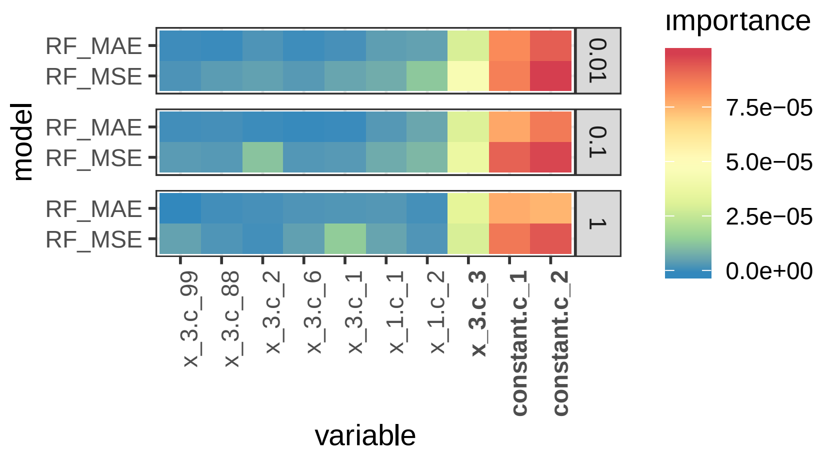


Figure 12: g2 BC VIMP

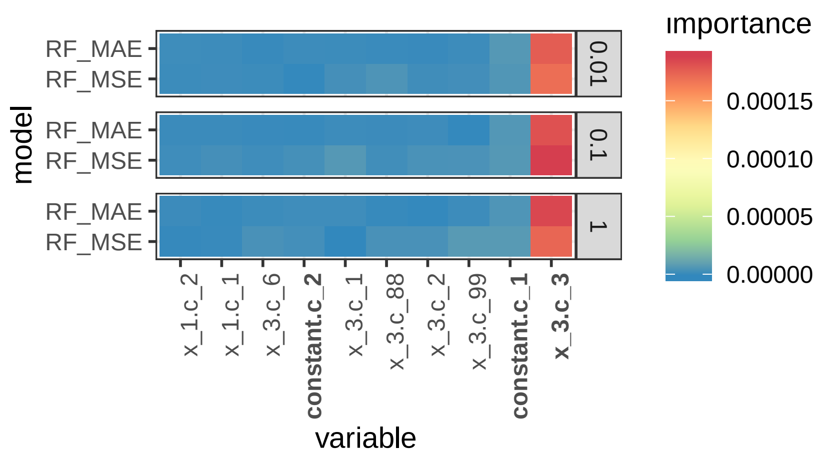


Figure 13: g3 BC VIMP

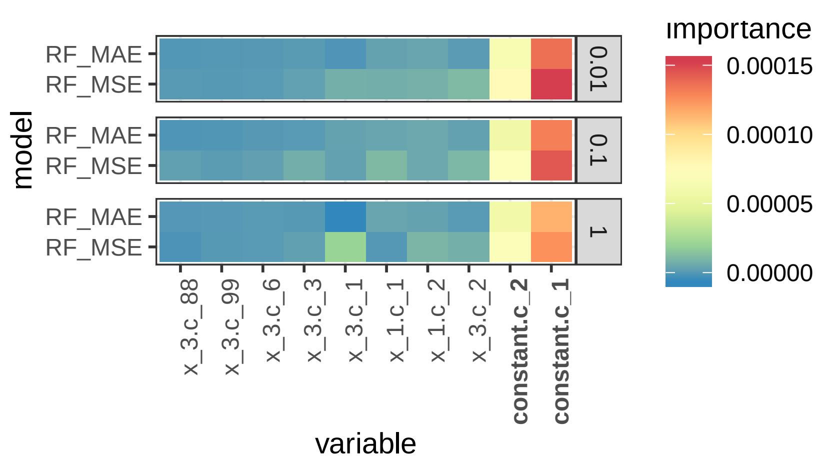


Figure 14: g1 IK VIMP

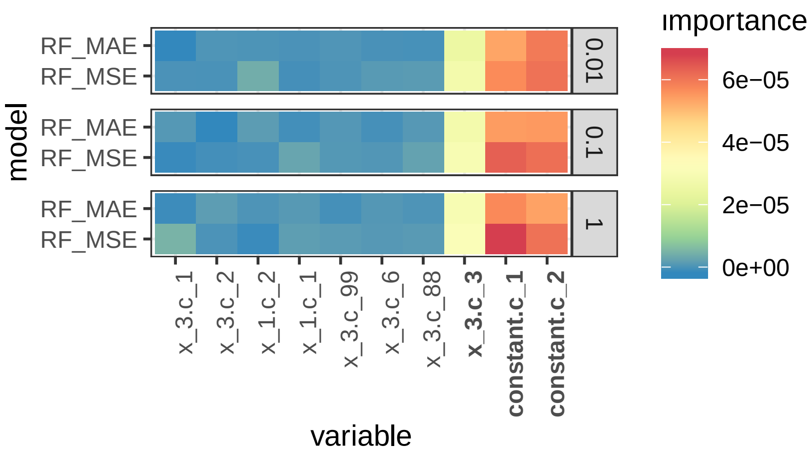


Figure 15: g2 IK VIMP

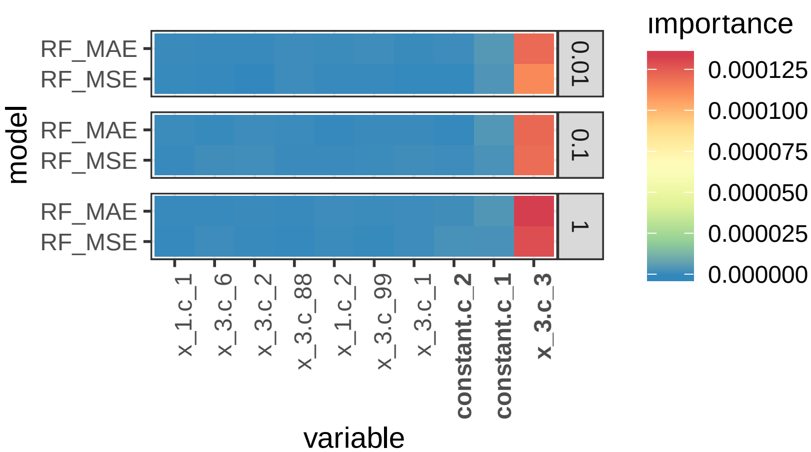
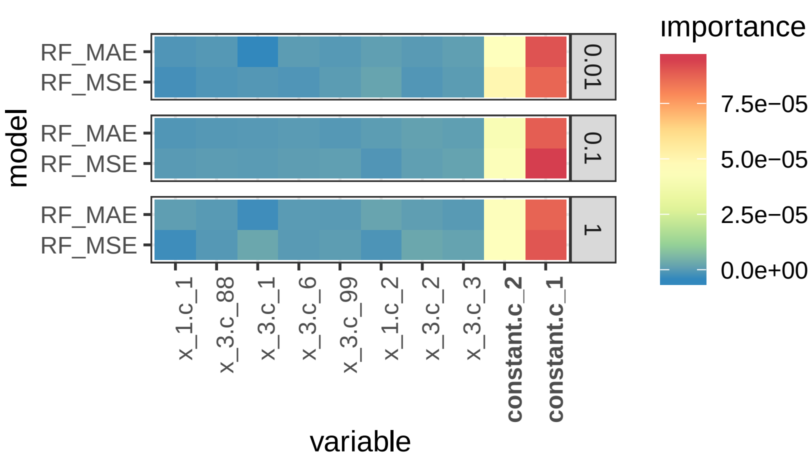


Figure 16: g3 IK VIMP



A Additional details: Empirical analysis

We conduct an empirical study as a final way to corroborate the findings of the properties of machine learning models which we observed in the simulation study. Though our simulation study was aimed at capturing the main features of observed data, the underlying data generating process for empirical returns is unknown. This study thus acts as a robustness check as to how machine learning performs on real world data, which can be significantly more complex and noisy than simulated contexts. Our two studies together can be thought of a repeated sampling exercise in exploring how machine learning methods perform on datasets which feature the “stylized facts" of empirical returns. This empirical study also acts as a final validation against what has been reported in the literature.

Importantly, we find that our findings from the simulation study are largely corroborated for empirical returns data.

A.1 Data

We begin by obtaining monthly individual price data from CRSP for all firms listed in the NYSE, AMEX and NASDAQ, starting from 1957 (starting date of the S&P 500) and ending in December 2016, totalling 60 years. To build individual factors, we construct a factor set based on the cross section of returns literature. This data was sourced from and is the same data used in [**?**]. Like our initial returns sample, it begins in March 1957 and ends in December 2016, totalling 60 years. It contains 94 stock level characteristics: 61 updated annually, 13 updated quarterly and 20 updated monthly, in addition to 74 industry dummies corresponding the the first two digits of the Standard Industrial Classification (SIC) codes. It is noted that this dataset so far contains all securities traded, including those with a CRSP share code other than 10 or 11 and thus includes instruments such as REITs and mutual funds, and those with a share price of less than $5.

We detail our cleaning procedure of this dataset. To reduce the size of the dataset and increase feasibility, the dataset was filtered such that only stocks traded primarily on NASDAQ were included (using the PRIMEXCH variable from WRDS). Then, penny stocks (also referred to as microcaps in the literature) with a stock price of less than $5 were filtered out, as is commonly done in the literature to reduce variability. Stocks without a share code of 10 or 11 (referring to equities) were filtered out, so that securities that are not equities were not included (such as REITs and trust funds). The dataset is provided in a monthly format, which means that many of the factors which are updated only quarterly or annually have very low levels of variability, which can lead to misleading results in the model fitting process. To achieve a balance between having a dataset with enough data points and variability among factors, the dataset was converted to a quarterly format. Quarterly returns were then constructed using the PRC variable according to actual returns (ie not logged differences):

$$RET_t = \frac{PRC_t - PRC_{t-1}}{PRC_{t-1}} \tag{30}$$

We allow all stocks which have a quarterly return to enter the dataset, even if they disappear from the dataset for certain periods, as opposed to only keeping stocks which appear continuously throughout the entire period. This was primarily done to reduce survivorship bias in the dataset, which can be very prevalent in financial data, and also allows for stocks which were unlisted and relisted again to feature in the dataset. This has the obvious drawback of introducing some bias in the dataset, as attrition in the dataset is likely to be non-random and correlated with the stocks' returns.

The sic2 variable, corresponding to the stocks' Standard Industrial Classification (SIC) codes was also dropped. The SIC code system suffers from inconsistent logic in classifying companies, and as a system built for pre-1970s traditional industries has been slow in recognizing new and emerging industries. Indeed, WRDS explicitly cautions the use of SIC codes beyond the use of rough grouping of industries, warning that SIC codes are not strictly enforced by government agencies for accuracy, in addition to most large companies belonging to multiple SIC codes over time. Because of this latter point in particular, there can be inconsistencies on the correct SIC code for the same company depending on the data source. Dropping the sic2 variable also reduced the dimensionality of the dataset by 74 columns, significant increasing computational feasibility.

There existed a significant amount of missing data in the dataset. The dataset's columns were first examined, and any characteristics that had over 20% of their data were removed. However, as the amount of missing data increases dramatically going further back in time, a balance between using more periods at the cost of removing more characteristics versus using less periods but keeping more characteristics was needed. 1993 Q3 was determined to be a reasonable time frame to begin the dataset, as there was a noticeable increase in data availability and quality after this time. Missing characteristics were then imputed using their cross sectional medians for each year.

We then follow [**?**] and construct eight macroeconomic factors following the variable definitions in [**?**]: dividend-price ratio (dp), earnings-price ratio (ep), book-to-market ratio (bm), net equity expansion (ntis), Treasury-bill

Table 6: Macroeconomic Factors, ([?])

No.	Acronym	Macroeconomic Factor
1	macro_dp	Dividend Price Ratio
2	macro_ep	Earnings Price Ratio
3	macro_bm	Book to Market Ratio
4	macro_ntis	Net Equity Expansion
5	macro_tbl	Treasury Bill Rate
6	macro_tms	Term Spread
7	macro_dfy	Default Spread
8	macro_svar	Stock Variance

rate (tbl), term spread (tms), default spread (dfy) and stock variance (svar). These factors were lagged by one period so as to be used to predict one period ahead quarterly returns. The treasury bill rate was also used from this source to proxy for the risk free rate in order to construct excess quarterly returns.

The two sets of factors were then combined to form a baseline set of covariates, which we define throughout all methods and analysis as:

$$z_{i,t} = (1, x_t)' \otimes c_{i,t} \tag{31}$$

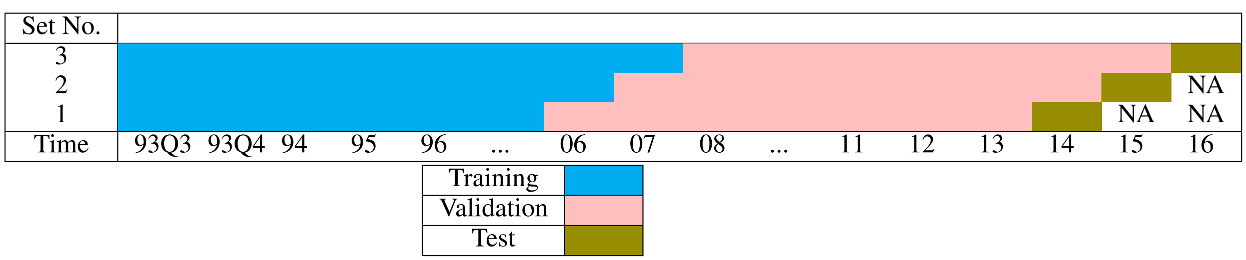
where $c_{i,t}$ is a P_c matrix of characteristics for each stock i, and $(1, x_t)'$ is a $P_x \times 1$ vector of macroeconomic predictors, , and $\otimes$ represents the Kronecker product. $z_{i,t}$ is therefore a $P_x P_c$ vector of features for predicting individual stock returns and includes interactions between stock level characteristics and macroeconomic variables. The total number of covariates in this baseline set is $61 \times (8 + 1) = 549$[14].

The dataset was not normalized for all methods, as only penalized regression and neural networks are sensitive to normalization. For these two methods, the dataset was normalized such that each predictor column had 0 mean and 1 variance.

The final dataset spanned from 1993 Q3 to 2016 Q4 with 202, 066 individual observations.

We mimic the procedure used in the simulation study. For the sample splitting procedure, the dataset was split such that the training and validation sets were split such that the training set was approximately 1.5 times the length of the validation set, in order to predict a test set that is one year in length.

Figure 17: Empirical Data Sample Splitting Procedure



[14]As the individual and macroeconomic factors can have similar names, individual and macroeconomic factors were prefixed with ind_ and macro_ respectively.

Table 7: Empirical Study Loss Statistics

	Sample 1			Sample 2			Sample 3		
model	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2
LM.MSE	0.229034	0.116015	-1.808481	0.397573	0.312653	-6.329935	0.566307	0.83804	-17.522476
LM.MAE	0.273452	0.15894	-2.8476	0.555673	0.742223	-16.400898	0.651614	1.225121	-26.077774
ELN.MSE	0.133887	0.039947	0.032956	0.140402	0.04277	-0.002712	**0.14433**	**0.043761**	**0.032789**
ELN.MAE	0.131369	0.040718	0.014306	**0.137092**	**0.041892**	**0.017875**	0.146251	0.045207	0.000835
RF.MSE	0.130366	**0.036629**	**0.113289**	0.195817	0.070642	-0.656158	0.157934	0.05122	-0.132066
RF.MAE	**0.126703**	0.036785	0.109505	0.173721	0.057546	-0.349132	0.14692	0.046037	-0.01752
NN1.MSE	0.169127	0.057044	-0.380909	0.207662	0.074751	-0.752494	0.192125	0.069738	-0.541369
NN1.MAE	0.157324	0.050418	-0.22052	0.191762	0.066746	-0.564818	0.18547	0.063053	-0.393606
NN2.MSE	0.168773	0.059436	-0.43883	0.181808	0.063232	-0.482433	0.180584	0.062745	-0.386797
NN2.MAE	0.162667	0.055447	-0.342256	0.194277	0.069386	-0.626702	0.185173	0.065186	-0.440746
NN3.MSE	0.154784	0.050152	-0.21408	0.180103	0.060193	-0.411175	0.177604	0.060404	-0.335065
NN3.MAE	0.146411	0.044901	-0.086967	0.18499	0.06461	-0.514744	0.184986	0.063861	-0.411475
NN4.MSE	0.153802	0.048641	-0.177503	0.193066	0.067515	-0.582833	0.172707	0.057774	-0.276929
NN4.MAE	0.157301	0.050286	-0.217308	0.168815	0.055711	-0.306102	0.167998	0.055129	-0.218463
NN5.MSE	0.149436	0.047279	-0.14452	0.183584	0.064137	-0.503653	0.170238	0.056992	-0.259652
NN5.MAE	0.140781	0.042832	-0.036882	0.181096	0.06216	-0.4573	0.164896	0.053458	-0.181528

Table 8: Missing Data Threshold Robustness Check Loss Statistics

	Sample 1			Sample 2			Sample 3		
model	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2
LM.MSE	0.247457	0.130166	-2.151058	0.541089	0.700574	-15.424468	0.615714	1.188991	-25.279238
LM.MAE	0.214055	0.102848	-1.489727	0.372683	0.259976	-5.094954	0.507397	0.766373	-15.93847
ELN.MSE	0.133887	0.039947	0.032956	0.140402	0.04277	-0.002712	**0.14433**	**0.043761**	**0.032789**
ELN.MAE	0.131338	0.040465	0.020421	**0.137083**	**0.041804**	**0.019938**	0.146589	0.045362	-0.002596
RF.MSE	0.129226	0.035869	0.131692	0.198914	0.072749	-0.705542	0.168068	0.05777	-0.276838
RF.MAE	**0.124319**	**0.035103**	**0.150229**	0.167845	0.053578	-0.256106	0.15463	0.051594	-0.140342
NN1.MSE	0.153785	0.048726	-0.179553	0.221019	0.084867	-0.98964	0.172557	0.058354	-0.289742
NN1.MAE	0.154534	0.048854	-0.18266	0.199647	0.073699	-0.727823	0.176348	0.061359	-0.356155
NN2.MSE	0.158513	0.057061	-0.381324	0.233631	0.095004	-1.227299	0.154083	0.048353	-0.068708
NN2.MAE	0.138489	0.043364	-0.049759	0.215253	0.078792	-0.847234	0.164459	0.055049	-0.216706
NN3.MSE	0.167392	0.058508	-0.416345	0.19754	0.071293	-0.671422	0.156873	0.049602	-0.096299
NN3.MAE	0.144457	0.045293	-0.096445	0.210372	0.077747	-0.822723	0.159841	0.05152	-0.138704
NN4.MSE	0.147989	0.047211	-0.142888	0.184277	0.064247	-0.506225	0.152214	0.048185	-0.064987
NN4.MAE	0.15851	0.052021	-0.259326	0.18643	0.063032	-0.477746	0.177651	0.064046	-0.415562
NN5.MSE	0.153187	0.050053	-0.211683	0.181622	0.060313	-0.413989	0.161028	0.051221	-0.132095
NN5.MAE	0.149496	0.050779	-0.229251	0.165726	0.053988	-0.265712	0.156151	0.049772	-0.100061

A.2 Empirical Data Robustness Checks

In addition to the main study, we provide four additional robustness checks for our empirical study, with regards to different training/validation splitting schemes, missing data imputation and additional regressors. Importantly, our overall results are consistent across all checks.

We consider training:validation length ratios of 1:1 and 1:2 in addition to 1:1.5 in the main study.

We consider changing the missing data threshold to be 10% - that is, any regressors with over 10% missing data were omitted.

We finally consider supplementing our macroeconomic regressor set with the five Fama-French factors.

A.3 Empirical Data Results

A.3.1 Prediction Accuracy

Figure 18: Individual Factor Importance

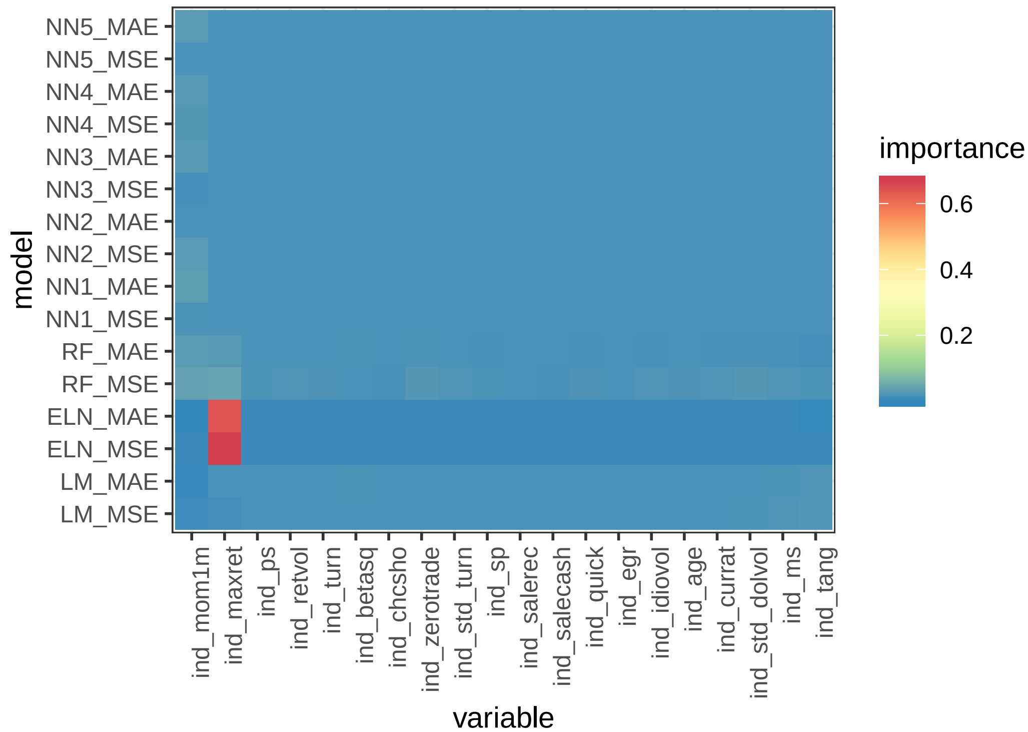


Figure 19: Macroeconomic Factor Importance

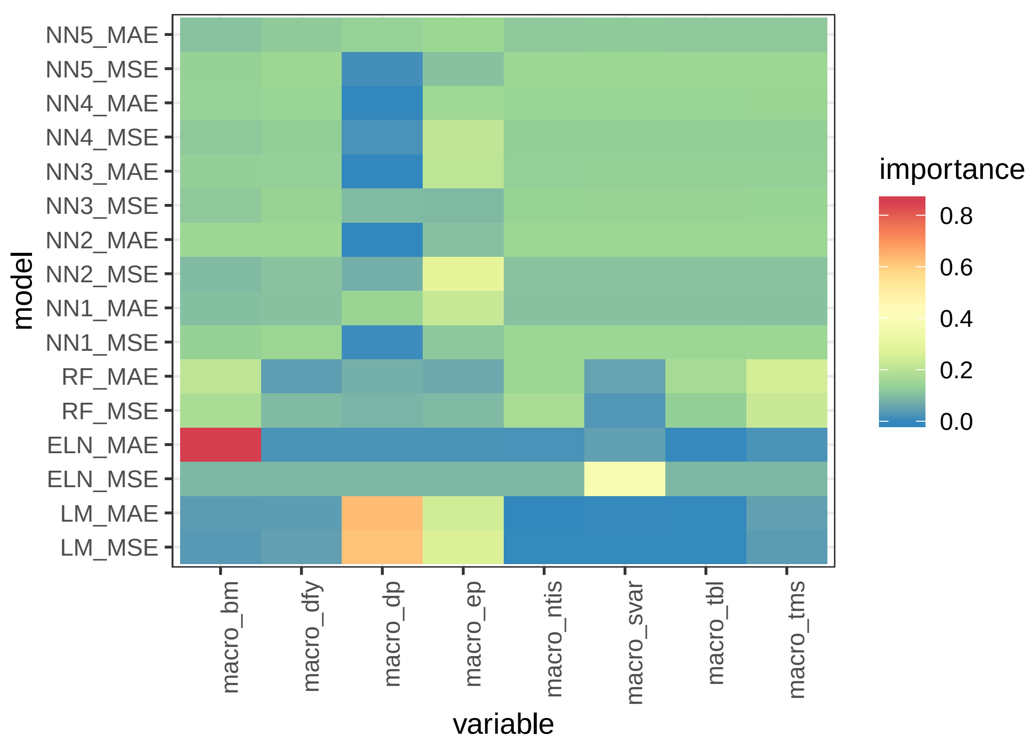


Figure 20: Robustness Check RF VIMP

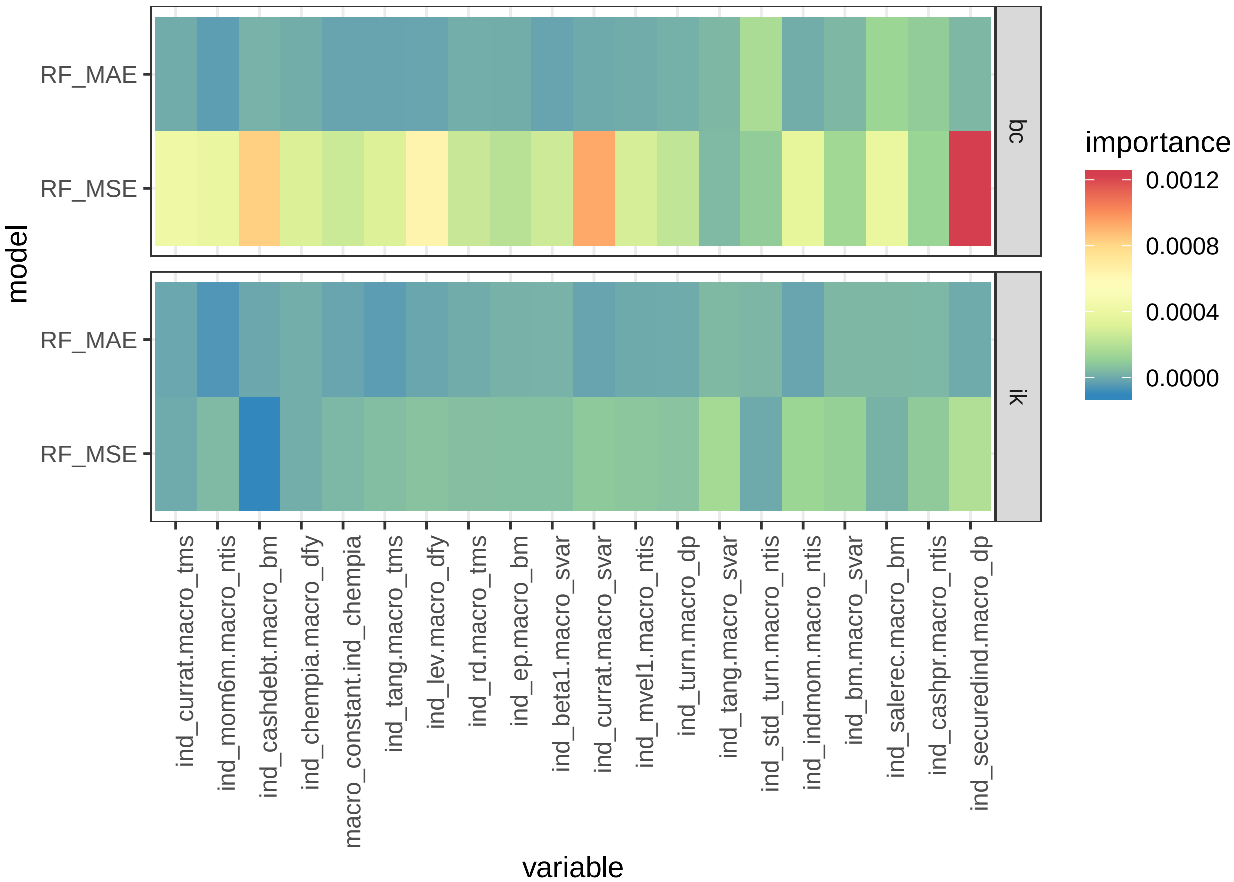


Table 9: Train:Validation 1:1 Robustness Check Loss Statistics

	Sample 1			Sample 2			Sample 3		
model	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2
LM.MSE	0.915703	2.495094	-59.401029	0.717	1.553454	-35.419641	0.451206	0.375505	-7.299459
LM.MAE	0.751551	1.583265	-37.32754	0.469831	0.524686	-11.300895	0.675112	1.105759	-23.43964
ELN.MSE	0.134609	**0.040072**	**0.029933**	0.141434	0.043169	-0.012055	**0.144375**	**0.043705**	**0.034019**
ELN.MAE	**0.131668**	0.040748	0.013583	**0.137494**	**0.042135**	**0.012178**	0.146776	0.045753	-0.01123
RF.MSE	0.155282	0.046655	-0.129427	0.210936	0.078006	-0.828784	0.229147	0.092622	-1.047155
RF.MAE	0.13882	0.04016	0.027805	0.185338	0.063217	-0.482087	0.182753	0.063873	-0.411736
NN1.MSE	0.218129	0.087699	-1.123002	0.238606	0.110201	-1.583582	0.260721	0.120908	-1.672321
NN1.MAE	0.202259	0.072844	-0.763409	0.205092	0.073567	-0.724721	0.239051	0.096477	-1.132346
NN2.MSE	0.239446	0.101312	-1.452556	0.206109	0.078412	-0.838305	0.228591	0.095126	-1.102488
NN2.MAE	0.19141	0.068261	-0.652455	0.184095	0.062366	-0.462125	0.220087	0.086888	-0.920403
NN3.MSE	0.193117	0.069206	-0.675336	0.193859	0.070747	-0.658609	0.205093	0.076497	-0.690745
NN3.MAE	0.191596	0.066926	-0.620138	0.176555	0.060022	-0.407183	0.234768	0.091003	-1.011359
NN4.MSE	0.191361	0.07068	-0.71101	0.175311	0.059253	-0.389136	0.18148	0.061718	-0.364096
NN4.MAE	0.139659	0.041096	0.005158	0.179318	0.05976	-0.401027	0.188921	0.066144	-0.461932
NN5.MSE	0.17209	0.056982	-0.379418	0.164756	0.054398	-0.275325	0.202012	0.074051	-0.636691
NN5.MAE	0.170945	0.056029	-0.356356	0.180669	0.059697	-0.399552	0.189149	0.065921	-0.456988

Figure 21: Missing Data Threshold Robustness Check Individual Factor Importance

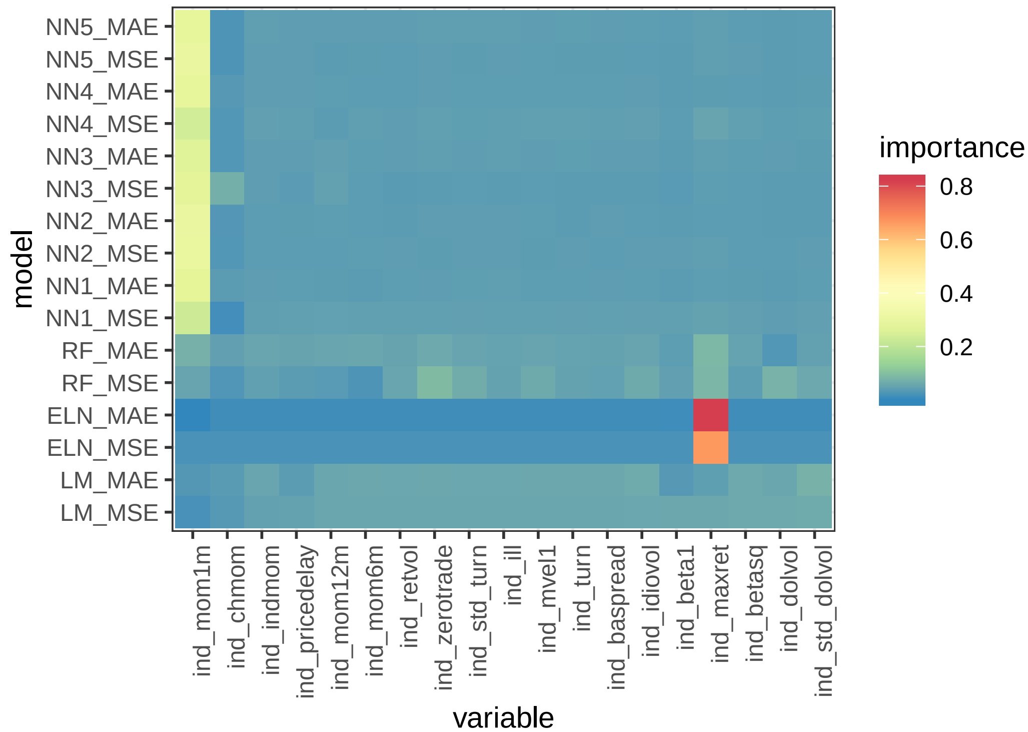


Figure 22: Missing Data Threshold Robustness Check Macroeconomic Factor Importance

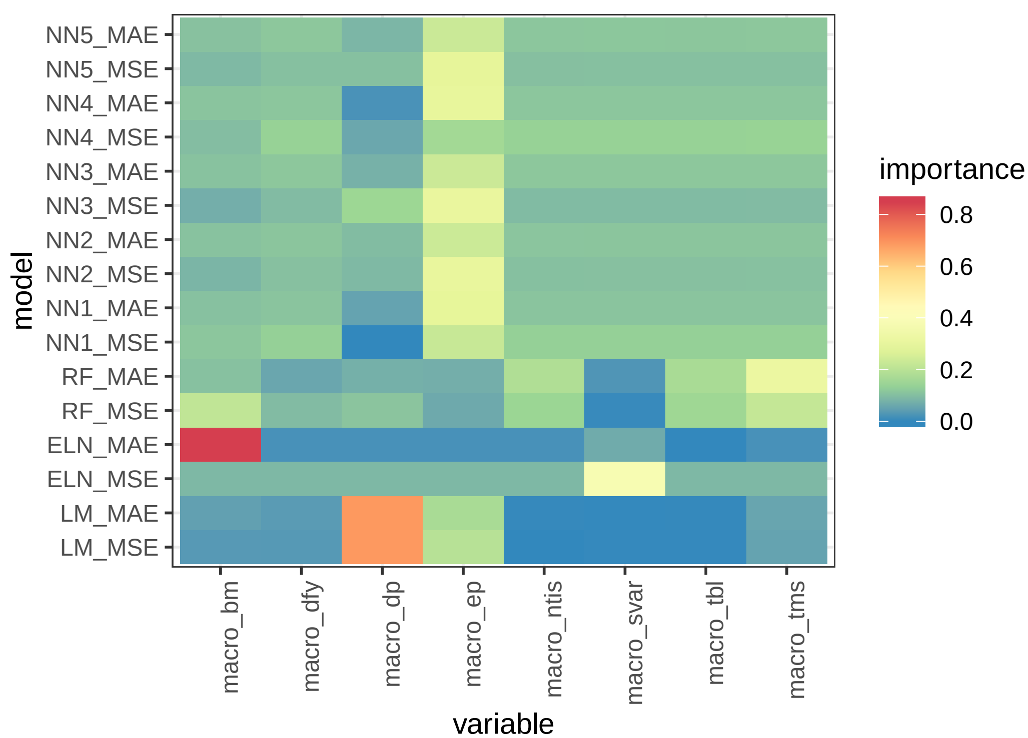


Figure 23: Missing Data Threshold Robustness Check RF VIMP

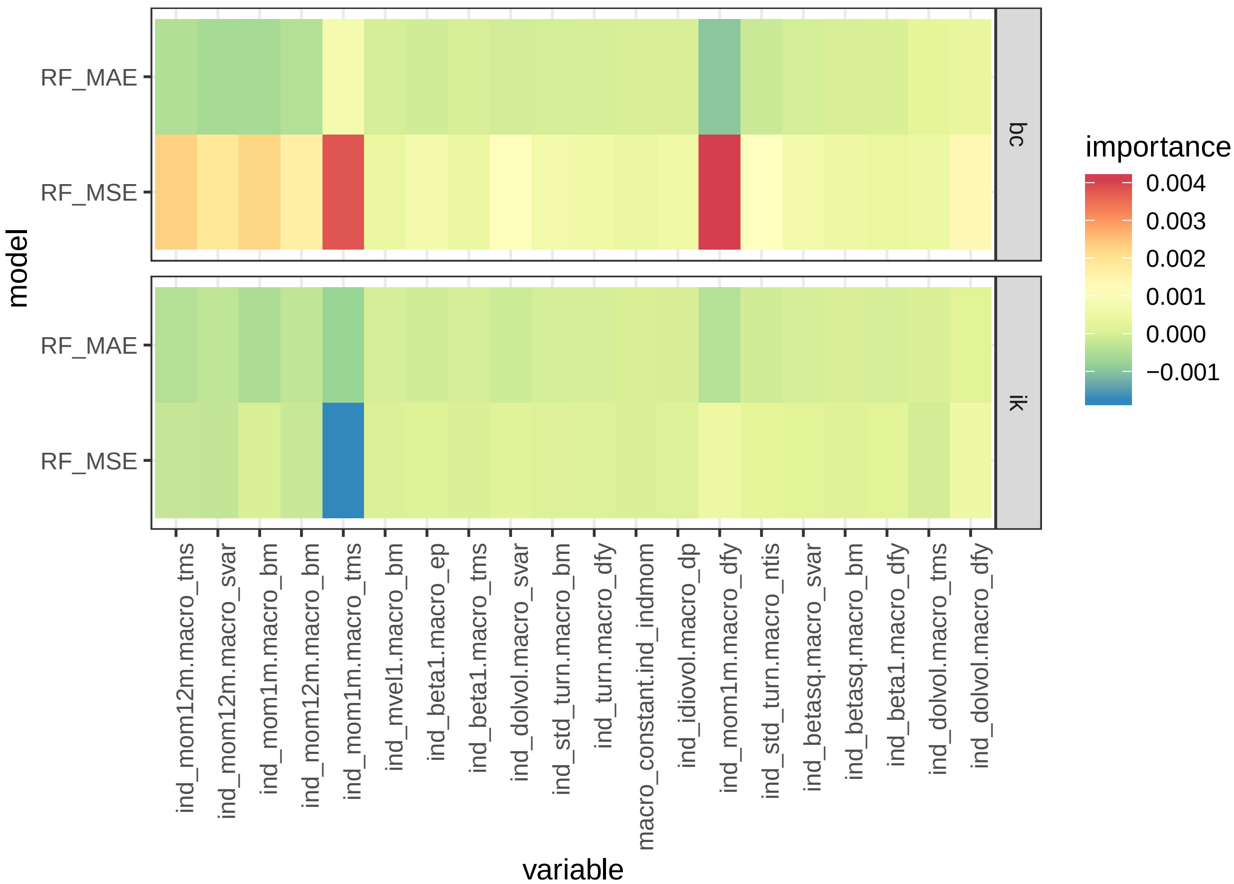


Table 10: Train:Validation 2:1 Robustness Check Loss Statistics

	Sample 1			Sample 2			Sample 3		
model	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2
LM.MSE	0.277087	0.164599	-2.98459	0.383421	0.31299	-6.337839	0.523418	0.740288	-15.361936
LM.MAE	0.246936	0.147979	-2.582262	0.277044	0.161215	-2.779579	0.487285	0.631575	-12.95915
ELN.MSE	0.133715	0.039919	0.033647	0.139723	0.042525	0.003028	**0.145034**	**0.044306**	**0.020752**
ELN.MAE	0.131237	0.040361	0.022952	**0.137205**	**0.041858**	**0.018674**	0.174408	0.064513	-0.425873
RF.MSE	0.130808	0.036982	0.104754	0.162762	0.051118	-0.198417	0.155264	0.048661	-0.075516
RF.MAE	**0.127013**	**0.036722**	**0.111033**	0.146758	0.043961	-0.030633	0.168905	0.055983	-0.237348
NN1.MSE	0.155088	0.050284	-0.217281	0.165871	0.053459	-0.253309	0.181984	0.064621	-0.428262
NN1.MAE	0.159797	0.050566	-0.224107	0.163397	0.052329	-0.226828	0.181636	0.062407	-0.379326
NN2.MSE	0.155815	0.050954	-0.233492	0.168576	0.055738	-0.306745	0.170991	0.057453	-0.269824
NN2.MAE	0.148149	0.047617	-0.152709	0.166334	0.054058	-0.26734	0.163141	0.052639	-0.163436
NN3.MSE	0.154141	0.04976	-0.204586	0.166218	0.053402	-0.251967	0.169539	0.05661	-0.251204
NN3.MAE	0.142464	0.043771	-0.059594	0.154233	0.048682	-0.141321	0.184217	0.064175	-0.418401
NN4.MSE	0.166547	0.056184	-0.360092	0.150748	0.047566	-0.115162	0.168447	0.056575	-0.250437
NN4.MAE	0.150167	0.046919	-0.135802	0.16197	0.05226	-0.225199	0.171676	0.057352	-0.267598
NN5.MSE	0.155784	0.052258	-0.265047	0.139699	0.043082	-0.010018	0.166166	0.055027	-0.216219
NN5.MAE	0.161161	0.053216	-0.28825	0.149207	0.046344	-0.086511	0.149424	0.047544	-0.050824

Figure 24: Train:Validation = 1:1 Robustness Check Individual Factor Importance

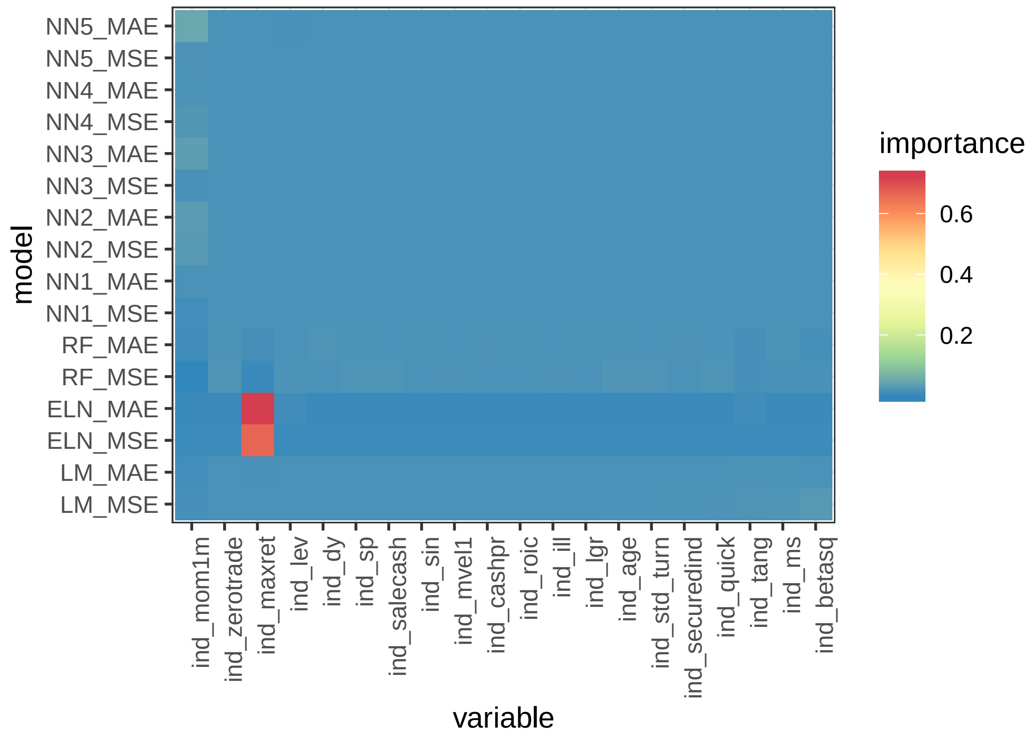


Figure 25: Train:Validation = 1:1 Robustness Check Macroeconomic Factor Importance

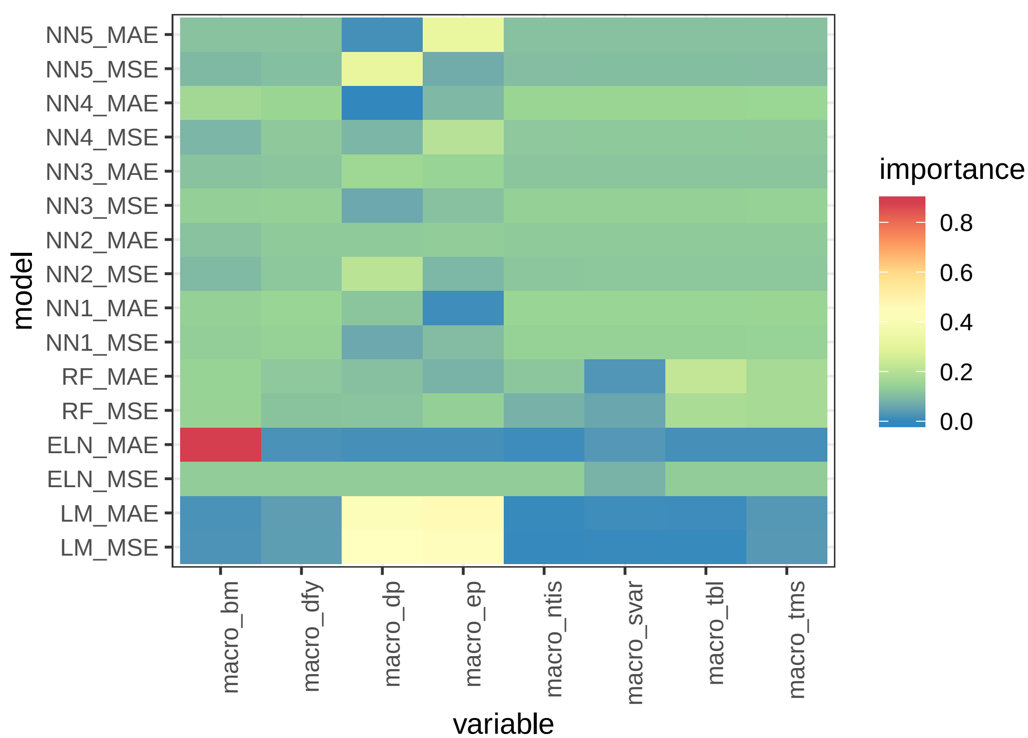


Figure 26: Train:Validation = 1:1 Robustness Check RF VIMP

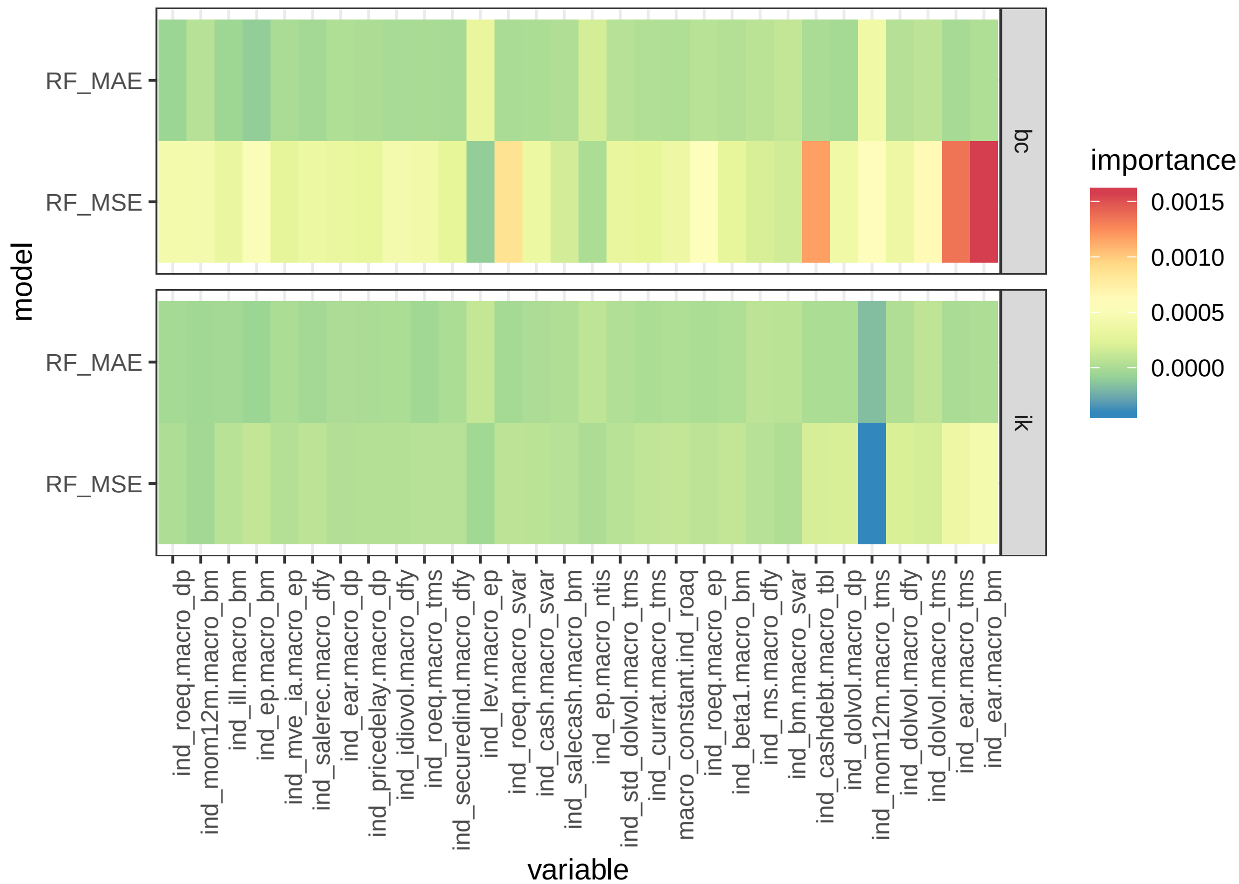


Table 11: Fama French Factor Robustness Check Loss Statistics

	Sample 1			Sample 2			Sample 3		
model	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2	Test MAE	Test MSE	Test R^2
LM.MSE	0.288636	0.182966	-3.42923	0.367636	0.264918	-5.210825	1.101604	5.012469	-109.78624
LM.MAE	0.280535	0.179777	-3.352038	0.376163	0.279476	-5.552114	1.25341	7.06036	-155.048996
ELN.MSE	0.13383	0.039956	0.032746	0.14022	0.0427	-0.00107	**0.144472**	**0.043852**	**0.030769**
ELN.MAE	**0.128936**	**0.039665**	**0.039798**	**0.13716**	**0.042144**	**0.011965**	0.172148	0.063154	-0.395841
RF.MSE	0.146318	0.042607	-0.031434	0.151137	0.047091	-0.104011	0.177125	0.064664	-0.429221
RF.MAE	0.138266	0.04005	0.030475	0.138714	0.042246	0.009583	0.152068	0.048488	-0.071698
NN1.MSE	0.168063	0.055354	-0.340017	0.192143	0.068904	-0.61541	0.275195	0.138165	-2.053731
NN1.MAE	0.161596	0.051507	-0.246873	0.199416	0.068181	-0.598444	0.23054	0.093434	-1.065082
NN2.MSE	0.169842	0.056899	-0.377415	0.179733	0.058966	-0.382416	0.252929	0.117102	-1.588199
NN2.MAE	0.155816	0.046809	-0.133147	0.185008	0.060854	-0.426679	0.219342	0.085115	-0.881213
NN3.MSE	0.1621	0.053165	-0.287008	0.182996	0.059643	-0.398278	0.232226	0.099353	-1.195903
NN3.MAE	0.161255	0.050737	-0.228237	0.191625	0.064676	-0.516291	0.218355	0.085297	-0.885238
NN4.MSE	0.166036	0.055575	-0.345349	0.191589	0.066207	-0.552182	0.23417	0.097348	-1.151607
NN4.MAE	0.148375	0.045227	-0.094843	0.168623	0.054176	-0.270114	0.20837	0.077667	-0.7166
NN5.MSE	0.147379	0.044503	-0.077315	0.166006	0.054935	-0.287914	0.20667	0.077866	-0.721013
NN5.MAE	0.150541	0.045723	-0.106868	0.172466	0.055402	-0.298865	0.218796	0.084938	-0.877301

Figure 27: Train:Validation = 2:1 Robustness Check Individual Factor Importance

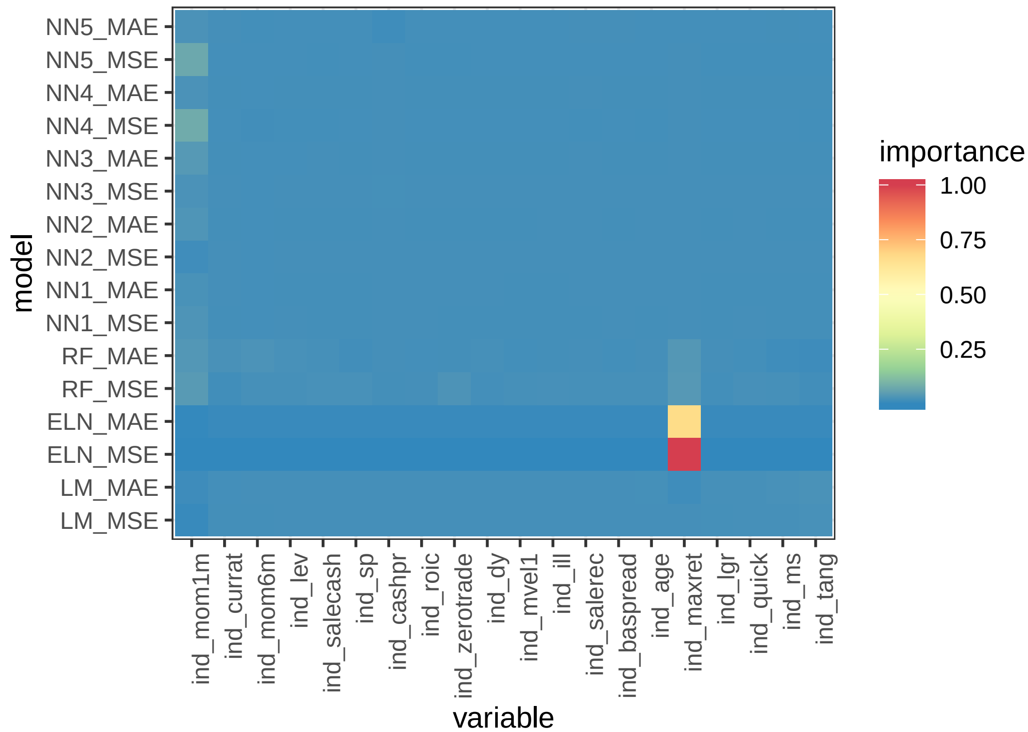


Figure 28: Train:Validation = 2:1 Robustness Check Macroeconomic Factor Importance

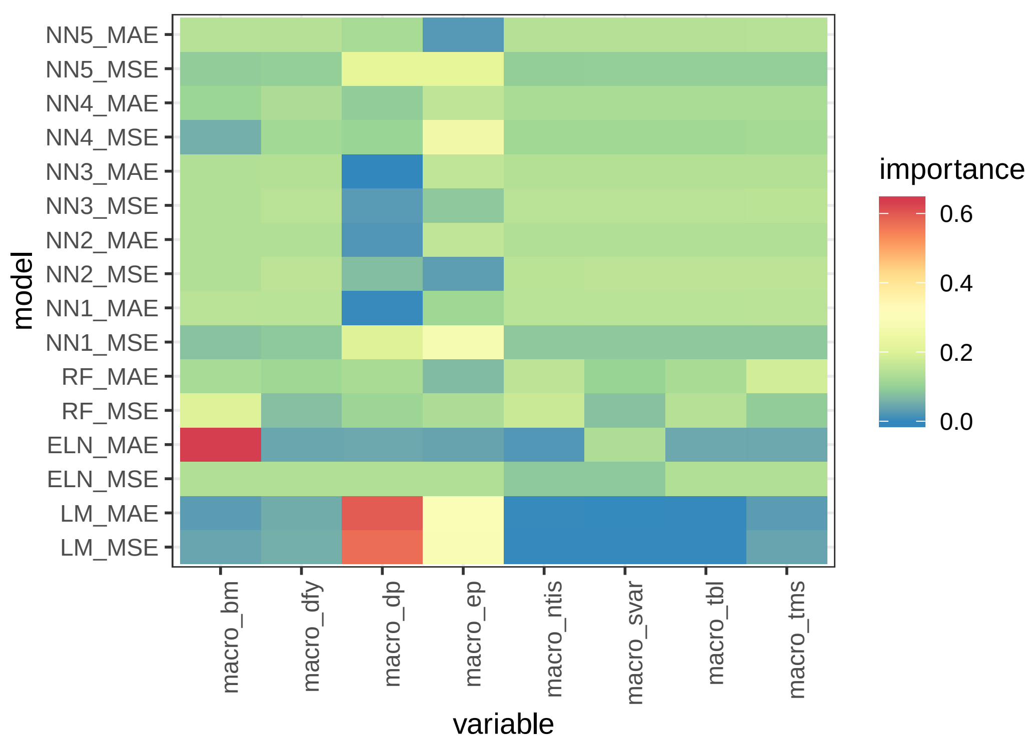


Figure 29: Train:Validation = 2:1 Robustness Check RF VIMP

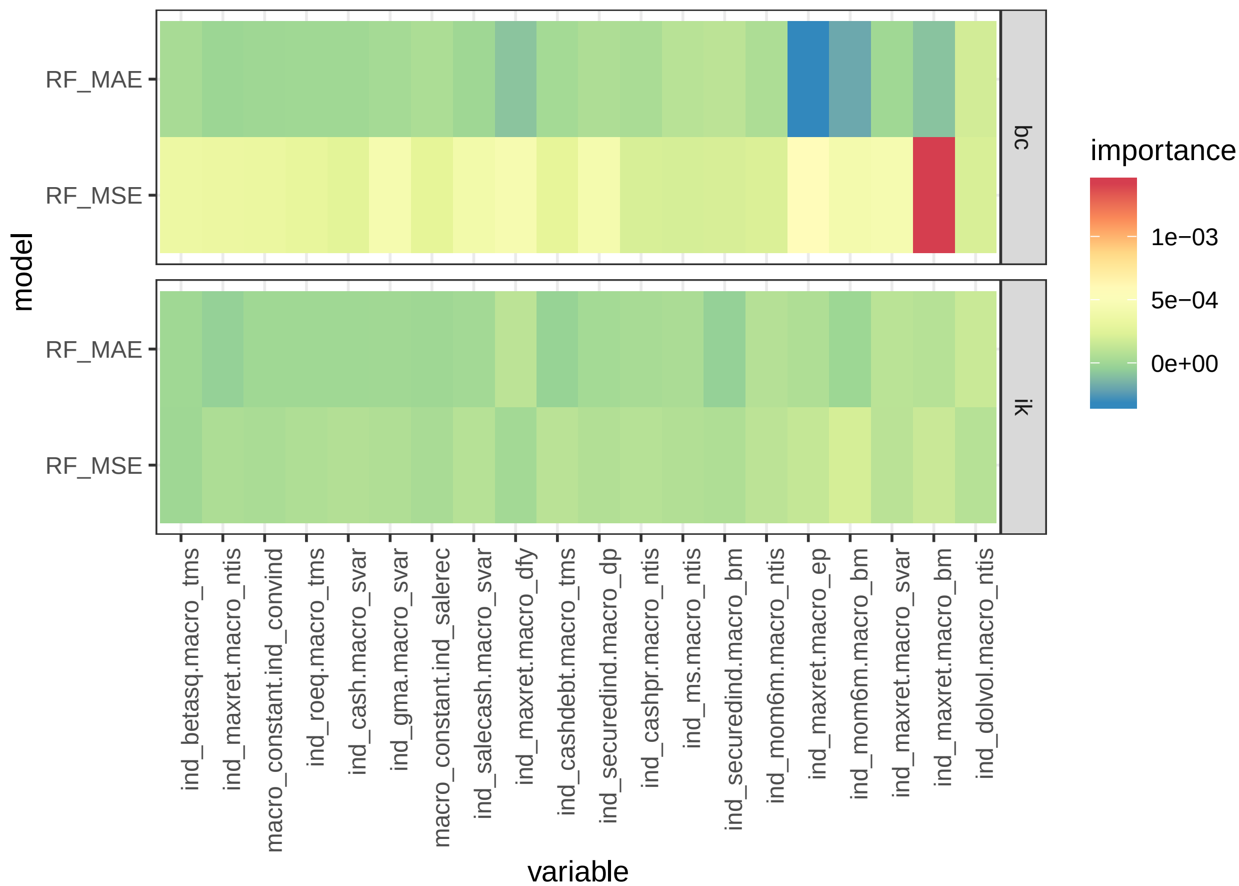


Figure 30: Fama French Factors Robustness Check Individual Factor Importance

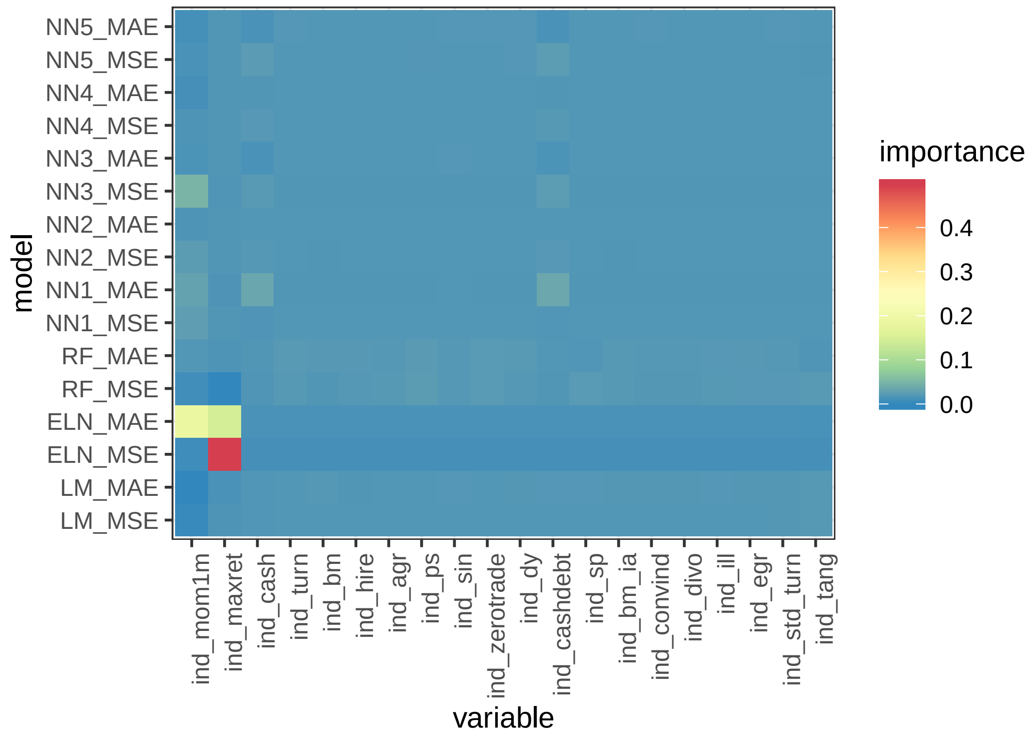


Figure 31: Fama French Factors Robustness Check Macroeconomic Factor Importance

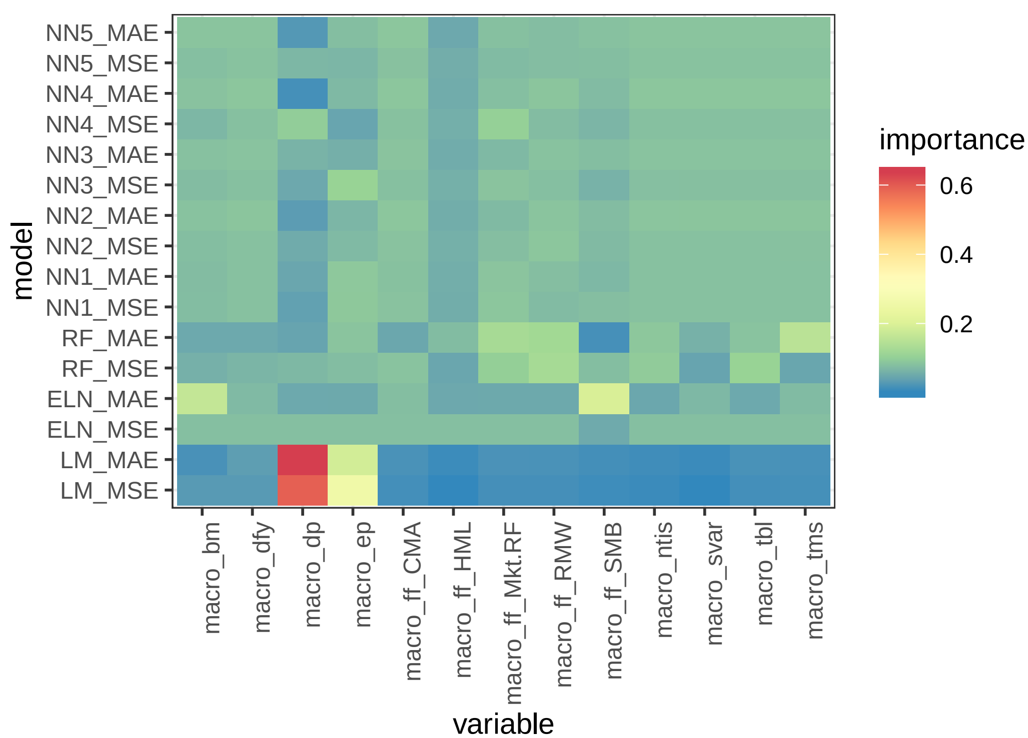


Figure 32: Fama French Factors Robustness Check RF VIMP