Geotechnical Summary Report — Bayesian Updating for 1-D Consolidation

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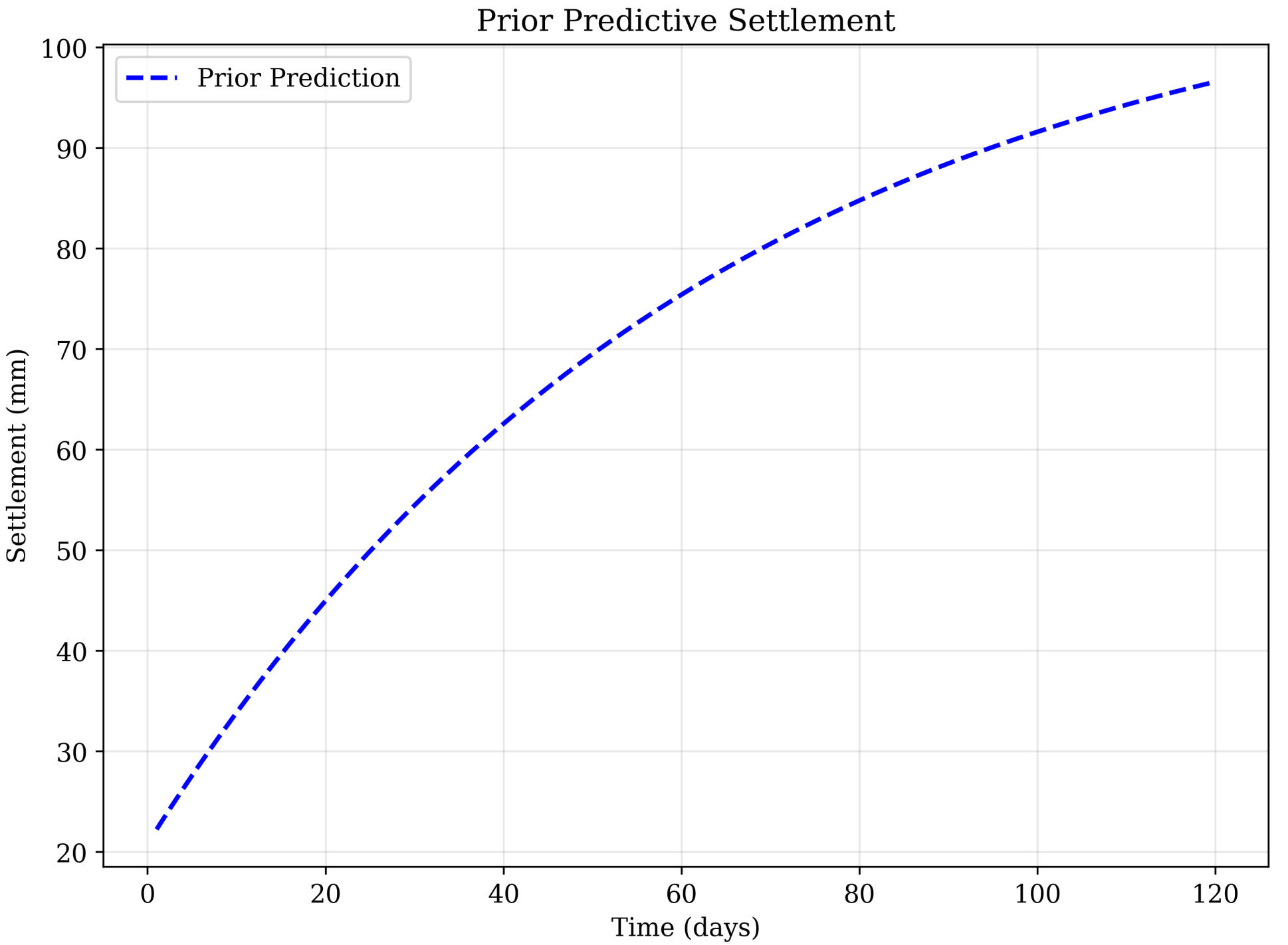
Key Assumptions:
• Soil model: Terzaghi 1-D, first-term approximation U(T_v)
• Layer thickness H = 5 m (double drainage ⇒ H/2 = 2.5 m)
• Surcharge Δσ' = 22 kPa → S_inf(mm) = 110000 · m_v
• Measurement noise σ_e = 3 mm (Gaussian, independent)
• Priors: independent lognormal on m_v and c_v
• Posterior: computed on a 161×161 log-spaced grid in (m_v, c_v)

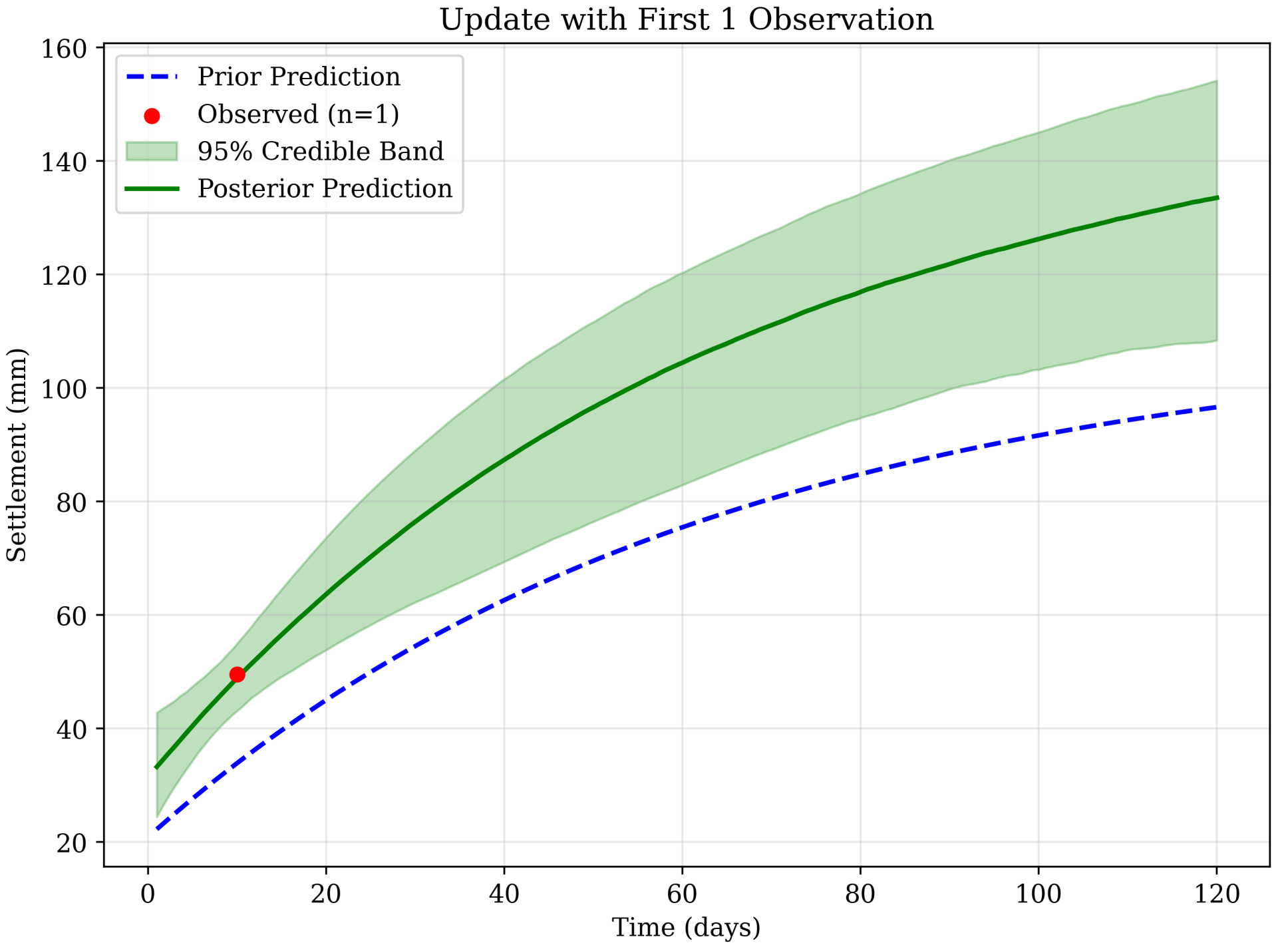
Analysis Workflow:
1) Define (m_v, c_v) log grids; evaluate prior p(m_v)p(c_v).
2) Compute s(t_i; θ) and likelihood φ((y_i − s)/σ_e) on the grid.
3) Form unnormalized posterior = prior × likelihood; normalize via log-sum-exp.
4) Summaries: posterior mean (m̂_v, ĉ_v), marginals, 95% credible intervals.
5) Sequential updates for k = 1..4; 3-D and joint posterior maps generated.

Deliverables:
• Sequential predictive figures (prior and k=1..4)
• Worked-example panels (prior, likelihood, posterior) per k
• Posterior joint heatmap (all data) and 3-D surfaces (k=1, k=4)
• Marginal prior vs posterior plots (m_v, c_v) with 95% CrI
• Summary equation page; parameter summary table

step0_prior_only.png



step1_first_obs.png



equation_obs1.png

Bayesian Update Equation (1 Observation)

$$p(\theta \mid y_{1:1}) \propto \left[\prod_{i=1}^{1} \phi\left(\frac{y_i - s(t_i;\theta)}{\sigma_e}\right)\right] p(\theta)$$

$$\phi\left(\frac{49.5\,\mathrm{mm} - s(10;\theta)}{3.0\,\mathrm{mm}}\right)$$

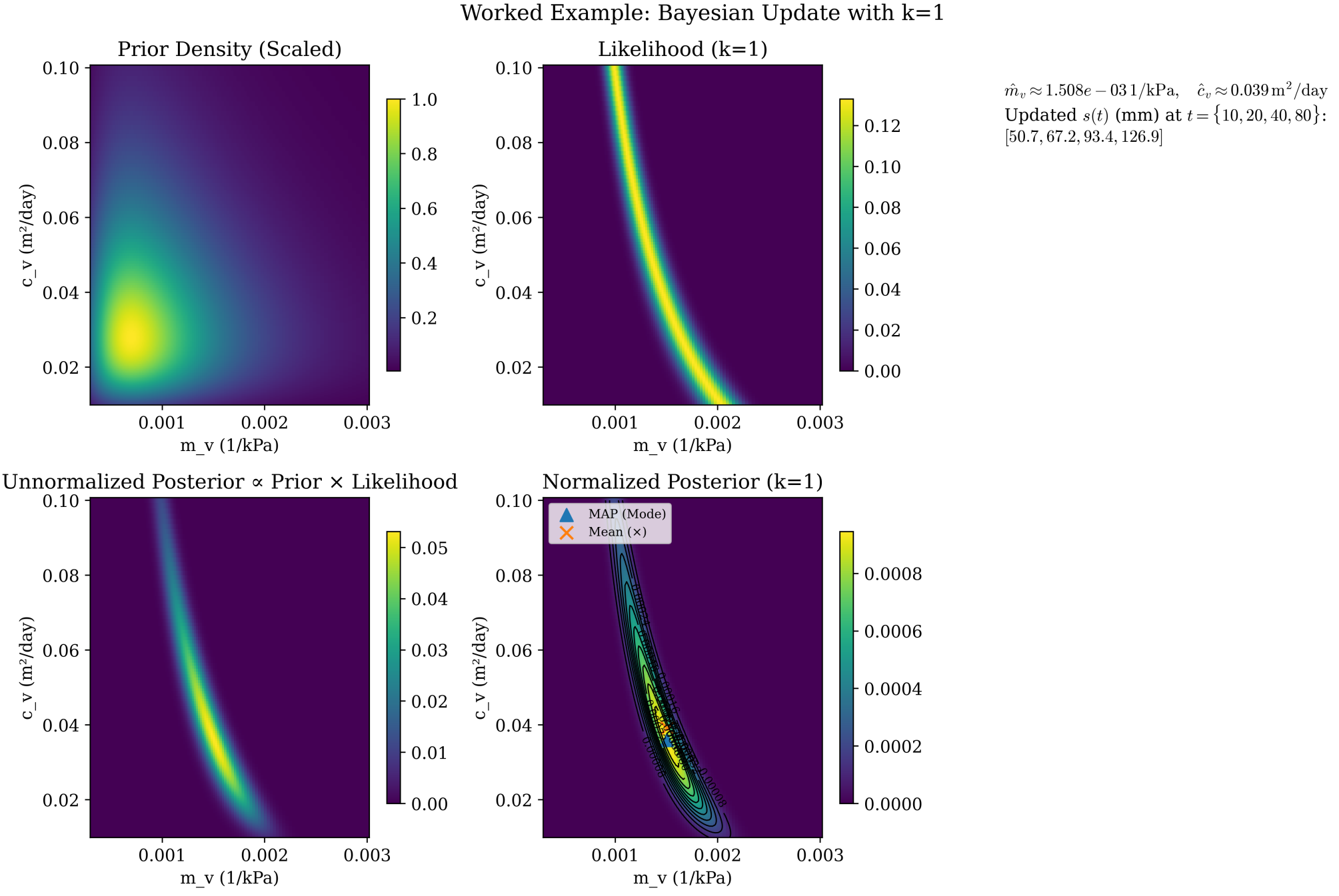
$$s(t;\theta) = m_v\,\Delta\sigma'\,H\,U\left(\frac{c_v\,t}{(H/2)^2}\right), U(T_v) \approx 1 - \frac{8}{\pi^2}\exp\left(-\frac{\pi^2}{4}T_v\right)$$

$$\Delta\sigma' = 22\,\mathrm{kPa},\ H = 5\,\mathrm{m},\ H/2 = 2.5\,\mathrm{m},\ \sigma_e = 3.0\,\mathrm{mm}$$

$$\hat{m_v} = 1.508e-03\,1/\mathrm{kPa},\ \hat{c_v} = 0.039\,\mathrm{m}^2/\mathrm{day}$$

Updated $s(t)$ (mm) at $t = [10, 20, 40, 80] : [50.7, 67.2, 93.4, 126.9]$

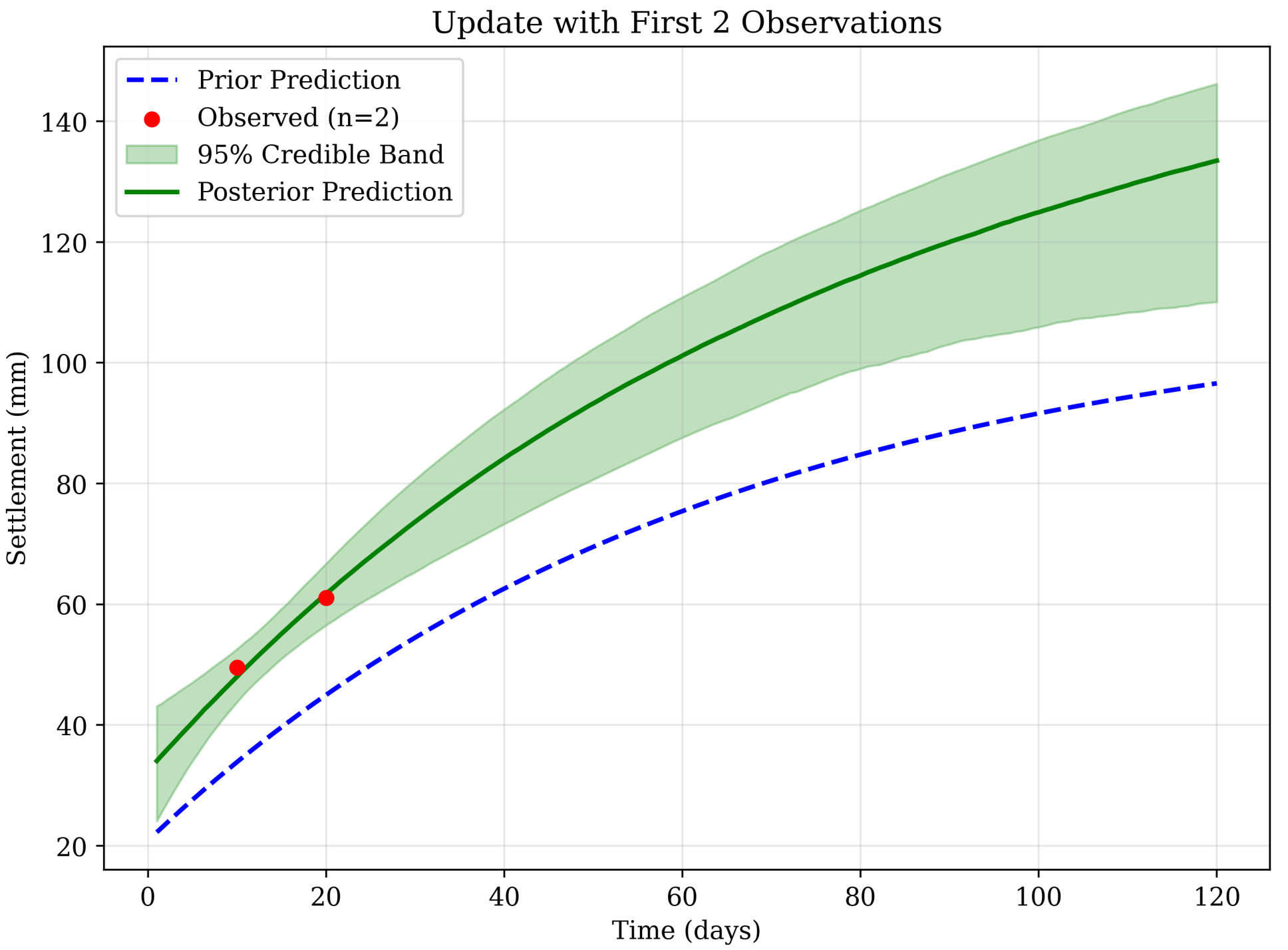
equation_obs1_worked_example.png



$\hat{m}_v \approx 1.508e-03\,1/\mathrm{kPa}, \quad \hat{c}_v \approx 0.039\,\mathrm{m}^2/\mathrm{day}$

Updated $s(t)$ (mm) at $t = \{10, 20, 40, 80\}$:
$[50.7, 67.2, 93.4, 126.9]$

step2_first_two.png



equation_obs2.png

Bayesian Update Equation (2 Observations)

$$\boxed{p(\theta \mid y_{1:2}) \propto \left[\prod_{i=1}^{2} \phi\left(\frac{y_i - s(t_i;\theta)}{\sigma_e}\right)\right] p(\theta)}$$

$$\phi\left(\frac{49.5\,\mathrm{mm} - s(10;\theta)}{3.0\,\mathrm{mm}}\right)$$
$$\phi\left(\frac{61.1\,\mathrm{mm} - s(20;\theta)}{3.0\,\mathrm{mm}}\right)$$

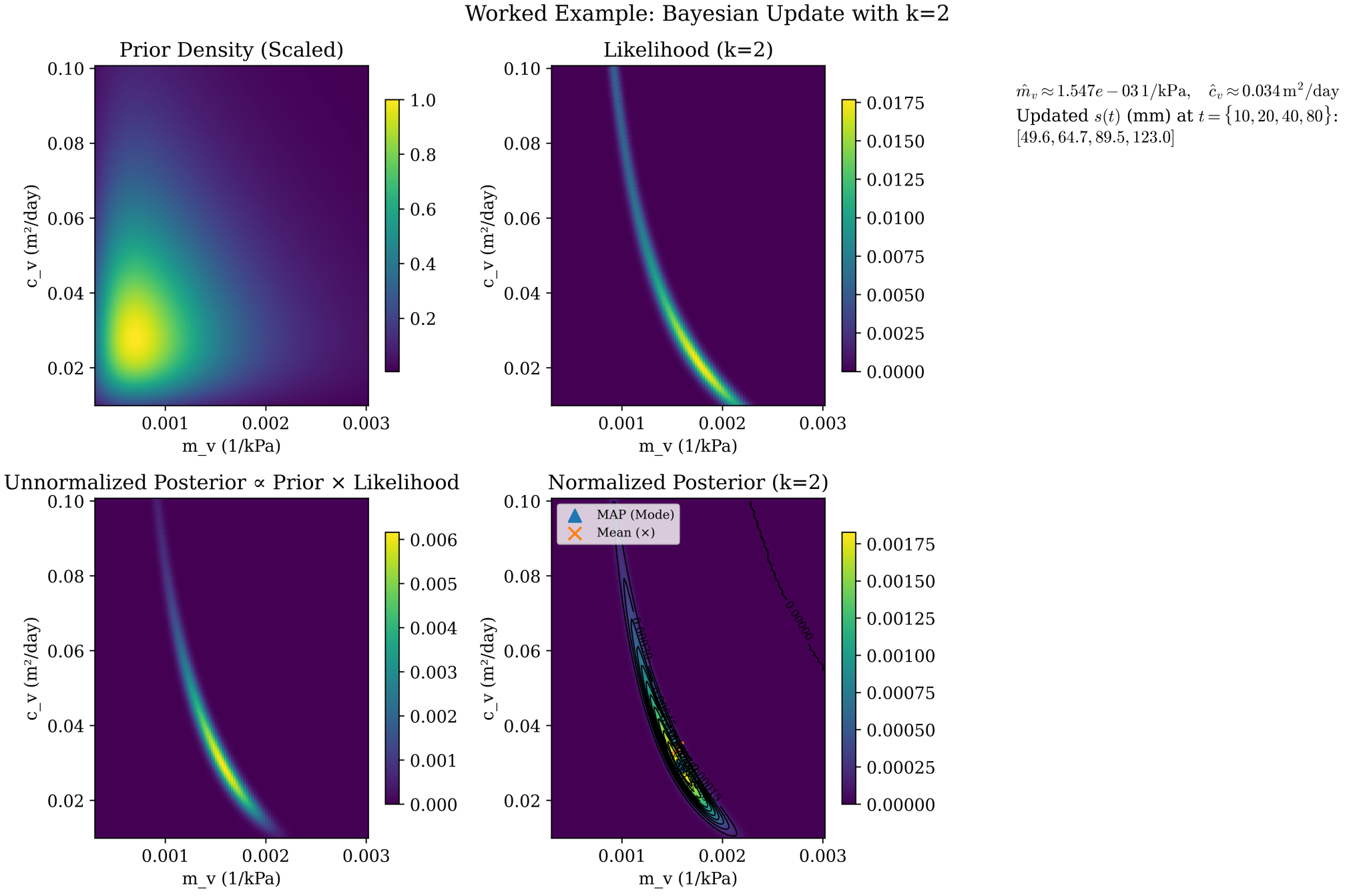
$$s(t;\theta) = m_v\,\Delta\sigma'\,H\,U\left(\frac{c_v\,t}{(H/2)^2}\right), U(T_v) \approx 1 - \frac{8}{\pi^2}\exp\left(-\frac{\pi^2}{4}T_v\right)$$

$$\Delta\sigma' = 22\,\mathrm{kPa},\ H = 5\,\mathrm{m},\ H/2 = 2.5\,\mathrm{m},\ \sigma_e = 3.0\,\mathrm{mm}$$

$$\hat{m_v} = 1.547e-03\,1/\mathrm{kPa},\ \hat{c_v} = 0.034\,\mathrm{m}^2/\mathrm{day}$$

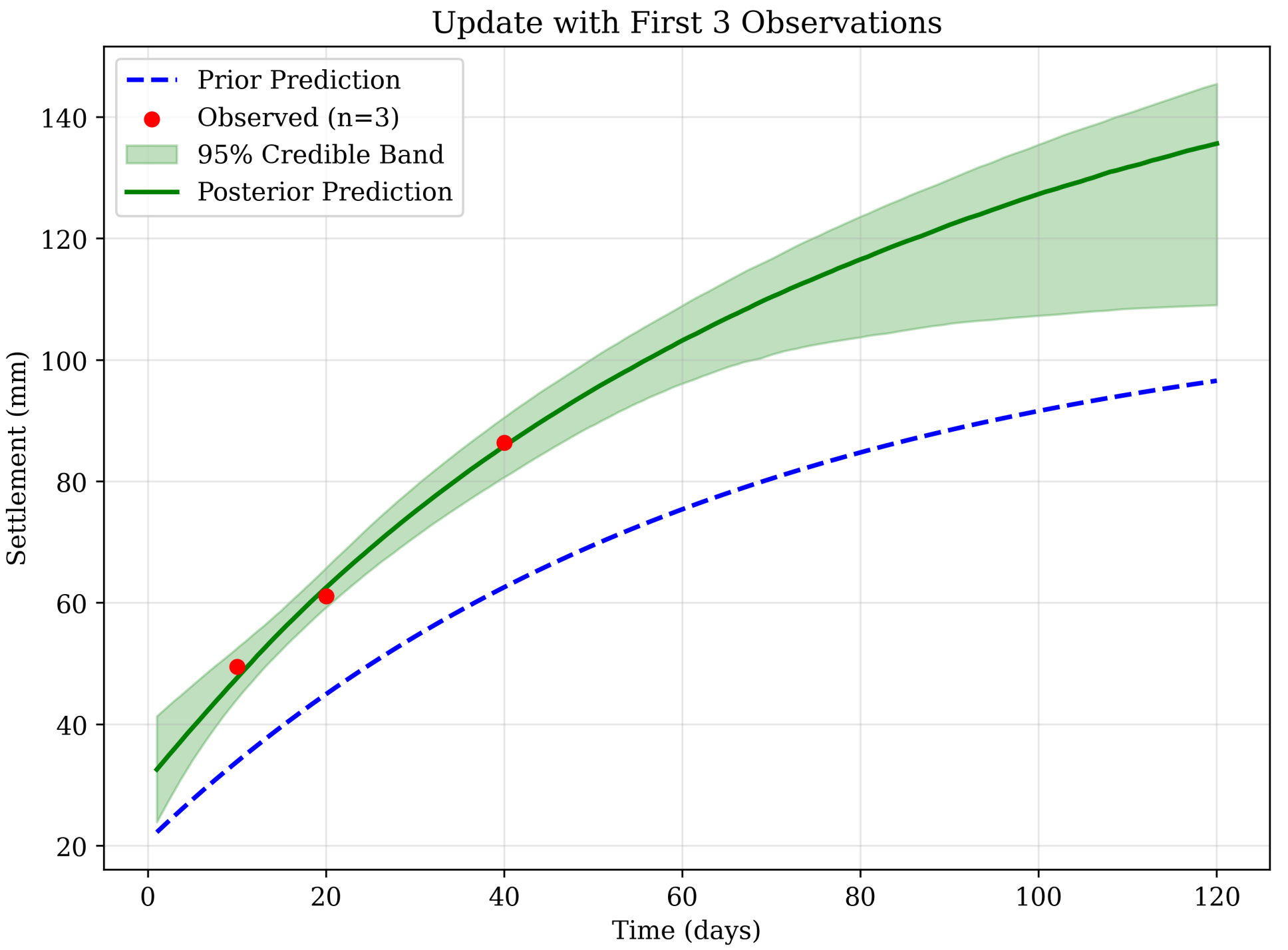
Updated $s(t)$ (mm) at $t = [10, 20, 40, 80] : [49.6, 64.7, 89.5, 123.0]$

equation_obs2_worked_example.png



$\hat{m}_v \approx 1.547e-03\,1/\mathrm{kPa}, \quad \hat{c}_v \approx 0.034\,\mathrm{m}^2/\mathrm{day}$
Updated $s(t)$ (mm) at $t = \{10, 20, 40, 80\}$:
$[49.6, 64.7, 89.5, 123.0]$

step3_first_three.png



Bayesian Update Equation (3 Observations)

$$p(\theta \mid y_{1:3}) \propto \left[\prod_{i=1}^{3} \phi\left(\frac{y_i - s(t_i;\theta)}{\sigma_e}\right)\right] p(\theta)$$

$$\phi\left(\frac{49.5\,\mathrm{mm} - s(10;\theta)}{3.0\,\mathrm{mm}}\right)$$
$$\phi\left(\frac{61.1\,\mathrm{mm} - s(20;\theta)}{3.0\,\mathrm{mm}}\right)$$
$$\phi\left(\frac{86.4\,\mathrm{mm} - s(40;\theta)}{3.0\,\mathrm{mm}}\right)$$

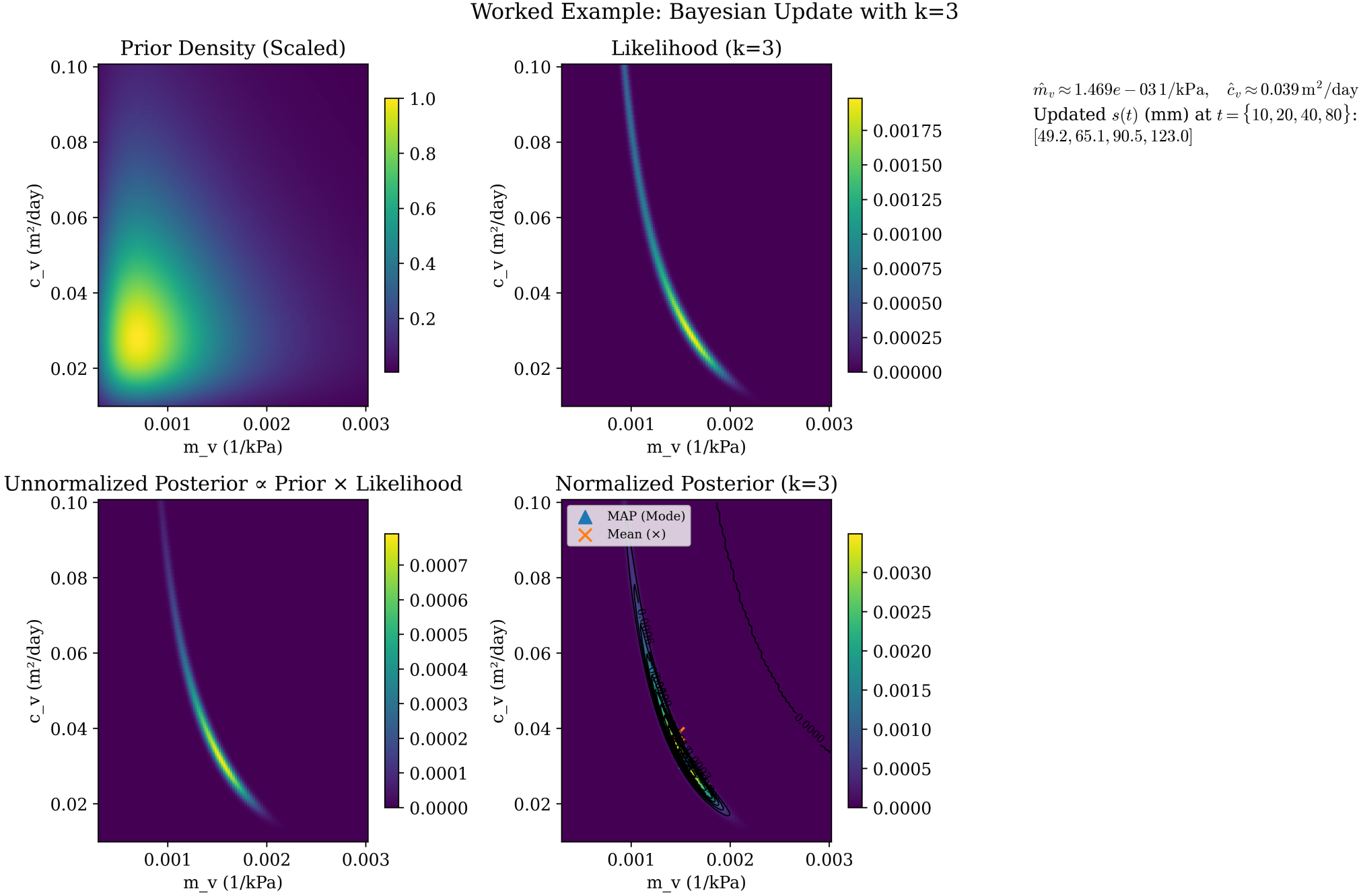
$$s(t;\theta) = m_v\,\Delta\sigma'\,H\,U\left(\frac{c_v\,t}{(H/2)^2}\right), U(T_v) \approx 1 - \frac{8}{\pi^2}\exp\left(-\frac{\pi^2}{4}T_v\right)$$

$$\Delta\sigma' = 22\,\mathrm{kPa},\ H = 5\,\mathrm{m},\ H/2 = 2.5\,\mathrm{m},\ \sigma_e = 3.0\,\mathrm{mm}$$

$$\hat{m_v} = 1.469e-03\,1/\mathrm{kPa},\ \hat{c_v} = 0.039\,\mathrm{m}^2/\mathrm{day}$$

Updated $s(t)$ (mm) at $t = [10, 20, 40, 80] : [49.2, 65.1, 90.5, 123.0]$

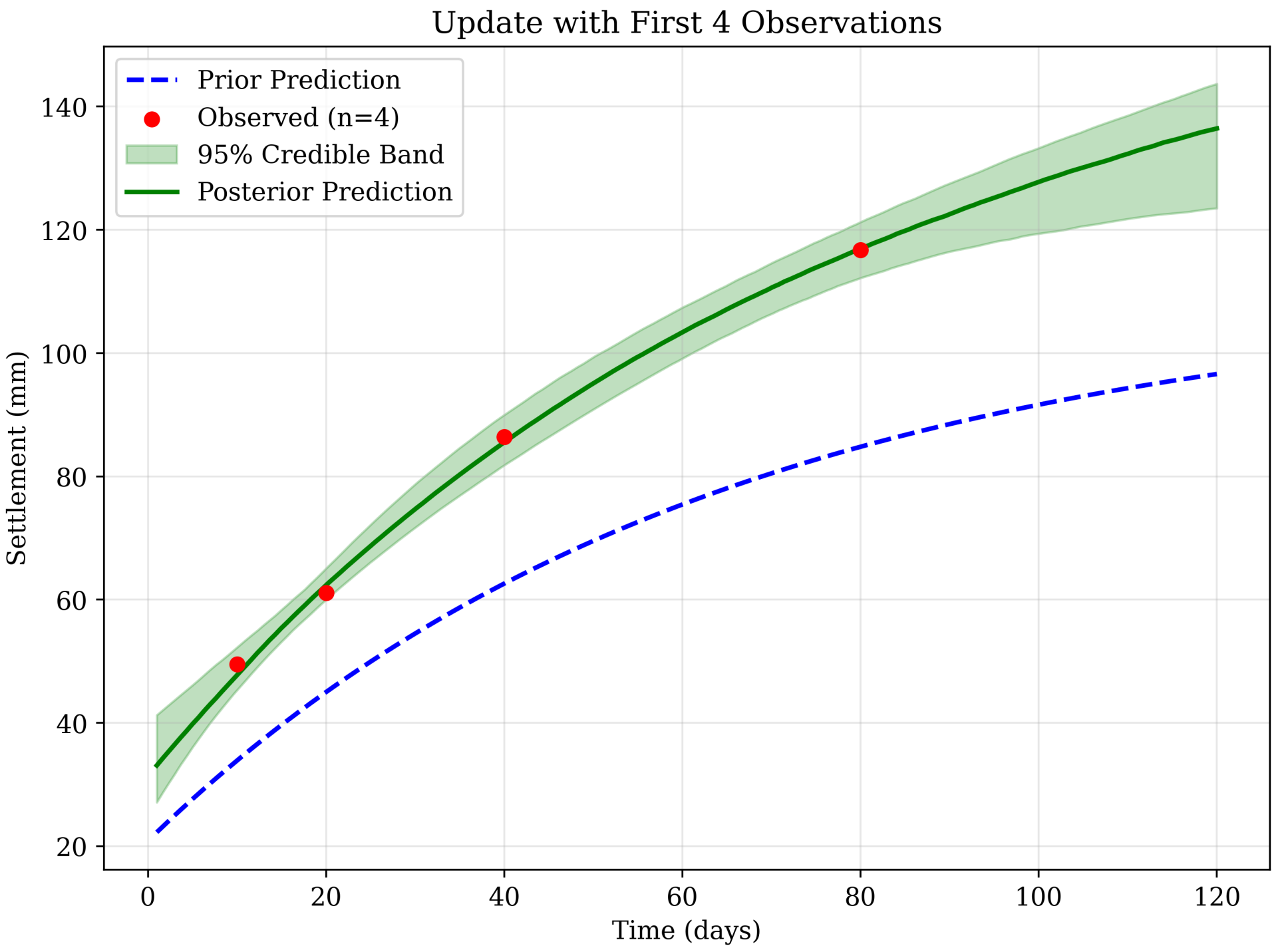
equation_obs3_worked_example.png



$\hat{m}_v \approx 1.469e-03\,1/\mathrm{kPa}, \quad \hat{c}_v \approx 0.039\,\mathrm{m}^2/\mathrm{day}$

Updated $s(t)$ (mm) at $t = \{10, 20, 40, 80\}$:
$[49.2, 65.1, 90.5, 123.0]$

step4_all_four.png



equation_obs4.png

Bayesian Update Equation (4 Observations)

$$\boxed{p(\theta \mid y_{1:4}) \propto \left[\prod_{i=1}^{4} \phi\left(\frac{y_i - s(t_i;\theta)}{\sigma_e}\right)\right] p(\theta)}$$

$$\phi\left(\frac{49.5\,\mathrm{mm} - s(10;\theta)}{3.0\,\mathrm{mm}}\right)$$
$$\phi\left(\frac{61.1\,\mathrm{mm} - s(20;\theta)}{3.0\,\mathrm{mm}}\right)$$
$$\phi\left(\frac{86.4\,\mathrm{mm} - s(40;\theta)}{3.0\,\mathrm{mm}}\right)$$
$$\phi\left(\frac{116.7\,\mathrm{mm} - s(80;\theta)}{3.0\,\mathrm{mm}}\right)$$

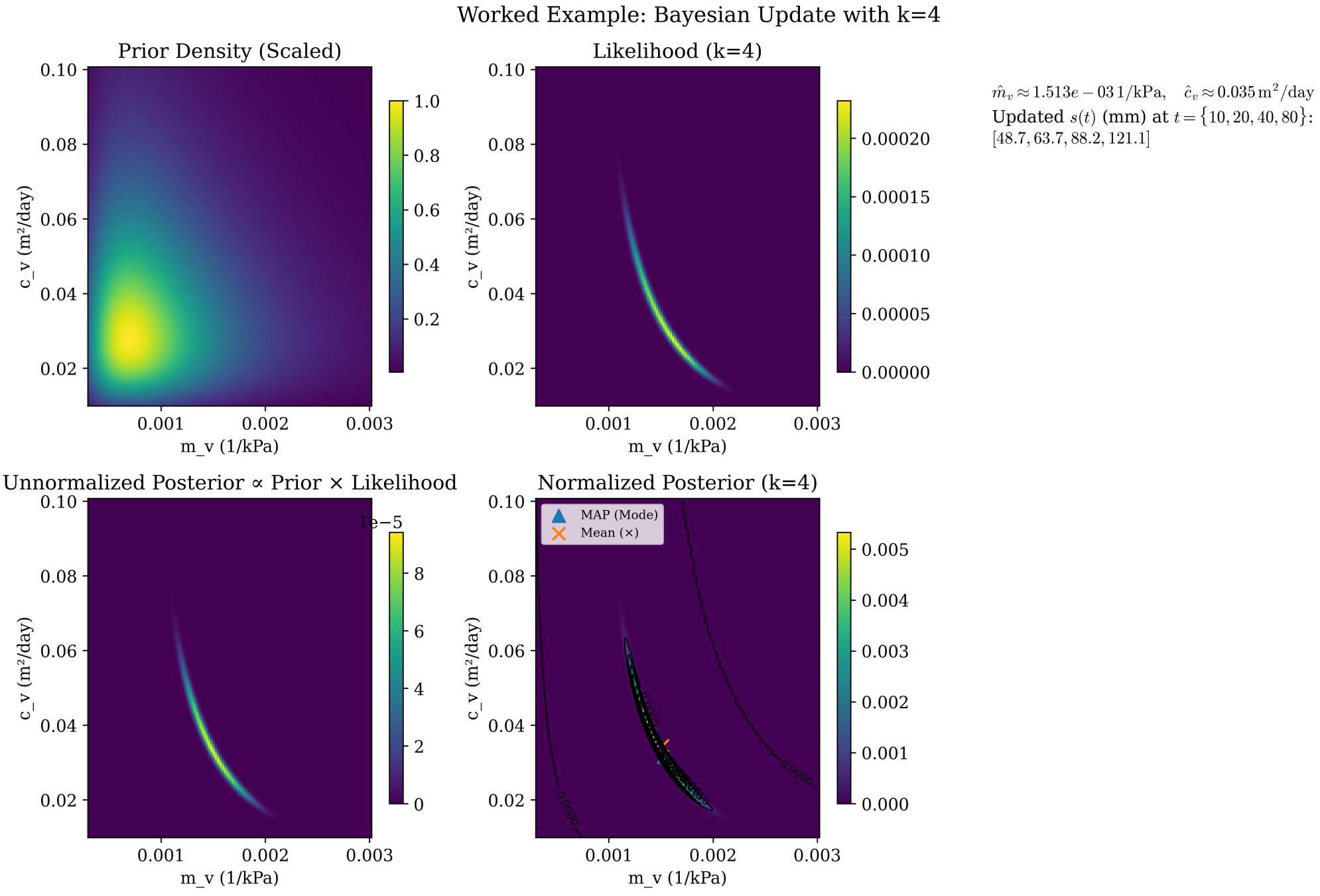
$$s(t;\theta) = m_v\,\Delta\sigma'\,H\,U\left(\frac{c_v\,t}{(H/2)^2}\right), U(T_v) \approx 1 - \frac{8}{\pi^2}\exp\left(-\frac{\pi^2}{4}T_v\right)$$

$$\Delta\sigma' = 22\,\mathrm{kPa},\ H = 5\,\mathrm{m},\ H/2 = 2.5\,\mathrm{m},\ \sigma_e = 3.0\,\mathrm{mm}$$

$$\hat{m_v} = 1.513e-03\,1/\mathrm{kPa},\ \hat{c_v} = 0.035\,\mathrm{m}^2/\mathrm{day}$$

Updated $s(t)$ (mm) at $t = [10, 20, 40, 80] : [48.7, 63.7, 88.2, 121.1]$

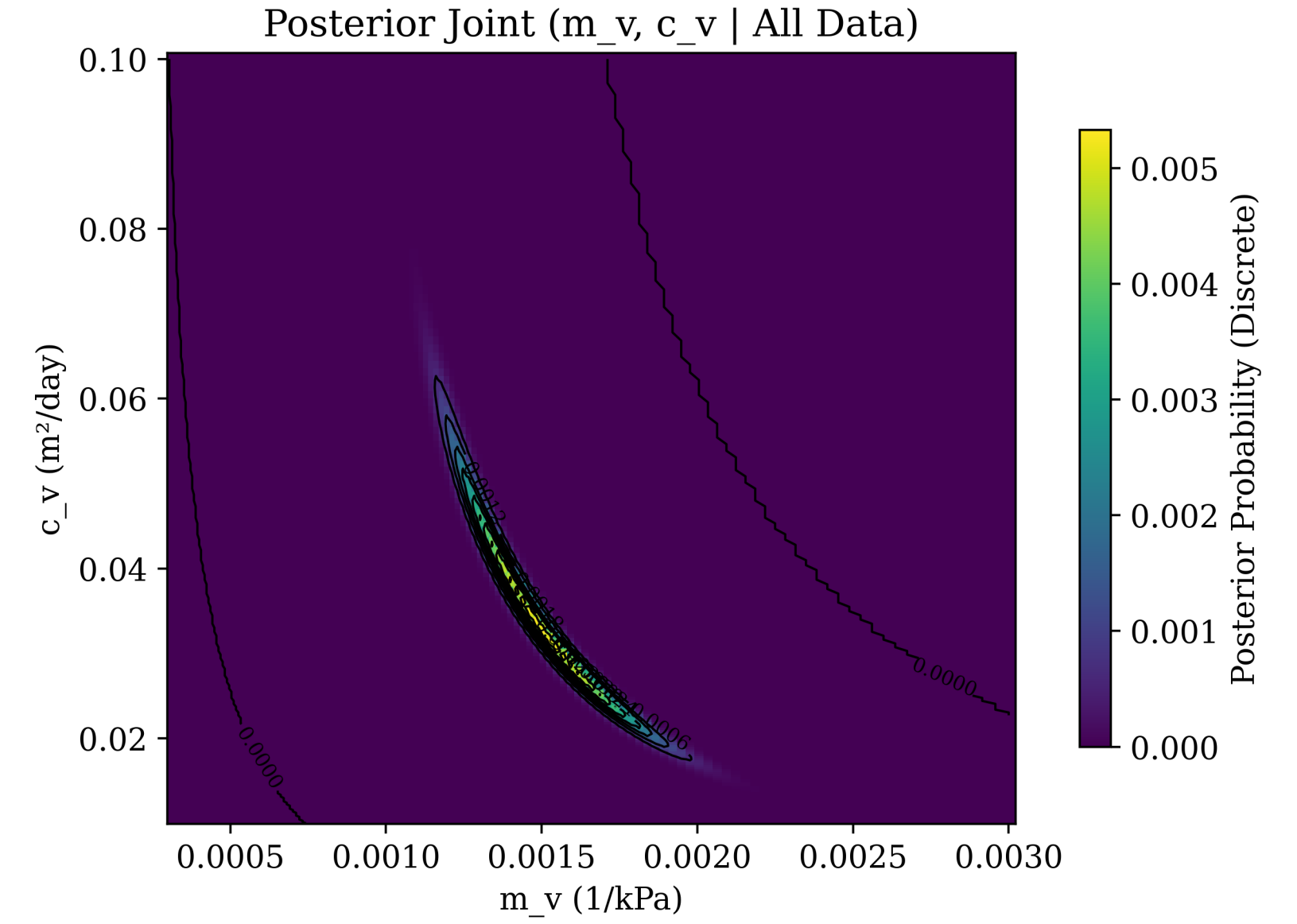
equation_obs4_worked_example.png



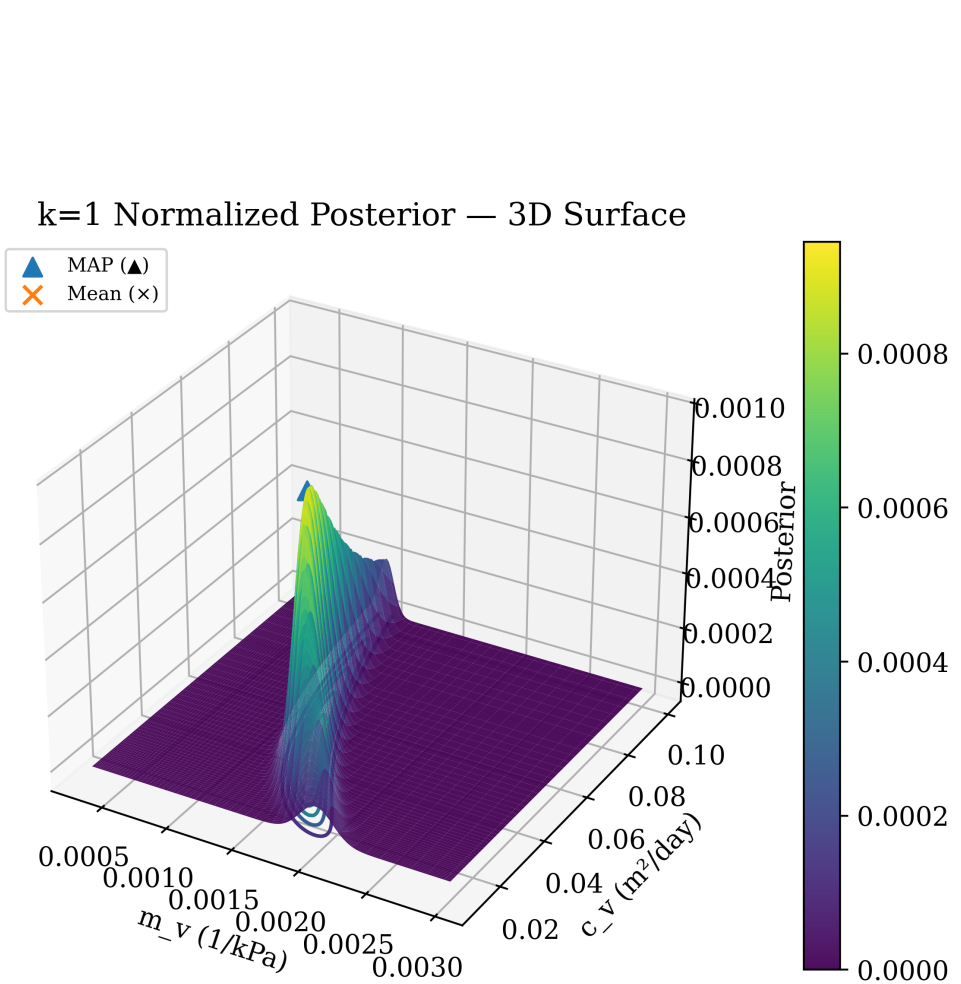
$\hat{m}_v \approx 1.513e-03\,1/\mathrm{kPa}, \quad \hat{c}_v \approx 0.035\,\mathrm{m}^2/\mathrm{day}$

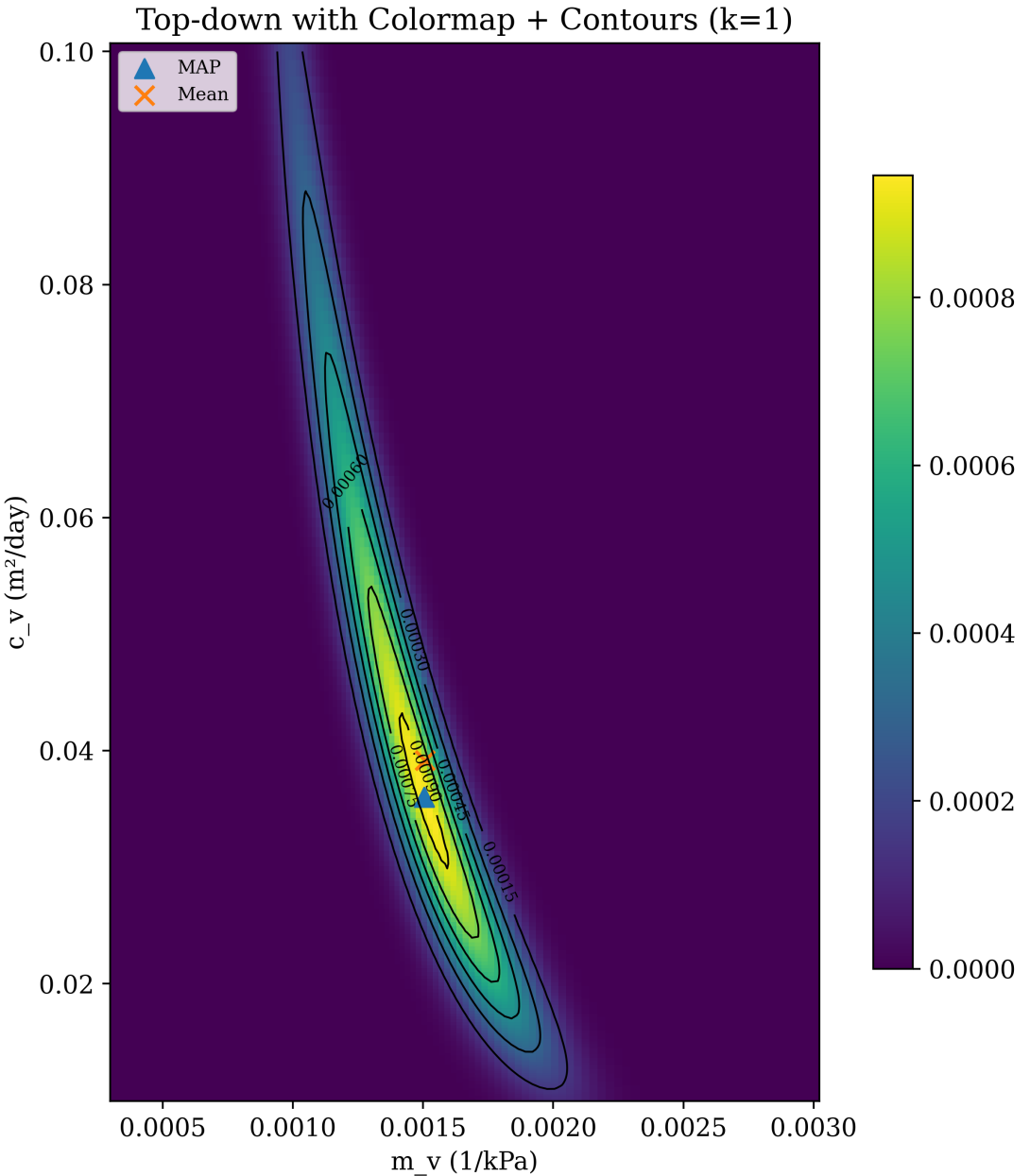
Updated $s(t)$ (mm) at $t = \{10, 20, 40, 80\}$:
$[48.7, 63.7, 88.2, 121.1]$

fig_joint_posterior_heatmap.png

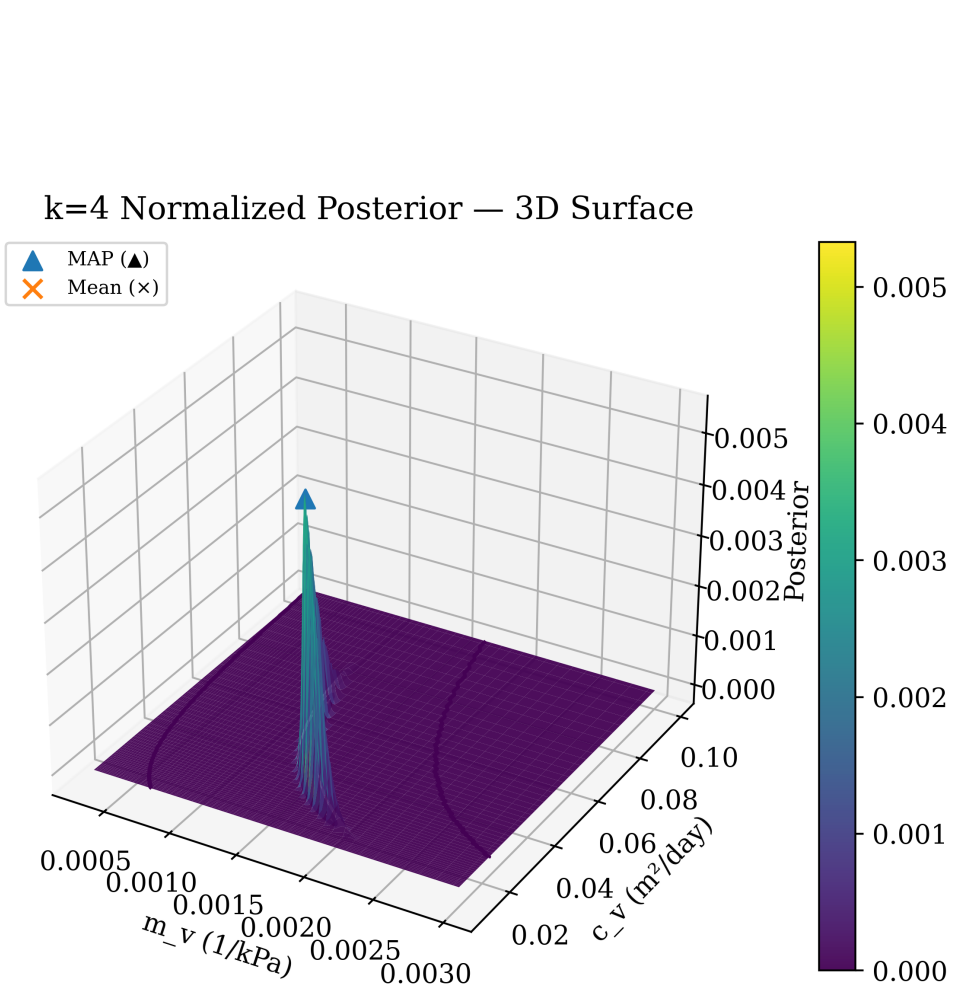


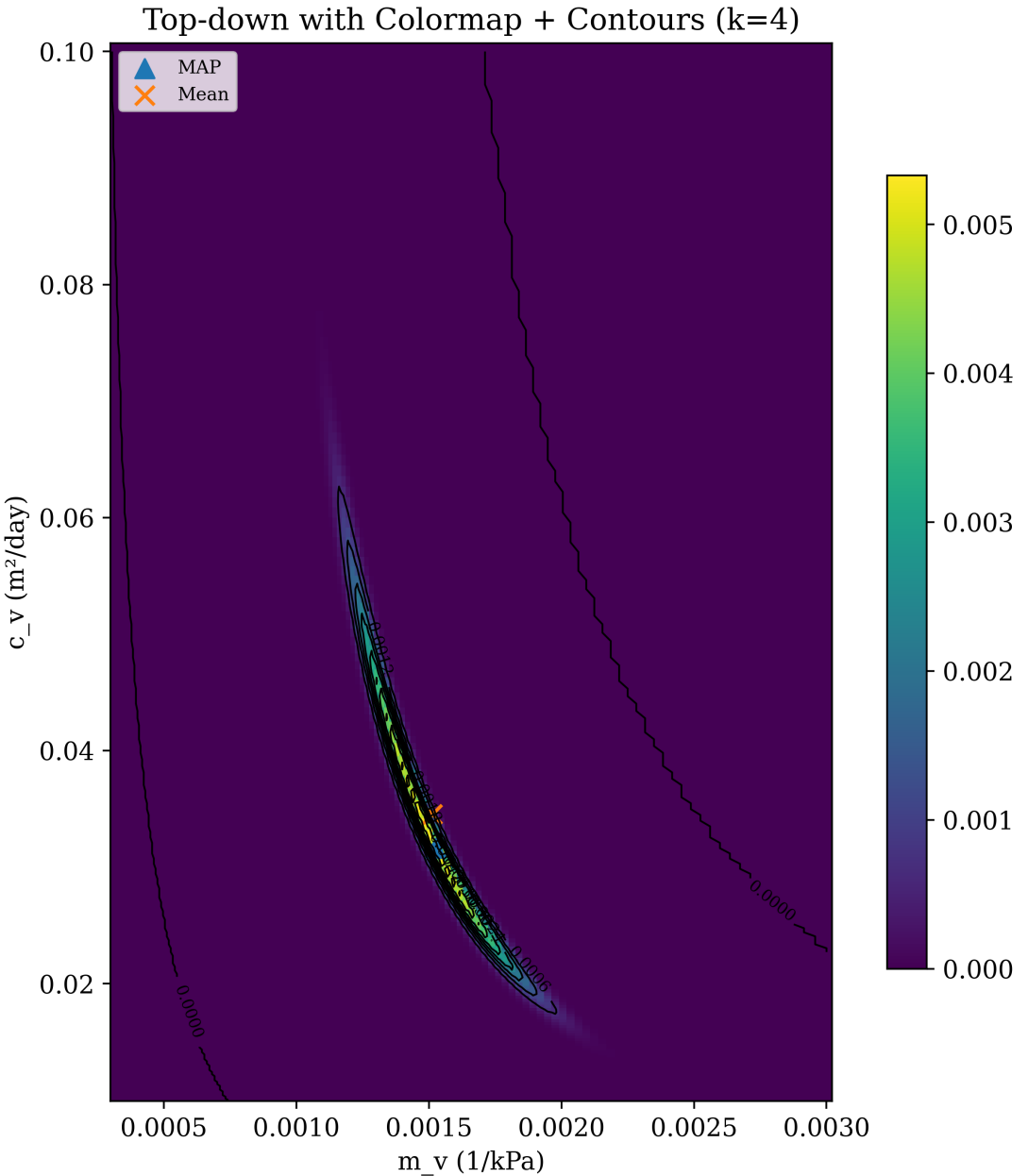
k1_post_surface.png



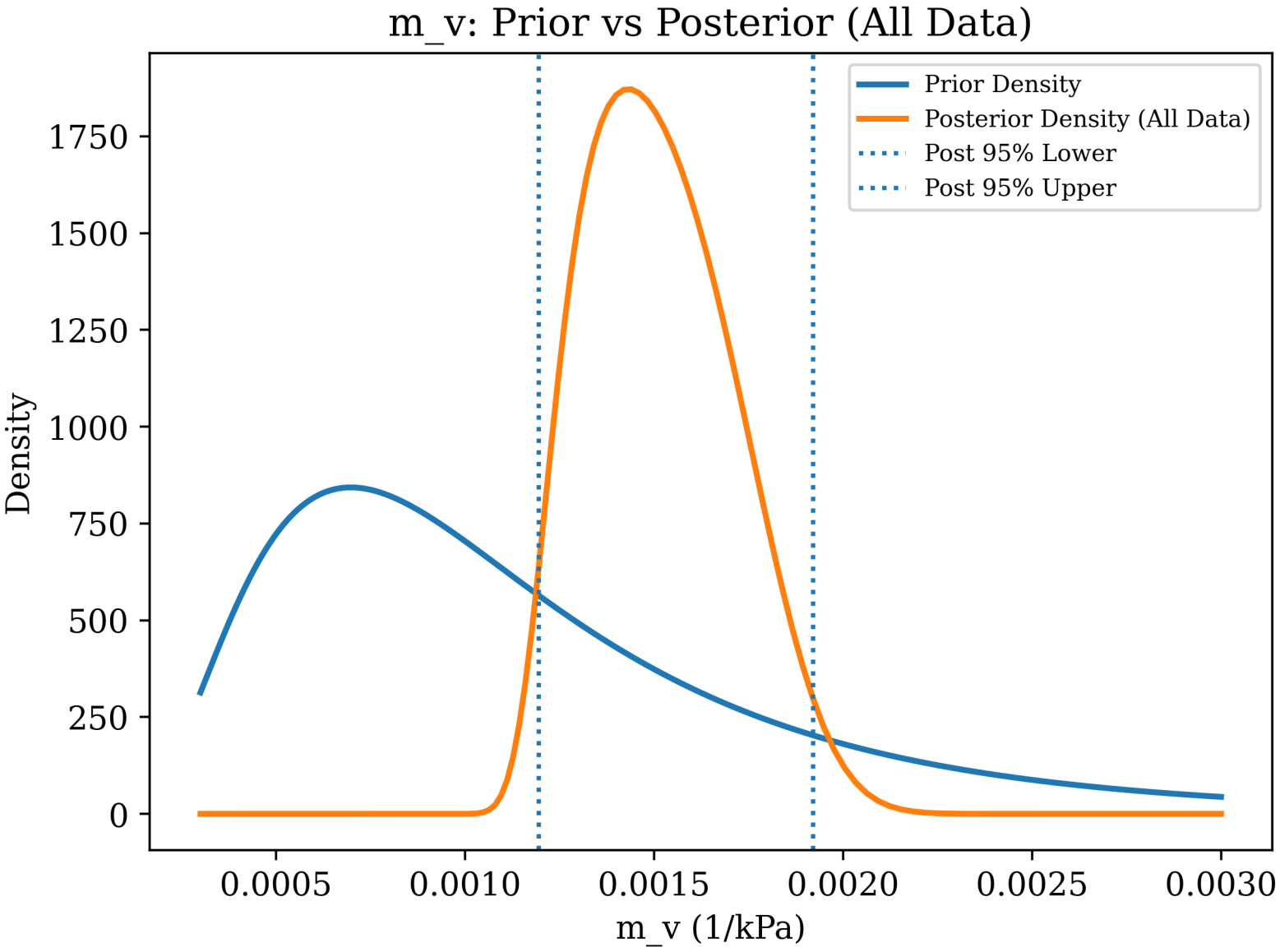


k4_post_surface.png

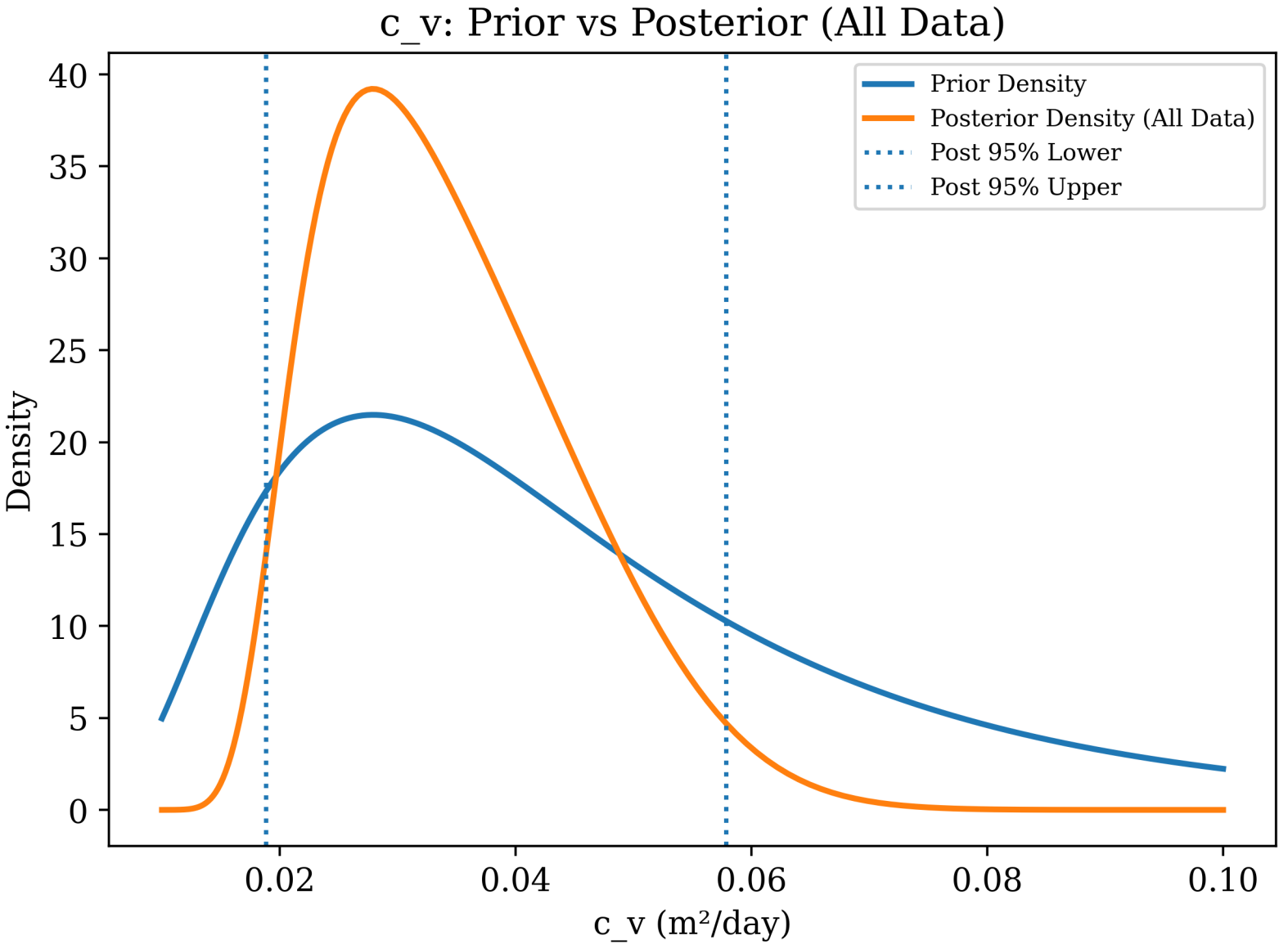




fig_mv_prior_posterior.png



fig_cv_prior_posterior.png



Bayesian Updating Equation (Summary)

$$\boxed{p(\theta \mid \mathbf{y}) \propto \left[\prod_i \phi\left(\frac{y_i - s(t_i;\theta)}{\sigma_e}\right)\right] p(\theta), \theta = (m_v, c_v)}$$

$$s(t_i;\theta) = m_v \, \Delta\sigma' \, H \, U\left(\frac{c_v \, t_i}{(H/2)^2}\right), U(T_v) \approx 1 - \frac{8}{\pi^2} \exp\left(-\frac{\pi^2}{4} T_v\right)$$

param_summary.png

Parameter Summary (Prior and Sequential Posteriors)

Stage	m_v_mean	m_v_95_lo	m_v_95_hi	c_v_mean	c_v_95_lo	c_v_95_hi
Prior	1.000e-03	3.085e-04	3.241e-03	0.040	0.012	0.130
Posterior (1 obs)	1.508e-03	1.019e-03	2.005e-03	0.039	0.012	0.088
Posterior (2 obs)	1.547e-03	1.019e-03	2.034e-03	0.034	0.013	0.078
Posterior (3 obs)	1.469e-03	1.005e-03	1.920e-03	0.039	0.019	0.082
Posterior (4 obs)	1.513e-03	1.194e-03	1.920e-03	0.035	0.019	0.058