November 30

① a) $y = a(x+5)^2 + 2$

b) An infinite ammount.

c) $-5 = a(5+5)^2 + 2$

$-5 = 100a + 2$

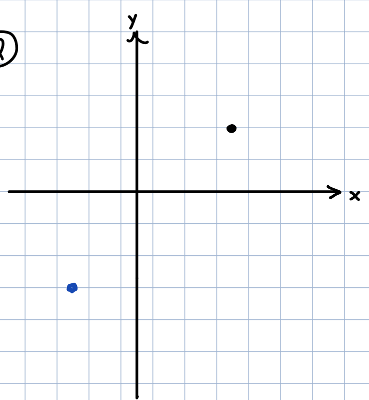
$-7 = 100a$

$\boxed{a = \frac{-7}{100}}$ $y = \frac{-7}{100}(x+5)^2 + 2$

e) 1 unique parabola (assuming point given is NOT vertex)

*- Alternative (easier approach)
- make one point vertex, then solve using method in Q1

②



A) $y = ax^2 + bx + c$

$(3,2)$: $2 = 9a + 3b + c$

$(-2,-3)$: $-3 = 4a - 2b + c$

- System of 2 equations with 3 variables will have infinite solutions or No solutions
- Choose arbitrary value for c, say c=0 to find a parabola.

$2 = 9a + 3b$

$-3 = 4a - 2b$

$4 = 18a + 6b$

$-9 = 12a - 6b$

$-5 = 30a$

$\boxed{a = \frac{-5}{30} = -\frac{1}{6}}$

$2 = 9(-\frac{1}{6}) + 3b$

$3b = 2 + \frac{3}{2} = \frac{7}{2}$

$\boxed{b = \frac{7}{6}}$

$y = -\frac{1}{6}x^2 + \frac{7}{6}x$

this is ONE possible solution, there are an infinite ammount!

B) we now require parabola to pass through $(0,-10)$.

$(3,2)$: $2 = 9a + 3b + c$

$(-2,-3)$: $-3 = 4a - 2b + c$

$(0,-10)$: $-10 = c$

thus $2 = 9a + 3b - 10$

$-3 = 4a - 2b - 10$

$12 = 9a + 3b$

$7 = 4a - 2b$

$24 = 18a + 6b$

$21 = 12a - 6b$

$45 = 30a$

$\boxed{a = \frac{45}{30} = \frac{3}{2}}$

$12 = 9(\frac{3}{2}) + 3b$

$3b = 12 - \frac{27}{2}$

$3b = -\frac{3}{2}$

$\boxed{b = -\frac{3}{6} = -\frac{1}{2}}$

so $y = \frac{3}{2}x^2 - \frac{1}{2}x - 10$

Only possible solution!

③ $y = ax^2 + bx + c$

$(-2,2)$: $2 = 4a - 2b + c$

$(0,2)$: $2 = c$

$(5,2)$: $2 = 25a + 5b + c$

Algebraically

$2 = 4a - 2b + 2$

$2 = 25a + 5b + 2$

$0 = 4a - 2b$ ×5

$0 = 25a + 5b$ ×2

$0 = 20a - 10b$

$0 = 50a + 10b$

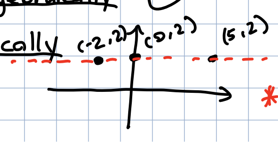
$70a = 0$

$\boxed{a = 0}$ Not a parabola!

if $a = 0$, then $0 = 4(0) - 2b$, thus $b = c$

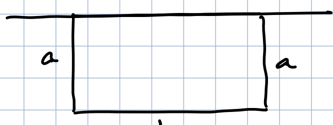
so the eqn is $y = 2$ (a horizontal line).

geometrically



* clearly can't draw parabola through

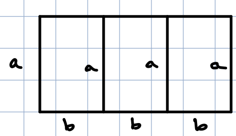
5.



$2a + b = 100$

$$\begin{aligned} \text{Area} &= a \cdot b \\ &= a(100 - 2a) \\ &= -2a^2 + 100a \\ &= -2(a-25)^2 + \underbrace{1250}_{\text{max area}} \end{aligned}$$

6.

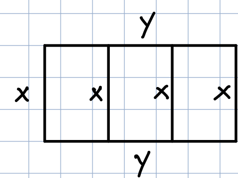


$$\begin{aligned} \text{Area} &= a \cdot 3b \\ &= a \cdot (500 - 2a) \\ &= -2a^2 + 500x \end{aligned}$$

$6b + 4a = 1000$

$3b + 2a = 500$

$3b = 500 - 2a$



$4x + 2y = 1000$

$2x + y = 500$

Area : $x \cdot y = x(500 - 2x)$

$= -2x^2 + 500x$

Same regaurdless

$$\text{Area} = -2(x-125)^2 + \underbrace{31250}_{\text{max area}}$$

$2h = 100 - 2\pi r$

7. $2\pi r + 2h = 100\,\text{cm}$

$$\begin{aligned} \text{Area} &= \pi r^2 + 2r \cdot h \\ &= \pi r^2 + (2h) \cdot r \\ &= \pi r^2 + (100 - 2\pi r) r \\ &= \pi r^2 - 2\pi r^2 + 100r \\ &= -\pi r^2 + 100r \end{aligned}$$

complete the square...

$$\text{Area} = -\pi\left(r - \frac{50}{\pi}\right) + \underbrace{\frac{2500}{\pi}}_{\text{max area.}}$$

8. let x be # of times ticket price is increased by \$20

Revenue = (# ticket holders) $\cdot$ (Price)

$= (900 - 15x)(400 + 20x)$

complete the square

⋮

$$y = -300(x - \underbrace{20}_{\substack{\text{\# of} \\ \text{increases} \\ \text{to achieve max.}}})^2 + \underbrace{480\,000}_{\text{max Revenue}}$$