Think It Through
September 21, 2022

★ indicates that the problem is considered 'more challenging' then what would normally be required in Math 20. Please make sure you give your best attempt on all of these problems.

1. $\dfrac{\sqrt{50}+7}{\sqrt{50}-7} + \dfrac{\sqrt{50}-7}{\sqrt{50}+7} = ?$

> **Solution:** 198

2. Solve $x + \sqrt{x-2} = 4$.

> **Solution:** $x = 3$

3. Solve the following equation: $\sqrt{2m+9} + \sqrt{1-2m} = 4$.

> **Solution:** $m = -4$ or $m = 0$

4. $\sqrt{3+2\sqrt{2}} - \sqrt{3-2\sqrt{2}} = a$. Find a.

> **Solution:** Squaring both sides of the equation arrives at $a^2 = 4$. Note that $a > 0$ as $\sqrt{3+2\sqrt{2}} > \sqrt{3-2\sqrt{2}}$ so we disregard $a = -2$. $a = 2$

5. Solve the following equation: $\sqrt{9x-2} = \sqrt{4x-3} + \sqrt{x+1}$

> **Solution:** $x = 3$

6. ★ What is the closed form of the below expression?

$$\sqrt{4+\sqrt{4-\sqrt{4+\sqrt{4-\sqrt{4+\sqrt{4-\cdots}}}}}}$$

Solution: Let

$$x = \sqrt{4+\sqrt{4-\sqrt{4+\sqrt{4-\sqrt{4+\sqrt{4-\cdots}}}}}}$$

$$y = \sqrt{4-\sqrt{4+\sqrt{4-\sqrt{4+\sqrt{4-\sqrt{4+\cdots}}}}}}$$

Then you can see that $\sqrt{4+y} = x$ and $\sqrt{4-x} = y$, which leads to $x^2 - y^2 = x + y$, and $x - y = 1$. Substituting $y = x - 1$ we into the second equation we have $x^2 - x - 3 = 0$, which has solution $\frac{1}{2} \pm \frac{\sqrt{13}}{2}$. x cannot be negative which leads only the positive root.

7. ★ Solve for x:

$$x + \sqrt{x + \frac{1}{2} + \sqrt{x + \frac{1}{4}}} = 1024$$

Solution:

$$x + \sqrt{x + \frac{1}{2} + \sqrt{x + \frac{1}{4}}} = \sqrt{\left(\sqrt{x + \frac{1}{4}} + \frac{1}{2}\right)^2}$$

$$= \left(\sqrt{x + \frac{1}{4}} + \frac{1}{2}\right)^2 = 1024 = 2^{10}$$

$$x = 992$$