

FACTORING BY GROUPING

Math 10 · Mr. Merrick · October 29, 2025

Goal. Recognize when to factor expressions by grouping and apply it in several settings.

General plan (four terms). Split into two groups, factor each group, then factor out the *common binomial*. If grouping fails, try a different order or pull a GCF first.

GCF Warm-Up

Factor out the greatest common factor.

1) $12x^3 - 18x^2$

3) $9a^2b - 6ab^2$

5) $25p^3q + 10pq$

2) $28y^4 + 21y^3$

4) $14m^2n^3 - 42mn^2$

6) $8r^5 - 12r^3$

Classic Four-Term Grouping

Factor completely by grouping.

7) $ax + ay + bx + by$

10) $4a^2 - 12a + 3a - 9$

13) $pq + 2p + q + 2$

8) $3x^2 + 6x + 2x + 4$

11) $6u^2 + 9u - 4u - 6$

14) $r^2 + 5r + 2r + 10$

9) $5m^2 - 15m + 2m - 6$

12) $8y^2 - 10y - 4y + 5$

15) $3x^2 - 2x - 6x + 4$

Rearrangement Before Grouping

Reorder terms (and/or factor GCFs) to make grouping work.

16) $2x - 6 + 3x^2 - 9x$

18) $5y^2 + 10y + 4y + 8$

20) $x^2 + 7x - 3x - 21$

17) $a^2 + 4a + 3b + 12$

19) $6m^2 - 8m + 9n - 12$

21) $9p^2 - 6pq + 3q^2 - 12$

Common-Binomial Pull-Out

Each group hides the same binomial factor.

$$22) (x+y)(3x-1) + (x+y)(2) \quad 24) (2m+n)^3 - (2m+n)^2 \quad 26) q(2q-1) + p(2q-1)$$

$$23) 4(a-b)^2 - 7(a-b) \quad 25) k(k+5) - 3(k+5) \quad 27) (u-v)(4u+1) - (u-v)(v+3)$$

Variables Inside and Outside (careful GCFs)

$$28) 3x^2y - 6xy + 2xy - 4y \quad 31) 4p^2q + 6pq^2 - 8pq - 12q$$

$$29) 5a^2b + 10ab^2 - 3ab - 6b^2 \quad 32) 9x^2y - 12xy + 6xy - 8y$$

$$30) 7m^2n - 14mn + 5mn - 10n \quad 33) 12r^2s - 18rs + 4rs - 6s$$

Decomposition Method (splitting the middle term)

For $ax^2 + bx + c$, find two numbers that multiply to ac and add to b , *split* b , then group.

$$34) x^2 + 7x + 12 \quad 37) 6y^2 + y - 12 \quad 40) 7a^2 - 9a + 2$$

$$35) 2x^2 + 5x + 2 \quad 38) 4m^2 - 11m - 3 \quad 41) 8t^2 - 14t + 3$$

$$36) 3x^2 - 10x - 8 \quad 39) 5p^2 + 13p + 6 \quad 42) 9z^2 + 12z + 4$$

Three-and-Two Groupings & Standalone Grouping

Sometimes five terms (or more) still factor by grouping.

43) $3x^2 + 6x + 2x + 4 + 5(x + 2)$

46) $m^2 - 9 + 2m(n - 3)$

44) $a^2 + ab + b^2 - 4a - 4b - 4$

47) $x^3 + 2x^2 + 3x + 6$

45) $pq + 2p + q + 2 + 3(p + 1)$

48) $4r^2 + 9 - 12r + 6(r - 1)$

Common Binomial “Hidden” by Signs

Watch for sign changes that still leave a common binomial.

49) $3(x - y) - 7(y - x)$

51) $(2p - q)(r + s) - (q - 2p)(r + s)$

53) $(m - n)^2 - (n - m)^2$

50) $(a + 2b) - 4(2b + a)$

52) $5(u + v) - 2(v + u)$

54) $k(1 - k) - (k - 1)$

Mixed Review

Factor completely:

55) $6x^2 + 19x + 10$

60) $x^2 - 5x - 24$

56) $10a^2 - 11a - 6$

61) $2r^2 - 7r + 3$

57) $12m^2 + 11m - 5$

62) $4t^3 + 6t^2 - 8t - 12$

58) $8p^3 + 4p^2 - 2p - 1$

63) $a^2b + ab^2 - 6a - 6b$

59) $9y^2 - 30y + 25$

64) $z^2 + z - 2 + 3(z + 2)$