Math 10 — Unit 2 Quick Check

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Instructions. Answer each question. For numeric response, write your final value clearly in the box.

A. Multiple Choice

Select *one* option.

1. Simplify $\dfrac{(3\,x^2y^{-1})^2}{(x^{-2}y^3)^2}$ and write with positive exponents.

(A) $\dfrac{9\,x^8}{y^8}$ (B) $\dfrac{9}{x^8y^8}$ (C) $9x^8y^8$ (D) $\dfrac{9\,x^4}{y^2}$

Solution: (A) $\dfrac{(3^2)x^4y^{-2}}{x^{-4}y^6} = \dfrac{9x^{4+4}}{y^{2+6}} = \dfrac{9x^8}{y^8}$

2. Which equals $\dfrac{1}{\left(a^{-3}b^2\right)^{-1}c^{-2}}$ for $a, b, c \neq 0$?

(A) $\dfrac{b^2c^2}{a^3}$ (B) $\dfrac{a^3}{b^2c^2}$ (C) $a^{-3}b^2c^{-2}$ (D) $\dfrac{b^{-2}}{a^3c^2}$

Solution: (A) $(a^{-3}b^2)^{-1} = a^3b^{-2}$, so $\frac{1}{a^3b^{-2}c^{-2}} = \frac{b^2c^2}{a^3}$

3. Compute $(3.20 \times 10^{-2})(4.50 \times 10^5)$ in scientific notation.

(A) 1.44×10^4 (B) 14.40×10^3 (C) 14.40×10^4 (D) 1.44×10^3

Solution: (A) $(3.20 \cdot 4.50) \times 10^{-2+5} = 14.40 \times 10^3 = 1.44 \times 10^4$

4. Compute $\dfrac{0.45 \times 10^{-6}}{6.0 \times 10^2}$ in scientific notation.

(A) 7.5×10^{-10} (B) 0.75×10^{-9} (C) 7.5×10^{-8} (D) 0.75×10^{-7}

Solution: (A) $\frac{0.45}{6.0} \times 10^{-6-2} = 0.075 \times 10^{-8} = 7.5 \times 10^{-10}$

5. $(32)^{\frac{3}{5}}$ equals

(A) 8 (B) 4 (C) 6 (D) 10

Solution: (A) $32^{1/5} = 2 \Rightarrow 2^3 = 8$

6. Simplify $\sqrt{72\,x^5y^4}$ for $x, y \geq 0$.

(A) $6x^2y^2\sqrt{2x}$ (B) $6xy^2\sqrt{2x}$ (C) $\sqrt{72\,x^5y^4}$ (D) $6x^2y^2\sqrt{2y}$

Solution: (A) $\sqrt{72} = 6\sqrt{2}$, $\sqrt{x^5} = x^2\sqrt{x}$, $\sqrt{y^4} = y^2$

7. Write $a^{3/4}b^{1/4}$ as a single radical $(a, b \geq 0)$.

(A) $\sqrt[4]{a^3b}$ (B) $\sqrt{ab}$ (C) $\sqrt[3]{ab}$ (D) $\sqrt[4]{ab}$

Solution: (A) $a^{3/4}b^{1/4} = \sqrt[4]{a^3b}$

8. Which is *not* in proper scientific notation?

(A) 9.99×10^0 (B) 1.01×10^9 (C) 12.0×10^{-4} (D) 6.2×10^{-3}

Solution: (C) Mantissa must be in $[1, 10)$

9. Simplify $\dfrac{x^{1/2}y^{-1}z^3}{x^{-3/2}y^2z^{-1}}$ and write with positive exponents.

(A) $\dfrac{x^2z^4}{y^3}$ (B) $\dfrac{xz^2}{y^3}$ (C) $\dfrac{x^2}{yz^4}$ (D) $\dfrac{x^2y^3}{z^4}$

Solution: (A) $x^{1/2-(-3/2)} = x^2$, $y^{-1-2} = y^{-3}$, $z^{3-(-1)} = z^4$

10. Simplify $(27\,a^9b^6)^{2/3}$ for $a, b > 0$.

(A) $9a^6b^4$ (B) $3a^6b^4$ (C) $9a^3b^2$ (D) $\sqrt[3]{27a^9b^6}$

Solution: (A) $(27)^{2/3} = 9$, $a^{(9)(2/3)} = a^6$, $b^{(6)(2/3)} = b^4$

11. Write $a^{2/5}b^{3/5}$ as a single radical $(a, b \geq 0)$.

(A) $\sqrt[5]{a^2b^3}$ (B) $\sqrt{ab}$ (C) $\sqrt[2]{a^5b^5}$ (D) $\sqrt[3]{ab}$

Solution: (A) Common denominator 5

12. Convert to a *simplified* mixed radical: $\sqrt{200\,x^3y}$ for $x, y \geq 0$.

(A) $10xy\sqrt{2x}$ (B) $10x\sqrt{2xy}$ (C) $5x\sqrt{8xy}$ (D) $4xy\sqrt{50x}$

Solution: (B) $\sqrt{200x^3y} = \sqrt{200}\,\sqrt{x^3}\,\sqrt{y} = 10\sqrt{2}\cdot x\sqrt{x}\cdot\sqrt{y} = 10x\sqrt{2xy}$ (simplified mixed radical)

13. Evaluate $\sqrt[3]{-125\,x^7}$ for $x \geq 0$ and write in simplest radical form.

(A) $-5x^2\sqrt[3]{x}$ (B) $-5x^2\sqrt{x}$ (C) $-5x^{7/3}$ (D) $5x^{7/3}$

Solution: (A) $\sqrt[3]{-125} = -5$, $x^{7/3} = x^2\sqrt[3]{x}$

14. Choose the simplest *equivalent* form of $(a^4b^5)^{1/2}$ for $a, b \geq 0$.

(A) $a^2b^2\sqrt{b}$ (B) $ab^2\sqrt{ab}$ (C) $a^2\sqrt{b^5}$ (D) $\sqrt{a^4b^5}$

Solution: (A) $a^{4/2} = a^2$, $b^{5/2} = b^2\sqrt{b}$

15. Which is **improper** scientific notation?

(A) 7.1×10^{-3} (B) 0.91×10^{7} (C) 9.99×10^{0} (D) 1.0×10^{-9}

Solution: (B) Mantissa must be in $[1, 10)$

16. Evaluate $\left(5.0 \times 10^{3}\right)\left(2.0 \times 10^{-3}\right) x^{0}$, $x > 0$.

(A) 10 (B) 1 (C) 0 (D) 10^{0}

Solution: (A) $(5.0 \cdot 2.0) \times 10^{3+(-3)} \cdot 1 = 10$ since $x^{0} = 1$ for $x \neq 0$.

B. Numeric Response

Write your final answer clearly in the box.

1. Evaluate $(64)^{\frac{3}{6}}$. $\boxed{}$

Solution: $(64)^{1/2} = 8$

2. Evaluate $(81)^{-\frac{1}{4}}$. $\boxed{}$

Solution: $\dfrac{1}{3}$

3. Compute $(3.20 \times 10^{6}) + (4.50 \times 10^{5})$ in scientific notation. $\boxed{}$

Solution: 3.65×10^{6}

4. Compute $\dfrac{7.2 \times 10^{3}}{8.0 \times 10^{-2}}$ in scientific notation. $\boxed{}$

Solution: 9.0×10^{4}

5. If $(2^{x-1})(2^{2x+5}) = 2^{25}$, find x. $\boxed{}$

Solution: $(x-1) + (2x+5) = 25 \Rightarrow 3x + 4 = 25 \Rightarrow x = 7$

6. Evaluate $\dfrac{(6 \times 10^{-1})^{2}}{3 \times 10^{-4}}$ in scientific notation. $\boxed{}$

Solution: $\frac{36 \times 10^{-2}}{3 \times 10^{-4}} = 12 \times 10^{2} = 1.2 \times 10^{3}$

C. Written Response

Show full reasoning; express final answers with positive exponents and/or radicals.

1. Simplify completely:

$$\frac{18\, x^{4} y^{-2}}{9\, x^{-1} y^{3}}.$$

Solution: $\frac{18}{9} \cdot x^{4-(-1)} \cdot y^{-2-3} = 2\, x^{5} y^{-5} = \dfrac{2x^{5}}{y^{5}}$

2. Write the product in scientific notation and name the exponent rule used:

$$(3.2 \times 10^3)(4.5 \times 10^2).$$

Solution: $(3.2 \cdot 4.5) \times 10^{3+2} = 14.40 \times 10^5 = 1.44 \times 10^6$; rule $10^m \cdot 10^n = 10^{m+n}$ with normalization

3. Write $4\sqrt{6}$ as an *entire* radical.

Solution: $4\sqrt{6} = \sqrt{4^2 \cdot 6} = \sqrt{96}$

4. Rewrite using radicals and simplify (assume $x, y \geq 0$):

$$x^{\frac{7}{3}}\, y^{\frac{5}{4}}.$$

Solution: $x^{7/3} = x^2 \sqrt[3]{x}$, $y^{5/4} = y\,\sqrt[4]{y}$, so $x^2 y\,\sqrt[3]{x}\sqrt[4]{y}$

5. Simplify $\left(144\, x^{10} y^7\right)^{1/2}$ for $x, y \geq 0$.

Solution: $12\, x^5 y^3 \sqrt{y}$

6. Rationalize and simplify: $\dfrac{7}{3\sqrt{5}}$.

Solution: $\dfrac{7}{3\sqrt{5}} = \dfrac{7\sqrt{5}}{3 \cdot 5} = \dfrac{7\sqrt{5}}{15}$

7. Order from least to greatest:

$$0.00034, \qquad 3.4 \times 10^{-4}, \qquad 5.0 \times 10^{-5}.$$

Solution: $5.0 \times 10^{-5} < 3.4 \times 10^{-4} = 0.00034$

8. Solve for $t > 0$: $\quad t^{3/2} = 16\sqrt{t}$.

Solution: Divide by $t^{1/2}$: $t = 16$

9. A bacterium has diameter 2.8×10^{-6} m. Using $\pi \approx 3.14$, express the area of its circular cross-section in scientific notation.

Solution: $r = 1.4 \times 10^{-6}$, $A \approx 3.14 \times (1.4)^2 \times 10^{-12} = 6.15 \times 10^{-12}$ m^2

10. Simplify

$$\frac{(27a^{-3}b^6)^{1/3}}{(3a^{-1}b^2)^2} \qquad (a, b \neq 0).$$

Solution: Num $= 3a^{-1}b^2$, Den $= 9a^{-2}b^4$, result $= \dfrac{a}{3b^2}$

11. If $x^3 y^2 = 2^7 3^6$ and $\dfrac{x}{y} = 2^{-1}3^2$, find x and y in prime-power form.

Solution: Let $x = 2^a 3^b$, $y = 2^c 3^d$. Then $3a + 2c = 7$, $3b + 2d = 6$, and $a - c = -1$, $b - d = 2$. Solve $\Rightarrow a = 1, b = 2, c = 2, d = 0$. So $x = 2^1 3^2 = 18$, $y = 2^2 = 4$

12. The power model $y = ax^r$ passes through $(2, 12)$ and $(5, 75)$. Find r.

Solution: $\dfrac{75}{12} = (\dfrac{5}{2})^r \Rightarrow r = 2$

13. Solve for integers x, y: $\quad 2^{x+1} + 2^x = 3^{y+1} - 3^y$.

Solution: $2^x(2+1) = 3^y(3-1) \Rightarrow 3 \cdot 2^x = 2 \cdot 3^y \Rightarrow 3^{1-y} = 2^{1-x} \Rightarrow x = 1,\ y = 1$

14. Let $f(x) = 2^{4x-6}$. Compute $f(x)\, f(3-x)$ and explain briefly.

Solution: $2^{4x-6} \cdot 2^{12-4x-6} = 2^0 = 1$ (exponents add to 0)

15. Solve: $\quad 3^{x+1} + 2^{x+2} + 2^x = 2^{x+3} + 3^x$.

Solution: $3^x(3-1) + 2^x(4+1-8) = 0 \Rightarrow 2 \cdot 3^x - 3 \cdot 2^x = 0 \Rightarrow \left(\frac{3}{2}\right)^x = \frac{3}{2} \Rightarrow x = 1$

16. Evaluate the following expression:

$$\frac{3^9 + 3^8}{3^6 + 3^6 + 3^6}.$$

Solution: $\dfrac{3^8(3+1)}{3 \cdot 3^6} = \dfrac{4 \cdot 3^8}{3^7} = 12$

17. Arrange the numbers below in ascending order:

$$2^{30}, \quad 3^{24}, \quad 6^{12}, \quad 5^{18}.$$

Solution: Write each as $(\cdot)^6$: $32^6, 36^6, 81^6, 125^6 \Rightarrow 32 < 36 < 81 < 125$; hence $2^{30} < 6^{12} < 3^{24} < 5^{18}$.

18. Simplify the following expressions:

(a) $\left(\dfrac{\sqrt[3]{x^3y^6}}{16\, x^8 y^{-2}}\right)^{-\frac{2}{5}}, \quad x, y > 0$

Solution: $\sqrt[3]{x^3y^6} = x\, y^2$, so inside $= \dfrac{x\, y^2}{16x^8y^{-2}} = \dfrac{y^4}{16x^7}$. Then raising to $-\frac{2}{5}$ gives

$$\left(\frac{y^4}{16x^7}\right)^{-\frac{2}{5}} = \left(\frac{16x^7}{y^4}\right)^{\frac{2}{5}} = 16^{\frac{2}{5}}\, x^{\frac{14}{5}}\, y^{-\frac{8}{5}} = \frac{2x^2}{y}\sqrt[5]{\frac{8x^4}{y^3}}.$$

(b) $\sqrt[3]{\dfrac{9a^2b}{4c^3}\left(\dfrac{4c^3}{9a^2b}\right)^4}, \quad a \neq 0,\ b \neq 0,\ c \neq 0$

Solution: $\dfrac{9a^2b}{4c^3} \cdot \dfrac{4^4c^{12}}{9^4a^8b^4} = \dfrac{4^3c^9}{9^3a^6b^3} = \left(\dfrac{4}{9}\right)^3 \cdot \dfrac{c^9}{a^6b^3}$, so the cube root is $\dfrac{4}{9} \cdot \dfrac{c^3}{a^2b} = \dfrac{4c^3}{9a^2b}$.

19. Simplify (assume $m \neq 0$).

(a) $y^5 \div y^{-2} \times y$

Solution: $y^{5-(-2)+1} = y^8$

(b) $\dfrac{42a^{-3}b^2c}{-14a^2b^{-1}c^{-2}}$

Solution: $\frac{42}{-14} \cdot a^{-3-2} \cdot b^{2-(-1)} \cdot c^{1-(-2)} = -3a^{-5}b^3c^3 = -\dfrac{3b^3c^3}{a^5}$

(c) $\dfrac{m \cdot m \cdot m \cdot m}{m+m+m+m+m}$

Solution: $\dfrac{m^4}{5m} = \dfrac{m^3}{5}$