Think It Through
October 5

1. Write two examples of arithmetic series, and two examples of geometric series

2. Find x so that the following is an arithmetic sequence

$$\{x+2, 3x-1, 2x+1\}$$

Solution: $x = \frac{5}{3}$

.

3. Determine the general formula for S_n for each of the following arithmetic sequences using the information below

   (a) $a_5 = 21, a_{10} = 41$

   Solution:
   $$S_n = \frac{n}{2}\left(10 + 4(n-1)\right)$$

   (b) $a_4 = -9, t_{15} = -31$

   Solution:
   $$S_n = \frac{n}{2}\left(-6 - 2(n-1)\right)$$

4. In a geometric sequence $a_2 = 3$ and $a_7 = 729$. Determine a_{10}.

Solution: 19683

5. The general term for the sequence $32, 16, 8, 4, \cdots$ can be expressed in the form $t_n = 2^{k-n}$. What is the value of k?

Solution: 6

6. ★ The sum of the first 2011 terms of a geometric sequence is 200. The sum of the first 4022 terms of the same geometric sequence is 380. Find the sum of the first 6033 terms of this geometric sequence.

Solution: The sum of the first 2011 terms can be written $\frac{a_1(1-k^{2011})}{1-k}$, and the first 4022 terms can be written as $\frac{a_1(1-k^{4022})}{1-k}$. Dividing these equations, we get $\frac{1-k^{2011}}{1-k^{4022}} = \frac{10}{19}$. Noticing that k^{4022} is just the square of k^{2011}, we substitute $x = k^{2011}$, so $\frac{1}{1-x} = \frac{10}{19}$. That means that $k^{2011} = \frac{9}{10}$. Since the sum of the first 6033 terms can be written as $\frac{a_1(1-k^{6033})}{1-k}$, dividing this gives $\frac{1-k^{2011}}{1-k^{6033}}$. Since $k^{6033} = \frac{729}{1000}$, plugging all the values gives 542.

7. Derive the general formula for the nth term in a geometric sequence.

8. Derive the general formula for the nth partial sum of a geometric series.

9. I have a glass of orange juice. I drink half of it and then give it to you. You drink half of what's remaining, and pass it back to me. I drink half of what's remaining and then give it to you. This process continues forever. How much of the glass will I drink?

Solution: $\frac{2}{3}$

10. ★ $1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \cdots + 100 \cdot 2^{99} = a \cdot 2^b + 1$. Determine a, and b.

Solution:

$$\begin{aligned}
S &= 1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \cdots + 100 \cdot 2^{99} \\
2S &= 2 + 2(2^2) + 3(2^3) + 4(2^4) + \cdots + 99(2^{99}) + 100(2^{100}) \\
2S - S &= 100\left(2^{100}\right) - (1 + 2 + 4 + \cdots 2^{99}) \\
&= 100\left(2^{100}\right) - \left(\frac{1(2^{100} - 1)}{2 - 1}\right) \\
&= 100\left(2^{100}\right) - 2^{100} + 1 \\
&= 99\left(2^{100}\right) + 1
\end{aligned}$$