① a) $f(g(x)) = (x^2+1)^2$

b) $g(f(x)) = (x^2)^2 + 1$

c) $\underbrace{(g \circ f \times h)(x)}_{\text{wont see this.}} = g(f(x)) \cdot h(x)$

$\qquad\qquad\qquad = ((x^2)^2 + 1)\sqrt{x}$

d) $g(x) + h(x) = x^2 + 1 + \sqrt{x}$

e) $g(x) - h(x) = x^2 + 1 - \sqrt{x}$

SKT when in this form ↘

vertical translation
k units up

② $\quad g(x) = a\, f(b(x-h)) + k \longrightarrow$ ⑥

— vertical
① stretch by
a factor of a
about the x-axis

horizontal
⑤ translation
h units
right

-horizontal stretch
② by a factor of
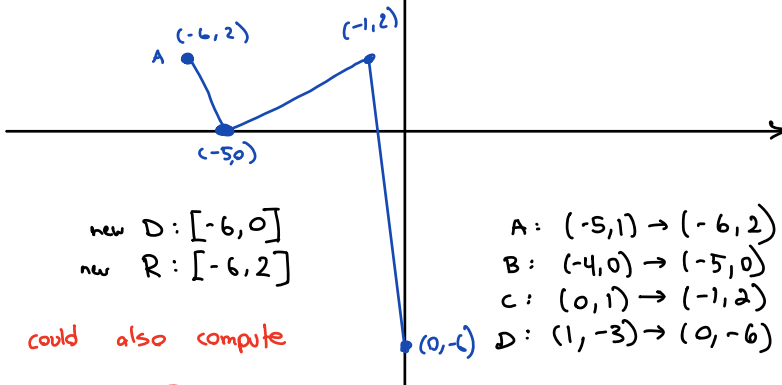$1/b$ in the y-axis

— reflection in x-axis
③ if $|a| < 0$

— reflection in y-axis
④ if $|b| < 0$.

③. a)

$g(x) = 2f(x+1)$

$(x,y) \rightarrow (x-1, 2y)$

(-6,2)   (-1,2)
A
(-5,0)
(0,-6)

new D: $[-6, 0]$
new R: $[-6, 2]$

could also compute

D: $[-5, 1] \rightarrow [-5-1, 1-1]$
$\qquad\qquad [-6, 0]$

R: $[-3, 1] \rightarrow [-6, 2]$

A: $(-5,1) \rightarrow (-6, 2)$
B: $(-4,0) \rightarrow (-5,0)$
C: $(0,1) \rightarrow (-1,2)$
D: $(1, -3) \rightarrow (0, -6)$

b)

$g(x) = -f(2x+2)$
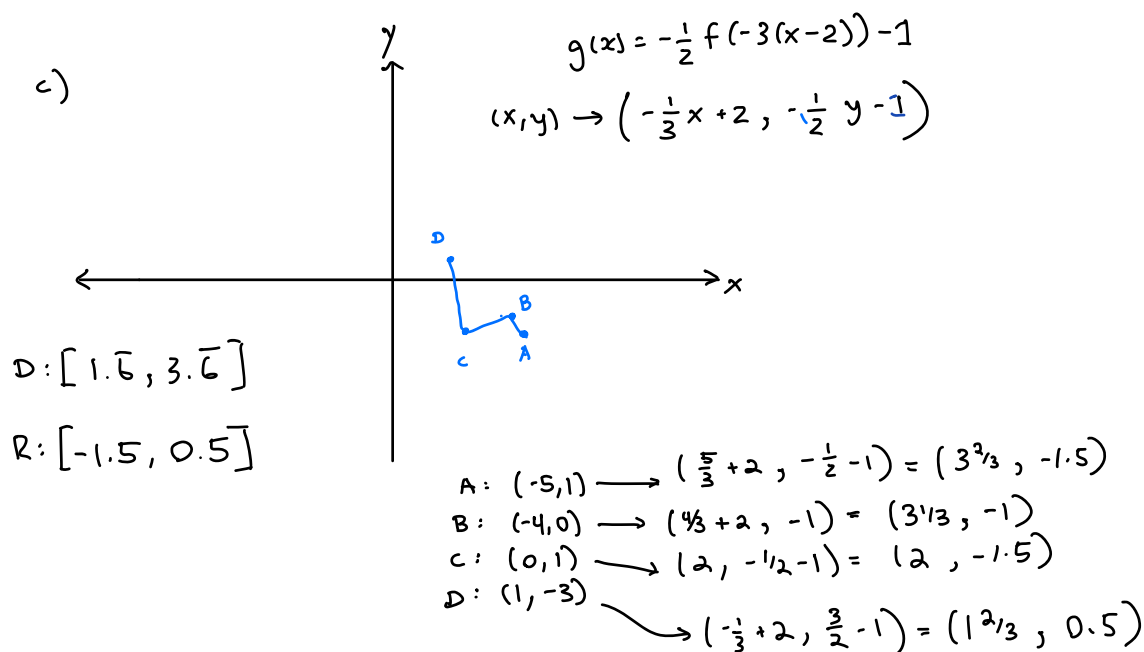$= -f(2(x+1))$
$(x,y) \rightarrow (\frac{1}{2}x-1, -y)$

D

B

A   C

new D: $[-3.5, -0.5]$
new R: $[-1, 3]$

A: $(-5,1) \rightarrow (-\frac{5}{2}-1, -1)$
B: $(-4,0) \rightarrow (-\frac{4}{2}-1, 0)$
C: $(0,1) \rightarrow (-1, -1)$
D: $(1,-3) \rightarrow (\frac{1}{2}-1, 3)$

c)

$g(x) = -\frac{1}{2} f(-3(x-2))-1$

$(x,y) \rightarrow (-\frac{1}{3}x+2, -\frac{1}{2}y-1)$

D

B

C   A

D: $[1.\overline{6}, 3.\overline{6}]$

R: $[-1.5, 0.5]$

A: $(-5,1) \rightarrow (\frac{5}{3}+2, -\frac{1}{2}-1) = (3\frac{2}{3}, -1.5)$
B: $(-4,0) \rightarrow (\frac{4}{3}+2, -1) = (3\frac{1}{3}, -1)$
C: $(0,1) \rightarrow (2, -\frac{1}{2}-1) = (2, -1.5)$
D: $(1,-3) \rightarrow (-\frac{1}{3}+2, \frac{3}{2}-1) = (1\frac{2}{3}, 0.5)$
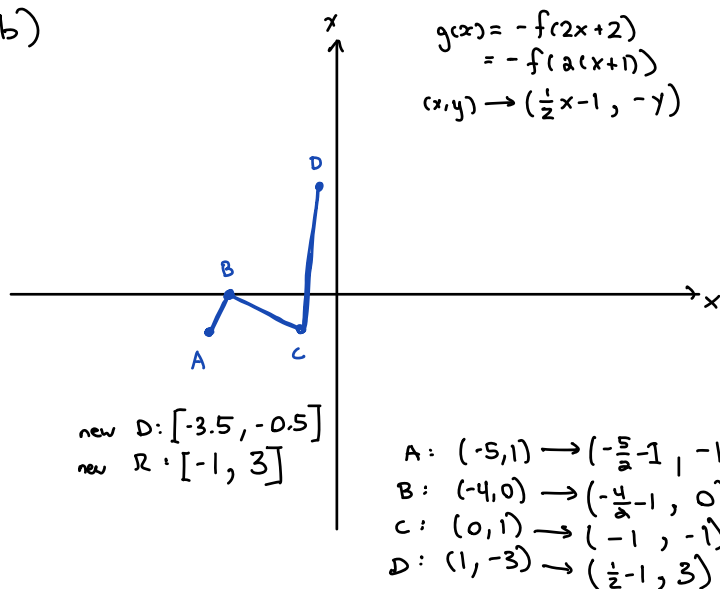
write out mapping step by step...
Start.

D) (x,y) 1) $(x, -y)$

2) $(2x, -y)$
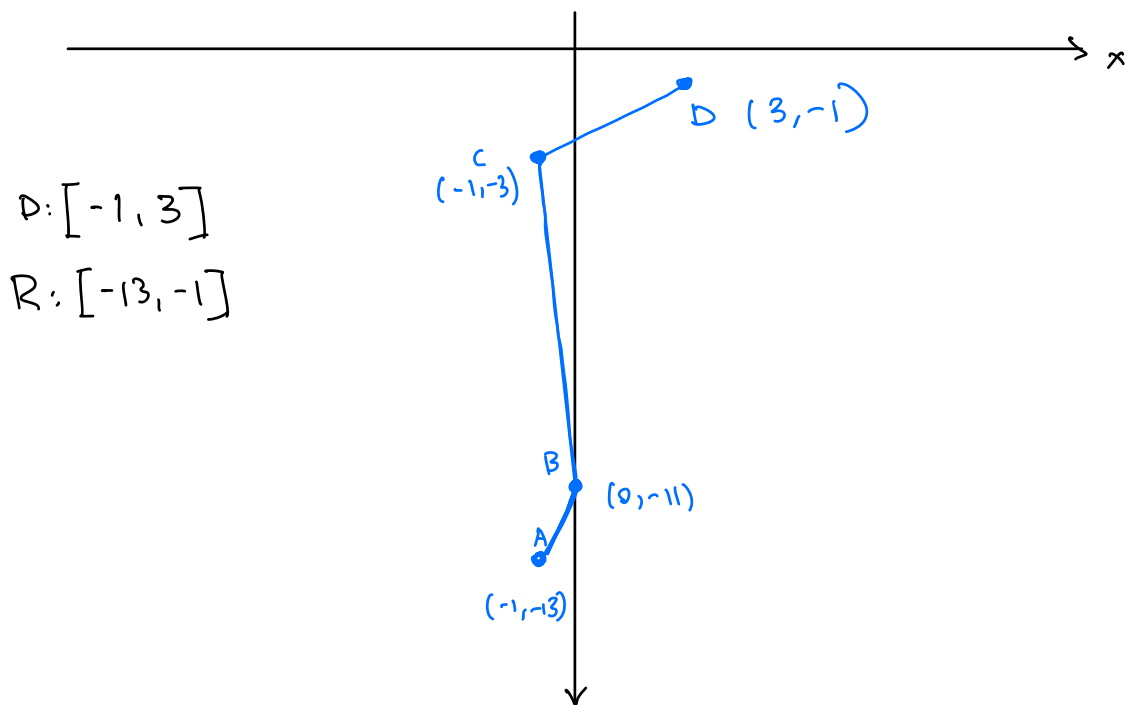
3) $(-y, 2x)$

4) $(-y, 2x-3)$

$(x,y) \rightarrow (-y, 2x-3)$

A: $(-5,1) \rightarrow (-1, -13)$
B: $(-4,0) \rightarrow (0, -11)$
C: $(0,1) \rightarrow (-1, -3)$
D: $(1,-3) \rightarrow (3, -1)$

$D: [-1, 3]$

$R: [-13, -1]$

Graph points: C $(-1, -3)$, D $(3, -1)$, B $(0, -11)$, A $(-1, -13)$

(4.) $f(x) = |x + 3|$ $x > 0$

$f(x) = x + 3$ ← so

$\underbrace{y = x + 3}$

(1.) $y - 3 = x + 3$, $y = x + 6$

(2) $y = (x - 4) + 6$, $y = x + 2$

(3.) $y = -x + 2$

(4) $y + 2 = -x + 2$, $y = -x$

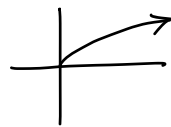(5) $y = -\frac{1}{a} x$, now $4 = -\frac{1}{a}(-20)$

$\boxed{a = 5}$

⑤ $f(x) = \sqrt{x}$ $y = \sqrt{x}$

$x \geq 0$
$y \geq 0$

① $x = \sqrt{y}$, $y = x^2$

only half.

② $y = \left(\frac{1}{2}x\right)^2$

③ $y - 3 = \left(\frac{1}{2}x\right)^2$, $y = \left(\frac{1}{2}x\right)^2 + 3$

④ $\frac{1}{2}y = \left(\frac{1}{2}x\right)^2 + 3$, $y = 2\left(\frac{1}{2}x\right)^2 + 6$

$$g(x) = 2\left(\frac{1}{2}x\right)^2 + 6 \;,\quad x \geq 0$$

⑥ $f(x) = \frac{1}{x}$ $g(x) = \dfrac{2}{-3(x+1)} + 1$

$y = 2f\left(-3(x+1)\right) + 1$
① ② ⑤ ④
③

① V.S. about x-axis by a factor of 2

② H.S about y-axis by a factor of $\frac{1}{3}$

③ reflection in y-axis

④ Vertical translation 1 unit up

⑤ horizontal translation 1 unit left.

⑦ $f(x) = \frac{1}{2}\left(3(x+1)\right)^2$ $g(x) = 3x^2 + 2$

Start: $y = \frac{1}{2}\left(3(x+1)\right)^2$

① Shift 1 unit right: $x \to (x-1)$ $y = \frac{1}{2}\left(3((x-1)+1)\right)^2$

$y = \frac{1}{2}(3x)^2$

② V.S. by 2 about x-axis: $y \to \frac{y}{2}$ $\frac{y}{2} = \frac{1}{2}(3x)^2$

$y = (3x)^2$

③ h.s. by a factor of 3 about y-axis:

$$x \to \frac{x}{3} \qquad y = \left(3\left(\frac{x}{3}\right)\right)^2 = x^2$$

④ v.s by a factor of 3 about x-axis:

$$y \to \frac{y}{3} \qquad \frac{y}{3} = x^2 \quad \boxed{1 \cdot y = 3x^2}$$

⑤. r.t. 2 units up.: $\qquad y - 2 = 3x^2, \quad \boxed{y = 3x^2 + 2}$

✱ NOTE: Answers may vary for this problem.
this is <u>one</u> possible sol'n.

⑧ $\qquad \underline{y = \sqrt{x} + 1}$

① $\qquad y = \sqrt{3(x-1)+1} + 1$ $\underset{\sim}{(x-1) \to 3(x-1)}$

② $\qquad x = \sqrt{3(y-1)+1} + 1$ $\begin{cases} x \to y \\ y \to x \end{cases}$

③ $\qquad x = \sqrt{3\left(\frac{1}{a}(y+2) - 2 - 1\right) + 1} + 1$

$\underset{\sim}{(y+2) \to \frac{1}{a}(y+2)}$ $\qquad x = \sqrt{3\left(\frac{1}{a}(y+2) - 3\right) + 1} + 1$

$$\underline{g(3) = 4}$$

$$3 = \sqrt{3\left(\frac{6}{a} - 3\right) + 1} + 1$$

$$2 = \sqrt{\frac{18}{a} - 8}$$

$$4 = \frac{18}{a} - 8$$

$$12 = \frac{18}{a} \qquad \boxed{a = \frac{18}{12} = \frac{3}{2}}$$