Think It Through
September 2022

Check in with your teacher after solving EACH problem.

1. Factor completely, wherever possible check your work by expanding. Solve each problem using decomposition.

   (a) $10p^2 - 27p + 5$

   Solution: $(5x - 1)(2x - 5)$

   (b) $5y^2 + 23y + 12$

   Solution: $(5y + 3)(y + 4)$

   (c) $3x^2 + 40x + 48$

   Solution: $(3x + 4)(x + 12)$

   (d) $7y^2 + 9y - 36$

   Solution: $(7y - 12)(y + 3)$

   (e) $25z^2 + 50z + 9$

   Solution: $(5z + 1)(5z + 9)$

   (f) $81y^2 - 9$

   Solution: $9(3y - 1)(3y + 1)$

   (g) $9x^2 - 1$

   Solution: $(3x + 1)(3x - 1)$

   (h) $x^2y^2 - 4x^2$

   Solution: $x^2(y - 2)(y + 2)$

   (i) $z^4 - 1$

   Solution: $(z^2 + 1)(z - 1)(z + 1)$

   (j) $x^2 + 4x + 4$

Solution: $(x+2)^2$

(k) $9x^2 + 24x + 16$

Solution: $(3x+4)^2$

(l) $16x^2 - 48x + 36$

Solution: $4(2x-3)^2$

(m) $4(x+2)^2 - 9(x-4)^2$

Solution: $(16-x)(5x-8)$

(n) $16e^{4x} - 4$

Solution: $4(2e^{2x}-1)(2e^{2x}+1)$

More Challenging: Check in with your teacher after solving EACH problem.

2. $a^3 + b^3 = 2593080$, $a + b = 210$, $ab =?$

Solution:

$$\begin{aligned} a^3 + b^3 &= (a+b)(a^2 - ab + b^2) \\ &= 210(a^2 - ab + b^2) \end{aligned}$$

$$\begin{aligned} a^2 - ab + b^2 &= 12348 \\ a^2 - ab + b^2 + 3ab &= 12348 + 3ab \\ (a+b)^2 &= 12348 + 3ab \\ 3ab + 12348 &= 44100 \\ 3ab &= 31752 \\ ab &= 10584 \end{aligned}$$

3. Given that $a + b = 1$ and $a^2 + b^2 = 2$, What is the value of $a^7 + b^7$.

Solution:

$$\begin{aligned} (a+b)^2 &= a^2 + 2ab + b^2 = 1 \\ ab &= -\frac{1}{2} \end{aligned}$$

Let $S_n = a^n + b^n$:

$$\begin{aligned} (a+b)S_n &= (a+b)(a^n + b^n) \\ &= a^{n+1} + b^{n+1} + a^n b + ab^n \\ &= a^{n+1} + b^{n+1} + ab(a^{n-1} + b^{n-1}) \end{aligned}$$

So we have $S_n = S_{n+1} - \frac{1}{2}S_{n-1}$, or $S_{n+1} = S_n + \frac{1}{2}S_{n-1}$ and from the problem we have $S_1 = 1$, and $S_2 = 2$. So we have:

$$
\begin{aligned}
S_3 &= S_1 + \frac{1}{2}S_2 = \frac{5}{2} \\
S_4 &= S_3 + \frac{1}{2}S_4 = \frac{7}{2} \\
&\vdots \\
S_7 &= \frac{71}{8}
\end{aligned}
$$

4. If $x^2 - 5x - 1 = 0$, then find the value of $x^2 + \dfrac{1}{x^2}$.

Solution:

$$
\begin{aligned}
x^2 - 1 &= 5x \\
x - \frac{1}{x} &= 5 \\
x^2 + \frac{1}{x^2} &= \left(x - \frac{1}{x}\right)^2 + 2 \\
5^2 + 2 &= 27
\end{aligned}
$$