Think It Through
September, 2022

★ indicates that the problem is considered 'more challenging' then what would normally be required in Math 20. Please make sure you give your best attempt on all of these problems.

1. Solve each of the following equations using factoring:
   (a) $5x^2 - 9x - 18 = 0$

   Solution: $x = -\frac{6}{5}$, $x = 3$

   (b) $4a^2 + 4a - 15 = 0$

   Solution: $a = -\frac{5}{2}$, $a = \frac{3}{2}$

   (c) $36n + n - 2n^2 = 0$

   Solution: $n = 0$, $n = \frac{37}{2}$

   (d) $-9 + 4r^2 = 0$

   Solution: $r = -\frac{3}{2}$, $n = \frac{3}{2}$

   (e) $x^4 - 4 = 0$

   Solution: $x = \pm\sqrt{2}$, $x = \pm\sqrt{2}i$

2. Rationalize the expression $\dfrac{y^2\sqrt{x^2-1}}{\sqrt{x+1}}$

   Solution: $y^2\sqrt{x-1}$

3. Rationalize the denominator $\dfrac{3}{\sqrt[3]{3}}$.

   Solution: $\sqrt[3]{9}$

4. Simplify the following expression: $\dfrac{3}{\sqrt{3}} + \dfrac{2}{\sqrt{2}+\sqrt{5}}$

   Solution: $\dfrac{3\sqrt{3} - 2\sqrt{2} + 2\sqrt{5}}{3}$

5. Simplify the following expression: $\dfrac{7\sqrt{6}+\sqrt{303}}{\sqrt{3}} \times (\sqrt{101}-\sqrt{98})$

Solution: 3

6. After rationalizing the denominator the expression $\dfrac{\sqrt{\sqrt[3]{2}}}{\sqrt[4]{\sqrt{5}}-\sqrt[4]{\sqrt{3}}}$ can be written as

$$\frac{2^a(5^b+3^b)(5^c+3^c)(5^d+3^d)}{2}$$

find a, b, c, and d.

Solution:

$$\begin{aligned}
\frac{\sqrt{\sqrt[3]{2}}}{\sqrt[4]{\sqrt{5}}-\sqrt[4]{\sqrt{3}}} &= \frac{2^{\frac{1}{6}}}{5^{\frac{1}{8}}-3^{\frac{1}{8}}} \\
&= \frac{2^{\frac{1}{6}}(5^{\frac{1}{8}}+3^{\frac{1}{8}})}{(5^{\frac{1}{8}}-3^{\frac{1}{8}})(5^{\frac{1}{8}}+3^{\frac{1}{8}})} \\
&= \frac{2^{\frac{1}{6}}(5^{\frac{1}{8}}+3^{\frac{1}{8}})(5^{\frac{1}{4}}+3^{\frac{1}{4}})}{(5^{\frac{1}{4}}-3^{\frac{1}{4}})(5^{\frac{1}{4}}+3^{\frac{1}{4}})} \\
&\vdots \\
&= \frac{2^{\frac{1}{6}}(5^{\frac{1}{8}}+3^{\frac{1}{8}})(5^{\frac{1}{4}}+3^{\frac{1}{4}})(5^{\frac{1}{2}}+3^{\frac{1}{2}})}{2}
\end{aligned}$$

7. ★ Simplify $\dfrac{\sqrt{5}^3-\sqrt{4}^3}{\sqrt{5}^3+\sqrt{4}^3}$

Solution: $\dfrac{189-80\sqrt{5}}{61}$

8. ★

$$x = \frac{4}{(\sqrt{5}+1)(\sqrt[4]{5}+1)(\sqrt[8]{5}+1)(\sqrt[16]{5}+1)}$$

Find the value of $(x+1)^{64}$

Solution:

$$\begin{aligned}
x &= \frac{4}{(\sqrt{5}+1)(\sqrt[4]{5}+1)(\sqrt[8]{5}+1)(\sqrt[16]{5}+1)} \\
&= \frac{4(\sqrt[16]{5}-1)}{(\sqrt{5}+1)(\sqrt[4]{5}+1)(\sqrt[8]{5}+1)(\sqrt[16]{5}+1)(\sqrt[16]{5}-1)} \\
&= \frac{4(\sqrt[16]{5}-1)}{(\sqrt{5}+1)(\sqrt[4]{5}+1)(\sqrt[8]{5}+1)(\sqrt[8]{5}-1)} \\
&= \frac{4(\sqrt[16]{5}-1)}{(\sqrt{5}+1)(\sqrt{5}-1)} \\
&= \frac{4(\sqrt[16]{5}-1)}{4} \\
&= \sqrt[16]{5}-1
\end{aligned}$$

So $x+1 = \sqrt[16]{5}$
$\therefore (\sqrt[16]{5})^{64} = 5^4 = 625$

9. ★

$$\sqrt{3+\sqrt{3}+\sqrt{2+\sqrt{3}+\sqrt{7+\sqrt{48}}}} = a+\sqrt{b}$$

is a and b are positive integers, find $a+b$.

Solution: First note that $\sqrt{a+b+2\sqrt{ab}} = \sqrt{a}+\sqrt{b}$

$$\begin{aligned}
\sqrt{3+\sqrt{3}+\sqrt{2+\sqrt{3}+\sqrt{7+\sqrt{48}}}} &= \sqrt{3+\sqrt{3}+\sqrt{2+\sqrt{3}+\sqrt{3+4+2\sqrt{3\cdot 4}}}} \\
&= \sqrt{3+\sqrt{3}+\sqrt{2+\sqrt{3}+\sqrt{3}+\sqrt{4}}} \\
&= \sqrt{3+\sqrt{3}+\sqrt{1+3+2\sqrt{3\cdot 1}}} \\
&= \sqrt{3+\sqrt{3}+\sqrt{1}+\sqrt{3}} \\
&= \sqrt{3+\sqrt{3}+\sqrt{1}+\sqrt{3}} \\
&= \sqrt{3+1+2\sqrt{3\cdot 1}} \\
&= 1+\sqrt{3}
\end{aligned}$$

So $a+b=4$