Dessert Isle Unit Analysis

Stranded Standards for Coconauts

Mr. Merrick

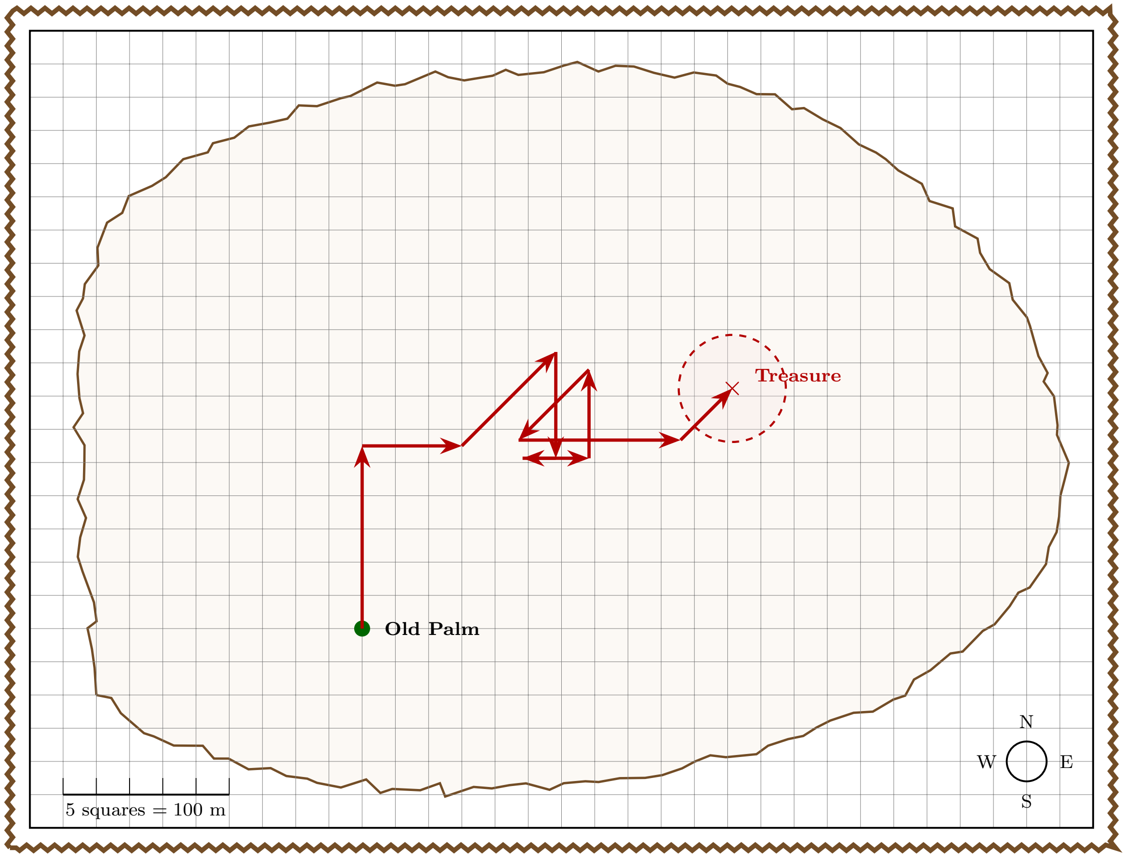
On the fabled *Dessert Isle*, shipwrecked scientist pirates standardized measurement using whatever they had: coconuts, bananas, ropes, hammocks, and drumbeats. These **Dessert Units** are *made up but internally consistent*. Your job is to apply unit analysis to convert between Dessert Units and familiar SI/US units, and to chain conversions through density, energy, power, and time. Keep the **Master Table** and the **Data Box** open while you work.

Master Table

Quantity	**Dessert Unit (symbol)**	**Equivalence (exact unless noted)**
Length		
Banana (ba)	1 ba	= 0.20 m
Palm (pl)	1 pl	= 0.50 m
Coconut rope (crp)	1 crp	= 2.00 m
Island mile (imi)	1 imi	= 800 m
Area		
Hammock (hmk)	1 hmk	= 2.0 m^2
Sandpatch (spd)	1 spd	= 1.5 m^2
Leaf-mat (lmt)	1 lmt	= 0.50 m^2
Volume		
Coconut shell (csh)	1 csh	= 0.60 L
Gourd (grd)	1 grd	= 2.4 L = 4 csh
Island barrel (ibr)	1 ibr	= 25 L ≈ 6.58 US gal
Mass		
Coconut (cn)	1 cn	= 1.40 kg
Mango (mgo)	1 mgo	= 0.35 kg = 350 g
Crab (crb)	1 crb	= 0.12 kg
Stone (stn)	1 stn	= 2.50 kg
Time		
Drumbeat (db)	1 db	= 0.75 s
Sunset (sst)	1 sst	= 12 min = 720 s
Nap (np)	1 np	= 20 min
Tide (td)	1 td	= 5 h = 300 min
Derived / Reference		
Scurry (scy)	1 scy	$= (1\text{ pl})/(1\text{ db}) = \dfrac{0.50\text{ m}}{0.75\text{ s}} = 0.667\text{ m/s}$
Firechip (fch)	1 fch	= 1.00 MJ = 10^6 J
Torch (trc)	1 trc	= 50 W = 50 J/s
Coco-milk density	ρ_{cmlk}	= 1050 kg/m^3 (use when cited)
Dry wood energy (ref)	e_{wood}	≈ 16 MJ/kg (use when cited)

Dessert Isle Data Box: Quick Equivalences
1 L = 1000 mL = 1000 cm^3; 1 m^3 = 1000 L
1 in = 2.54 cm; 1 ft = 0.3048 m; 1 mi = 1609 m
1 US gal = 3.785 L; 1 lb = 0.4536 kg
1 kWh = 3.6×10^6 J; 1 BTU ≈ 1055.06 J

Captain Pi-rate Gaussbeard has left movement instructions. Convert each instruction to *squares*. Mark the treasure with an ×.



1. From the **Old Palm**, walk **260 pl** straight **north**.

$$260\ \text{pl} \times \frac{0.50\ \text{m}}{\cancel{1\ \text{pl}}} \times \frac{1\ \text{square}}{20\ \cancel{\text{m}}} = \mathbf{5.5}\ \text{squares (N)}$$

2. Then go **300 ba east**.

$$300\ \text{ba} \times \frac{0.20\ \text{m}}{\cancel{1\ \text{ba}}} \times \frac{1\ \text{square}}{20\ \cancel{\text{m}}} = \mathbf{3.0}\ \text{squares (E)}$$

3. Next, head **40 crp** toward the **northeast** at 45°.

$$40\ \text{crp} \times \frac{2.00\ \text{m}}{\cancel{1\ \text{crp}}} \times \frac{1\ \text{square}}{20\ \cancel{\text{m}}} = \mathbf{4.0}\ \text{squares @ } 45^\circ \text{ (NE)}$$

4. Travel **0.08 imi south**.

$$0.08\ \text{imi} \times \frac{800\ \text{m}}{\cancel{1\ \text{imi}}} \times \frac{1\ \text{square}}{20\ \cancel{\text{m}}} = \mathbf{3.2}\ \text{squares (S)}$$

5. Go **100 ba west**.

$$100\ \text{ba} \times \frac{0.20\ \text{m}}{\cancel{1\ \text{ba}}} \times \frac{1\ \text{square}}{20\ \cancel{\text{m}}} = \mathbf{1.0}\ \text{squares (W)}$$

6. Move **east** at **25 crp/min** for **60 s**.

$$\frac{25\ \text{crp}}{1\ \text{min}} \times \frac{2.00\ \text{m}}{1\ \text{crp}} \times \frac{1\ \text{min}}{60\ \text{s}} \times 60\ \text{s} \times \frac{1\ \text{square}}{20\ \text{m}} = \mathbf{2.0}\ \text{squares (E)}$$

7. Go **north** for **0.08 sst** at **1.4 scy**.

$$0.08\ \text{sst} \times \frac{720\ \text{s}}{1\ \text{sst}} \times 1.4\ \text{scy} \times \frac{0.667\ \text{m/s}}{1\ \text{scy}} \times \frac{1\ \text{square}}{20\ \text{m}} \approx \mathbf{2.668}\ \text{squares (N)}$$

8. Head **30 crp** toward the **southwest** at 45°.

$$30\ \text{crp} \times \frac{2.00\ \text{m}}{1\ \text{crp}} \times \frac{1\ \text{square}}{20\ \text{m}} = \mathbf{3.0}\ \text{squares @ } 45^\circ \text{ (SW)}$$

9. For **180 db** at **1.3 scy**, move **east**.

$$180\ \text{db} \times \frac{0.75\ \text{s}}{1\ \text{db}} \times 1.3\ \text{scy} \times \frac{0.667\ \text{m/s}}{1\ \text{scy}} \times \frac{1\ \text{square}}{20\ \text{m}} \approx \mathbf{4.877}\ \text{squares (E)}$$

10. Finally, advance **22 crp** toward the **northeast** at 45°.

$$22\ \text{crp} \times \frac{2.00\ \text{m}}{1\ \text{crp}} \times \frac{1\ \text{square}}{20\ \text{m}} = \mathbf{2.2}\ \text{squares @ } 45^\circ \text{ (NE)}$$

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Practice — Dessert Isle Conversions

Use only the Master Table and the Data Box. Show unit cancellation at every step.

1. **Banana highway.** The beach loop is 1.75 imi. Express its length in palms (pl) and in bananas (ba).

   Solution.

$$1.75 \text{ imi} \times \frac{800 \text{ m}}{1 \cancel{\text{imi}}} = 1400 \text{ m}.$$

$$1400 \text{ m} \times \frac{1 \text{ pl}}{0.50 \cancel{\text{m}}} = 2800 \text{ pl}, \qquad 1400 \text{ m} \times \frac{1 \text{ ba}}{0.20 \cancel{\text{m}}} = 7000 \text{ ba}.$$

2. **Hammock zoning with conversion.** A rectangular lot is 18 pl by 35 ba. Compute its area in m^2, hmk, and ft^2.

   Solution.

$$18 \text{ pl} \times \frac{0.50 \text{ m}}{1 \cancel{\text{pl}}} = 9.0 \text{ m}, \qquad 35 \text{ ba} \times \frac{0.20 \text{ m}}{1 \cancel{\text{ba}}} = 7.0 \text{ m}.$$

$$A = 9.0 \text{ m} \times 7.0 \text{ m} = 63.0 \text{ m}^2.$$

$$63.0 \text{ m}^2 \times \frac{1 \text{ hmk}}{2.0 \cancel{\text{m}^2}} = 31.5 \text{ hmk}, \qquad 63.0 \text{ m}^2 \times \frac{10.764 \text{ ft}^2}{1 \cancel{\text{m}^2}} = 6.78 \times 10^2 \text{ ft}^2.$$

3. **Coco-milk mass from volume.** A keg holds 12 grd of coco-milk. Using ρ_{cmlk}, find the mass in kg and in mangos (mgo).

   Solution.

$$12 \text{ grd} \times \frac{2.4 \text{ L}}{1 \cancel{\text{grd}}} = 28.8 \text{ L} \times \frac{1 \text{ m}^3}{1000 \cancel{\text{L}}} = 2.88 \times 10^{-2} \text{ m}^3.$$

$$m = \rho V = \left(1050 \frac{\text{kg}}{\cancel{\text{m}^3}}\right)\left(2.88 \times 10^{-2} \cancel{\text{m}^3}\right) = 30.2 \text{ kg}.$$

$$30.2 \text{ kg} \times \frac{1 \text{ mgo}}{0.35 \cancel{\text{kg}}} = 86.3 \text{ mgo}.$$

4. **Flow over a tide.** A still produces 4.2 ibr every tide. Report the average flow in L/s and in US gal/min.

   Solution.

$$4.2 \text{ ibr} \times \frac{25 \text{ L}}{1 \cancel{\text{ibr}}} = 105 \text{ L per td}. \qquad 1 \text{ td} \times \frac{5 \text{ h}}{1 \cancel{\text{td}}} \times \frac{3600 \text{ s}}{1 \cancel{\text{h}}} = 18{,}000 \text{ s}.$$

$$\frac{105 \text{ L}}{18{,}000 \text{ s}} = 5.83 \times 10^{-3} \text{ L/s}.$$

$$105 \text{ L} \times \frac{1 \text{ US gal}}{3.785 \cancel{\text{L}}} = 27.7 \text{ US gal per td} \quad \Rightarrow \quad \frac{27.7 \text{ US gal}}{300 \text{ min}} = 9.23 \times 10^{-2} \text{ US gal/min}.$$

5. **Pace to mph.** A runner holds 2.0 scy for one sunset (sst). Give the distance in meters and the average speed in mph.

Solution.

$$2.0\ \text{scy} \times \frac{0.667\ \text{m/s}}{\cancel{1\ \text{scy}}} = 1.334\ \frac{\text{m}}{\text{s}}. \qquad 1\ \text{sst} \times \frac{12\ \text{min}}{\cancel{1\ \text{sst}}} \times \frac{60\ \text{s}}{\cancel{1\ \text{min}}} = 720\ \text{s}.$$

$$d = \left(1.334\ \frac{\text{m}}{\cancel{\text{s}}}\right)(720\ \cancel{\text{s}}) = 960\ \text{m}. \qquad 1.334\ \frac{\text{m}}{\text{s}} \times \frac{2.237\ \text{mph}}{1\ \cancel{\text{m/s}}} = 2.98\ \text{mph}.$$

6. **Fire to light (energy chain).** The beacon runs at 3.5 trc for 1.5 td. How many firechips (fch), kWh, and BTU is that?

Solution.

$$3.5\ \text{trc} \times \frac{50\ \text{J/s}}{\cancel{1\ \text{trc}}} = 175\ \frac{\text{J}}{\text{s}}. \qquad 1.5\ \text{td} \times \frac{5\ \text{h}}{\cancel{1\ \text{td}}} \times \frac{3600\ \text{s}}{\cancel{1\ \text{h}}} = 27{,}000\ \text{s}.$$

$$E = \left(175\ \frac{\text{J}}{\cancel{\text{s}}}\right)(27{,}000\ \cancel{\text{s}}) = 4.73 \times 10^6\ \text{J}.$$

$$4.73\times 10^6\ \text{J} \times \frac{1\ \text{fch}}{10^6\ \cancel{\text{J}}} = 4.73\ \text{fch}, \quad 4.73\times 10^6\ \text{J} \times \frac{1\ \text{kWh}}{3.6 \times 10^6\ \cancel{\text{J}}} = 1.31\ \text{kWh}, \quad 4.73\times 10^6\ \text{J} \times \frac{1\ \text{BTU}}{1055.06\ \cancel{\text{J}}} = 4.48\times 10^3\ \text{B}$$

7. **Wood-to-heat estimate.** If dry wood has $e_{\text{wood}} \approx 16$ MJ/kg, how many kilograms of wood are equivalent to 18 fch? Also report in pounds.

Solution.

$$18\ \text{fch} \times \frac{1.00\ \text{MJ}}{\cancel{1\ \text{fch}}} = 18\ \text{MJ} \times \frac{1\ \text{kg}}{16\ \cancel{\text{MJ}}} = 1.125\ \text{kg}. \qquad 1.125\ \text{kg} \times \frac{2.20462\ \text{lb}}{1\ \cancel{\text{kg}}} = 2.48\ \text{lb}.$$

8. **Raft displacement (density + geometry).** A rectangular raft is 40 pl long and 60 ba wide. It sits so that an average of 1.5 lmt of underside area per meter of length is in contact with water. Approximate the displaced water volume if the draft is 0.12 m. (Answer in L.)

Solution.

$$40\ \text{pl} \times \frac{0.50\ \text{m}}{\cancel{1\ \text{pl}}} = 20\ \text{m}, \qquad 60\ \text{ba} \times \frac{0.20\ \text{m}}{\cancel{1\ \text{ba}}} = 12\ \text{m}.$$

$$\text{Contact area} = 1.5\ \frac{\text{lmt}}{\text{m}} \times 20\ \text{m} \times \frac{0.50\ \text{m}^2}{\cancel{1\ \text{lmt}}} = 15\ \text{m}^2.$$

$$V \approx A \times \text{draft} = (15\ \text{m}^2)(0.12\ \text{m}) = 1.8\ \text{m}^3 = 1.8\ \text{m}^3 \times \frac{1000\ \text{L}}{1\ \cancel{\text{m}^3}} = 1800\ \text{L}.$$

9. **Market basket (mixed units).** A trader brings 18 crb, 12 mgo, and 6 cn. Find the total mass in kg and in stones (stn).

Solution.

$$18\ \text{crb} \times \frac{0.12\ \text{kg}}{\cancel{1\ \text{crb}}} = 2.16\ \text{kg}, \quad 12\ \text{mgo} \times \frac{0.35\ \text{kg}}{\cancel{1\ \text{mgo}}} = 4.20\ \text{kg}, \quad 6\ \text{cn} \times \frac{1.40\ \text{kg}}{\cancel{1\ \text{cn}}} = 8.40\ \text{kg}.$$

$$m_{\text{total}} = 2.16 + 4.20 + 8.40 = 14.76\ \text{kg}, \qquad 14.76\ \text{kg} \times \frac{1\ \text{stn}}{2.50\ \cancel{\text{kg}}} = 5.90\ \text{stn}.$$

10. **Lagoon in bananas (two ways).** A lagoon is labelled 0.62 imi. Compute its length in bananas (ba) using (i) direct conversion, and (ii) via meters then bananas.

Solution.

$$\text{(i)} \quad 0.62 \text{ imi} \times \frac{800 \text{ m}}{\cancel{1 \text{ imi}}} \times \frac{1 \text{ ba}}{0.20 \cancel{\text{m}}} = 2480 \text{ ba.}$$

$$\text{(ii)} \quad 0.62 \text{ imi} \times \frac{800 \text{ m}}{\cancel{1 \text{ imi}}} = 496 \text{ m}, \qquad 496 \text{ m} \times \frac{1 \text{ ba}}{0.20 \cancel{\text{m}}} = 2480 \text{ ba.}$$

11. **From barrels to pace.** A drip system delivers 0.85 ibr per nap. How many csh per minute is that? Then, if each person drinks 3 csh per sunset, how many people can you serve continuously?

Solution.

$$0.85 \text{ ibr} \times \frac{25 \text{ L}}{\cancel{1 \text{ ibr}}} = 21.25 \text{ L per np}, \qquad 1 \text{ np} = 20 \text{ min.}$$

$$\frac{21.25 \text{ L}}{20 \text{ min}} \times \frac{1 \text{ csh}}{0.60 \cancel{\text{L}}} = 1.77 \text{ csh/min.}$$

$$1.77 \frac{\text{csh}}{\text{min}} \times 12 \text{ min per sst} = 21.2 \text{ csh/sst}, \qquad 21.2 \text{ csh} \times \frac{1 \text{ person}}{3 \cancel{\text{csh}}} \approx 7 \text{ people.}$$

12. **Energy to distance.** A cart (plus rider) of 95 kg rolls with negligible loss. If the cook burns 9.0 fch running a 2.0 trc motor for acceleration, and the cart reaches a steady 2.5 scy and maintains it for one tide, estimate the total mechanical energy used by the motor (J), then *if* all of it became kinetic energy initially, what speed (m/s) would that imply (compare to 2.5 scy)?

Solution.

$$9.0 \text{ fch} \times \frac{10^6 \text{ J}}{\cancel{1 \text{ fch}}} = 9.0 \times 10^6 \text{ J.}$$

Assuming (unrealistically) all becomes kinetic:

$$\tfrac{1}{2}mv^2 = 9.0 \times 10^6 \text{ J} \;\Rightarrow\; v = \sqrt{\frac{2 \times 9.0 \times 10^6 \cancel{\text{J}}}{95 \cancel{\text{kg}}}} = 4.35 \times 10^2 \text{ m/s.}$$

Compare to steady 2.5 scy $\times \dfrac{0.667 \text{ m/s}}{\cancel{1 \text{ scy}}} = 1.667$ m/s: the computed v shows the energy mostly goes to non-KE sinks in reality.