November 30

1. a) $y = a(x+5)^2 + 2$

b) An infinite ammount.

c) $-5 = a(5+5)^2 + 2$

$-5 = 100a + 2$

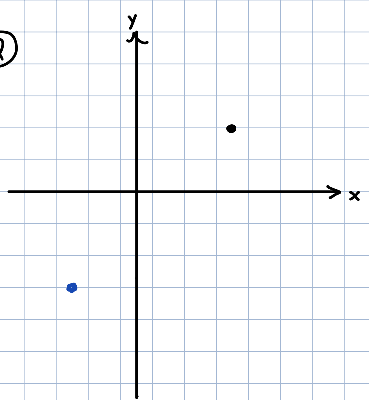
$-7 = 100a$

$\boxed{a = \frac{-7}{100}}$

$y = \frac{-7}{100}(x+5)^2 + 2$

e) 1 unique parabola (assuming point given is NOT vertex)

* - Alternative (easier approach)
- make one point vertex, then solve using method in Q1

2. 

A) $y = ax^2 + bx + c$

$(3,2)$: $2 = 9a + 3b + c$

$(-2,-3)$: $-3 = 4a - 2b + c$

- System of 2 equations with 3 variables will have infinite solutions or No solutions
- Choose arbitrary value for c, say $c=0$ to find a parabola.

$2 = 9a + 3b$

$-3 = 4a - 2b$

$4 = 18a + 6b$

$-9 = 12a - 6b$

$-5 = 30a$

$\boxed{a = \frac{-5}{30} = -\frac{1}{6}}$

$2 = 9\left(-\frac{1}{6}\right) + 3b$

$3b = 2 + \frac{3}{2} = \frac{7}{2}$

$\boxed{b = \frac{7}{6}}$

$$\underbrace{y = -\frac{1}{6}x^2 + \frac{7}{6}x}$$

this is ONE possible solution, there are an infinite ammount!

B) we now require parabola to pass through $(0,-10)$.

$(3,2)$: $2 = 9a + 3b + c$

$(-2,-3)$: $-3 = 4a - 2b + c$

$(0,-10)$: $-10 = c$

thus $2 = 9a + 3b - 10$, $-3 = 4a - 2b - 10$

$12 = 9a + 3b$

$7 = 4a - 2b$

$24 = 18a + 6b$

$21 = 12a - 6b$

$45 = 30a$

$\boxed{a = \frac{45}{30} = \frac{3}{2}}$

$12 = 9\left(\frac{3}{2}\right) + 3b$

$3b = 12 - \frac{27}{2}$

$3b = -\frac{3}{2}$

$\boxed{b = -\frac{3}{6} = -\frac{1}{2}}$

so $\underbrace{y = \frac{3}{2}x^2 - \frac{1}{2}x - 10}$

Only possible solution!

3. $y = ax^2 + bx + c$

$(-2,2)$: $2 = 4a - 2b + c$

$(0,2)$: $2 = c$

$(5,2)$: $2 = 25a + 5b + c$

Algebraically

$2 = 4a - 2b + 2$

$2 = 25a + 5b + 2$

$0 = 4a - 2b$ $\times 5$

$0 = 25a + 5b$ $\times 2$

$0 = 20a - 10b$

$0 = 50a + 10b$

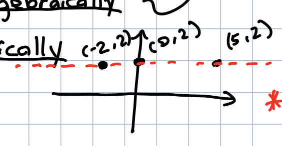
$70a = 0$

$\boxed{a = 0}$ Not a parabola!

if $a = 0$, then $0 = 4(0) - 2b$, thus $b = c$

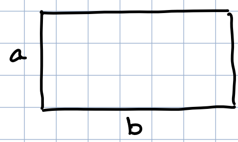
so the eqn is $y = 2$ (a horizontal line).

geometrically



* clearly can't draw parabola through

(5.)



$$2a+2b=100$$

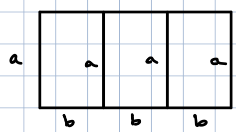
so $a+b=50$

$$\text{Area} = a\cdot b = a(50-a)$$
$$= -a^2+50a$$

$$y=-x^2+50x$$
$$y=-(x^2-50x)$$
$$=-(x^2-50x+25^2-25^2)$$
$$=-(x-25)^2+625$$

maximized when $x=25$,
Area is 625.

(6)



$$\text{Area} = a\cdot 3b$$
$$= \underbrace{3a(500-2a)}_{\text{need to 'minimize'}}$$

$$6b+4a=1000$$
$$3b+2a=500$$
$$3b=500-2a$$

$$= -6a^2+1500a$$

complete square

⋮

$$\text{Area} = -6(a-125)^2 + \underbrace{93\,750}_{\text{max area.}}$$

$$2h=100-2\pi r$$

(7) $2\pi r+2h=100\text{ cm}$

$$\text{Area} = \pi r^2 + 2r\cdot h$$
$$= \pi r^2 + (2h)\cdot r$$

$$= \pi r^2 + (100-2\pi r)r$$
$$= \pi r^2 - 2\pi r^2 + 100r$$
$$= -\pi r^2 + 100r$$

complete the square...

$$\text{Area} = -\pi\left(r-\frac{50}{\pi}\right) + \underbrace{\frac{2500}{\pi}}_{\text{max area.}}$$

(8) let x be # of times ticket price is increased by \$20

$$\text{Revenue} = (\#\text{ ticket holders})\cdot(\text{Price})$$
$$=(900-15x)(400+20x)$$

complete the square

⋮

$$y = -300(\underbrace{x-20}_{\substack{\#\text{ of increases}\\ \text{to achieve max.}}})^2 + \underbrace{480\,000}_{\text{max Revenue}}$$