Think It Through
September 9, 2022

★ indicates that the problem is considered 'more challenging' then what would normally be required in Math 20. Please make sure you give your best attempt on all of these problems.

1. Find ♡ such that $x^2 - 11x + ♡$ is a perfect square trinomial. Write your answer as a fraction, and factor the trinomial.

Solution: $♡ = \frac{121}{4}$, $\left(x - \frac{11}{2}\right)^2$

2. if a and b are positive numbers such that

$$x^3 + 2x^2y - 9x - 18y = (x + a)(x - b)(x + cy)$$

what is the value of $a + b + c$?

Solution:

$$\begin{aligned} x^3 + 2x^2y - 9x - 18y &= x^2(x + 2y) - 9(x + 2y) \\ &= (x^2 - 9)(x + 2y) \\ &= (x + 3)(x - 3)(x + 2y) \end{aligned}$$

$a + b + c = 8$

3. ★ Factorize: $(x + 2y)(x + 2y + 2) - 8$

Solution: Let $x + 2y = A$. Then the given expression can be factorized as follows:

$$\begin{aligned} (x + 2y)(x + 2y + 2) - 8 &= A(A + 2) - 8 \\ &= A^2 + 2A - 8 \\ &= (A - 2)(A + 4) \\ &= (x + 2y - 2)(x + 2y + 4) \end{aligned}$$

4. ★ $x^4 - 40x^2 + 4 = (x^2 + ax - b)(x^2 - cx - d)$. What is the value of $a + b + c + d$?

Solution:

$$\begin{aligned} x^4 - 40x^2 + 4 &= x^4 - 4x^2 + 4 - 36x^2 \\ &= (x^2 - 2)^2 - (6x)^2 \\ &= (x^2 - 2)^2 - (6x)^2 \\ &= (x^2 + 6x - 2)(x^2 - 6x - 2) \end{aligned}$$

5. ★ $a^2 + b^2 + 2c^2 - 4a + 2c - 2bc + 5 = 0$, $a \in \mathbb{R}$, $b \in \mathbb{R}$, $c \in \mathbb{R}$. What is the value of $a + b - c$?

Solution: We can factorize the left hand side into $(a-2)^2 + (c-b)^2 + (c+1)^2 = 0$. This is a sum of squares so we require each square to be zero. This implies $a = 2$, $b = c$, and that $c = -1$. Therefore $a + b - c = 2 - 1 + 1 = 2$.