# Extra Practice: Prime Factors, Applications, Rational/Irrational, Number Systems & Radicals

Math 10 · Mr. Merrick

## Prime Factors

1. State *all* positive divisors of the following.
   a) 84 $1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84$
   b) 75 $1, 3, 5, 15, 25, 75$
   c) 96 $1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96$
   d) 105 $1, 3, 5, 7, 15, 21, 35, 105$
2. In each case, determine the *number* of factors of the given whole number.
   a) 96 $96 = 2^5 \cdot 3 \Rightarrow (5+1)(1+1) = 12$
   b) 131 *Prime* $\Rightarrow$ 2 *factors*
   c) 225 $225 = 3^2 \cdot 5^2 \Rightarrow (2+1)(2+1) = 9$
   d) 256 $256 = 2^8 \Rightarrow 9$ *factors*
   e) 374 $374 = 2 \cdot 11 \cdot 17 \Rightarrow 2 \cdot 2 \cdot 2 = 8$
3. From the list in Question 2, state which numbers are prime and which are composite. *Prime:* 131. *Composite:* $96, 225, 256, 374$.
4. Classify each whole number as prime or composite.
   (a) 47 (b) 91 (c) 101 (d) 143 (e) 221 (f) 257
   *(a) prime; (b)* $7 \cdot 13$ *composite; (c) prime; (d)* $11 \cdot 13$ *composite; (e)* $13 \cdot 17$ *composite; (f) prime*
5. Twin primes are consecutive odd primes (e.g. $5, 7$). List seven other twin-prime pairs $< 120$. $(11, 13), (17, 19), (29, 31), (41, 43), (59, 61), (71, 73), (101, 103)$
6. a) State the factors of 48.
   $1, 2, 3, 4, 6, 8, 12, 16, 24, 48$
   b) State the *prime* factors of 48. $2, 3$
   c) Express 72 as a product of prime factors. $72 = 2^3 \cdot 3^2$
7. State the *prime factors* of:
   a) 18 $2, 3$
   b) 40 $2, 5$
   c) 63 $3, 7$
   d) 90 $2, 3, 5$
8. Explain why the numbers 0 and 1 have no prime factors. 0 *is divisible by every integer (no unique prime factorization).* 1 *has only one factor (itself) and no prime divisors.*
9. Use a *division table* to determine the prime factorization of:
   a) 252 $2^2 \cdot 3^2 \cdot 7$
   b) 378 $2 \cdot 3^3 \cdot 7$
   c) 2025 $3^4 \cdot 5^2$
   d) 2926 $2 \cdot 7 \cdot 11 \cdot 19$
10. Use a *factor tree* to determine the prime factorization of:
    a) 784 $2^4 \cdot 7^2$
    b) 960 $2^6 \cdot 3 \cdot 5$
    c) 4725 $3^3 \cdot 5^2 \cdot 7$
    d) 8400 $2^4 \cdot 3 \cdot 5^2 \cdot 7$
11. In each case, write the number as a product of prime factors.
    a) 3315 $3 \cdot 5 \cdot 13 \cdot 17$
    b) 8085 $3 \cdot 5 \cdot 7^2 \cdot 11$
    c) 9990 $2 \cdot 3^3 \cdot 5 \cdot 37$
    d) 7980 $2^2 \cdot 3 \cdot 5 \cdot 7 \cdot 19$
12. Which of the following numbers is *not* a prime factor of 2079?
    **A.** 3 **B.** 7 **C.** 11 **D.** 13 $2079 = 3^3 \cdot 7 \cdot 11$; *not a factor:* ***13***.
13. How many numbers in the list $2, 3, 9, 13$ are *not* prime factors of 2592? $2592 = 2^5 \cdot 3^4$; *not prime factors:* $9, 13 \Rightarrow 2$
14. The sum of all *distinct* prime factors of 462 462 is _______. $462\,462 = 2 \cdot 3 \cdot 7 \cdot 11 \cdot 13 \Rightarrow 36$
15. There is only one set of *prime triplets* (three consecutive odd primes). If the triplets are $a, b, c$, find $abc$. $(3, 5, 7) \Rightarrow 105$
16. The number 375 can be expressed as $p \times q^r$ in primes. Find $p+q+r$. $375 = 3 \cdot 5^3 \Rightarrow 3+5+3 = 11$

## Applications of Prime Factors

1. State the greatest common factor (GCF) of:
   a) 18 and 27 9
   b) 32 and 56 8
   c) $36, 48, 90$ 6
2. Use prime factorization to determine the GCF of:
   a) 180 and 420 $180 = 2^2 \cdot 3^2 \cdot 5,\ 420 = 2^2 \cdot 3 \cdot 5 \cdot 7 \Rightarrow GCF = 60$
   b) 294 and 385 $294 = 2 \cdot 3 \cdot 7^2,\ 385 = 5 \cdot 7 \cdot 11 \Rightarrow GCF = 7$
   c) 252 and 756 $252 = 2^2 \cdot 3^2 \cdot 7,\ 756 = 2^2 \cdot 3^3 \cdot 7 \Rightarrow GCF = 252$
3. Use prime factorization to determine the GCF of each pair.
   a) 528 and 780 $528 = 2^4 \cdot 3 \cdot 11,\ 780 = 2^2 \cdot 3 \cdot 5 \cdot 13 \Rightarrow 12$
   b) 616 and 840 $616 = 2^3 \cdot 7 \cdot 11,\ 840 = 2^3 \cdot 3 \cdot 5 \cdot 7 \Rightarrow 56$
   c) 1870 and 2210 $1870 = 2 \cdot 5 \cdot 11 \cdot 17,\ 2210 = 2 \cdot 5 \cdot 13 \cdot 17 \Rightarrow 170$
   d) 714 and 1050 $714 = 2 \cdot 3 \cdot 7 \cdot 17,\ 1050 = 2 \cdot 3 \cdot 5^2 \cdot 7 \Rightarrow 42$
   e) 128 and 320 $2^7$ *and* $2^6 \cdot 5 \Rightarrow 64$
   f) 735 and 980 $735 = 3 \cdot 5 \cdot 7^2,\ 980 = 2^2 \cdot 5 \cdot 7^2 \Rightarrow 245$
4. Determine the GCF of:
   a) 84, 420, 1008 84
   b) 128, 984, 1496, 3080 8
5. State the lowest common multiple (LCM) of:
   a) 8 and 12 24
   b) 7 and 9 63
   c) 12 and 20 60
   d) 15 and 18 90
6. Use prime factorization to determine the LCM of:
   a) 18 and 24 $2^3 \cdot 3^2 = 72$
   b) 45 and 84 $2^2 \cdot 3^2 \cdot 5 \cdot 7 = 1260$
   c) 96 and 144 $2^5 \cdot 3^2 = 288$
   d) 55 and 143 $5 \cdot 11 \cdot 13 = 715$
   e) 72 and 252 $2^3 \cdot 3^2 \cdot 7 = 504$
7. Determine the LCM of:
   a) $8, 12, 18$ 72
   b) $6, 14, 35$ 210
   c) $9, 10, 25$ 450
   d) $12, 30, 105$ 420
8. In each case, decide whether the number is a perfect square (give the root if so).
   a) 9801 $99^2$
   b) 7776 *not a square* $(2^5 \cdot 3^5)$
   c) 4900 $70^2$
   d) 1089 $33^2$
9. Consider 103 823.
   a) Evaluate $\sqrt[3]{103\,823}$. 47
   b) Explain why 103 823 is a perfect cube. $47^3$*; prime exponents are all multiples of* 3.
10. Use prime factorization to test for perfect cubes (give the cube root if so).
    a) 2744 $14^3$
    b) 110 592 $48^3$
    c) 35 937 $33^3$
    d) 421 875 $75^3$
11. Explain how to tell if a number is *both* a perfect square and cube. *All prime exponents multiples of* 6 *(a perfect* 6*th power).*
12. The greatest common factor of 425 and 595 is
    **A.** 5 **B.** 7 **C.** 17 **D.** 85 $425 = 5^2 \cdot 17,\ 595 = 5 \cdot 7 \cdot 17 \Rightarrow$ ***85***.
13. Two whole numbers $x, y$ have $\gcd(x, y) = 14$. Which statement must be false?
    **A.** $x, y$ both even **B.** $xy$ divisible by 98 **C.** $x, y$ both multiples of 7 **D.** Neither $x$ nor $y$ can be prime *(B) is false;* $\gcd = 14$ *does not force* $7^2$ *in* $xy$.
14. The LCM of $36, 231, 275$ is ________. $36 = 2^2 \cdot 3^2,\ 231 = 3 \cdot 7 \cdot 11,\ 275 = 5^2 \cdot 11 \Rightarrow LCM = 2^2 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11 = 69{,}300$
15. An encyclopedia has 840 pages. Page 12 and every 12th page is green; page 21 and every 21st is orange. How many pages are both? *Multiples of* $\operatorname{lcm}(12, 21) = 84$ *up to* 840*:* 10 *pages.*

## Rational and Irrational Numbers

1. For each, state repeating/non-repeating and terminating/non-terminating.
   a) $\dfrac{7}{20}$ 0.35*; terminating*
   b) $0.742742742\ldots$ *repeating, non-terminating*
   c) $\dfrac{19}{22}$ *repeating, non-terminating*
   d) $\sqrt{\dfrac{196}{400}}$ $\dfrac{7}{10}$*; terminating*
   e) $-\sqrt{31}$ *non-repeating, non-terminating (irrational)*
   f) $\sqrt{0.36}$ 0.6*; terminating*
   g) $-4\dfrac{5}{11}$ $-4.\overline{45}$*; repeating*
   h) $\pi$ *non-repeating, non-terminating (irrational)*
2. True/False.
   a) Every terminating decimal is rational. *T*
   b) A repeating decimal cannot be written as a fraction. *F*
   c) Only terminating decimals are rational. *F*
   d) Every rational decimal is either terminating or repeating. *T*
   e) A decimal cannot be both repeating and non-repeating. *T*
   f) $\pi$ is irrational. *T*
3. Rational or irrational? Briefly justify.
   a) $-\dfrac{17}{8}$ *rational; ratio of integers*
   b) 0.605 *rational; terminating*
   c) $\sqrt{196}$ *rational;* 14
   d) $0.305305305\ldots$ *rational; repeating*
4. Order on a number line: $\sqrt{14},\ \sqrt{\pi},\ \sqrt{0.2},\ \sqrt{98},\ 2\sqrt{11},\ 3\sqrt{5}$.
   *Approximations:* $\sqrt{0.2} \approx 0.447$,
   $\sqrt{\pi} \approx 1.772$,
   $\sqrt{14} \approx 3.742$,
   $2\sqrt{11} \approx 6.633$,
   $3\sqrt{5} \approx 6.708$,
   $\sqrt{98} \approx 9.899$.
   *So* $\sqrt{0.2} < \sqrt{\pi} < \sqrt{14} < 2\sqrt{11} < 3\sqrt{5} < \sqrt{98}$.
5. Identify as rational or irrational; if rational, simplest fraction.
   a) 0.92 $\frac{23}{25}$
   b) $\sqrt{\frac{9}{121}}$ $\frac{3}{11}$
   c) $\sqrt{0.0121}$ $0.11 = \frac{11}{100}$
   d) $-\sqrt{97}$ *irrational*
   e) $-0.\overline{8}$ $-\frac{8}{9}$
   f) $-\sqrt{\frac{49}{81}}$ $-\frac{7}{9}$
   g) $4.612612\ldots$ $\frac{512}{111}$
   h) $\sqrt{\frac{361}{529}}$ $\frac{19}{23}$
   i) $5.\overline{0}$ 5
6. Convert to improper fraction (simplest form).
   a) $0.\overline{7}$ $\frac{7}{9}$
   b) $0.1\overline{6}$ $\frac{1}{6}$
   c) $1.2\overline{3}$ $\frac{37}{30}$
   d) $0.\overline{204}$ $\frac{204}{999} = \frac{68}{333}$
   e) $-2.4\overline{545}$ $-\frac{245}{99}$
7. Convert the repeating decimal to a fraction (algebraic method).
   a) $0.\overline{3}$ $\frac{1}{3}$
   b) $0.7\overline{2}$ $\frac{13}{18}$
   c) $0.00\overline{981}$ $\frac{27}{2750}$
8. Convert each terminating decimal to an improper fraction (lowest terms).
   a) 3.007 $\frac{3007}{1000}$
   b) $-2.125$ $-\frac{17}{8}$
   c) 4.0625 $\frac{65}{16}$
9. The decimal for $\dfrac{7}{12}$ is
   **A.** terminating & repeating
   **B.** terminating & non-repeating
   **C.** non-terminating & repeating
   **D.** non-terminating & non-repeating
   *Correct:* ***C****.*
10. Which is irrational?
    **A.** $\sqrt{256}$ **B.** $\sqrt{0.09}$ **C.** $\frac{25}{6}$ **D.** $\sqrt{50}$
    *Correct:* ***D****.*
11. $9.\overline{9}$ is equal to
    **A.** $\frac{99}{10}$ **B.** $\frac{999}{100}$ **C.** 10 **D.** 9 *Correct:* ***C*** *(since* $9.\overline{9} = 10$*).*
12. Write $0.\overline{27} = \dfrac{a}{b}$ in lowest terms and compute $b - a$. $\frac{3}{11} \Rightarrow 8$

## Number Systems

1. Place each into the appropriate nested sets ($N \subset W \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$ and $\mathbb{R} \setminus \mathbb{Q}$): $-3,\ \sqrt{81},\ \dfrac{29}{11},\ \sqrt{2},\ 0,\ \pi$.
   $-3 : \mathbb{Z}$; $\sqrt{81} = 9 : N$; $\frac{29}{11} : \mathbb{Q}$; $\sqrt{2} : \mathbb{R} \setminus \mathbb{Q}$; $0 : W$; $\pi : \mathbb{R} \setminus \mathbb{Q}$.
2. List all sets (largest $\to$ smallest) each belongs to.
   a) $-8$ $\mathbb{R}, \mathbb{Q}, \mathbb{Z}$
   b) $\sqrt{64}$ $\mathbb{R}, \mathbb{Q}, \mathbb{Z}, W, N$
   c) $3.2727\ldots$ $\mathbb{R}, \mathbb{Q}$
   d) $-\dfrac{12}{7}$ $\mathbb{R}, \mathbb{Q}$
   e) $0$ $\mathbb{R}, \mathbb{Q}, \mathbb{Z}, W$
   f) $\sqrt{11}$ $\mathbb{R} \setminus \mathbb{Q}$
   g) non-repeating $-2.1345218\ldots$ $\mathbb{R} \setminus \mathbb{Q}$
   h) $\pi$ $\mathbb{R} \setminus \mathbb{Q}$
3. Why does $-7$ belong to more sets than $-\dfrac{7}{2}$? *$-7$ is an integer (hence rational, real); $-\dfrac{7}{2}$ is not an integer.*
4. Indicate membership in $\mathbb{N}, \mathbb{W}, \mathbb{Z}, \mathbb{Q}, \overline{\mathbb{Q}}$ (irrationals), and $\mathbb{R}$ for each.
   a) $\frac{1}{5}$ $\mathbb{Q}, \mathbb{R}$
   b) $123\,987$ $\mathbb{N}, \mathbb{W}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$
   c) $-4$ $\mathbb{Z}, \mathbb{Q}, \mathbb{R}$
   d) $7.534$ $\mathbb{Q}, \mathbb{R}$
   e) $9.5$ $\mathbb{Q}, \mathbb{R}$
   f) $\sqrt{75}$ $\overline{\mathbb{Q}}, \mathbb{R}$
   g) $-\pi$ $\overline{\mathbb{Q}}, \mathbb{R}$
   h) $-\frac{355}{113}$ $\mathbb{Q}, \mathbb{R}$
   i) $-\sqrt{49}$ $\mathbb{Z}, \mathbb{Q}, \mathbb{R}$
   j) $0.000005$ $\mathbb{Q}, \mathbb{R}$
   k) non-repeating $2.232425\ldots$ $\overline{\mathbb{Q}}, \mathbb{R}$
5. Find a number that satisfies each condition.
   a) Integer but not whole. $-3$
   b) Rational but not integer. $\frac{7}{5}$
   c) Real but not rational. $\sqrt{5}$
   d) Whole but not natural. $0$
6. Fill with *always/sometimes/never*.
   a) A whole number is ______ a natural number. *Sometimes (0 excluded)*
   b) The quotient of two integers is ______ an integer. *Sometimes*
   c) A whole number is ______ a rational number. *Always*
   d) The difference between two integers is ______ an integer. *Always*
   e) The square root of a number is ______ irrational. *Sometimes*
   f) A negative number is ______ in $W$. *Never*
   g) A number in $N$ is ______ in $\mathbb{R}$. *Always*
7. True/False.
   a) All natural numbers are integers. *T*
   b) Real numbers consist of rationals and irrationals. *T*
   c) Integers are nested within rationals. *T*
   d) All integers are rational. *T*
   e) All irrationals are real. *T*
   f) $\mathbb{R}$ is contained in $N$. *F*
   g) $\mathbb{Q}$ is contained in $W$. *F*
   h) Exactly one element of $W$ is not in $N$. *T (0)*
8. More about roots (True/False).
   a) Every positive number has two square roots but one cube root. *T (real roots)*
   b) Every negative number has one real cube root but no real square roots. *T*
9. Short explanations (estimation).
   a) $\sqrt{8} + \sqrt{17} \neq \sqrt{25}$ $2.83 + 4.12 \approx 6.95 \neq 5$
   b) $\sqrt{2} + \sqrt{3} + \sqrt{4} \neq \sqrt{9}$ $1.41 + 1.73 + 2 = 5.14 \neq 3$
10. Determine true or false.
   a) $\sqrt{9} + \sqrt{4} = \sqrt{9+4}$. *F*
   b) $\sqrt{9} - \sqrt{4} = \sqrt{9-4}$. *F*
   c) $\sqrt{9} \cdot \sqrt{4} = \sqrt{36}$. *T*
   d) $\sqrt{9} + \sqrt{4} = \sqrt{9} + 4$. *F*

11. For each, (i) estimate mentally; (ii) use a calculator (nearest tenth) and judge the estimate.
   a) $\sqrt{21}$ *(i) 4–5; (ii)* $\approx 4.6$
   b) $\sqrt{27.4}$ *(i) 5–6; (ii)* $\approx 5.2$
   c) $4\sqrt{48} - 3\sqrt{63} \approx 3.9$
   d) $\frac{3}{4}\sqrt{14.2} + \frac{1}{2}\sqrt{5} \approx 4.0$
   e) $\sqrt{123} \approx 11.1$
   f) $\sqrt{\sqrt{90}} \approx 3.08$
   g) $\sqrt{10} + \sqrt{24.5} \approx 8.11$
   h) $\sqrt{\sqrt{2601}} \approx 7.14$
12. Estimate to one significant digit.
   a) $\sqrt{507.1} \approx 20$
   b) $\sqrt{7991} \approx 90$
   c) $\sqrt{10\,389} \approx 100$
   d) $\sqrt{823\,775} \approx 900$
   e) $\sqrt{0.501} \approx 0.7$
   f) $\sqrt{0.0501} \approx 0.2$
   g) $\sqrt{0.0876} \approx 0.3$
   h) $\sqrt{0.000\,397\,2} \approx 0.02$
13. (i) estimate; (ii) calculator (nearest tenth).
   a) $\sqrt[3]{25} \approx 2.9$
   b) $\sqrt[3]{2} \approx 1.3$
   c) $\sqrt[3]{202} \approx 5.9$
   d) $\sqrt[3]{999.9} \approx 10.0$
   e) $2\sqrt[3]{58.7} - 3\sqrt[3]{7.62} \approx 1.9$
   f) $\frac{2}{3}\sqrt{40} - \frac{1}{2}\sqrt{60} \approx 0.3$
   g) $\sqrt[3]{3\sqrt{10}} \approx 2.1$
14. Order on the number line: $\sqrt{50}$, $\sqrt[3]{50}$, $5\sqrt{10}$, $\sqrt[3]{10^3}$, $10\sqrt{5}$, $10\sqrt[3]{5}$.
   $\sqrt[3]{50} \approx 3.68 < \sqrt{50} \approx 7.07 < \sqrt[3]{1000} = 10 < 5\sqrt{10} \approx 15.81 < 10\sqrt[3]{5} \approx 17.10 < 10\sqrt{5} \approx 22.36.$
15. Which nesting statement is *false*?
   **A.** Integers $\subset$ rationals
   **B.** Wholes $\subset$ naturals
   **C.** Irrationals $\subset$ reals
   **D.** Reals $\subset$ naturals *Correct:* ***D***.
16. How many of $-\sqrt{6}$, $\sqrt[3]{-6}$, $-\sqrt[3]{6}$, $\sqrt{-6}$ are not real? $\sqrt{-6}$ *only* $\Rightarrow 1$
17. How many of $\sqrt{49}$, $\sqrt{49/100}$, $\sqrt{0.49}$, $\sqrt{\frac{4}{9}}$ can be written as $\frac{a}{b}$ with $a, b \in \mathbb{N}$? *All 4.*
18. To the nearest hundredth, evaluate $5\sqrt[3]{7}$. $\approx 9.57$
19. Evaluate the absolute values.
   a) $|-4|$ 4
   b) $|13|$ 13
   c) $|3-9|$ 6
   d) $\big||3| - |9|\big|$ 6
   e) $|-\sqrt[3]{27}|$ 3
   f) $|\sqrt[3]{-27}|$ 3
20. Decide whether the statement is true or false.
   a) $|x| = x$ if $x > 0$. *T*
   b) $|x| = -x$ if $x < 0$. *T*
21. Sketch solution sets on a number line.
   a) $|x| < 5$ $(-5, 5)$
   b) $|a| \geq 3$ $(-\infty, -3] \cup [3, \infty)$

## Radicals

1. Mentally evaluate where possible (real numbers).
   a) $\sqrt{81}$ 9
   b) $\sqrt[4]{81}$ 3
   c) $5\sqrt[3]{27}$ 15
   d) $\sqrt[5]{100\,000}$ 10
   e) $\sqrt{\frac{16}{25}}$ $\frac{4}{5}$
   f) $\sqrt[4]{\frac{1}{16}}$ $\frac{1}{2}$
   g) $4\sqrt[4]{\frac{1}{16}}$ 2
   h) $-\sqrt{1}$ $-1$
   i) $\sqrt{-1}$ *not real*
   j) $\sqrt[5]{-1}$ $-1$
   k) $7\sqrt[3]{-125}$ $-35$
   l) $-\sqrt[4]{\frac{1}{16}}$ $-\frac{1}{2}$
   m) $3\sqrt{144}$ 36
   n) $\frac{5}{2}\sqrt[5]{32}$ 5
   o) $-\sqrt[11]{-1}$ 1 *(since* $\sqrt[11]{-1} = -1$*, so* $-(-1) = 1$*)*
   p) $\sqrt[3]{\frac{8}{27}}$ $\frac{2}{3}$
2. True/False.
   a) The square roots of 25 are $\pm 5$. *T*
   b) $\sqrt{25} = \pm 5$. *F (principal root* $= +5$*)*
   c) If $x^2 = 25$ and $x \in \mathbb{R}$, then $x = \pm 5$. *T*
3. Use a calculator to evaluate (state sign first, then value as needed).
   a) $\sqrt[4]{4096}$ 8
   b) $\sqrt[5]{-243}$ $-3$
   c) $-\sqrt[4]{2401}$ $-7$
   d) $-\sqrt[3]{729}$ $-9$
   e) $\sqrt[3]{-729}$ $-9$
   f) $-8\sqrt[4]{\frac{1}{256}}$ $-1$
   g) $\sqrt[6]{0.015625}$ 0.5
   h) $\sqrt[4]{-6561}$ *not real*
   i) $\frac{3}{2}\sqrt[4]{\frac{16}{81}}$ 1
4. Evaluate to the nearest hundredth.
   a) $\sqrt[4]{10} \approx 1.78$
   b) $\sqrt[8]{29} \approx 1.54$
   c) $\frac{3}{2}\sqrt[3]{-527} \approx -12.12$
5. Evaluate to the nearest tenth.
   a) $\sqrt[5]{-25} \approx -1.9$
   b) $-5\sqrt[4]{169} \approx -18.0$
   c) $\frac{1}{2}\sqrt[3]{-81} \approx -2.2$
6. Identify the *index* and the *radicand* in each radical.
   a) $\sqrt[3]{42}$
      index: ______
      radicand: ________
      *index* 3*, radicand* 42
   b) $\sqrt[4]{36}$
      index: ______
      radicand: ________
      *index* 4*, radicand* 36
   c) $5\sqrt{17}$
      index: ______
      radicand: ________
      *index* 2*, radicand* 17*; 5 is a coefficient*
7. Explain the meaning of the index 4 in the radical $\sqrt[4]{36}$. *It means the* fourth root*: the number which, raised to the 4th power, equals* 36*.*
8. Determine whether each statement is **true** or **false**.
   a) $\sqrt{30} = \sqrt{5}\sqrt{6}$ *True*
   b) $\sqrt{6-4} = \sqrt{6} - \sqrt{4}$ *False*
   c) $\sqrt{3} = \frac{\sqrt{45}}{\sqrt{15}}$ *True*
   d) $\frac{\sqrt{20}}{\sqrt{10}} = \sqrt{10}$ *False*
   e) $\sqrt{2} + \sqrt{2} = \sqrt{4}$ *False*
   f) $\sqrt{2} \times \sqrt{2} = \sqrt{4}$ *True*
   g) $\sqrt{\frac{1}{2} \cdot 30} = \sqrt{15}$ *True*
   h) $\frac{1}{2}\sqrt{30} = \sqrt{15}$ *False*

9. Write as a *single* radical in the form $\sqrt{x}$ (simplify $x$).
   a) $\sqrt{5}\sqrt{7}$ $\sqrt{35}$
   b) $\sqrt{14}\sqrt{2}$ $\sqrt{28}$
   c) $\sqrt{3}\cdot\sqrt{8}$ $\sqrt{24}$
   d) $\sqrt{6}\cdot\sqrt{11}$ $\sqrt{66}$
   e) $\dfrac{\sqrt{20}}{\sqrt{10}}$ $\sqrt{2}$
   f) $\dfrac{\sqrt{25}}{\sqrt{5}}$ $\sqrt{5}$
   g) $\dfrac{\sqrt{10}\sqrt{6}}{\sqrt{2}}$ $\sqrt{30}$
   h) $\dfrac{\sqrt{81}}{\sqrt{9}}$ $\sqrt{9}$
10. Express each as a product of radicals (split into two square roots).
   a) $\sqrt{35}$ $\sqrt{5}\sqrt{7}$
   b) $\sqrt{33}$ $\sqrt{3}\sqrt{11}$
   c) $\sqrt{65}$ $\sqrt{5}\sqrt{13}$
   d) $\sqrt{49}$ $\sqrt{7}\sqrt{7}$
11. Consider the statements:
   I. The cube root of $-27$ (over the reals) is $\pm3$.
   II. The fourth roots of 81 (over the reals) are $\pm3$.
   III. $-\sqrt[3]{1000} = \sqrt[3]{-1000}$.
   IV. $-\sqrt[4]{16} = \sqrt[4]{-16}$.
   Which are true?
   **A.** II and III only
   **B.** I, II, and III only
   **C.** I, II, III, and IV
   **D.** Some other combination *Correct:* ***A****. (I false, II true, III true, IV false).*
12. In the radical $\sqrt[4]{18}$, the index and radicand are
   **A.** index 2, radicand $\sqrt{18}$
   **B.** index 1, radicand 1
   **C.** index 18, radicand 1
   **D.** index 4, radicand 18 *Correct:* ***D****.*
13. To the nearest hundredth, evaluate $\sqrt{\frac{7}{8}} + 2\sqrt[4]{\frac{7}{8}}$.
   ________ $\sqrt{0.875} \approx 0.9354$*,* $\sqrt[4]{0.875} \approx 0.9672$*, sum* $\approx 2.87$*.*

## Entire Radicals and Mixed Radicals — Part One

1. Without a calculator, arrange in order from *greatest to least*:
   $3\sqrt{5},\ 5\sqrt{3},\ \sqrt{15},\ 2\sqrt{8},\ 8\sqrt{2}$.
   $8\sqrt{2}$ (11.31) $>$ $5\sqrt{3}$ (8.66) $>$ $3\sqrt{5}$ (6.71) $>$ $2\sqrt{8} = 4\sqrt{2}$ (5.66) $>$ $\sqrt{15}$ (3.87).
2. Two students find the hypotenuse $PQ$ of a right triangle with legs $\sqrt{34}$ and $\sqrt{38}$. Louis rounds each leg; Asia simplifies radicals first.
   a) Compute each to the nearest hundredth. *Louis* $\approx 8.48$*; Asia* $6\sqrt{2} \approx 8.49$.
   b) Which is more accurate? *Asia (kept exact form before rounding).*
   c) Exact mixed radical. $6\sqrt{2}$.
3. Convert each to a *mixed radical* (simplest form).
   a) $\sqrt{96}$ $4\sqrt{6}$
   b) $\sqrt{242}$ $11\sqrt{2}$
   c) $\dfrac{2}{3}\sqrt{180}$ $4\sqrt{5}$
   d) $\dfrac{1}{8}\sqrt{320}$ $\sqrt{5}$
   e) $\sqrt{245}$ $7\sqrt{5}$
   f) $4\sqrt{338}$ $52\sqrt{2}$
   g) $\sqrt{1250}$ $25\sqrt{2}$
   h) $\sqrt{66}$ *cannot convert (no perfect-square factor* $> 1$*)*
   i) $-\dfrac{5}{6}\sqrt{304}$ $-\frac{10}{3}\sqrt{19}$
   j) $\sqrt{980}$ $14\sqrt{5}$
   k) $4\sqrt{272}$ $16\sqrt{17}$
   l) $-3\sqrt{288}$ $-36\sqrt{2}$
   m) $2\sqrt{369}$ $6\sqrt{41}$
   n) $\sqrt{364}$ $2\sqrt{91}$
   o) $\dfrac{2}{5}\sqrt{450}$ $6\sqrt{2}$
   p) $\dfrac{7}{11}\sqrt{341}$ *cannot convert* $(341 = 11 \cdot 31)$
4. Convert to a *mixed radical* where the radicand is a whole number.
   a) $\sqrt{\dfrac{2}{9}}$ $\dfrac{1}{3}\sqrt{2}$
   b) $\sqrt{\dfrac{5}{4}}$ $\dfrac{\sqrt{5}}{2}$
   c) $\sqrt{\dfrac{18}{25}}$ $\dfrac{3\sqrt{2}}{5}$
   d) $7\sqrt{\dfrac{20}{49}}$ $2\sqrt{5}$
5. Convert to *entire radical* form.
   a) $2\sqrt{6}$ $\sqrt{24}$
   b) $3\sqrt{7}$ $\sqrt{63}$
   c) $5\sqrt{15}$ $\sqrt{375}$
   d) $12\sqrt{2}$ $\sqrt{288}$
   e) $3\sqrt{25}$ $\sqrt{225}$
   f) $-8\sqrt{3}$ $-\sqrt{192}$
   g) $9\sqrt{10}$ $\sqrt{810}$
   h) $-4\sqrt{5}$ $-\sqrt{80}$
6. Convert the following to *entire radical* form.
   a) $\dfrac{1}{3}\sqrt{27}$ $\sqrt{3}$
   b) $15$ $\sqrt{225}$
   c) $\dfrac{3}{2}\sqrt{8}$ $\sqrt{18}$
   d) $3^2\sqrt{21}$ $\sqrt{1701}$
7. Given $\sqrt{6} \approx 2.45$ and $\sqrt{60} \approx 7.75$, approximate:
   a) $\sqrt{600}$ 24.5
   b) $\sqrt{6000}$ 77.5
   c) $\sqrt{600\,000}$ 775
   d) $\sqrt{0.06}$ 0.245
   e) $\sqrt{0.6}$ 0.775
   f) $\sqrt{24}$ 4.90
   g) $\sqrt{540}$ 23.25
   h) $\sqrt{\dfrac{6}{25}}$ 0.49
8. Arrange from *greatest to least*:
   $3\sqrt{7},\ 5\sqrt{3},\ \sqrt{60},\ 2\sqrt{11},\ \dfrac{1}{2}\sqrt{200}$.
   $5\sqrt{3}$ (8.66) $>$ $3\sqrt{7}$ (7.94) $>$ $\sqrt{60}$ (7.75) $>$ $\frac{1}{2}\sqrt{200}$ (7.07) $>$ $2\sqrt{11}$ (6.63).
9. In right $\triangle XYZ$ with legs 19 cm and 5 cm, find hypotenuse $XY$:
   a) entire radical $\sqrt{386}$
   b) mixed radical *(already simplest)*
   c) decimal (nearest hundredth) 19.65
10. Find the missing side in simplest mixed radical form.
    a) legs $4, 8$; hypotenuse $x$ $x = \sqrt{80} = 4\sqrt{5}$
    b) legs $5, 6$; hypotenuse $x$ $x = \sqrt{61}$
    c) hypotenuse 8, leg 6; other leg $x$ $x = \sqrt{28} = 2\sqrt{7}$

11. The length of $\overline{KL}$ for legs $\sqrt{6}$ and $\sqrt{24}$ is
**A.** $\sqrt{540}$ **B.** $3\sqrt{2}$ **C.** $\sqrt{30}$ **D.** $9\sqrt{2}$ *Correct:* ***C.***

12. Without a calculator, which radical is *not* equal to the others?
**A.** $12\sqrt{2}$ **B.** $\sqrt{288}$ **C.** $6\sqrt{8}$ **D.** $4\sqrt{72}$ ***D.*** *(* $\sqrt{288} = 12\sqrt{2}$, $6\sqrt{8} = 12\sqrt{2}$, *but* $4\sqrt{72} = 24\sqrt{2}$.*)*

13. On a clear day, $d = \sqrt{13h}$ (km), where $h$ metres is eye level above ground. From a 698.2 m building with eye level 1.8 m above the roof, write $d = a\sqrt{b}$ and find $a + b$. $h = 700$, $d = \sqrt{9100} = 10\sqrt{91}$; $a + b = 101$.

14. Using Heron's formula, a triangle with sides $14, 15, 25$ has area $A = p\sqrt{26}$. Find $p$. $p = 18$.

15. A square of side 8 cm is inscribed in a larger square by joining midpoints. If larger side is $p\sqrt{q}$, find $pq$. $8\sqrt{2} \Rightarrow pq = 16$.

## Entire Radicals and Mixed Radicals — Part Two

1. Convert the following radicals to mixed radicals in simplest form.
   a) $\sqrt[3]{48}$ $2\sqrt[3]{6}$
   b) $\sqrt[3]{128}$ $4\sqrt[3]{2}$
   c) $\sqrt[3]{2000}$ $10\sqrt[3]{2}$
   d) $5\sqrt[3]{-81}$ $-15\sqrt[3]{3}$
   e) $\frac{5}{6}\sqrt[3]{108}$ $\frac{5}{2}\sqrt[3]{4}$
   f) $5\sqrt[4]{162}$ $15\sqrt[4]{2}$
   g) $5\sqrt{192}$ $40\sqrt{3}$
   h) $-2\sqrt[3]{625}$ $-10\sqrt[3]{5}$
2. Convert the following mixed radicals to entire radicals.
   a) $2\sqrt[5]{2}$ $\sqrt[5]{32}$
   b) $3\sqrt[3]{4}$ $\sqrt[3]{108}$
   c) $-3\sqrt[4]{3}$ $-\sqrt[4]{243}$
   d) $-10\sqrt[3]{5}$ $-\sqrt[3]{5000}$
   e) $2\sqrt[5]{6}$ $\sqrt[5]{192}$
   f) $\frac{1}{2}\sqrt[3]{16}$ $\sqrt[3]{2}$
   g) $\frac{3}{10}\sqrt[4]{100000}$ $\sqrt[4]{810}$
   h) $-5\sqrt[3]{9}$ $-\sqrt[3]{1125}$
3. Arrange, least to greatest (no calculator): $7\sqrt[6]{1},\ -3\sqrt[3]{-27},\ \frac{5}{2}\sqrt[4]{16},\ 3\sqrt[3]{64}$.
   $-3\sqrt[3]{-27} < 7\sqrt[6]{1} < \frac{5}{2}\sqrt[4]{16} < 3\sqrt[3]{64}$.
4. Consider $2\sqrt[3]{11},\ 3\sqrt[3]{3},\ 4\sqrt[3]{2},\ 2\sqrt[3]{6}$.
   a) Explain how to compare without a calculator. *Cube each expression; compare the resulting values.*
   b) Order least to greatest.
   $2\sqrt[3]{6} < 3\sqrt[3]{3} < 4\sqrt[3]{2} < 2\sqrt[3]{11}$.
5. $\sqrt[3]{240}$ is equivalent to
   **A.** $2\sqrt[3]{40}$ **B.** $4\sqrt[3]{15}$ **C.** $2\sqrt[3]{30}$ **D.** $8\sqrt[3]{30}$
   *Correct:* ***B***.
6. Consider the statements:
   1) $-3\sqrt[4]{8} = 3\sqrt[4]{-8}$,
   2) $-2\sqrt[3]{7} = 2\sqrt[3]{-7}$.
   **A.** Both true **B.** Both false **C.** 1 true, 2 false **D.** 1 false, 2 true *Correct:* ***D***.
7. The mixed radical $\dfrac{1}{12}\sqrt[3]{128}$ equals $a\sqrt[3]{b}$ in simplest form. Find $a+b$ (nearest tenth). $\frac{1}{3}\sqrt[3]{2} \Rightarrow 2.3$.
8. $\sqrt{3x}\cdot\sqrt{2x}$ is equivalent to
   **A.** $\sqrt{6x}$ **B.** $\sqrt{36x^2}$ **C.** $6\sqrt{x}$ **D.** $x\sqrt{6}$
   $\sqrt{3x}\sqrt{2x} = \sqrt{6x^2} = x\sqrt{6}$ $(x \geq 0)$.
   *Answer:* ***D***.
9. Express as an *entire* radical.
   a) $6\sqrt{y}$ $\sqrt{36y}$
   b) $8\sqrt{c^2}$ $\sqrt{64c^2}$
   c) $10\sqrt{2yz^3}$ $\sqrt{200yz^3}$
   d) $-3\sqrt[3]{x^2}$ $\sqrt[3]{-27x^2}$
   e) $c\sqrt{c}$ $\sqrt{c^3}$
   f) $x\sqrt{3y^3}$ $\sqrt{3x^2y^3}$
   g) $11c^2\sqrt{c^2d}$ $\sqrt{121c^6d}$
   h) $5a^3b\sqrt{3a^2b}$ $\sqrt{75a^8b^3}$
10. Express as a mixed radical in simplest form.
    a) $\sqrt{a^5}$ $a^2\sqrt{a}$
    b) $\sqrt{t^3}$ $t\sqrt{t}$
    c) $\sqrt{x^{11}}$ $x^5\sqrt{x}$
    d) $\sqrt[3]{x^4}$ $x\sqrt[3]{x}$
    e) $\sqrt[3]{b^8}$ $b^2\sqrt[3]{b^2}$
    f) $\sqrt[4]{x^6}$ $x\sqrt[4]{x^2}$
11. Express as a mixed radical.
    a) $\sqrt{8y^2}$ $2y\sqrt{2}$
    b) $\sqrt{16p^3}$ $4p\sqrt{p}$
    c) $\sqrt{75y^3z^4}$ $5yz^2\sqrt{3y}$
    d) $\sqrt{300a^9w^7}$ $30a^4w^3\sqrt{3aw}$
    e) $5\sqrt{28c^4d^3}$ $10c^2d\sqrt{7d}$
    f) $-6\sqrt{29a^4b^8}$ $-6a^2b^4\sqrt{29}$

## Quick Check

1. Which is not a prime factor of 14 014?
   **A.** 7 **B.** 11 **C.** 13 **D.** 17 *D*
2. How many numbers in the list $7, 11, 17, 21$ are prime factors of 3234?
   **A.** 1 **B.** 2 **C.** 3 **D.** 4 *B*
3. The sum of prime factors of 160797 is ______. *73*
4. The GCF of 6699 and 8265 is ______. *87*
5. LCM of 14 and 105 equals GCF of $P, Q$. Which must be false?
   **A.** $P$ multiple of 7
   **B.** $Q$ multiple of 21
   **C.** $P < 200$
   **D.** $Q > 2000$ *C*
6. If $x$ is a perfect square, minimum value of $d$ (in the given factor tree) is
   **A.** 2 **B.** 3 **C.** 6 **D.** 9 *D*
7. If $x$ is a perfect cube, the minimum value of $x$ is ______. *216*
8. $\otimes$ is irrational. Its decimal representation is
   **A.** terminating & repeating
   **B.** terminating & non-repeating
   **C.** non-terminating & repeating
   **D.** non-terminating & non-repeating *D*
9. Which are rational? (I) $1.0100100001\ldots$ (non-repeating), (II) $\sqrt[3]{\frac{8}{27}}$, (III) 0.04, (IV) 0.29
   **A.** III, IV
   **B.** II, III, IV
   **C.** I, II, III, IV
   **D.** Other *B*
10. The rational number $1.\overline{54}$ as $\frac{c}{d}$; the value of $c$ is
    **A.** 17 **B.** 11 **C.** 6 **D.** 4 *A*
11. $M, N$ irrationals with $30 < M < 40$, $3 < N < 4$. The value of $\sqrt{M} + \sqrt{N}$ is best represented by
    **A.** P **B.** Q **C.** R **D.** S *B*
12. Largest among $\sqrt[3]{67}$, $\sqrt[4]{98}$, $\sqrt{19}$, $\sqrt[5]{201}$ is
    **A.** $\sqrt{19}$ **B.** $\sqrt[4]{98}$ **C.** $\sqrt[3]{67}$ **D.** $\sqrt[5]{201}$ *A*
13. The length $12\sqrt[4]{4000}$ m has index and radicand
    **A.** 4 and 4000
    **B.** 12 and 4000
    **C.** 4 and 12
    **D.** 4000 and 4 *A*
14. When $7\sqrt[3]{6}$ is written as an entire radical, the radicand is ______. *2058*
15. Which statements are true? 1) $35 = 7\sqrt{5}$ 2) $\sqrt{28} = 2\sqrt{7}$ 3) $4\sqrt{3} = 48$
    **A.** 1 only
    **B.** 2 only
    **C.** 1 and 2 only
    **D.** 2 and 3 only *C*
16. Three students rewrote $\sqrt{4050}$. Who is correct?
    **A.** I only
    **B.** II and III
    **C.** All three
    **D.** Other *B*
17. Circle area: if area $120\pi$ cm$^2$, radius is
    **A.** 60 **B.** $12\sqrt{10}$ **C.** $2\sqrt{30}$ **D.** $2\sqrt{15}$ *C*
18. A cube with volume 720 mm$^3$ has edge length $a\sqrt[3]{b}$ mm. Find $a + b$. *92*
19. Consider $4\sqrt[3]{3}$, $5\sqrt{x}$, $16\sqrt{y}$. Which is correct?
    **A.** $x < y < z$
    **B.** $z < x < y$
    **C.** $y < z < x$
    **D.** $z < y < x$ *C*