Unit 1: Extra Practice

September 8, 2025

1. Number of positive divisors. Find the number of positive divisors for each.

   a) 12

   b) 24

   c) 26

   d) 54

2. Find the number of positive divisors for each.

   a) 2025

   b) 384

   c) 945

   d) 2310

3. Find the number of positive divisors for each.

   a) 81

   b) 256

   c) 420

   d) 8192

4. Counting integers with divisibility conditions.

   a) How many positive integers < 2025 are multiples of 3 or 4 but not 5?

   b) How many positive integers ≤ 1000 are multiples of 6 or 10 but not 15?

   c) How many integers $1 \leq n \leq 500$ are multiples of 4 or 9 but not both?

   d) How many integers ≤ 2025 are divisible by 12 but not by 18?

5. Babylonian (Newton) square-root approximations. Approximate to 4 decimal places and give as an improper fraction.

   a) $\sqrt{15}$

   b) $\sqrt{7}$

   c) $\sqrt{2}$

   d) $\sqrt{19}$

6. Consider the sets (from 0 to 2025, *inclusive*):

$$A : \{\text{multiples of } 5\}, \qquad B : \{\text{multiples of } 2\}, \qquad C : \{\text{multiples of } 3\}.$$

(Assume 0 is included wherever it qualifies.)

a) Find $\sum_{a \in A} a$.

b) Find $\sum_{x \in A \cap B} x$.

c) Find $\sum_{x \in A \cup B} x$.

d) How many numbers are in $B \cap C$ but not in A?

e) Find $\sum_{x \in B \setminus (A \cup C)} x$.

f) Compute $\sum_{x \in A \cup B \cup C} x$.

g) What is the average of the numbers in A?

7. Divisors of 360.

a) List all positive divisors of 360.

b) If a divisor from your list is chosen uniformly at random, what is the probability it is even?

c) With replacement, pick two divisors. What is the probability both are multiples of 4?

d) Without replacement, what is the probability both are multiples of 9?

e) With replacement, what is the probability that at least one is a multiple of 2?

f) What is the expected value (mean) of a uniformly random divisor of 360?

g) How many divisors of 360 are relatively prime to 10?