

Think It Through
September 2022

Check in with your teacher after solving EACH problem.

1. Factor completely, wherever possible check your work by expanding. Solve each problem using decomposition.

(a) $10p^2 - 27p + 5$

Solution: $(5x - 1)(2x - 5)$

(b) $5y^2 + 23y + 12$

Solution: $(5y + 3)(y + 4)$

(c) $3x^2 + 40x + 48$

Solution: $(3x + 4)(x + 12)$

(d) $7y^2 + 9y - 36$

Solution: $(7y - 12)(y + 3)$

(e) $25z^2 + 50z + 9$

Solution: $(5z + 1)(5z + 9)$

(f) $81y^2 - 9$

Solution: $9(3y - 1)(3y + 1)$

(g) $9x^2 - 1$

Solution: $(3x + 1)(3x - 1)$

(h) $x^2y^2 - 4x^2$

Solution: $x^2(y - 2)(y + 2)$

(i) $z^4 - 1$

Solution: $(z^2 + 1)(z - 1)(z + 1)$

(j) $x^2 + 4x + 4$

Solution: $(x + 2)^2$

(k) $9x^2 + 24x + 16$

Solution: $(3x + 4)^2$

(l) $16x^2 - 48x + 36$

Solution: $4(2x - 3)^2$

(m) $4(x + 2)^2 - 9(x - 4)^2$

Solution: $(16 - x)(5x - 8)$

(n) $16e^{4x} - 4$

Solution: $4(2e^{2x} - 1)(2e^{2x} + 1)$

More Challenging: Check in with your teacher after solving EACH problem.

2. $a^3 + b^3 = 2593080$, $a + b = 210$, $ab = ?$

Solution:

$$\begin{aligned}a^3 + b^3 &= (a + b)(a^2 - ab + b^2) \\&= 210(a^2 - ab + b^2)\end{aligned}$$

$$\begin{aligned}a^2 - ab + b^2 &= 12348 \\a^2 - ab + b^2 + 3ab &= 12348 + 3ab \\(a + b)^2 &= 12348 + 3ab \\3ab + 12348 &= 44100 \\3ab &= 31752 \\ab &= 10584\end{aligned}$$

3. Given that $a + b = 1$ and $a^2 + b^2 = 2$, What is the value of $a^7 + b^7$.

Solution:

$$\begin{aligned}(a + b)^2 &= a^2 + 2ab + b^2 = 1 \\ab &= -\frac{1}{2}\end{aligned}$$

Let $S_n = a^n + b^n$:

$$\begin{aligned}(a + b)S_n &= (a + b)(a^n + b^n) \\&= a^{n+1} + b^{n+1} + a^n b + a b^n \\&= a^{n+1} + b^{n+1} + ab(a^{n-1} + b^{n-1})\end{aligned}$$

So we have $S_n = S_{n+1} - \frac{1}{2}S_{n-1}$, or $S_{n+1} = S_n + \frac{1}{2}S_{n-1}$ and from the problem we have $S_1 = 1$, and $S_2 = 2$. So we have: