Assignment #7
January 12, 2023

1. Let A an U be as shown below.

$$A = \begin{bmatrix} 5 & -2 \\ -1 & 1 \\ 1 & 1 \end{bmatrix} \; U = \begin{bmatrix} -5 \\ 2 \end{bmatrix}$$

   And define a transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ by

$$T(\vec{v}) = A\vec{v}$$

   Find $T(U)$.

2. Suppose $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation. Let $\vec{u}$ and $\vec{v}$ be the vectors given below and suppose that $T(\vec{u})$ and $T(\vec{v})$ are as given. Find $T(\vec{u} - 3\vec{v})$.

$$\vec{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \; \vec{v} = \begin{bmatrix} -3 \\ -2 \end{bmatrix} \; T(\vec{u}) = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} \; T(\vec{v}) = \begin{bmatrix} -3 \\ -2 \\ 4 \end{bmatrix}$$

3. If $T : \mathbb{R}^3 \to \mathbb{R}^2$ Let $\vec{u}$, $\vec{v}$, and $\vec{w}$ be the vectors given below and suppose that $T(\vec{u})$, $T(\vec{v})$, are given. Find $T(\vec{w})$.

$$\vec{u} = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \; \vec{v} = \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix} \; \vec{w} = \begin{bmatrix} 8 \\ 6 \\ 12 \end{bmatrix} \; T(\vec{u}) = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \; T(\vec{v}) = \begin{bmatrix} 12 \\ -6 \end{bmatrix}$$

4. Recall that the standard bases of $\mathbb{R}^3$ is $\{E_1, E_2, E_3\}$. If $T : \mathbb{R}^3 \to \mathbb{R}^2$ is a transformation and the action of T on the vectors E_i is as given, find the formula for $T(\vec{x})$, $\vec{x} \in \mathbb{R}^3$.

$$T(E_1) = \begin{bmatrix} 10 \\ -8 \end{bmatrix} \; T(E_2) = \begin{bmatrix} 8 \\ 8 \end{bmatrix} \; T(E_3) = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

5. Let S be the transformation whose matrix is A and let T be the transformation whose matrix is B, where A and B are the matrices below. Find the matrix C for the transformation resulting from S followed by T.

$$A = \begin{bmatrix} 7 & 10 \\ -8 & -10 \\ -4 & 5 \end{bmatrix} \; B = \begin{bmatrix} 3 & -7 & -9 \\ 10 & -3 & 7 \end{bmatrix}$$

6. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a transformation and the action of T is as given find the formula $T^{-1}(\vec{v})$, where $\vec{v}$ is any vector in $\mathbb{R}^2$.

$$T(\vec{x}) = \begin{bmatrix} 5x + 5y \\ -3x - 2y \end{bmatrix}$$

7. Let A be the matrix below and define a transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ by $T(\vec{u}) = A\vec{u}$. For each of the vectors $\vec{v}_i$ below, find a vector $\vec{u}$ such that T maps $\vec{u}$ to $\vec{v}$, if possible. Otherwise state that there is no such $\vec{u}$.

$$A = \begin{bmatrix} 2 & 4 & -6 \\ 3 & 6 & -9 \\ 2 & 5 & -10 \end{bmatrix}$$

$$\vec{v}_1 = \begin{bmatrix} -21 \\ -31 \\ -25 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} -20 \\ -30 \\ -24 \end{bmatrix}$$