

Assignment #5
December 11, 2022

1. Find a unit vector \vec{u} with the same direction as the given vector \vec{v} .

$$\vec{v} = \begin{bmatrix} -3 \\ 6 \\ -10 \end{bmatrix}$$

2. Find the distance d between the points $P_1 = (-3, -1, 3)$ and $P_2 = (6, 1, -5)$ by first finding the vector \vec{v} from P_1 to P_2 , then finding the length of \vec{v} .
3. If \vec{u} and \vec{v} are the vectors below, solve the following equation for the vector \vec{x} .

$$\vec{u} = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix}$$

$$-2\vec{v} + 2(-2\vec{x} - 2\vec{u}) = 3\vec{v} - 3\vec{u} + 2\vec{x}$$

4. If \vec{u} and \vec{v} are the vectors below, find the vector \vec{w} whose tail is at the point halfway from the tip of \vec{v} to the tip of \vec{u} and whose head is at the point halfway from the tip of \vec{u} to the tip of $\vec{v} + \vec{u}$. Assume all vectors are in standard position.

$$\vec{u} = \begin{bmatrix} 4 \\ 3 \\ -1 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 4 \\ -2 \\ 2 \end{bmatrix}$$

5. Find the vector equation for the line passing through the points $P_1 = (-6, -8, -5)$ and $P_2 = (-15, -10, -13)$.
6. The parametric equation for the line L_1 is as follows

$$\begin{aligned} x &= 5 + 3t \\ y &= -2 - 3t \\ z &= -2 - 9t \end{aligned}$$

Let L_2 be the line parallel to L_1 passing through the point $(-3, -2, 5)$. Find the point P on L_2 whose x -coordinate is -8 .

7. Let L_1 be the line passing through the points $Q_1 = (-3, -2, -3)$ and $Q_2 = (-4, 1, -4)$ and let L_2 be the line passing through the point $P_1 = (-37, -10, 7)$ with direction vector $\vec{d} = (-9, -3, 3)^T$. Determine whether L_1 and L_2 intersect. If so, find the point of intersection Q .
8. Compute the orthogonal projection of \vec{u} onto \vec{v} . Use the square root symbol where needed to give an exact value for your answer.

$$\vec{u} = \begin{bmatrix} -3 \\ -9 \\ -8 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$$

9. Express the vector \vec{u} below as a sum of two vectors \vec{u}_1 and \vec{u}_2 , where \vec{u}_1 is parallel to the vector \vec{v} given below and \vec{u}_2 is perpendicular to \vec{v} .

$$\vec{v} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \quad \vec{u} = \begin{bmatrix} -5 \\ 1 \\ -2 \end{bmatrix}$$

10. Let L be the line with parametric equations:

$$\begin{aligned} x &= 6 + 2t \\ y &= -7 + t \\ z &= -1 + 2t \end{aligned}$$

Find the vector equation for a line that passes through the point $P = (6, -9, 4)$ and intersects L at a point that is distance 3 from the point $Q = (6, -7, -1)$ (Point Q is on the line L). Note that there are two possible correct answers.