Assignment #6

January 12, 2023

1. Let L be the line with the parametric equations

$$\begin{aligned} x &= 4 - t \\ y &= 4 + 2t \\ z &= 4 - 3t \end{aligned}$$

   Find the shortest distance d from the point $P_0 = (4, 5, 4)$ to L and the point Q on L that is closest to P_0.

2. Let T be the plane $-3x+y-2z = -2$. Find the shorted distance d from the point $P_0 = (1, -3, 5)$ to T and the point Q in T that is closest to P_0.

3. Let L_1 be the line passing through the point $P_1 = (-4, -4, 3)$ with direction vector $\vec{d} = [-3, 1, -1]^T$ and let L_2 be the line passing through the point $P_2 = (4, -1, -5)$ with the same direction vector.

   Find the shortest distance 'd' between these two lines and find a point Q_1 on L_1 and a point Q_2 on L_2 so that there distance apart is d.

4. Let L_1 be the line passing through the point $P_1 = (5, 8, -2)$ with direction vector $d_1 = [-2, -3, -2]^T$ and let L_2 be the line passing through the point $P_2 = (-5, 3, 4)$ with direction vector $d_2 = [-2, -1, -2]^T$.

   Find the shortest distance 'd' between these two lines and find a point Q_1 on L_1 and a point Q_2 on L_2 so there distance apart is d.

5. Let L_1 be the line passing through the point $P = (-1, 1, 2)$ with direction vector $d = [-2, 4, -4]^T$ and let T be the plane defined by $-5x+2y+2z = 7$. Find the point Q where L and T intersect.

6. Let S be the plane defined by $3x+6y-6z = 15$ and let T be the plane defined by $2x+5y-5z = 12$. Find the vector equation for the line where S and T intersect.

7. Determine the volume of the parallelepiped with one vertex at the origin and the three vertices adjacent to it at $(2, 3, 1)$, $(4, 9, 1)$ and $(6, 6, 5)$.

8. Find the scalar equation for the plane passing through the points $P_1 = (1, -1, -3)$, $P_2 = (-3, -1, -4)$ and $P_3 = (3, -4, -1)$.

9. Find the scalar equation for the plane passing through the point $P = (4, -4, 5)$ and containing the line L defined by:

$$\begin{aligned} x &= t \\ y &= -5 + t \\ z &= 6t \end{aligned}$$

10. Determine the area of the triangle with vertices $(5, -7, -8)$, $(8, -6, -10)$ and $(11, 0, -9)$.