PRACTICE XIV

1. An administrator at a large university is interested in determining whether the residential status of a student is associated with level of participation in extracurricular activities. Residential status is categorized as on campus for students living in university housing and off campus otherwise. A simple random sample of 100 students in the university was taken, and each student was asked the following two questions.

   - Are you an on campus student or an off campus student?
   - In how many extracurricular activities do you participate?

   The responses of the 100 students are summarized in the frequency table shown:

	Residential Status		
Level of Participation in Extracurricular Activities	On campus	Off campus	Total
No activities	9	30	39
One activity	17	25	42
Two or more activities	7	12	19
Total	33	67	100

   (a) Calculate the proportion of on campus students in the sample who participate in at least one extracurricular activity and the proportion of off campus students in the sample who participate in at least one extracurricular activity.

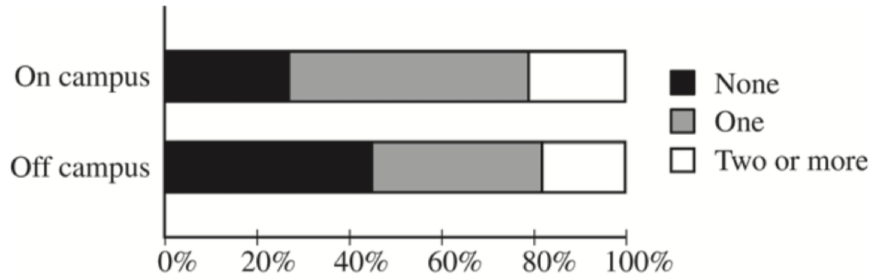
   - The proportion of on campus residents who participate in at least one extracurricular activity is

$$\frac{17+7}{33} = \frac{24}{33} \approx 0.727$$

   - The proportion of off campus residents who participate in at least one extra-curricular activity is

$$\frac{25+12}{67} = \frac{37}{67} \approx 0.552.$$

The responses of the 100 students are summarized in the segmented bar graph shown.



(b) Write a few sentences summarizing what the graph reveals about the association between residential status and level of participation in extracurricular activities among the 100 students in the sample.

The graph revals that on campus students in this sample are more likely to participate in extra-curricular activities than off campus residents.

The proportions who participate in two or more extra curricular activities are similar between the two groups but slightly greater for on campus residents (0.212, vs. 0.179).

On campus residents have a greater proportion who participate in one activity (0.515 vs. 0.373) and a smaller proportion who participate in to extra curricular activities (0.273 vs. 0.448) than off campus residents.

2. Schools in a certain state receive funding based on the number of students who attend the school. To determine the number of students who attend a school, one school day is selected at random and the number of students in attendance that day is counted and used for funding purposes. The daily number of absences at High School A in the state is approximately normally distributed with mean of 120 students and standard deviation of 10.5 students.

(a) If more than 140 students are absent on the day the attendance count is taken for funding purposes, the school will lose some of its state funding in the subsequent year. Approximately what is the probability that High School A will lose some state funding?

The distribution of daily number of absences is approximately normal with $\mu = 120$ and $\sigma = 10.5$. We would like:

$$P\left(Z > \frac{140-120}{10.5}\right) = 1 - \text{normalcdf}(1.90) = \boxed{0.0287}$$

(b) The principals' association in the state suggests that instead of choosing one day at random, the state should choose 3 days at random. With the suggested plan, High School A would lose some of its state funding in the subsequent year if the mean number of students absent for the 3 days is greater than 140. Would High School A be more likely, less likely, or equally likely to lose funding using the suggested plan compared to the plan described in part (a)? Justify your choice.

High school A would be <u>less likely</u> to lose state funding.

With a random sample of 3 days, the distribution of the sample mean number of students absent would have less variability than that of a single day.

With less variability the distribution of the sample mean would concentrate more narrowly around the mean of 120 students, resulting in a smaller probability that the mean number of students absent would exceed 140.

In particular $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{10.5}{\sqrt{3}} = 6.602$

so $P\left(Z > \frac{140-120}{6.0602}\right) = 1 - 0.9995 = 0.0005$

which is less than 0.0287 described in

part a.

(c) A typical school week consists of the days Monday, Tuesday, Wednesday, Thursday, and Friday. The principal at High School A believes that the number of absences tends to be greater on Mondays and Fridays, and there is concern that the school will lose state funding if the attendance count occurs on a Monday or Friday. If one school day is chosen at random from each of 3 typical school weeks, what is the probability that none of the 3 days chosen is a Tuesday, Wednesday, or Thursday?

For any particular school week, the probability is $\frac{2}{5} = 0.4$ that the selected day is not Tuesday, not Wednesday, or not Thursday. Therefore, because the days are selected independently across the three weeks, the probability that none of the three days is T, W, or R is

$$(0.4)^3 = \boxed{0.064}$$