

SAMPLE SIZE FOR ESTIMATING A POPULATION MEAN

AP Statistics · Mr. Merrick · February 21, 2026

When planning a confidence interval for a population mean μ , we often want to know how large a sample is needed to guarantee a desired margin of error.

Margin of error for a mean

For a one-sample confidence interval for a population mean using a known (or planned) standard deviation σ , the margin of error is

$$\text{ME} = \text{critical value} \cdot \frac{\sigma}{\sqrt{n}}$$

If the population standard deviation σ is known (or we are planning using an estimate of it), we use a z^* critical value:

$$\text{ME} = z^* \frac{\sigma}{\sqrt{n}}$$

To guarantee a margin of error no larger than a chosen value ME, solve for n :

$$\text{ME} = z^* \frac{\sigma}{\sqrt{n}} \Rightarrow \sqrt{n} = \frac{z^* \sigma}{\text{ME}} \Rightarrow n = \left(\frac{z^* \sigma}{\text{ME}} \right)^2$$

Always round **up** to the nearest whole number.

Explainer

In practice, we nearly always use a z critical value when planning sample size for a mean — even if we will ultimately use a t interval — because the required t^* depends on n , which we are trying to determine.

Using z^* gives a very good approximation and slightly overestimates the needed sample size for small n , making the plan safe

What if σ is unknown?

If σ is unknown, we must use a **planning estimate** for the population standard deviation.

Common choices include:

- A previous study's sample standard deviation
- A pilot study
- An educated guess based on experience

The formula remains:

$$n = \left(\frac{z^* \sigma}{\text{ME}} \right)^2$$

There is no “most conservative” universal choice like $p = 0.50$ for proportions. The required sample size grows as σ increases.

Practice

1. Known standard deviation

A soda company wants to estimate the mean amount of soda in its cans. Suppose $\sigma = 0.12$ ounces. They want a 95% confidence interval with $ME \leq 0.02$. Find the required sample size.

2. Using a prior estimate of σ

A previous study found the standard deviation of commute times to be about 8 minutes. Find the required sample size for a 90% confidence interval with $ME \leq 2$ minutes.

3. Comparing different variability levels

A manufacturer wants a 99% confidence interval with $ME \leq 1.5$ units.

1. Find n if $\sigma = 4$.
2. Find n if $\sigma = 7$.