PRACTICE VI

Probability

1. MLB signing bonuses for different position players with respective probabilities are given in the table below.

	Position		
Bonus	Pitcher	Infielder	Outfielder
\$2000000	0.30	0.30	0.20
\$5000000	0.10	0.05	0.05

(a) What is the probability a given bonus was for a pitcher?

$P(\text{pitcher}) = 0.30 + 0.10 = 0.40$

(b) What is the expected value for a bonus?

$E[B] = \sum_{\text{all } b} b \cdot p(b) = 2\,000\,000(0.3+0.3+0.2) + 5000000(0.1+0.5+0.05)$

$\underline{\$2\,600\,000}$

(c) Are position and bonus independent? Explain.

if A, B are independent $P(A) \cdot P(B) = P(A \cap B)$

$P(A|B) = P(A)$

$P(B|A) = P(B)$

In this case

$P(2\,000\,000 \mid \text{pitcher}) = \dfrac{0.3}{0.3+0.1} = 0.75$

whereas $P(\text{pitcher}) = 0.8$

$\therefore$ These events are <u>not</u> independent, The probability clearly changes after conditioning.

2. The weights of individual apples are approximately normally distributed with a mean of 8 ounces and a standard deviation of 0.5 ounces. The weights of individual oranges are approximately normally distributed with a mean of 6 ounces and a standard deviation of 0.4 ounces. The weights of individual pieces of fruit are independent.

   (a) What is the distribution of the total fruit weight of fruit gift boxes containing 6 randomly selected apples and 6 randomly selected oranges?

   Let T be total weight

   $T = A+A+A+A+A+A+O+O+O+O+O+O$ where A is apple weight and O is orange weight.

   $E[T] = 6E[A] + 6E[O] = 6(8) + 6(6) = 84$ ounces.

   $Var[T] = 6(0.5)^2 + 6(0.4)^2 = 2.46$ → by independence...

   $sd[T] = \sqrt{2.46} = 1.568$ ounces.

   Note: we are summing 12 random variables not taking $6A + 6O$, this seriously effects variance.

   $T \sim Normal(\mu = 84, \sigma = 1.568)$

   write in words...

   (b) The gift boxes are advertised as containing at least 5 pounds of fruit. What is the probability that a gift box contains at least 5 pounds of fruit?

   first, 5 lbs is 80 ounces.

   $$P(X \geq 80) = P\left(z \geq \frac{80-84}{1.568}\right) = \boxed{0.995}$$

   (c) An empty gift box weighs exactly 12 ounces. What is the distribution of total weights (box plus fruit) of this gift offering?

   doesnt vary at all? LOL doesnt seem to realistic...

   $E[T + 12] = E[T] + E[12] = 84 + 12 = 96$ ounces.

   $Var[T+12] = Var[T]$

   The distribution is normal with $\mu = 96$ ounces and sd $= 1.568$ ounces.

3. Suppose that women's times for the 200-meter sprint have a roughly normal distribution with a mean of 25.2 seconds and a standard deviation of 1.2 seconds. Suppose that men's times have a roughly normal distribution with a mean of 22.8 seconds and a standard deviation of 0.9 seconds. A male and female sprinter are picked at random. Assume their times are independent.

(a) What is the probability the sum of their sprints is over 50 seconds?

Let M be men's times

Let W be women's times.

$E[M+W] = E[M] + E[W] = 22.8 + 25.2 = 48.0$ seconds.

The variables <u>are independent</u> so

$Var[M+W] = Var[M] + Var[W]$

<u>Let $T = M+W$</u>

$= (0.9)^2 + (1.2)^2 = 2.25$.

$T \sim normal(\mu = 48, \sigma = 1.5)$

so $SD[M+W] = \sqrt{2.25} = 1.5$ seconds.

$P(T > 50) = P\left(z > \frac{50-48}{1.5}\right) = \boxed{0.0912.}$

(b) What is the probability that the man sprinted faster than the woman?

$P(M < W) = P(M - W < 0)$

Let $D = M - W$

$\mu_D = E[D] = E[M] - E[W] = 22.8 - 25.2 = -2.4$ seconds.

$\sigma_D^2 = Var[D] = Var[M] - Var[W] = (0.9)^2 + (1.2)^2 = 2.25$

$\sigma_D = SD[D] = \sqrt{2.25} = 1.5$ seconds.

If two independent r.v are normal so is their difference so

$D \sim normal(\mu = -2.4\,s, \sigma = 2.25)$

$P(D < 0) = P\left(z < \frac{0-(-2.4)}{2.25}\right) = \boxed{0.9452}$