Two Sample tests for $\mu_1 - \mu_2$

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Last year, A High School had 30 students take the AP Statistics exam. They were informed later that the college board gave two forms of the exam, which were randomly assigned to the students. The results are shown below:

Form A: $3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 5$

Form B: $2, 2, 3, 3, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5$

Conduct a two sample t-test for the difference between average scores on form A and average scores on Form B.

Solution: **State:** $\mu_1 - \mu_2$: The difference between the true average scores on form A and average scores on Form B.
$\overline{x}_1 - \overline{x}_2 = 4.27 - 3.93 = 0.34$

$$H_0 : \mu_1 - \mu_2 = 0$$
$$H_a : \mu_1 - \mu_2 \neq 0$$

Plan: We are conducting a two sample t-test for $\mu_1 - \mu_2$. Check conditions:

- Random: Since the tests are randomly assigned to students we can imply causation. Notice that this is different for the experimentation case.
- Independence: We aren't actually sampling here, so we don't need to check the 10% condition.
- Normality: Here we When graphing the data it is strongly skewed so the condition is not satisfied. We will make the assumption that $\overline{x}_1 - \overline{x}_2$ is normally distributed.

  Do: Test Statistic:

$$\begin{aligned} t &= \frac{(\overline{x}_1 - \overline{x}_2) - (0)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \\ &= \frac{0.34}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \\ &= 0.5640760748 \end{aligned}$$

  P Value:

$$\begin{aligned} p\text{-value} &= 2P(t > 0.5640760748) \\ &= 0.5773938634 \end{aligned}$$

Conclude: Our p-value$= 0.5773938634 < \alpha = 0.05$ so we fail to reject the null hypothesis.
Assuming that there is no difference in the average scores between the two AP statistics forms, there is a roughly 57% chance of observing a difference in sample means of 0.34 or greater. This is not convincing enough evidence to reject the null hypothesis.