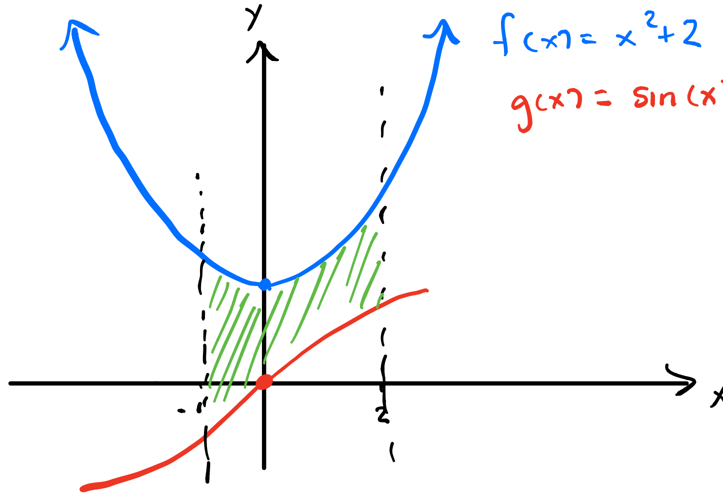
PRACTICE IV

1. Determine the area of the region bound by $y = x^2 + 2$, $y = \sin(x)$, $x = -1$, and $x = 2$.



$f(x) = x^2 + 2$

$g(x) = \sin(x)$

$$A = \int_{-1}^{2} (f(x) - g(x))\,dx$$

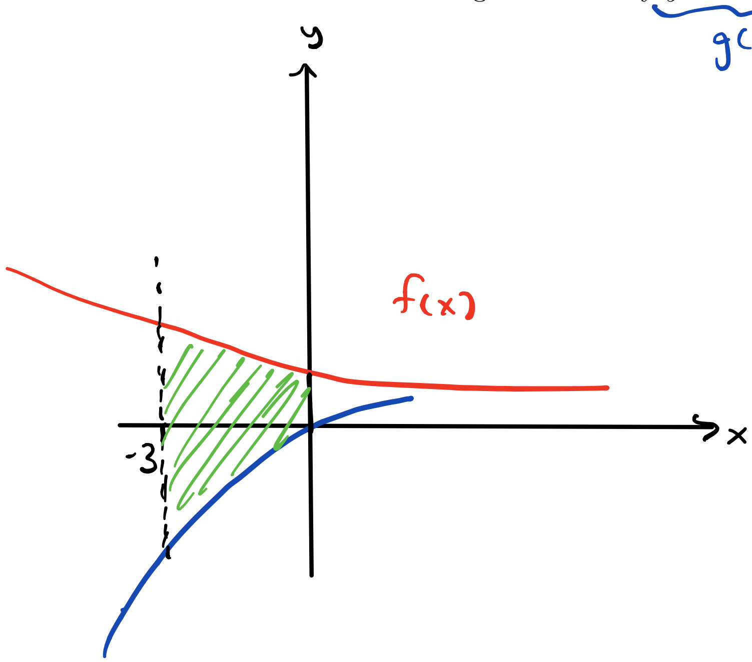
$$= \int_{-1}^{2} x^2 + 2 - \sin(x)\,dx$$

$$= \left[\frac{1}{3}x^3 + 2x + \cos(x)\right]\Big|_{-1}^{2}$$

$$= \left(\frac{1}{3}(2)^3 + 4 + \cos(4)\right) - \frac{1}{3}(-1)^3 - 2 + \cos(-1)$$

$$\boxed{= 8.04}$$

2. Determine the area of the region bound by $\underbrace{y = x\sqrt{x^2+1}}_{g(x)}$, $y = e^{-\frac{1}{2}x}$, $x = -3$, and the y-axis.



$$A = \int_{-3}^{0} e^{-\frac{1}{2}x} - x\sqrt{x^2+1}\, dx$$

$$= \left(-2e^{-\frac{1}{2}x} - \frac{1}{3}(x^2+1)^{3/2} \right) \Bigg|_{-3}^{0}$$

$$= \boxed{17.17}$$

$$\int x\sqrt{x^2+1}\, dx = \frac{1}{2} \int u^{\frac{1}{2}}\, du$$

$u = x^2 + 1$

$du = 2x\, dx$

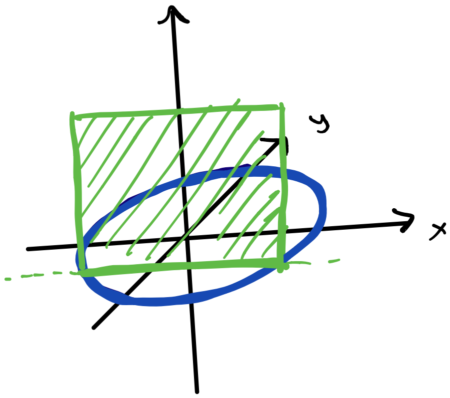
$\frac{1}{2} du = x\, dx$

$$= \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3} u^{\frac{3}{2}} \right) + C$$

$$= \frac{1}{3} u^{3/2}.$$

$$= \frac{1}{3}(x^2+1)^{3/2}.$$

3. Find the volume of the solid whose base is a disk of radius r and whose cross-sections are squares.

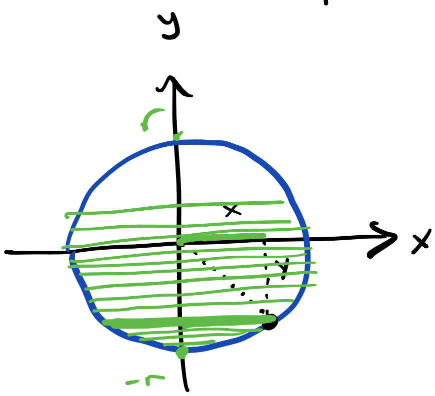


circle:

$$x^2 + y^2 = r^2 \qquad y^2 = r^2 - x^2$$

Each square section will have an area $(2y)^2 = 4y^2$

$$= 4(r^2 - x^2)$$



$$V = \int_{-r}^{r} 4(r^2 - x^2)\,dx = 4\int_{-r}^{r} r^2 - x^2\,dx$$

$$= 4\left(r^2x - \frac{1}{3}x^3 \Big|_{-r}^{r}\right)$$

$$= 4\left(\left(r^3 - \frac{1}{3}r^3\right) - \left(-r^3 + \frac{1}{3}r^3\right)\right)$$

$$= 4\left(\frac{2}{3}r^3 + \frac{2}{3}r^3\right)$$

$$\boxed{= \frac{16}{3}r^3}$$

4. Determine f_{avg} for $f(x) = 8x - 3 + 5e^{2-x}$ on $[0, 2]$.

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

where $[a,b]$ is interval of interest.

$$= \frac{1}{2}\int 8x - 3 + 5e^{2-x}\,dx$$

$$= \frac{1}{2}\left[4x^2 - 3x - 5e^{2-x}\right]\Big|_0^2$$

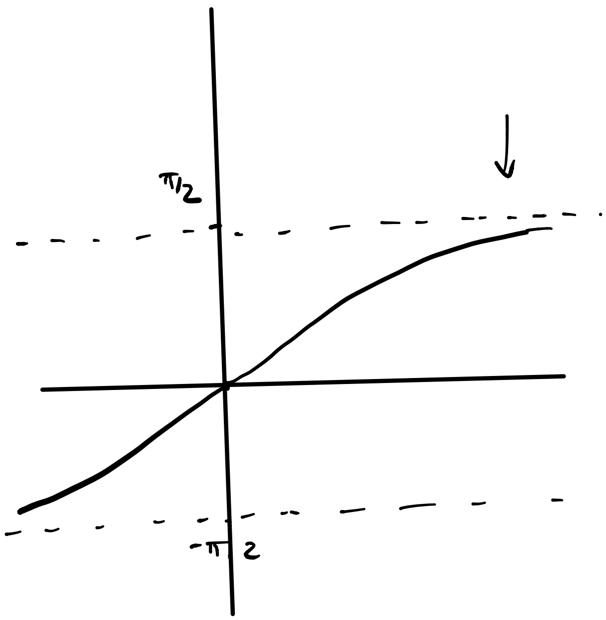
$$= \frac{1}{2}\left[4(2)^2 - 3(2) - 5e^{2-2} - (-5e^2)\right]$$

$$\boxed{= \frac{1}{2}(5 + 5e^2)}$$

5. Evaluate

$$\lim_{x\to\infty} \arctan\left(\frac{3w^2 - 9w^4}{4w - w^3}\right)$$

take $\lim_{x\to\infty} \frac{3w^2 - 9w^4}{4w - w^3} = \lim_{x\to\infty} \frac{\left(\frac{3}{w^2}\right) - 9}{\left(\frac{4}{w^3}\right) - \left(\frac{1}{w^2}\right)} = \infty$



as $x \to \infty$ we have $\frac{3w^2 - 9w^4}{4w - w^3} \to \infty$

$\therefore \arctan\left(\frac{3w^2 - 9w^4}{4w - w^3}\right) \to \boxed{\pi/2}$

6. Evaluate

$$\lim_{x\to-\infty} \ln\left(\frac{3z^4-8}{2+z^2}\right)$$

$$\lim_{z\to-\infty} \frac{3z^4-8}{2+z^2} = \lim_{z\to-\infty} \frac{\left(3-\frac{8}{z^4}\right)}{\left(\frac{2}{z^4}+\frac{1}{z^2}\right)} = \infty$$

$$\frac{3z^4-8}{2+z^2} \to \infty \quad \text{as} \quad z\to-\infty$$

$$\ln\left(\frac{3z^4-8}{2+z^2}\right) \to \boxed{\infty}$$

7. Differentiate the following function

$$y = \frac{x^5}{(1-10x)\sqrt{x^2+2}}$$

$$\ln(y) = \ln\left(\frac{x^5}{(1-10x)\sqrt{x^2+2}}\right)$$

$$\ln(y) = 5\ln(x) - \ln(1-10x) - \frac{1}{2}\ln(x^2+2)$$

$$\frac{y'}{y} = \frac{5}{x} + \frac{10}{1-10x} - \frac{1}{2}\,\frac{2x}{x^2+2}$$

$$\boxed{y' = \left(\frac{x^5}{(1-10x)\sqrt{x^2+2}}\right)\left(\frac{5}{x} + \frac{10}{1-10x} - \frac{2x}{2(x^2+2)}\right)}$$

8. Differentiate x^x

$$y = x^x$$

$$\ln(y) = x\ln(x)$$

$$\frac{y'}{y} = \ln(x) + 1$$

$$\boxed{y' = x^x(\ln(x)+1)}$$