PRACTICE IX

Inference

1. A company advertises it has a process that can extract a mean of 35 grams of dissolved salts from 1 liter of seawater. A geologist believes the true figure is lower. Use this process, a sample of fifteen 1 liter containers of seawater from 15 random locations yields a mean of 34.82 grams of dissolved salts with a standard deviation of 0.65 grams. Assume the sample distribution is symmetric and unimodal with no outliers.

   (a) Is there sufficient evidence for the geologist to dispute the advertisement? Justify your answer.

   hypothesis: $H_0: \mu = 35$

   $H_a: \mu < 35$

   procedure: one somple t-test for a mean.

   Conditions: random sample from roughly normol population.

   n clearly < 10% of N.

   Test Statistic and P-value: T-test gives $t = -1.0725$ and $P = 0.1508$

   or.. $t = \dfrac{34.82 - 35}{\left(\frac{0.64}{\sqrt{12}}\right)} = -1.703$ with $df = 15-1 = 14$

   $P(t_{15} < -1.703) = 0.1508$

   Conclusion: With this large of a p-value, $0.151 > 0.05$ there is not sufficient evidence to reject H_0, that is, there is ~~not evidence~~ sufficient evidence for a geologist to dispute the advertised claim of extraction of a mean 35 grams of dissolved salt per 1 liter of saltwater.

   (b) A large-scale test of a second company's process shows yields of dissolved salts that are roughly normally distributed with a mean of 34.75 grams and a standard deviation of 0.83 grams. What is the probability that using this second process, a 1 liter container of seawater will yield at least 35 grams of dissolved salts?

   With $S \sim N(\mu = 34.75,\ \sigma = 0.83)$

   $P(S \geq 35) = 0.3816$

(c) What is the probability that when using this second process on 10 randomly selected 1 liter containers of seawater, at least 2 of them yield at least 35 grams of dissolved salts?

Let x be number that yield ≥ 35 g of dissolved salts

$$X \sim \text{binomial}(n=10,\ p=0.3816)$$

$$P(X \geq 2) = 1 - [P(X=0) + P(X=1)]$$
$$= 1 - [(0.6184)^{10} + 10(0.6184)^{9}(0.3816)]$$
$$= 0.9414$$

or

$$1 - \text{binomcdf}(10, 0.3816, 1) = 0.9414$$

2. Can a particular video game improve a batter's reaction time? Batters' reaction times (fraction of a second between the ball leaving a pitcher's hand and the start of a swing) are measured before and after playing the video game for 25 hours.

   (a) What is the appropriate test, the hypothesis, and the conditions to check?

   Test: A paired t-test for comparing means. (A one-sample t-test for differences before - After, reaction times)

   Hypothesis: $H_0: \mu_d = 0$
   $H_a: \mu_d > 0$

   where μ_d is the mean of the set of differences between reaction times of batters who have not trained and those who have trained on the video game.

   Conditions: · Need random sample of batters.
   · Unless batters reaction times come from roughly normal distribution, require large enough sample size ($n \geq 30$) for central limit theorem to apply.

   (b) Suppose the test is run and no statistically significant improvement is detected in batter reaction times after the video game training. If the researcher plans a second test, name two specific changes that can be made to increase the power of the test. Explain your choices.

   Power = P(reject H_0 | H_0 false)

   - Power can be increased by increasing sample size, which in turn decreases standard error of the sampling distribution.
   - A smaller standard error results in a more extreme test statistic, which makes it easier to detect that the null hypothesis is false.
   - A second change to increase power is to use a greater significance level, which also makes it easier to reject a null hypothesis.