PRACTICE

1. Determine The following indefinite integrals:

   (a) $\int \frac{e^{\tan\theta}}{\cos^2(\theta)} d\theta = \int e^u du = e^u + C = \underbrace{e^{\tan\theta} + C}$

   $u = \tan\theta$

   $du = \sec^2\theta \, d\theta = \frac{1}{\cos^2\theta} d\theta$

   $e^{\tan\theta} \cdot \sec^2\theta$

   (b) $\int \frac{e^x}{1+e^{2x}} d\cancel{\theta} = \int \frac{e^x}{1+e^{2x}} dx = \int \frac{1}{1+u^2} du$

   $u = e^x$

   $du = e^x dx$

   $= \arctan(u) + C$

   $\boxed{= \arctan(e^x) + C}$

   (c) $\int \cos^3(\theta)\sin^2(\theta) d\theta = \int u^2 \cos^2\theta \, du$

   $u = \sin\theta$

   $du = \cos\theta \, d\theta$

   $= \int u^2(1-\sin^2\theta) du$

   $= \int u^2(1-u^2) du = \int u^2 - u^4 du$

   $= \frac{1}{3}u^3 - \frac{1}{5}u^5 + C$

   $\boxed{= \frac{1}{3}\sin^3\theta - \frac{1}{5}\sin^5(\theta) + C}$

2. For the following curves determine the tangent line for the curve at a given point:

(a) $x^2y^2 = 3x - 2y^3$ at $(1, 1)$

$$2xy^2 + 2x^2y\cdot y' = 3 - 6y^2\cdot y'$$

$$2x^2yy' + 6y^2\cdot y' = 3 - 2xy^2$$

$$y'(2x^2y + 6y^2) = 3 - 2xy^2$$

$$y' = \frac{3 - 2xy^2}{2x^2y + 6y^2} \qquad y'(1,1) = \frac{3 - 2(1)(1)^2}{2(1)^2(1) + 6(1)^2}$$

$$= \frac{1}{8}$$

$$\boxed{y - 1 = \frac{1}{8}(x-1)}$$

(b) $y^4 = e^{x^2-y^2}$ at $(-1, 1)$

$$4y^3\cdot y' = e^{(x^2-y^2)}\cdot(2x - 2yy')$$

$$4y^3\cdot y' = 2xe^{x^2-y^2} - 2yy'e^{x^2-y^2}$$

$$4y^3\cdot y' + 2yy'e^{x^2-y^2} = 2xe^{x^2-y^2}$$

$$e^{\cancel{(-1)^2-1^2}} = \cancel{e^0}\ 1$$

$$y'(4y^3 + 2ye^{x^2-y^2}) = 2xe^{x^2-y^2}$$

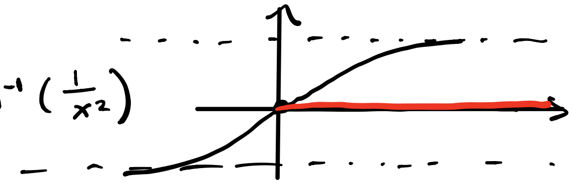
$$y' = \frac{2xe^{x^2-y^2}}{(4y^3 + 2ye^{x^2-y^2})} \qquad y'\big|_{(-1,1)} = \frac{2(-1)e^{(-1)^2-(1)^2}}{4(1)^3 + 2(1)\cancel{e^{(-1)^2-(1)^2}}}$$

$$= \frac{-2}{6} = -\frac{1}{3}$$

$$\boxed{y - 1 = -\frac{1}{3}(x+1)}$$

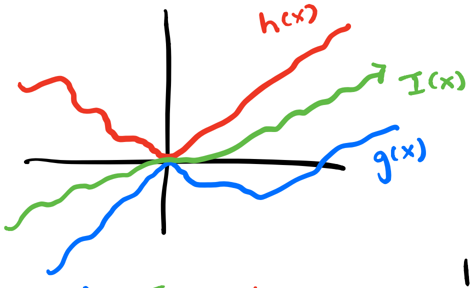
3. Compute: $\cancel{\lim_{x\to\infty}}\, x^3 \tan^{-1}(\frac{1}{x^2})$

$\lim_{x\to 0^+} x^3 \tan^{-1}\left(\frac{1}{x^2}\right)$



Notice: on the interval $[0,\infty)$, we know that $0 \le \tan^{-1}(x) < \pi/2$.

In particular, we have



$g(x) \le I(x) \le h(x)$

$$0 \le \tan^{-1}\left(\frac{1}{x^2}\right) < \pi/2$$

$$0 \le x^3 \tan^{-1}\left(\frac{1}{x^2}\right) < \frac{\pi x^3}{2}$$

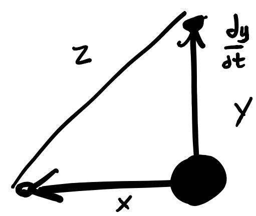
$$\lim_{x\to 0} 0 = 0 = \lim_{x\to 0} \frac{\pi x^3}{2}$$

so by squeeze theorem

$$\lim_{x\to 0^+} x^3 \tan^{-1}\left(\frac{1}{x^2}\right) = 0$$

4. Mr. Merrick and Dr. Vince are standing in the same location. Mr. Merrick begins walking north at 5 km/h and Dr. Vince begins walking west at 4 km/h. How quickly are they moving apart at time $t = 2$ hours?

$\frac{dz}{dt} = ?$



$\frac{dy}{dt} = 5$ km/h

$\frac{dx}{dt} = 4$ km/h.

look at distances:

$\to x^2 + y^2 = z^2 \leftarrow$

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 2z \cdot \frac{dz}{dt}$$

$$x \cdot \underbrace{\frac{dx}{dt}}_{4} + y \underbrace{\frac{dy}{dt}}_{5} = z \underbrace{\frac{dz}{dt}}_{?}$$

$x(2) = 4 \cdot 2 = 8$

$y(2) = 5 \cdot 2 = 10$

$z(2) = \sqrt{64 + 100} = \sqrt{164}$

$$8(4) + 10(5) = \sqrt{164} \cdot \frac{dz}{dt}$$

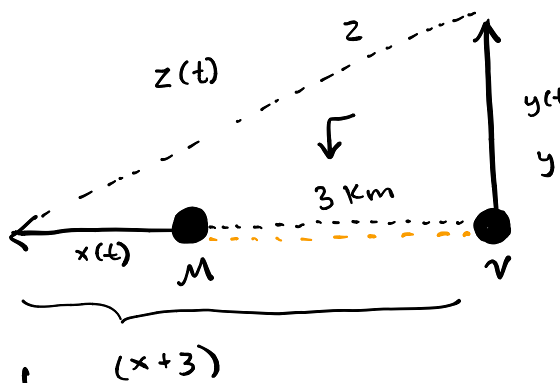
$$\boxed{\frac{dz}{dt} = \sqrt{41}}$$

5. Mr. Merrick is standing 3km west of Dr. Vince. Dr. Vince begins walking north at 5 km/h and Mr. Merrick begins walking West at 4 km/h. How quickly are they moving apart at time ~~$t = 3$ hours?~~

$t = 3$ hours?

$$\frac{dy}{dt} = 5 \text{ km/h}$$

$$\frac{dx}{dt} = 4 \text{ km/h}$$



$x(t) = 4 \cdot t$, $x(3) = 12$ km

$y(t) = 5 \cdot t$, $y(3) = 15$ km

$z(3) = \sqrt{2 \cdot 15^2} = 15\sqrt{2}$

$$(x+3)^2 + y^2 = z^2$$

$$2(x+3) \cdot \left(\frac{dx}{dt}\right) + 2y \cdot \frac{dy}{dt} = 2z\left(\frac{dz}{dt}\right)$$

$$(x+3)\left(\frac{dx}{dt}\right) + y\left(\frac{dy}{dt}\right) = z\left(\frac{dz}{dt}\right)$$

$$(12+3)4 + 15(5) = 15\sqrt{2}\left(\frac{dz}{dt}\right)$$

$$\frac{dz}{dt} = \frac{1}{\cancel{15}\sqrt{2}}(9 \cdot \cancel{15}) = \frac{9}{\sqrt{2}}$$