PRACTICE VI

1. The length of a vine during a 12-hour period is given by a twice-differentiable function L, where $L(t)$ is measured in feet and t is measured in weeks for $0 \le t \le 12$. The graph of L is concave down on the interval $0 \le t \le 12$. Selected values of the derivative of L, $L'(t)$, are given in the table below. At time $t = 4$, the length of the vine is 5 feet.

t	2	4	5	8	10
$L'(t)$	1.0	0.8	0.7	0.4	0.2

   (a) Use the tangent line approximation for L at time $t = 4$ to estimate $L(4.3)$, the length of the vine at time $t = 4.3$. Is the approximation an overestimate or an underestimate for $L(4.3)$? Give a reason for your answer.

   (b) Use a left Riemann sum with four subintervals indicated by the data in the table to approximate $\int_2^{10} L'(t)dt$. Indicate the units of measure

   (c) Is the approximation in part (b) an overestimate or an underestimate for $\int_2^{10} L'(t)dt$? Give a reason for your answer.

2. Find two positive numbers whose sum is 300 and whose product is a maximum

3. Find two positive numbers whose product is 750 and for which the sum of one and 10 times the other is a minimum

4. Let x and y be two positive numbers such that $x + 2y = 50$ and $(x + 1)(y + 2)$ is a maximum. Find x and y

5. Find the linear approximation to $g(z) = \sqrt[4]{z}$ at $z = 2$. Use the linear approximation to approximate the value of $\sqrt[4]{3}$ and $\sqrt[4]{10}$. Compare the approximate values to the exact values.

6. Verify that $y = -t\cos t - t$ is a solution of the initial value problem

$$t\frac{dy}{dx} = y + t^2 \sin t, \quad y(\pi) = 0$$

7. Find a solution to the initial-value problem

$$y' = -y^2, \;\; y(0) = \frac{1}{2}$$

8. Find a solution to the initial-value problem

$$y' = xy^3, \;\; y(0) = 2$$

9. **Without using technology** sketch the following:

(a) The solid formed when the region bound by $x = \sqrt{y}$, $x = \sqrt{-y}$, and $x = 4$, is revolved around the y-axis

(b) The solid formed when the region bound by $y = e^x$, $y = e^{-x} + 4$ and the y-axis is revolved around the line $x = 4$