Binomial

$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$

Geometric Probability distribution.

$$P(X=x) = (1-p)^{x-1} \cdot p$$

Suppose that the probability of engine malfunction during any one-hour period is $p = .02$. Find the probability that a given engine will survive two hours.

p(x)

P(X=x)

$$P(X=x) = \underbrace{(1-p)^{x-1} \cdot p}, \quad x = \underbrace{1, 2, 3, \ldots\ldots}$$

$$E[X] = \sum_{x=1}^{\infty} (1-p)^{x-1} \cdot p \cdot x \qquad (1-p) = q$$

$$= \sum_{x=1}^{\infty} q^{x-1} \cdot p \cdot x$$

$$= p \sum_{x=1}^{\infty} \underbrace{x \cdot q^{x-1}} = p \sum_{x=1}^{\infty} \frac{d}{dq} q^x$$

Geometric Series.

$$= p \frac{d}{dq} \sum_{x=1}^{\infty} q^x$$

$$= p \frac{d}{dq} \left(\frac{q}{1-q} \right)$$

$$= \frac{p((1-q) + q)}{(1-q)^2}$$

$$= \frac{p}{(1-q)^2} = \frac{p}{(1-(1-p))^2} = \frac{p}{p^2} = \boxed{\frac{1}{p}}$$

$$m_x(t) = E\left[e^{tx}\right] = \sum_{x=1}^{\infty} e^{tx} \cdot p(x) = \sum_{x=1}^{\infty} e^{tx} q^{x-1} \cdot p$$

$$= p \sum_{x=1}^{\infty} e^{tx} q^{x-1}$$

$$= p \sum_{x=1}^{\infty} (e^t)^x q^{x-1}$$

$$= \frac{p}{q} \sum_{x=1}^{\infty} e^{tx} \cdot q^x$$

$$= \frac{p}{q} \sum_{x=1}^{\infty} (qe^t)^x$$

$$= \frac{p}{\cancel{q}} \left(\frac{\cancel{q}e^t}{1 - qe^t} \right)$$

$$m_x(t) = \frac{pe^t}{1 - qe^t}$$

$$m_x'(t) = \frac{pe^t(1-qe^t) + pe^t qe^t}{(1-qe^t)^2}$$

$$m_x'(0) = \frac{p(1-q) + pq}{(1-q)^2} = \frac{p - \cancel{pq} + \cancel{pq}}{p^2} = \frac{p}{p^2} = \frac{1}{p}.$$

$E[x^2] = m''(0)$ $\quad$ $E[x] = \frac{1}{p}$ $\quad$ $(E[x])^2 = \frac{1}{p^2}$