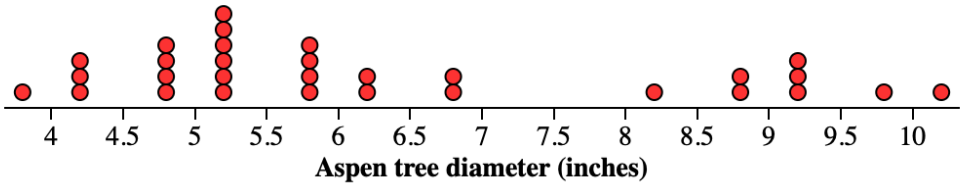
Question 2

Allotted time: 15 minutes (plus 5 minutes to submit)

A park ranger at a large national park wants to estimate the mean diameter of all the aspen trees in the park. The park ranger believes that due to environmental changes, the aspen trees are not growing as large as they were in 1975.

(a) Data collected in 1975 indicate that the distribution of diameter for aspen trees in this park was approximately normal with a mean of 8 inches and a standard deviation of 2.5 inches. Find the approximate probability that a randomly selected aspen tree in this park in 1975 would have a diameter less than 5.5 inches.

The park ranger selects a random sample of 30 aspen trees from the park in 2020 and measures their diameters. A dotplot of the diameters is shown below.



(b) Describe the distribution of aspen tree diameters from the sample.

(c) Assume the conditions for inference have been met. The park ranger uses the sample data to construct a 95% confidence interval for the mean diameter of all aspens in the park in 2020 as 5.6 to 7.0 inches. Calculate the point estimate and the margin of error.

(d) Based on the confidence interval, does the park ranger have convincing evidence that the mean diameter for all aspen trees in the park is different than 8 inches, as it was in 1975?

(e) Aspen trees tend to be smaller in the highlands of the park because they are subject to strong winds. While the number of aspen trees in the highlands of the park is about the same as the number of aspen trees in the lowlands of the park, the park ranger is concerned that 24 out of the 30 trees in the random sample came from the highlands. Propose a sampling method to address this concern and describe the benefit of using that sampling method.

Soutions

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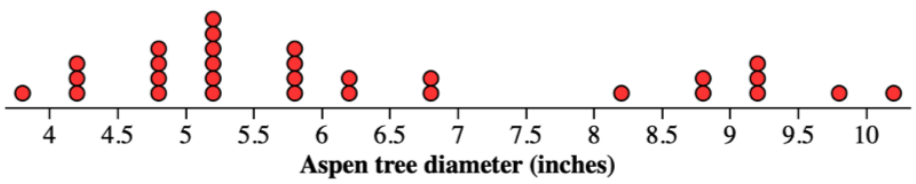
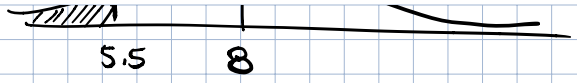


a) $N(8, 2.5)$



$$z = \frac{5.5 - 8}{2.5} = -1$$

$P(z < -1) = 0.1587$

or

use calculator

or

use table.

(b) Describe the distribution of aspen tree diameters from the sample.

S O C S

Shape outlier center spread

The shape of the distribution of aspen tree diameters appears to be skewed right. It has a gap between 7 and 8.

The center (median) of the distribution is around 5.8 inches with no obvious outliers.

The range of the distribution is $10.2 - 3.8 = 6.4$ inches.

(c) Assume the conditions for inference have been met. The park ranger uses the sample data to construct a 95% confidence interval for the mean diameter of all aspens in the park in 2020 as 5.6 to 7.0 inches. Calculate the point estimate and the margin of error.

(5.6, 7.0)

so the estimate is simply the midpoint or

$$\frac{5.6 + 7}{2} = 6.3 \text{ inches}$$

the margin of error is $7 - 6.3 = 0.7$

(d) Based on the confidence interval, does the park ranger have convincing evidence that the mean diameter for all aspen trees in the park is different than 8 inches, as it was in 1975?

Yes, all plausible values for the mean diameter are less than 8 inches. 8 is not contained in the interval. The park ranger has convincing evidence (at $\alpha = 0.05$) that the 2020 mean diameter does not equal 8.

(e) Aspen trees tend to be smaller in the highlands of the park because they are subject to strong winds. While the number of aspen trees in the highlands of the park is about the same as the number of aspen trees in the lowlands of the park, the park ranger is concerned that 24 out of the 30 trees in the random sample came from the highlands. Propose a sampling method to address this concern and describe the benefit of using that sampling method.

- we may use a stratified random sample. Take a SRS of 15 trees from highlands and a SRS of 15 trees from the lowlands.
- This will reduce the variability of the estimates of the mean (increasing precision).