One Sample Confidence Interval for μ practice

February 2, 2022

1. City council members want to estimate how many pounds of trash households in their community produce per week. To determine an estimate for the standard deviation of the weight of trash produced a small random sample of households was selected and their trash was weighed on garbage day. This produces an estimated standard deviation of 36 pounds.

   (a) How many households need to surveyed to estimate μ at the 95% confidence level with a margin of error of at most 3 lbs?

   Solution: Here we are setting our margin of error as 3 and solving for n:

   $$t^\star \frac{s}{\sqrt{n}} = 3$$

   What's weird here is that we do not know what *df* the t distribution has, so we don't know $t^\star$. In this case we use $z^\star$ instead.

   $$z^\star \frac{s}{\sqrt{n}} = 3$$

   Solving we arrive at $n = 553.19$, so we would need to sample 554 households trash.

   (b) After solving part (a), the city council realizes that it would be too much work to weigh the garbage of that many households. They give up on their hopes of estimating the true mean weight of trash produced within 3 pounds and select a random sample of 15 households and weigh the trash for each of these households on garbage day. Here are the results:

   $$114.8, 74.3, 80.1, 41.5, 99.1, 31.0, 93.1, 118.9, 26.533.1, 88.3, 46.1, 119.7, 46.3, 19.8$$

   Calculate and interpret a 95% confidence interval for the mean weight of trash for all households in this community.

   Solution: State: μ: The mean weight of all trash for households in this community. We are using a 95% confidence level.

   Plan: One sample t interval for μ.
   Random sample: Stated in the problem
   Independence: Our sample of 15 people is clearly less that 10% of the entire population so we may assume points are independent of each other
   normality: Here $n < 30$, but our sample shows little to no skew and is free from outliers, so we will assume it is appropriate to assume $\overline{x} \sim \text{Normal}\,(\mu, \sqrt{s}\sqrt{n})$.

   Do:

   $$\begin{aligned} \text{Point Estimate} &\pm \text{Margin of Error} \\ \overline{x} &\pm t_1^\star 4 \frac{s}{\sqrt{n}} \\ 68.85 &\pm 2.145 \frac{35.78}{\sqrt{15}} \end{aligned}$$

Our inteval for μ is $[49.03, 88.66]$. We are 95% confident that the interval from 49.03 tp 88.66 pounds contains the true mean for trash for all in the community.