One-Sample z and t Intervals for a Population Mean

AP Statistics · Mr. Merrick · February 27, 2026

Assume we have a random sample of size n from a population with an unknown distribution. We want to estimate the parameter μ (the population mean).

From the sample, we compute $\overline{x}$ and s. Sometimes the population standard deviation σ is known. When σ is unknown (which is most common), we estimate it using s.

Constructing confidence intervals for μ

Both methods follow the same general structure: $\boxed{\text{statistic} \pm \text{(critical value)(standard error)}}$

z-interval for μ (when σ is known)

Check conditions:

- Random: data come from a random sample or randomized experiment.
- Independence: $n \le 0.10N$ if sampling without replacement.
- Normal/Large Sample: population is Normal, or n is large enough for the CLT to apply.

If conditions are satisfied, then the sampling distribution of $\overline{x}$ is approximately

$$\overline{x} \sim N\left(\mu,\ \frac{\sigma^2}{n}\right), \qquad Z = \frac{\overline{x}-\mu}{\sigma/\sqrt{n}} \sim N(0,1).$$

For a 95% confidence level,

$$P(-1.96 < Z < 1.96) = 0.95.$$

Substitute:

$$P\left(-1.96 < \frac{\overline{x}-\mu}{\sigma/\sqrt{n}} < 1.96\right) = 0.95.$$

Multiply through by $\frac{\sigma}{\sqrt{n}}$ and rearrange:

$$P\left(\overline{x} - 1.96\frac{\sigma}{\sqrt{n}} < \mu < \overline{x} + 1.96\frac{\sigma}{\sqrt{n}}\right) = 0.95.$$

Thus the interval is

$$\boxed{\overline{x} \pm z^\star \frac{\sigma}{\sqrt{n}}}$$

t-interval for μ (when σ is unknown)

Check conditions:

- Random: data come from a random sample or randomized experiment.
- Independence: $n \le 0.10N$ if sampling without replacement.
- Normal/Large Sample: population is Normal, or n is large enough for the CLT to apply.

When σ is unknown, we estimate it with s.

$$T = \frac{\overline{x}-\mu}{s/\sqrt{n}}.$$

If the conditions are met, then $T \sim t_{n-1}$. For a 95% confidence level,

$$P(-t^\star < T < t^\star) = 0.95,$$

where $t^\star$ is the critical value from a t distribution with $df = n-1$. Substitute:

$$P\left(-t^\star < \frac{\overline{x}-\mu}{s/\sqrt{n}} < t^\star\right) = 0.95.$$

Multiply through by $\frac{s}{\sqrt{n}}$ and rearrange:

$$P\left(\overline{x} - t^\star\frac{s}{\sqrt{n}} < \mu < \overline{x} + t^\star\frac{s}{\sqrt{n}}\right) = 0.95.$$

Thus the interval is

$$\boxed{\overline{x} \pm t^\star \frac{s}{\sqrt{n}}, \quad df = n-1}$$

When n is small and the population distribution is unknown

The t-procedure requires that the population distribution be approximately Normal when n is small.

In AP Statistics, when n is small, we examine the sample data directly. If the sample distribution is:

- roughly symmetric,
- free of strong skewness,
- and free of outliers,

then it is reasonable to proceed with a t-interval.

If the sample shows strong skewness or clear outliers and n is small, the t-procedure may not be reliable.

AP Exam Tip

When n is small, and the population isn't stated as normal, describe the sample distribution.

If a graph is provided, reference it directly in your explanation.
If raw data are given, use your calculator to create a graph (such as a histogram), then describe the shape.

A strong AP response includes words such as:

- roughly symmetric
- no obvious outliers
- not strongly skewed

Do not simply write "the calculator says it is Normal." Always justify using the shape of the distribution.

In more advanced statistics courses or in professional practice, other approaches may be used, including:

- nonparametric methods (such as bootstrap procedures),
- Bayesian methods.

These approaches are beyond the scope of AP Statistics, but they address situations where the Normality assumption is questionable. For this course:

$$\text{Small } n \Rightarrow \text{Check the sample shape carefully.}$$

Example 1

A botanist is studying a rare species of plant that grows in a protected greenhouse. Because the species is uncommon, only a small number of seedlings are available for measurement. The botanist randomly selects 10 seedlings and records their heights (in centimeters):

$$12.1,\ 11.8,\ 12.5,\ 12.0,\ 11.9,\ 12.3,\ 12.2,\ 11.7,\ 12.4,\ 12.1$$

The botanist would like to estimate the true mean height of all seedlings of this species grown under these conditions. Construct and interpret a 95% confidence interval for the true mean seedling height.

Solution.

Step 1 — State

Let μ = the true mean height (in cm) of all seedlings of this species grown under these greenhouse conditions.

We will construct a 95% confidence interval for μ.

Step 2 — Justify

We will use a one-sample t interval for a population mean.

- Random: The 10 seedlings were randomly selected.
- Independence: $n = 10$ is less than 10% of all seedlings grown in the greenhouse.
- Normal: A histogram and a boxplot of the sample data show a roughly symmetric distribution with no obvious outliers. So we will assume the population of heights is approximately normal.

Since the conditions are met, a t interval is appropriate.

Step 3 — Carry Out

Formula:

$$\overline{x} \pm t^{\star} \frac{s}{\sqrt{n}}$$

From the data:

$$\overline{x} = 12.10, \quad s \approx 0.258, \quad n = 10$$

$df = 9$, $t^{\star}_{0.975,9} = 2.262$

$$SE = \frac{s}{\sqrt{n}} = \frac{0.258}{\sqrt{10}} \approx 0.0816$$

$$ME = (t^{\star})(SE) = (2.262)(0.0816) \approx 0.1847$$

Confidence interval:

$$12.10 \pm 0.1847 = (11.915,\ 12.285)$$

Step 4 — Interpret

We are 95% confident that the true mean height of all seedlings grown under these conditions is between 11.915 cm and 12.285 cm.

Example 2

A food packaging company produces 50-pound bags of flour. Historical quality-control data indicate that the standard deviation of bag weights is $\sigma = 0.6$ pounds. To check the current production line, a quality-control technician randomly selects 25 bags and records a sample mean weight of 50.8 pounds. Construct and interpret a 99% confidence interval for the true mean weight of bags produced by this machine.

Solution.

Step 1 — State

Let $\mu =$ the true mean weight (in pounds) of all flour bags produced by this machine.
We will construct a 99% confidence interval for μ.

Step 2 — Justify

We will use a one-sample z interval for a population mean.

- Random: The 25 bags were randomly selected.
- Independence: $n = 25$ is less than 10% of all bags produced.
- Normal/Large Sample: $n = 25$ is not large enough for the Central Limit Theorem. We will proceed with caution since the sample data is not given.

Since the conditions are met and σ is known, a z interval is appropriate.

Step 3 — Carry Out

Formula:

$$\overline{x} \pm z^{\star}\frac{\sigma}{\sqrt{n}}$$

$z^{\star}_{0.995} = 2.576$

$$SE = \frac{0.6}{\sqrt{25}} = 0.12$$

$$ME = (2.576)(0.12) = 0.309$$

Confidence interval:

$$50.8 \pm 0.309 = (50.49,\ 51.11)$$

Step 4 — Interpret

We are 99% confident that the true mean weight of flour bags produced by this machine is between 50.49 and 51.11 pounds.