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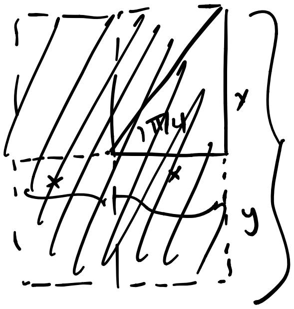
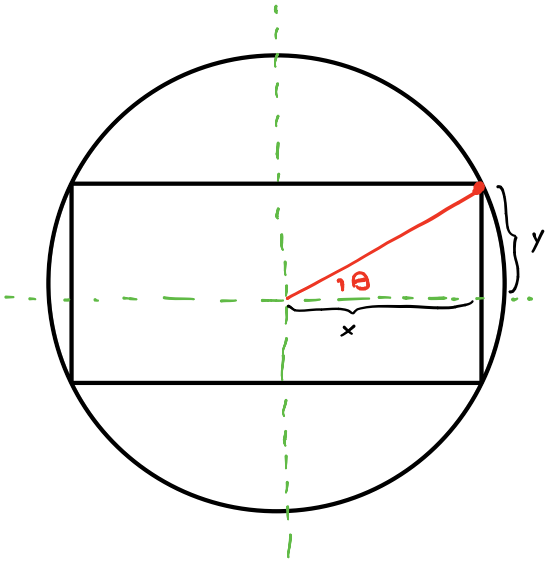
PRACTICE V

1. Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius r





$x(\theta) = r\cos\theta$

$y(\theta) = r\sin\theta$

$$A = 2x \cdot 2y = 4xy = 4r\cos\theta \cdot r\sin\theta$$

$$= 4r^2 \underbrace{\cos\theta\sin\theta}$$

$$A' = 4r^2\left[-\sin^2\theta + \cos^2\theta\right]$$

$$A' = \underbrace{4r^2\left[\cos^2\theta - \sin^2\theta\right]}$$

$$0 = 4r^2\left[\underbrace{\cos^2\theta - \sin^2\theta}\right]$$

$$\cos^2\theta = \sin^2\theta$$

$$\tan^2\theta = 1 \iff \tan\theta = 1$$

$$\theta = \frac{\pi}{4}$$

$y = r\sin\theta$

$= r\sin(\pi/4) = r \cdot \frac{1}{\sqrt{2}}$

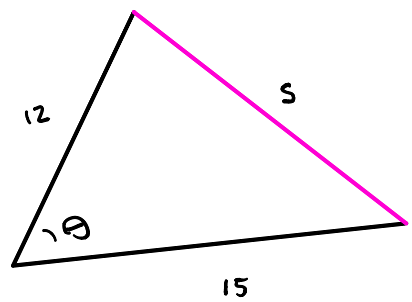
$x = r\cos\frac{\pi}{4} = r \cdot \frac{1}{\sqrt{2}}$

$$A = \overbrace{(2x)}\overbrace{(2y)}$$

$$A\left(x = \frac{r}{\sqrt{2}}, y = \frac{r}{\sqrt{2}}\right) = \left(\frac{2r}{\sqrt{2}}\right)\left(\frac{2r}{\sqrt{2}}\right) = \frac{4r^2}{2} = 2r^2$$

Speed   correctness

2. Two sides of a triangle have lengths 12m and 15m. The angle between them is increased at a rate of 2°/min. How fast is the length of the third side increasing when the angle between the sides of fixed length is 60°?



$$\frac{d\theta}{dt} = 2^\circ/\text{min}$$

$$s^2 = 12^2 + 15^2 - 2(12)(15)\cos\theta$$

$$2s \cdot \frac{ds}{dt} = 360\sin\theta \cdot \frac{d\theta}{dt}$$

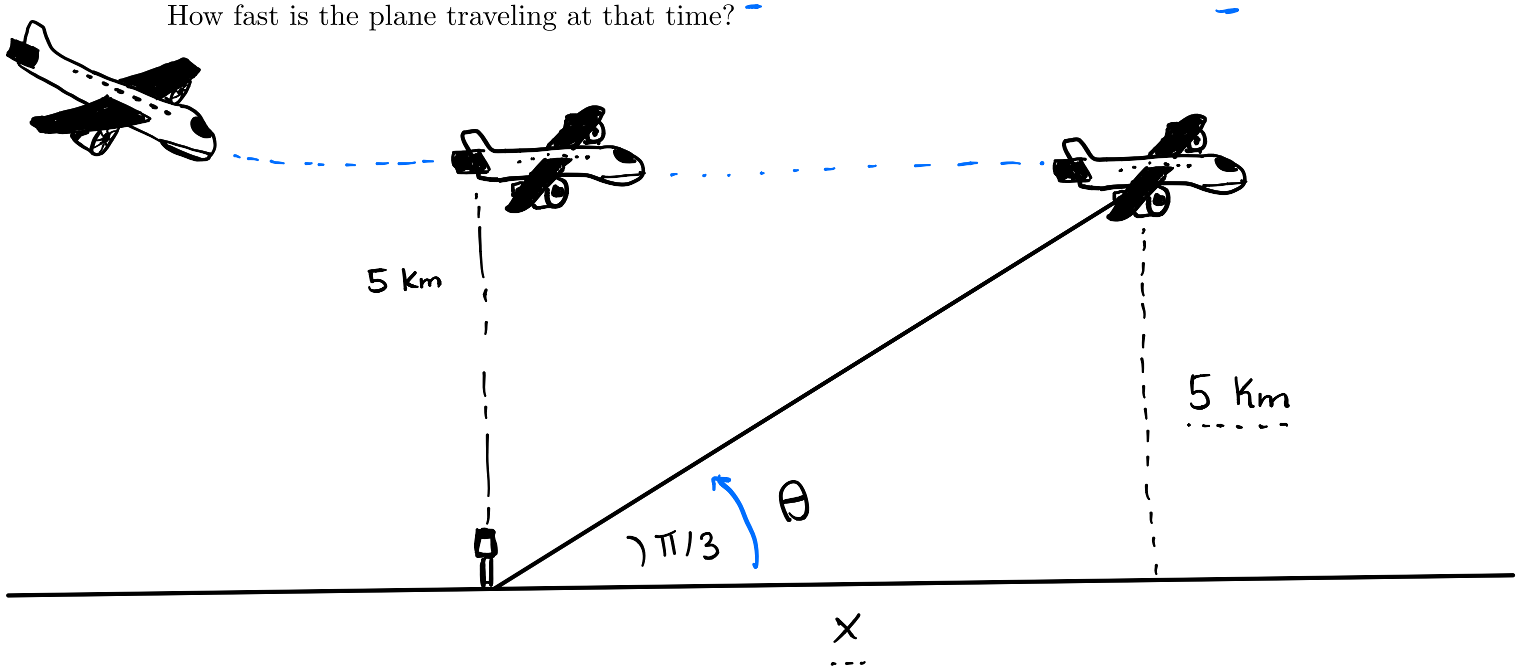
note: $s(60) = 369 - 360\cos(60)$
$= \sqrt{189}$

$$s\frac{ds}{dt} = 180\sin\theta \, \frac{d\theta}{dt}$$

$$\frac{ds}{dt} = \frac{1}{s} \, 180\sin\theta \, \frac{d\theta}{dt}$$

$$\left.\frac{ds}{dt}\right|_{\theta=60} = \frac{180}{\sqrt{189}}\sin(60)\cdot 2 \approx 0.396 \, \frac{m}{min}.$$

3. A plane flies horizontally in at an altitude of 5km and passes directly over a tracking telescope to the ground. When the angle of elevation is $\frac{\pi}{3}$, this angle is decreasing at a rate of $\frac{\pi}{6}$ rad/min. How fast is the plane traveling at that time?



$$\frac{d\theta}{dt} = -\frac{\pi}{6} \text{ rad/min}$$

$$\tan(\theta) = \frac{5}{x}$$

$$x = 5/\tan\theta = 5\cot\theta$$

$$\frac{dx}{dt} = -5\csc^2\theta \frac{d\theta}{dt}$$

$$= -5\csc^2(\pi/3)(-\pi/6)$$

$$= \frac{5\pi}{6}\csc^2(\pi/3)$$

$$= \frac{5\pi}{6}\left(\frac{2}{\sqrt{3}}\right)^2 = \frac{10\pi}{9} \text{ Km/min.}$$

4. Sketch the volume formed by each of the following integralsL:

(a)

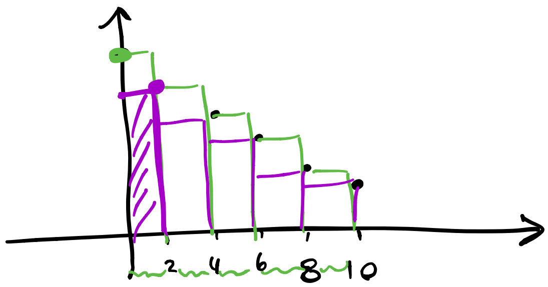
$$\pi \int_0^\pi \sin x dx$$

(b)

$$\pi \int_0^1 (y^4 - y^8) dy$$

5. Oil leaked from a tank at rate $r(t)$ litres per hour. The rate decreased as time passed and values of the rates at two hour time intervals are shown in the table. Find the lower and upper estimates for the total amount of oil that leaked out.

$t(h)$	0	2	4	6	8	10
$r(t)$ (L/h)	8.7	7.6	6.8	6.2	5.7	5.3



upper: $2(8.7) + 2(7.6) + 2(6.8) + 2(6.2) + 2(5.7) = \boxed{70\text{ L}}$

$2(8.7 + 7.6 + \cdots 5.7) = 70\text{ L}$

Lower: $2(7.6) + 2(6.8) + 2(6.2) + 2(5.7) + 2(5.3) = \boxed{63.2\text{ L}}$