Assignment #4
December 4, 2022

1. You purchase a certain product. The manual states that the lifetime T of the product, defined as the amount of time (in years) the product works properly until it breaks down, satisfies

$$P(T \geq t) = e^{-\frac{t}{5}}, \text{ for all } t \geq 0$$

   For example, the probability that the product lasts more than (or equal to) 2 years is $P(T \geq 2) = e^{-\frac{2}{5}}$. I purchase the product and use it for two years without any problems. What is the probability that it breaks down in the third year?

2. You toss a fair coin three times:

   (a) What is the probability of three heads, HHH?

   (b) What is the probability that you observe exactly one heads?

   (c) Given that you have observed at least one heads, what is the probability that you observe at least two heads?

3. For events A, B, and C, we know that

   - A and C are independent
   - B and C are independent
   - A and B are disjoint
   - $P(A \cup C) = \frac{2}{3}$, $P(B \cup C) = \frac{3}{4}$, and $P(A \cup B \cup C) = \frac{11}{12}$.

   Find $P(A)$, $P(B)$, and $P(C)$.

4. In my town, it's rainy one third of the days. Given that it is rainy, there will be heavy traffic with probability $\frac{1}{2}$, and given that it is not rainy, there will be heavy traffic with probability $\frac{1}{4}$. If it's rainy and there is heavy traffic, I arrive late for work with probability $\frac{1}{2}$. On the other hand, the probability of being late is reduced to $\frac{1}{8}$ if it is not rainy and there is no heavy traffic. In other situations (rainy and no traffic, not rainy and traffic) the probability of being late is 0.25. You pick a random day.

   (a) What is the probability that it's not raining and there is heavy traffic and I am not late?

   (b) What is the probability that I am late?

   (c) Given that I arrived late at work, what is the probability that it rained that day?

5. A box contains three coins: two regular coins and one fake two-headed coin ($P(H) = 1$)),

   - You pick a coin at random and toss it. What is the probability that it lands heads up?

   - You pick a coin at random and toss it, and get heads. What is the probability that it is a two headed coin?