PRACTICE IV

Probability

1. You are asked to choose between two envelopes, one of which has twice as much money as the other. You arbitrarily pick one, open it, and find \$2. You are given the chance to switch envelopes. You reason that the other envelope has either \$1 or \$4, each with probability 0.5. Applying your understanding of expected value, you calculate:

	x	$2x$
$P(X=x)$	0.5	0.5

$$0.5(\$1) + 0.5(\$4) = \$2.50$$

and conclude that you should switch envelopes. Comment on this reasoning.

- Clearly something is wrong. If you follow this line of reasoning you should always switch envelopes.

However, there is a probability of 0.5 that you picked the envelope with the larger money to start with.

The correct line of reasoning is as follows: One envelope has \$$x$, while the other has \$$2x$. If you stick with the origional choice your pay off is

$(0.5)(x)+(0.5)(2x) = 1.5x$. If however, you switch envelopes, your expected payoff is
$(0.5)(2x) + (0.5)(x) = 1.5x$.

There is exactly the same payoff whether or not you switch envelopes.

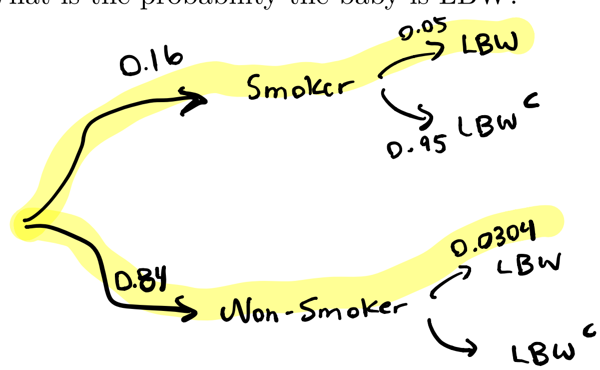
2. The weights of babies born to nonsmokers have a normal distribution with a mean of 7.0 pounds and a standard deviation of 0.8 pounds. Babies are considered low birth weight if they weight less than 5.5 pounds. Note that 5 percent of babies born to smokers are LBW and that 16 percent of pregnant woman are smokers.

(a) What is the probability that a nonsmoker will have an LBW baby?

With $N(\mu = 7.0, \sigma = 08)$, $P(X < 5.5) = P\left(z < \frac{5.5 - 7.0}{0.8}\right)$

$\boxed{= 0.0304}$

(b) What is the probability the baby is LBW?



$P(LBW) = P(\text{Smoker} \cap LBW) + P(\text{nonsmoker} \cap LBW)$

$= (0.16)(0.05) + (0.84)(0.0304)$

$\boxed{= 0.03354}$

(c) Given that a baby is LBW, what is the probability that the baby was born to a nonsmoker?

$$P(\text{non-smoker} \mid LBW) = \frac{P(\text{nonsmoker} \cap LBW)}{P(LBW)} = \frac{(0.84)(0.0304)}{(0.03354)}$$

$\boxed{= 0.761}$

3. Die A has three 5s, two 3s, and one 1 on its six faces. Die B has two 2s and ~~four~~ four 4s on its six faces. Both dice are fair. Each player simultaneously rolls one of the dice, and the winner is the player with the higher number showing.

(a) If you want to win, would you rather roll die A or die B? Explain.

Die A : $\{5,5,5,3,3,1\}$, $P(5)=\frac{1}{2}$, $P(3)=\frac{1}{3}$, $P(1)=\frac{1}{6}$

Die B: $\{2,2,4,4,4,4\}$, $P(2)=\frac{1}{3}$, $P(4)=\frac{2}{3}$

$$P(A \text{ wins}) = P(A=5) + P(A=3 \cap B=2)$$
$$= \frac{1}{2} + \left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = \frac{11}{18}$$

$$P(B \text{ wins}) = P(A=1 \cap B=4) + P(A=3 \cap B=4) + P(A=1 \cap B=4)$$
$$= \left(\frac{2}{3}\right)\left(\frac{1}{6}\right) + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{6}\right)$$
$$= \frac{2}{18} + \frac{2}{9} + \frac{1}{18} = \frac{7}{18}$$

Probability of A winning is higher $\therefore$ choose A.

(b) If the winner receives whatever shows on his/her winning die, what is the expected value for one roll to each player? Explain.

$E[x] = \sum x \cdot p(x)$.

Let A be dice A.

$$E[A] = 5\left(\frac{1}{2}\right) + 3\left(\frac{1}{9}\right) = \boxed{2\frac{5}{6}}$$

Let B be dice B.

$$E[B] = 2\left(\frac{1}{18}\right) + 4\left(\frac{1}{3}\right) = \boxed{1\frac{4}{9}}$$

- note: you make \$0 when you lose hense exclusion