PRACTICE VII

Inference

1. In a random sample of 150 adults over the age of 45, 30 say they have played in a band at least one time in their lives.

   (a) Construct a 99% confidence interval for the proportion of all adults over the age of 45 who have played in a band at least one time in their lives.

   1. Procedure : one sample z-interval for a population proportion.

   2. Conditions: Independence $n < 10\% N$ as 150 adults over 45 is small in relation to all adults over 45.

      Normality $n\hat{p} = 30 \geq 10$ and $n(1-\hat{p}) = 120 \geq 10$

   3. Mechanics : 1-PropZInt gives (0.116, 0.284)

      (alternatively, $0.2 \pm 2.576\sqrt{\frac{(0.2)(0.8)}{150}} = 0.2 \pm 0.084$).

   4. Conclusion in context : we are 99% confident that the proportion of all adults over the age of 45 who have played a band at least one time in their lives is between 0.116 and 0.284.

   (b) Suppose that all adults over the age of 45 who have played in a band at least one time in their lives each send in a donation of $10 to the organization Musicians without Borders, which aims to empower musicians as social activists. Assuming thee are 125 000 000 adults over the age of 45 in the U.S., What is a 99% confidence interval for what these donations would total for this worthwhile charity?

   a 99% estimate for the number of adults sending in donations is between 0.116(125 000 000) = 14 500 000 and 0.284(125 000 000) = 35 500 000. At $10 each, a 99% confidence interval for the total donations from this group of adults to Musicians without borders is $145 000 000 to $355 000 000.

2. In a random sample of 915 adults, 366 say that they believe in ghosts.

   (a) With what margin of error can we find a 95% confidence interval for the proportion of adults who believe in ghosts?

• first off sample is random, $n\hat{p} = 366$ and $n(1-\hat{p}) = 549$ are both greater than 10. The sample of 915 adults is also clearly smaller than 10% of the population of all adults.

$$\hat{p} = \frac{366}{915} = 0.4 \quad \text{and} \quad \sigma_{\hat{p}} = \sqrt{\frac{0.4(0.6)}{915}} = 0.0162.$$

for a 95% confidence interval:

$$\hat{p} \pm \underbrace{z^* \sigma_{\hat{p}}}_{\text{margin of error.}}$$

where $z^* = 1.96$ for a 95% C.I. so our margin of error is $\pm\ 1.46(0.0162) = \boxed{\pm\ 0.032 \text{ percent.}}$

   (b) With what confidence can we report a margin of error of ± 2 percent in giving a confidence interval of the proportion of adults who believe in ghosts.

$z(0.0162) = 0.02$, which gives $z = 1.235$.

The resulting confidence level is

$$P(-1.235 < z < 1.235) = 0.783 \quad \boxed{\text{or } 78.3\% \text{ confidence.}}$$

3. A senator's approval rating stood at 65 percent before she took a crucial vote.

(a) Her staff believes the rating is still around 65 percent. To confirm this, how large of a simple random sample (SRS) should the staff sample to obtain a 94% confidence interval estimate with a margin of error $\leq 3.5\%$.

$$\hat{p} \pm \underbrace{z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}_{\text{margin of error}}$$

$z^* = \text{invNorm}(0.03)$
$= -1.88$

want $z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq 0.035$

$1.88\sqrt{\frac{(0.65)(0.35)}{n}} \leq 0.035$ gives

$\sqrt{n} \geq 25.62$, and $n \geq 656.4$ so 657 people must be surveyed. Note that we use 0.65 in the calculation because we believe $p \approx 0.65$. we would otherwise use 0.5.

(b) The senator's staff randomly samples 700 people and find 432 people approve of the senator's job performance. Is there evidence that the rating has changed from 65 percent? Perform an appropriate statistical test.

from sample of $n = 700$ find $\hat{p} = \frac{432}{700}$

Procedure : two-sided one-sample z-test for p.

Condition check : Independence: random sample and 700 people is clearly 10% smaller than all people in population (N).

Normality : $np = 700(0.65) = 455$ and $n(1-p) = 700(0.35) = 245$ are both greater than 10.

Hypothesis : $H_0: p = 0.65$.
$H_a: p \neq 0.65$

Test Statistic and P-value : 1-PropZTest gives $z = 1.8226$ and $P = 0.0684$ (alternatively with $\hat{p} = \frac{432}{700} = 0.6171$ we get

Conclusion : with this large of a p-value, $0.068 > 0.05$

$z = \frac{0.6171 - 0.65}{\quad} = -1.825$

there is not evidence that the senator's approval rating has slipped.

$$\sqrt{\frac{(0.65)(0.35)}{700}}$$

P-value : $2 \cdot P(Z < -1.825) = 2(0.034) = \boxed{0.068}$.

(c) The senator's staff randomly samples 700 people and finds 432 people approve of the senator's job performance. Is there evidence that the rating has changed from 65 percent? Perform an appropriate statistical test.

Whoops...

(d) In part (b) above, suppose the staff suspects the rating has gone down. Is there evidence that the approval rating has slipped down from the 65 percent? Perform an appropriate statistical test

- Test has changed from one tailed to two tailed.
- With $H_0: p = 0.65$ and $H_a: p < 0.65$, $P = 0.034$. with $0.034 < 0.05$ there is sufficent evidence that the senators approval has slipped.

(e) Are the answers from (b) and (c) contradictory? Explain.

- There is no contradiction.
- The tail probability of the test statistic is doubled when finding the P-value in a two-sided test. Thus, what might be a small enough tail probability to reject H_0 for a one-sided test might no longer be small enough when doubled for a two tailed test.