Two Sample t-test for $\mu_1 - \mu_2$

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According to a Stanford Business article, Americans may eat fewer calories at restaurants if the calories of the food items are labeled on the menu. To investigate this, researchers compared Starbucks receipts from locations where the menus were labeled to receipts from the stores where the menus were not labeled. A random sample of 30 receipts from stores with the menus labeled had an average of 225 calories with a standard deviation of 100 calories. A random sample of 40 receipts from stored without menus labeled showed an average of 265 calories per receipt with. a standard deviation of 75 calories. Does this provide convincing evidence that the average calories per receipt at Starbucks with a labeled menu is less than that at a Starbucks without labeled menus?

Solution: State: $mu_1 - \mu_2$: the true difference in mean calories per receipt at Starbucks for labeled and non labeled menus.
$\overline{x}_1 - \overline{x}_2 = 225 - 226 = -40.$

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_0 : \mu_1 - \mu_2 < 0$$

Plan: Here we are conducting a two sample t test for $\mu_1 - \mu_2$.
Conditions:

- Random: Independent random samples stated in problem.
- Independence: Each sample is clearly less than 10% of their respective populations.
- Normality: Both samples are greater than 30 so the central limit theorem applies and we may assume that $\overline{x}_1 - \overline{x}_2$ is normal.

Do: Test Statistic:

$$\begin{aligned}
\text{Test Statistic} &= \frac{\text{Statistic} - \text{Parameter}}{\text{SD Statistic}} \\
&= \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \\
&= -1.84
\end{aligned}$$

p-value:

$$\begin{aligned}
p\text{-value} &= P(t < -1.84) \\
&= 0.036
\end{aligned}$$

Conclude: Since our p-value$= 0.036 < \alpha = 0.05$ we reject the null hypothesis in favour of the alternative. Assuming that the null hypothesis is true (state context). There is a 0.038 percent chance of observing a difference in sample means of -40 calories or lower. This is very condemning evidence against the null hypothesis.

Can Balloons hold more air or water before bursting? A student purchased a large bag of 12-inch filled with air until bursting and the other half to be filled with water until bursting. He used devices to measure the amount of air and water dispensed until the balloons burst. The data is shown below: Do the data give

Air (cubic feet)	0.52	0.58	0.50	0.55	0.61
Water (cubic feet)	0.44	0.41	0.45	0.46	0.38

convincing evidence air filled balloons can attain a greater volume than water filled balloons?

Solution: State: $mu_1 - \mu_2$: The true difference in mean columes for air and water filled balloons $\overline{x}_1 - \overline{x}_2 = .124$.

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_0 : \mu_1 - \mu_2 > 0$$

Plan: Here we are conducting a two sample t test for $\mu_1 - \mu_2$.
Conditions:

- Random: Random selection of balloons and random assignment to treatments. This allows us tp generalize our results to all balloons, and imply causation.
- Independence: Each sample is clearly less than 10% of their respective populations.
- Normality: Samples show no strong skew or outliers so we may assume that the difference in sample means is normally distributed

Do: Test Statistic:

$$\begin{aligned} \text{Test Statistic} &= \frac{\text{Statistic} - \text{Parameter}}{\text{SD Statistic}} \\ &= \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \\ &= 5.04 \end{aligned}$$

p-value:

$$\begin{aligned} p\text{-value} &= P(t > 5.04) \\ &= 0.0036 \end{aligned}$$

Conclude: Since our p-value$= 0.0036 < \alpha = 0.05$ we reject the null hypothesis in favour of the alternative. Assuming that the null hypothesis is true (state context). There is a roughly .36 percent chance of observing a difference in sample means of .124 cubic feet or higher. This is very condemning evidence against the null hypothesis.