Practice - Sampling Distributions

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1. In one town 51% of the voters are conservative and in a second town 44% of the voters are conservative. Suppose 100 voters are surveyed from each town.

   (a) Is a normal model appropriate for the difference in proportions of conservative voters from the two samples?

   Solution: We need to check our requirements for the normal model

   - **Simple Random Sample:** here we will assume the samples are drawn randomly
   - **Independence:** We will assume both towns have more than 1000 people, so $n_1 < 10\% N_1$ and $n_2 < 10\% N_2$. This means we will assume sample points are independent.
   - **Normality:** We have 51 expected successes and 49 expected failures in the first town, and 44 expected successes and 56 expected failures in the second town. This means we can assume there will be normality in the sampling distribution for the difference of proportions.

   (b) What is the probability that the first sample will yield a lower sample proportion of conservative voters than the second town?

   Solution:

$$\begin{aligned} P\left(\hat{p}_1 - \hat{p}_2 < 0\right) &= P\left(\frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}} < \frac{0 - (.51 - .44)}{\sqrt{\frac{(.51)(0.49)}{100} + \frac{(0.44)(0.56)}{100}}}\right) \\ &= P(Z < -0.9936328) \\ &= 0.1602008 \end{aligned}$$

2. The starship Enterprise is exploring a new planet. Spock is inspecting two different alien species. Suppose the first species has a mean weight of 5 kg with a standard deviation of 1 kg, and the second species has a mean weight of 6 kg with a standard deviation of 2 kg. Spock randomly samples 63 of species one, and 80 of species 2.

   (a) Is a normal model appropriate for a difference in sample means?

   Solution: Let's look at the requirements for the normal model:

   - **Random Sampling:** It is clearly stated that Spock takes a simple random sample
   - **Independence:** The population sizes for the two types of aliens are not clearly stated but we will assume that $n_1 < 10\% N_1$ and $n_2 < 10\% N_2$.
   - **Normality:** Here we have a sample of $n_1 = 62$ and $n_2 = 80$ which are both greater than 30.

   As the sample fulfills all requirements the normal model is appropriate.

   (b) What is the probability that species one is greater than 1 kg heavier than species two?

   Solution:

$$\begin{aligned} P\left(\overline{x}_1 - \overline{x}_2 > 1\right) &= P\left(\frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} > \frac{1 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right) \\ &= P\left(Z > \frac{1 - (5 - 6)}{\sqrt{\frac{1^2}{63} + \frac{2^2}{80}}}\right) \\ &= P(Z > 7.792489) \approx 0 \end{aligned}$$