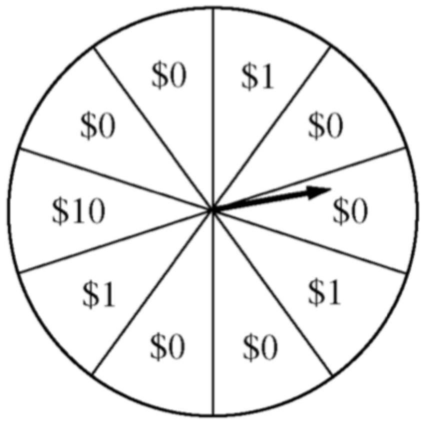
PRACTICE XV

1. A charity fundraiser has a Spin the Pointer game that uses a spinner like the one illustrated in the figure below. A donation of \$2 is required to play the game. For each \$2 donation, a



player spins the pointer once and receives the amount of money indicated in the sector where the pointer lands on the wheel. The spinner has an equal probability of landing in each of the 10 sectors.

(a) Let X represent the net contribution to the charity when one person plays the game once. Complete the table for the probability distribution of X.

x	\$2	\$1	–\$8
$P(x)$	0.6	0.3	0.1

$\frac{2}{10}$ $\frac{3}{10}$ $\frac{1}{10}$

(b) What is the expected value of the net contribution to the charity for one play of the game?

$$E[x] = 2(0.6) + 1(0.3) + (-8)(0.1)$$
$$= 0.70 \quad \text{or} \quad \boxed{\$0.70}$$

(c) The charity would like to receive a net contribution of $500 from this game. What is the fewest number of times the game must be played for the expected value of the net contribution to be at least $500?

The expected contribution after n plays is $0.70n$. Setting this to be $500, we have

$$0.70n \geq 500$$

so

$$n \geq 714.286$$

so 715 plays are needed for the expected contribution to be at least $500

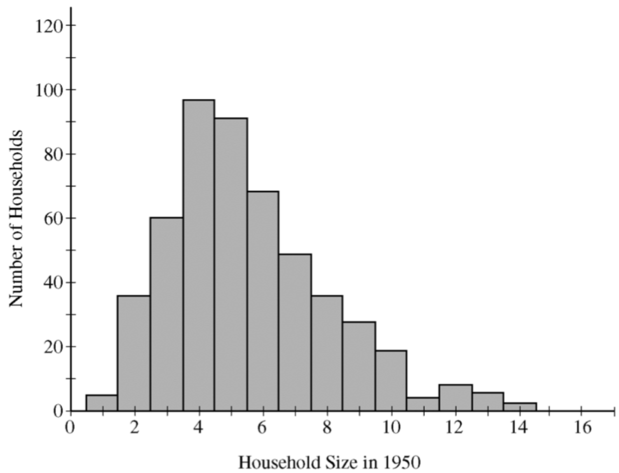
(d) Based on last year's event, the charity anticipates that the Spin the Pointer game will be played 1,000 times. The charity would like to know the probability of obtaining a net contribution of at least $500 in 1,000 plays of the game. The mean and standard deviation of the net contribution to the charity in 1,000 plays of the game are $700 and $92.79, respectively. Use the normal distribution to approximate the probability that the charity would obtain a net contribution of at least $500 in 1,000 plays of the game.

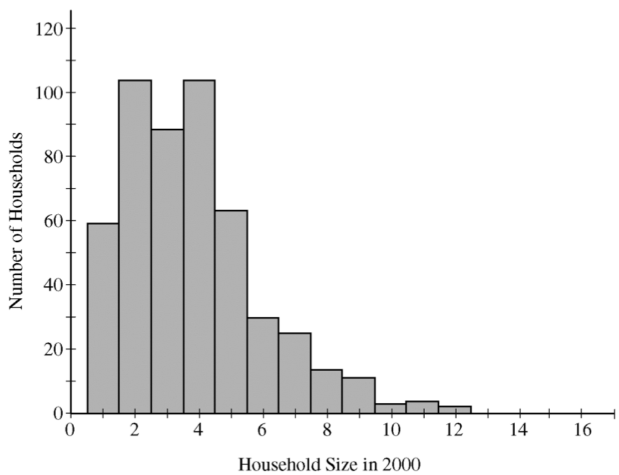
- The normal approximation is appropriate because the very larg sample size ($n=1000$) ensures that the central limit theorem holds.
- The sample mean of the contributions will be be approximately normal
- The Z-score is $Z = \dfrac{500-700}{92.79} = -2.155$.

$P(Z > -2.155) = \text{normalCDF}(\ldots) = 0.9844$.

- The charity can be very confident about gaining a net contribution of at least $500 from 1000 plays of the game.

2. Independent random samples of 500 households were taken from a large metropolitan area in the United States for the years 1950 and 2000. Histograms of household size (number of people in a household) for the years are shown below.





(a) Compare the distributions of household size in the metropolitan area for the years 1950 and 2000

Household sizes tend to be larger in 1950 than in 2000. There appears to be more small (1, 2, 3, person) households in 2000 than in 1950. Similarly the histograms show smaller proportion of large houses (5+ people and higler) in 2000 than in 1950.

The median of 1950 housholds appears to be higher at 5 people a househdd in 1950 when compared to 2000 which has a median of 3-4 people.

It appears that the 1950 households have a larger range and thus a larger standard deviation than the 2000 households.

Both distributions are skewed right. Households with very large families (12+ people) may be seen as outliers.

(b) A researcher wants to use these data to construct a confidence interval to estimate the change in mean household size in the metropolitan area from the year 1950 to the year 2000. State the conditions for using a two-sample t-procedure, and explain whether the conditions for inference are met.

Conditions: - we have a simple random somple. ✓

- we would need a somple of sufficient size, 1000, 500 being from each group satisfies this condition ✓

- we require the population size to be 10 times the sample size. ✓ It is clear that $n = 500$ for each group is very small in relation to population of all houses.

3. A recent report stated that less than 35 percent of the adult residents in a certain city will be able to pass a physical fitness test. Consequently, the city's Recreation Department is trying to convince the City Council to fund more physical fitness programs. The council is facing budget constraints and is skeptical of the report. The council will fund more physical fitness programs only if the Recreation Department can provide convincing evidence that the report is true. The Recreation Department plans to collect data from a sample of 185 adult residents in the city. A test of significance will be conducted at a significance level of $\alpha = 0.05$ for the following hypotheses.

$$H_o : p = 0.35$$

$$H_a : p < 0.35$$

Where p is the proportion of adult resident in a city who are able to pass the physical fitness test.

(a) Describe what a Type II error would be in the context of the study, and also describe a consequence of making this type of error.

• Type II error means failing to reject Ho and assume the proportion of adult residents able to pass the test is ~0.35, when in reality, the proportion is <0.35.

• A consequence here, would be that the council will fail to fund the program, and the city will continue to have a smaller proportion of physically fit residents than desired.

(b) The Recreation Department recruits 185 adult residents who volunteer to take the physical fitness test. The test is passed by 77 of the 185 volunteers, resulting in a p-value of 0.97 for the hypotheses stated above. If it was reasonable to conduct a test of significance for the hypotheses stated above using the data collected from the 185 volunteers, what would the p-value of 0.97 lead you to conclude?

Because the p-value of 0.97 is larger than $\alpha = 0.05$ we fail to reject the null hypothesis. There is not sufficient evidence to suggest that the proportion of all adult residents able to pass the test is less than 0.35.

The sample proportion is actually higher than the null value here.

(c) Describe the primary flaw in the study described in part (b), and explain why it is a concern.

- This is not a randomly selected sample because the sample was selected by recruiting volunteers.
- It seems reasonable to think that volunteers would be more physically fit than the population of adults as a whole.
- For this reason the sample proportion will likely <u>overestimate</u> the proportion of adult residents in the city who are able to pass the physical fitness test.