PRACTICE V

Probability

1. The probability distribution function for the number of days per week that college students get a good night's sleep is as follows:

Number of Days	0	1	2	3	4
Relative Frequency	0.12	0.41	0.25	0.15	0.07

(a) Calculate and give a brief description of the mean of this probability distribution.

$$\mu = \sum_{\text{all } x} x \cdot p(x) = 0(.12) + 1(0.41) + 2(0.25) + 3(0.15) + 4(0.07)$$

$$= 1.64.$$

· On average, college students get a good nights sleep on 1.64 days of the week.

(b) In a random sample of 10 college students, there are a total of 20 good nights of sleep. A new random sample of 50 students is planned. How do you expect the average number of good nights sleep for this new sample to compare to that of the first sample? Explain.

· random sample gives $\hat{p} = \frac{10}{20} = 0.5$, the next random sample has $n = 50$.

· The average number of sleeps for the second sample will be a better estimate as it is sampling more students. For this reason the second sample will 'land' closer to the true mean of 1.64.

· If many samples are carried out the second sampling method will lead to less variation in the sampling distribution.

(c) Find the median of the above distribution, where the median M is defined to be a value such that $P(x \geq M) \geq 0.5$ and $P(x \leq M) \geq 0.5$

We see that $P(x \leq 1) = 0.12 + 0.41 = 0.53 \geq 0.5$ and $P(x \geq 1) = 0.41 + 0.25 + 0.15 + 0.07 = 0.88 \geq 0.5$, so the median here is <u>one</u>.

2. A school ~~myst~~ must choose between two snow removal services. Service A charges \$2500 yearly plus \$2000 for every month with over three snowfalls necessitating their services. Service B charges \$450 per snow removal. Relevant probabilities are shown in the following tables: Which service

Month	Oct	Nov	Dec	Jan	Feb	Mar	Apr
$P(> 3$ snowfall services)	0.01	0.02	0.18	0.28	0.22	0.13	0.02

Annual Number of Snow services	6	7	8	9	10	11	12
Probability	0.05	0.10	0.20	0.25	0.20	0.15	0.05

should the school use to minimize expected costs? Justify your answer.

<u>Let A be cost of service A</u>

$E[A] = 2500 + 2000\,(0.01 + 0.02 + 0.18 + 0.28 + 0.22 + 0.13 + 0.02) = \$4220$

<u>Let B be cost of service B</u>

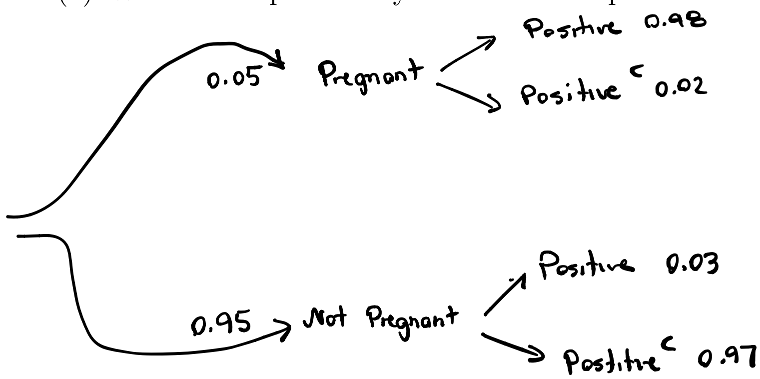
$E[B] = 450 \cdot \big(6(0.05) + 7(0.10) + 8(0.20) + 9(0.25) + 10(0.20) + 11(0.15) + 12(0.05) \big)$

$= \$4095.$

- The school should choose service B which will be less expensive on average.

3. Suppose a pregnancy test correctly tests positive for 98 percent of pregnant woman but also gives a false positive reading for 3 percent of the women who are not pregnant. Suppose 5 percent of woman who purchase this over-the-counter test are actually pregnant.

(a) What is the probability a women tests positive?



$P(\text{Positive}) = P(\text{Pregnant} \cap \text{Positive}) + P(\text{Not pregnant} \cap \text{Positive})$

$= 0.05(0.98) + 0.95(0.03)$

$\boxed{= 0.0775}$

(b) If a woman tests positive, what is the probability she is pregnant?

$$P(\text{pregnant} \mid \text{positive}) = \frac{P(\text{pregnant} \cap \text{positive})}{P(\text{positive})}$$

$$= \frac{0.05(0.98)}{0.0775} = \boxed{0.6323}$$

(c) If two women purchase the test and both test positive, what is the probability that exactly one of the two is pregnant?

$P(\text{pregnant} \mid \text{positive}) = 0.6323$

Binomial R.V. with $p = 0.6323$, $n=2$

Let $X \sim \text{binomial}(n=2, p=0.6323)$

$$P(X = 1) = 2(0.6323)(1-0.6323)$$

$$\boxed{= 0.4650}$$