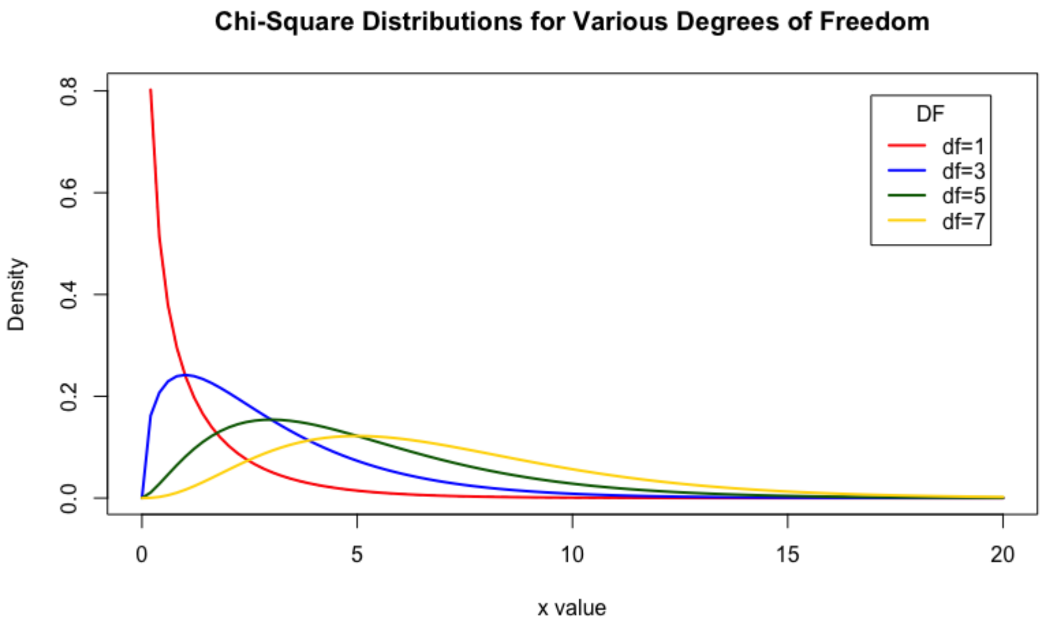
3.6.7 The χ^2 Distribution

χ^2 (Chi-Square) distributions are used frequently in statistical theory. It will be utilized later when we study statistical inference. For now we will take a look at the shape of the distribution. The pdf is a rather complicated function with one parameter, k, the **degrees of freedom**. We can denote a χ^2 distribution with k degrees of freedom as χ^2_k. If $X \sim \chi^2_k$, we have $E(X) = k$ and $Var(X) = 2k$. Shown below are several χ^2 distributions with various degrees of freedom.



The distributions are all **skewed right**, but as the degrees of freedom increase the distribution appears to be getting more and more symmetric. It turns out the mean for each distribution is the respective degrees of freedom. Like with the normal distribution, we can calculate probabilities for a χ^2_k distribution using a computer, tables, or our calculators.

Example 9: Let $Y \sim \chi^2_5$

(a) Find $P(Y < 3)$

Using a computer, tables, or our calculator we obtain $P(Y < 3) = 0.3000142$.

(b) Find $P(3 \leq Y \leq 7)$

Now we have $P(3 \leq Y \leq 7) = 0.4793455$

(c) Find $P(Y > 4)$

Using complimentary probability we have $P(Y > 4) = 1 - P(Y \leq 4) = 0.549416$

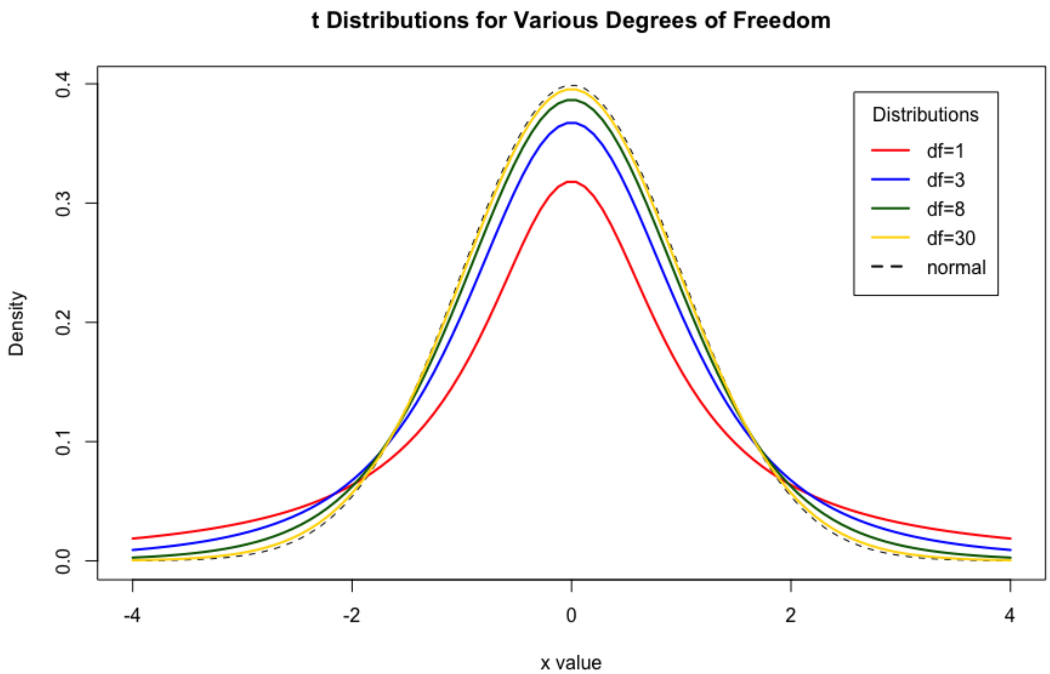
A deeper exploration of the mathematical definition, and proofs relating to the χ^2 distribution are left for a more advanced text.

3.6.8 The t Distribution

The t distribution, or student's t-distribution is another fundamental distribution used in statistical inference. Like with the χ^2 distribution the t distribution has one parameter; the **degrees of freedom**. We may denote a t distribution with ν degrees of freedom as t_ν. For this text we are less interested with the pdf for the distribution and more interested with the shape. However, we may take a look at how a t distribution is defined:

$$t = \frac{Z}{\sqrt{\frac{\chi_k^2}{k}}}$$

The distribution can be derived by taking this ratio involving a standard normal and chi-square distribution with k degrees of freedom. Below are some examples of t distributions with various degrees of freedom.



The distributions are all symmetric, approximately bell shaped, and centered at 0. It also seems as the degrees of freedom continue to get larger, the distribution gets closer and closer to being a normal distribution. Probabilities for the t-distribution can be found using a computer of a calculator.

Example 10: Let $Y \sim t_{13}$

(a) Find $P(Y < 12)$

Using a computer, tables, or our calculator we obtain $P(Y < 1) = 0.8322194$.

(b) Find $P(-2 \leq Y \leq 0.5)$

Now we have $P(-2 \leq Y \leq 0.5) = 0.6538642$.

(c) Find $P(Y > -\frac{1}{3})$

Now using complimentary probability $P(Y > -\frac{1}{3}) = 1 - P(Y \leq -\frac{1}{3}) = 0.6279012$

A deeper exploration of the mathematical definition, and proofs relating to the t distribution are left for a more advanced text.