

## Paired $t$ test for $\mu_d$

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1. Recall when constructing paired  $t$  intervals, the example where students used 2 different memory strategies to see on average how many words they could recall.

Suppose that 32 students are randomly selected to try both memory strategies. Each subject calculated the difference in the number of words. The average of the differences was  $\bar{x}_d = 1.5$  and the standard deviation of the difference was  $s_d = 2.7$ . Do these data give convincing evidence that strategy 2 improves memory at the  $\alpha = 0.05$  significance level?

**Solution: State:** True mean difference in words.

$$H_0 : \mu_d = 0$$

$$H_a : \mu_d > 0$$

**Plan:** Paired  $t$  test for  $\mu_d$ , or one sample  $t$  test for  $\mu_d$ . We may also want to randomize the order of which strategy is used here. Conditions:

Random Sample: Clearly stated so we may generalize

independence: Our sample is clearly less than 10% of the population of all students.

Normality: We have a large enough sample for the CLT to apply.

**Do:** (Calculations need to be shown here)  $t = 3.14$ ,  $p$ -value = 0.001.

**Conclude:** (Needs full explanation with context here). We reject  $H_0$  in favour of the alternative.

2. In each of the following settings, decide whether you should use two-sample  $t$  procedures to perform inference about a difference in means or paired  $t$  procedures to perform inference about a mean difference. Explain your choice.

- (a) A random sample of 30 adults were selected. Each adult reported the number of pieces of junk mail they received in their mailbox at home that day as well as the number of junk emails they received in their inbox that day. A researcher would like to know if adults receive significantly more junk email than junk mail in their mailbox.

**Solution:** Paired  $t$  procedures - there is one sample of 30 adults and the two measurements are paired for each of the adults

- (b) A random sample of 100 people who live on the west coast and 100 people who live on the east coast are selected. Each person reports how many spam calls they receive in a one-week time period. We would like to know if people who live on the west coast receive significantly more spam calls than those who live on the east coast.

**Solution:** Two sample  $t$  procedures - we have two independent samples. Here 100 from the West coast and 100 from the East coast

- (c) A researcher randomly selects a variety of 15 cell phones from different manufacturers and different service providers. The researcher measures the signal strength from two different remote areas: one in the state of Pennsylvania and one in the state of Virginia. The researcher would like to know if the signal strength is significantly weaker in the remote area in the state of Virginia.

**Solution:** Paired  $t$  procedures - there is one sample of 15 cell phones and the two measurements are paired for each cell phone.