# Hypothesis test for the slope of a regression line
March 23, 2022

1. A teacher is wondering if there is a relationship between GPA and ACT score. She took a random sample of 9 out of her 101 students and recorded their GPA and ACT score. The data are below.

GPA	3.7	2.3	4.0	3.8	3.0	1.8	2.0	2.3	3.9
ACT	23	20	35	33	22	13	17	20	29

Predictor	Coef	SE Coef	T	P
Constant	1.201	0.0874	13.72	0
GPA	7.507	1.29	5.82	0.0006511
S = 3.252686	R-Sq = 82.8%		R-Sq(adj) = 76.5%	
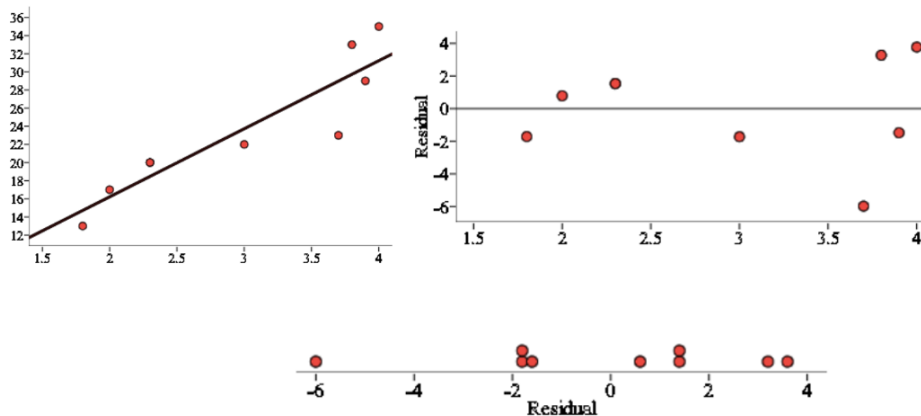
The the data provide significant evidence that there is a positive relationship between GPA and ACT?

Solution: 1. State: β is the true slope of the LSRL for x is GPA and ACT scores (y).

$$H_0 : \beta = 0 \text{ (No linear relationship)}$$

$$H_a : \beta > 0 \text{ (Positive linear relationship)}$$

Statistic: $b = 7.507$
Significance level: $\alpha = 0.05$.

2. Plan: We are conducting a 1 sample t test for the slope. Let's check conditions:

- Linear: Scatterplot is fairly linear and residual plot shows no problems.

- Independent: 9 students is less than 10% of all students so individual observations are independent.

- Equal σ: Residual plot shows similar variability for each x-value.

- Random: Clearly stated in problem.

3. Do:

$$\text{Test Statistic} = \frac{\text{Statistic-Parameter}}{\text{SD}}$$

$$t = \frac{b - 0}{SE_b}$$

$$= \frac{7.507 - 0}{1.29}$$

$$= 5.819$$

$$p\text{-value} = P(t_{n-2} > \text{Test Stat.})$$

$$= P(t_7 > 5.819)$$

$$= 0.000325$$

4. Conclude: Assuming H_0 is true ($\beta = 0$), there is a 0.000325 probability of getting a sample b of 7.507 or greater purely by chance. Because $0.000325 < 0.05$ we reject H_0 and have convincing evidence of a positive linear relationship between GPA and ACT.