One Sample Confidence Interval for μ

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1. Suppose you are interested in estimating the average weight of an Oreo cookie. A random sample of 30 cookies is selected with a mean weight of 11.1921 grams, and standard deviation 0.0817 grams. Construct a 95% confidence interval to estimate the true mean weight of an Oreo cookie.

   (a) What is the point estimate for the true mean?

   Solution:
   $$\overline{x} = 11.1921$$

   (b) Identify the population, parameter being estimated, sample, and statistic for this example.

   Solution: The population is all Oreos. The parameter is the true mean weight of all Oreo cookies μ. The sample is the 30 Oreo cookies selected. The statistic is the sample mean $\overline{x}$.

   (c) What is the formula for the standard deviation of $\overline{x}$?

   Solution:
   $$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

   (d) Are the conditions to assume $\overline{x} \sim \text{Normal}(\mu, \frac{\sigma}{\sqrt{n}})$ met here?

   Solution: Random Sampling: ✓.
   Independence: ✓.
   Normality: ✓.

   (e) We do not know σ, what do we use as an estimate?

   Solution: We use the standard error to estimate the standard deviation.
   $$SE_{\overline{x}} = \frac{s}{\sqrt{n}}$$

   (f) Under these circumstances, what is the distribution for $t = \dfrac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}}$.

   Solution: t follows a t-distribution with $n - 1$ degrees of freedom.

2. Construct a 95% confidence interval for μ, the population mean.

Solution:

$$\overline{x} \pm t^\star \frac{s}{\sqrt{n}}$$

Here we have

$$11.1921 \pm 0.0305$$

3. Interpret the interval.

Solution: We are 95% **confident** that the interval from 11.1616 to 11.2226 **grams** captures the **true mean weight of all Oreos**.

4. According to Nabisco, an Oreo weighs 11.3 grams. Does the confidence interval provide convincing evidence that the true average weight is less than 11.3 grams?

Solution: Yes. 11.3 grams is not contained within our interval.