Mini Math
January 28

1. A teacher believes that 85% of students will want to go on a field trip to the local zoo. She performs a hypothesis test to determine if the percentage is the same or different from 85%. The teacher randomly samples 100 students and 78 reply that they would want to go to the zoo. For the hypothesis test, use a 1% level of significance.

Solution: **State**: p: proportion of students who want to go to the zoo
$\hat{p}$: our sample proportion of $\frac{78}{100}$
α: 0.05 significance level

$$H_0 : p = 0.85$$
$$H_a : p \neq 0.85$$

We use the two tailed test here because it specifically states 'differ' in the problem.

Plan: We would like to conduct a one sample z-test for p. First I will check conditions:
Random Sampling: It is not explicitly stated in the problem, but we will assume the teachers sample is random and thus representative of the population.
Independence: Our sample of $n = 50$ students is clearly less than 10 percent of the the population of all students.
Normality: $np_0 > 10$ and $n(1 - p_0) > 10$ so is is reasonable to assume the sampling distribution for $\hat{p}$ is normal with a mean of $p_0 = 0.85$ and a standard deviation of $\sqrt{\frac{0.85(0.15)}{50}}$.

Do:

$$\begin{aligned} z &= \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} \\ &= \frac{.78 - .85}{\sqrt{\frac{0.85(0.15)}{100}}} \\ &= -1.960392 \\ p\text{-value} &= 2P(Z < -1.960392) \\ &= 0.04994999 \end{aligned}$$

Conclude: Here our p-value is 0.04994999 which is less than our significance level of 0.01, so we fail to reject the null hypothesis.

Assuming the true probability of zoo enthused children is 0.85, the probability of observing a sample proportion more extreme than our own is around 5%. This is not enough evidence to suggest the proportion differs significantly from 0.85.

2. Suppose a consumer group suspects that the proportion of households that have three cell phones is 30%. A cell phone company has reason to believe that the proportion is not 30%. Before they start a big advertising campaign, they conduct a hypothesis test. Their marketing people survey 150 households with the result that 43 of the households have three cell phones. Conduct an appropriate hypothesis test.

Solution: State: p: proportion of households that have three cell phones
$\hat{p}$: our sample proportion of $\frac{43}{150}$
α: 0.05 significance level

$$H_0 : p = 0.3$$

$$H_a : p \neq 0.3$$

We use the two tailed test here because it specifically states 'differ' in the problem.

Plan: We would like to conduct a one sample z-test for p. First I will check conditions:
Random Sampling: It is not explicitly stated in the problem, but we will assume the sample is random and thus representative of the population.
Independence: Our sample of $n = 150$ households is clearly less than 10 percent of the the population of all households.
Normality: $np_0 > 10$ and $n(1 - p_0) > 10$ so is is reasonable to assume the sampling distribution for $\hat{p}$ is normal with a mean of $p_0 = 0.3$ and a standard deviation of $\sqrt{\frac{0.3(0.7)}{150}}$.

Do:

$$\begin{aligned}
z &= \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} \\
&= \frac{0.2866 - .3}{\sqrt{\frac{0.3(0.7)}{150}}} \\
&= -0.3563474 \\
p\text{-value} &= 2P(Z < -0.3563474) \\
&= 0.7215804
\end{aligned}$$

Conclude: Here our p-value is 0.7215804 which is less than our significance level of 0.05, so we fail to reject the null hypothesis.
Assuming that the proportion of households with three cell phones is 0.3 there is an approximately 72% chance of observing a sample that differs the same or more than our own. This is not enough evidence to claim any significant difference.

3. State probabilistically what power is, and state the ways we can increase the power of a statistical test.

Solution: Power is the probability of rejecting H_0 given that H_0 is indeed false. Several ways to increase power are shown below:

- Increase alpha
- Conduct a one-tailed test
- Increase the effect size (distance between null parameter and true parameter)
- Decrease variability in sampling distribution
- Increase the sample size

4. A local fast food restaurant constructs a 95% interval for the proportion of customers that steal drinks when they ordered water. The interval is from 0.1883 to 0.3867.

   (a) Interpret the confidence interval that the fast food restaurant constructed.

   Solution: We are 95% confident that the proportion of customers that steal soft drinks when they order water is between 0.1883 and 0.3867.

   (b) The manager estimates that each customer who asks for a water cup but fills it with a soft drink costs the restaurant \$0.25. Suppose that in the month of June 3,000 customers asked a water cup when placing an order. Use the confidence interval given to give an interval estimate for the cost to the restaurant for the month of June from the customers who ask for a water cup but fill the cup a soft drink.

   Solution: Here we get an interval estimate for 0.1883(3000)(0.25) to 0.3857(3000)(0.25) or \$141.25 to \$290.00.