**Practice - Sampling Distribution for Sample Proportion**

January 5, 2022

1. The proportion of a population with a characteristic of interest is $p = 0.37$. Find the mean and standard deviation of the sample proportion $\hat{p}$ obtained from random samples of size 1,600.

   **Solution:** $\mu_{\hat{p}} = 0.37$, $\sigma_{\hat{p}} = 0.012$

2. Random samples of size 225 are drawn from a population in which the proportion with the characteristic of interest is 0.25. Decide whether or not the sample size is large enough to assume that the sample proportion $\hat{p}$ is normally distributed.

   **Solution:** Yes, there are 10 successes and 10 failures

3. Random samples of size n produced sample proportions $\hat{p}$ as shown. In each case decide whether or not the sample size is large enough to assume that the sample proportion is normally distributed.

   (a) $n = 50, \hat{p} = 0.72$

   **Solution:** Yes

   (b) $n = 50, \hat{p} = 0.12$

   **Solution:** No

   (c) $n = 100, \hat{p} = 0.12$

   **Solution:** Yes

4. A random sample of size 900 is taken from a population in which the proportion with the characteristic of interest is p = 0.62. Find the indicated probabilities.

   (a) $P\left(0.60 < \hat{p} < 0.64\right)$

   **Solution:** 0.7850

   (b) $P\left(0.57 \leq \hat{p} \leq 0.67\right)$

   **Solution:** 0.9980

5. Some countries allow individual packages of prepackaged goods to weigh less than what is stated on the package, subject to certain conditions, such as the average of all packages being the stated weight or greater. Suppose that one requirement is that at most 4% of all packages marked 500 grams can weigh less than 490 grams. Assuming that a product actually meets this requirement, find the probability that in a random sample of 150 such packages the proportion

weighing less than 490 grams is at least 3%. You may assume that the normal distribution applies.

**Solution:** 0.7357

6. A state public health department wishes to investigate the effectiveness of a campaign against smoking. Historically 22% of all adults in the state regularly smoked cigars or cigarettes. In a survey commissioned by the public health department, 279 of 1,500 randomly selected adults stated that they smoke regularly.

   (a) Find the sample proportion

   **Solution:** 0.186

   (b) Find the probability that, when a sample of size 1,500 is drawn from a population in which the true proportion is 0.22, the sample proportion will be no larger than the value you computed in part (a). You may assume that the normal distribution applies.

   **Solution:** 0.0007

   (c) Give an interpretation of the result in part (b). How strong is the evidence that the campaign to reduce smoking has been effective?

   **Solution:** In a population in which the true proportion is 22% the chance that a random sample of size 1500 would produce a sample proportion of 18.6% or less is only 7/100 of 1%. This is strong evidence that currently a smaller proportion than 22% smoke.

7. An ordinary die is "fair" or "balanced" if each face has an equal chance of landing on top when the die is rolled. Thus the proportion of times a three is observed in a large number of tosses is expected to be close to 1/6 or $0.1\overline{6}$. Suppose a die is rolled 240 times and shows three on top 36 times, for a sample proportion of 0.15.

   (a) Find the probability that a fair die would produce a proportion of 0.15 or less. You may assume that the normal distribution applies.

   **Solution:** 0.2451

   (b) Give an interpretation of the result in part (b). How strong is the evidence that the die is not fair?

   **Solution:** We would expect a sample proportion of 0.15 or less in about 24.5% of all samples of size 240, so this is practically no evidence at all that the die is not fair.

   (c) Suppose the sample proportion 0.15 came from rolling the die 2,400 times instead of only 240 times. Rework part (a) under these circumstances.

   **Solution:** 0.0139

   (d) Give an interpretation of the result in part (c). How strong is the evidence that the die is not fair?

   **Solution:** We would expect a sample proportion of 0.15 or less in only about 1.4% of all samples of size 2400, so this is strong evidence that the die is not fair.