One Sample tests for μ

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It's recommended that students 8 hours of sleep per night. A study is conducted where a random sample of 10 students from a university campus record the number of hours they sleep per night. The scores are shown below. Do the scores provide convincing evidence that the sample of students get less than 8 hours of sleep per night? (Use $alpha = 0.05$)

$$6.5, 4.75, 5, 8, 7, 7.25, 4.5, 5.5, 7, 6$$

1. Calculate the sample mean and standard deviation.

Solution: $\overline{x} = 6.15$ and $s = 1.18$.

2. State the appropriate hypothesis for a significance test, and significant test name. Be sure to define the parameter of interest.

Solution: H We will be conducting a one sample t-test for μ. μ here is the true mean hours of sleep of the Universities students.

$$H_0 : \mu = 8$$
$$H_a : \mu < 8$$

3. What conditions myst be met?

Solution: Random: Random sample is clearly stated in the problem
Independence: Here our sample of 10 students is clearly less than 10% of the population of all students attending the university.
Normality: Here we have a small sample (< 30), but the sample data is roughly symmetrical and free from outliers, so we will assume that the sampling distribution for $\overline{x}$ is normal. $\overline{x} \sim \text{Normal}\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$.

4. What is the difference between the standard deviation and standard error for $\overline{x}$ here?

Solution: Standard deviation refers to the true standard deviation of the mean whereas standard error refers to an estimate of the true standard deviation of the mean.

5. Visualize our test using a distribution and calculate the test statistic and p-value.

Solution: Draw a normal distribution with center at μ_0. Our rejection region is anything in the lower 5% tail.

$t = \frac{6.15-8}{0.376} = -4.92$, and p-value$= P(t_9 < -4.92) = 0.0004$.

6. Conclude the result of the t-test.

Solution: Because $0.0004 < 0.05$ we reject the null hypothesis suggesting that students likely aren't getting 8 hours of sleep per night.

Assuming that the true hours of sleep students get per night is 8, the probability of observing a sample with a mean of 6.15 hours or less in future samples is 0.0004, or 0.04%. This is very condemning evidence against the null hypothesis.