Assignment 9

Due Wednesday May 6

1. A complex electronic system is built with a certain number of backup components in its subsystems. One subsystem has four identical components, each with a probability of 0.2 of failing in less than 1000 hours. Assume that the components operate independently. Find the probability that:
   (a) exactly two of the four components last longer than 1000 hours.
   (b) the subsystem operates longer than 1000 hours.

2. A multiple choice exam has 15 questions, each with five possible answers, only one of which is correct. Suppose that one of the students, who takes the examination answers each of the questions with an independent random guess. What is the probability that he answers at least ten questions correctly?

3. Many utility companies promote energy conservation by offering discount rates to consumers who keep their energy usage below certain established subsidy standards. A recent EPA report notes that 70% of the island resident of Puerto Rico have reduced their electricity usage sufficiently to qualify for discounted rates. If five residential subscribers are randomly selected from San Juan, Puerto Rico, find the probability of each of the following events.
   (a) All five qualify for the favourable rates
   (b) At leas four qualify for the favourable rates

4. Show that for a discrete random variable Y that $E[Y(Y-1)(Y-2)] = E[Y^3] - 3E[Y^2] + 2E[Y]$.

5. Suppose that Y is a binomial random variable with $n > 2$ trials and success probability p. Use the fact that $E[Y(Y-1)(Y-2)] = E[Y^3] - 3E[Y^2] + 2E[Y]$ to derive $E[Y^3]$.

6. For a binomial random variable X derive the moment generating function $m_x(t)$, and find $E[Y^i]$ for $i = 1, 2, 3$